Enumeration of Kinematic Chains with Zero Variety for Epicyclic Gear Trains with One and Two Degrees of Freedom

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Abstract. The enumeration of kinematic chains for epicyclic gear trains allows to obtain all the possible configurations from specified structural characteristics. However, the more complex becomes the desired structure, the greater the number of options to analyse in order to select the most suitable one, what makes necessary a criterion to reduce this range of choices. The concept of variety and the conditions when its value equals to zero can be used for this purpose. This paper enumerates the kinematic chains with zero variety for epicyclic gear trains with one and two degrees of freedom, and with up to four independent loops. It is presented afterward two examples of transmissions obtained from the chains enumerated. Finally, the advantages of using zero variety chains are discussed.

Keywords: Enumeration of kinematic chains · Number synthesis · Epicyclic gear trains · Variety · Minimal sets

1 Introduction

The enumeration of kinematic chains or number synthesis for epicyclic gear trains is a prolific field of study and it is an important stage of design of system's transmissions, enabling to obtain all the possible configurations from specified structural characteristics. It allows to identify existing gear sets and to generate new ones, thereby avoiding a possible patent infringement. However, the more complex the desired structure, the greater the number of options to analyse in order to select the most suitable one, what renders impractical the task for verifying individually each one, and makes necessary a criterion to reduce this range of choices. The concept of *variety* and the conditions to force that its value being equal to zero can be used for this purpose [6,11,12].

This paper aims to enumerate and to present the kinematic chains with zero variety and with up to four independent loops for epicyclic gear trains with one and two degrees of freedom. The remainder of this paper is structured as follows. Section 2 introduces the concept of mobility and how it can be calculated. Section 3 briefly reviews the definition of number synthesis and the main tools

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employed on it. Section 4 introduces the definition of variety, minimal sets and the advantage of selecting a variety zero kinematic chain. In Sect. 5, the results of the enumeration are presented and for each mobility, a possible application is shown. The results found are compared to the current status of enumeration of non-fractionated chains with the specified structural characteristics [10]. Finally, Sect. 6 presents the final considerations about the results achieved in this paper.

2 Mobility

The mobility M, or the number of degrees of freedom (DoF) of a kinematic chain, is the number of independent parameters needed to define completely its configuration in the space, with respect to a specified link chosen as reference [6, 10]. The mobility is used to determine the number of necessary actuators to drive a mechanism and to verify its existence [10]. The mobility can be calculated by the Kutzbach-Chebyshev-Gruebler equation, formulated as:

$$M = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

Where λ is the order of screw system to which all joints of the kinematic chain belong, n is the number of links, j the number of joints and f_i are the degrees of relative motion allowed by the joint i.

Through the replacement of the joints with more than 1-DoF by a combination of single DoF joints, the Eq. 1 becomes:

$$M = \lambda(n - j - 1) + j \tag{2}$$

The number of independent loops ν of a kinematic chain can be expressed as a function of the number of links n and the number of joints j, as defined as:

$$\nu = j - n + 1 \tag{3}$$

The combination of Eqs. 2 and 3 yields:

$$M = j - \lambda\nu \tag{4}$$

Equation 4 is known as the loop mobility criterion [13] and it permits to quickly calculate the mobility of a kinematic chain, besides being the main parameter used in number synthesis process. The mobility criterion, Eq. 4, has some known limitations when applied to a few classical mechanisms and parallel robots, e.g. the Tripteron manipulator. Normally these discrepancies occur only with high values of λ, for example in the general spatial screw-system ($\lambda = 6$) [5]. Nevertheless, Eq. 4 has been proved suitable for lower values of λ such as $\lambda = 2$ and $\lambda = 3$, i.e. those applied to gear trains and the only systems considered in this paper.

3 Number Synthesis

The number synthesis or enumeration of kinematic chains consists of the generation of a complete list of kinematic chains that satisfy Eq. 2, excluding isomorphic and degenerated chains (kinematic chain where at least one subchain has mobility less than or equal to zero [10]). The main tools employed on the enumeration methodologies developed so far are: graph theory, group theory and matricial representation [9]. In this paper, only the first and second ones will be adopted, for further information about the number synthesis, see [9,10,13].

Kinematic chains can be represented in a univocal form by graphs, where the vertices correspond to the links and the edges correspond to the joints. Therefore, the number synthesis of kinematic chains is equivalent to the enumeration of graphs [9].

The McKay's algorithm and his graph generator [7,8] allow an isomorph-free enumeration of graphs/kinematic chains and the elimination of those which contain independent loops whose mobility is not desired. The graph generator will be used in Sect. 5 to achieve the purpose of this paper.

4 Variety

4.1 Definition

As defined by Tischler et al. [11], a kinematic chain is *variety* V if it does not contain any loop, or subset of loops, with a mobility less than $M - V$, but does contain at least one loop, or a subset of loops, which has a mobility of $M - V$. A kinematic chain is variety $V = 0$ if it contains no loop, or a subset of loops, with a mobility less than the mobility of the whole chain.

The concept of variety can be interpreted as a relationship between inputs and outputs of a kinematic chain, thus it must be considered in the selection of the joints that are going to be actuated [6]. In a kinematic chain of mobility M and variety V, $M - V$ joints may be selected at random for actuation, the other V actuated joints being selected so that not more than $M - V$ actuated joints belong to any subset of loops with mobility of $M - V$ [11].

For example, a variety $V = 1$ kinematic chain with mobility M contains a closed subset of one or more loops with mobility of $M - 1$, where only $M - 1$ actuated joints belonging to this subset are necessary to reduce its mobility to zero. Also, for this same kinematic chain, $M - 1$ joints can be randomly selected to be actuated, but the choice of the M^{th} actuated joint is restricted to the joints which do not belong to the subset with mobility $M - 1$.

Consequently, in a kinematic chain with j joints, mobility M and variety $V = 0$, M from those j joints must be selected and locked to decrease its mobility to zero. As $V = 0$, this selection of all the M actuated joints can be random. However, kinematic chains with lesser values of variety, like $V = 0$, are scarcer than those with higher values [12], what makes the use of the concept of null variety as a criterion to reduce the possible candidates of the enumeration process a very effective strategy, which may be employed in the synthesis of any parallel mechanism.

4.2 Minimal Sets

To understand the concept of minimal set, introduced in [11], consider the atlas of kinematic chains with $\lambda = 3$, $M = 0$ and $\nu = \{1, 2\}$, represented in Fig. 1. Note that all of them are variety $V = 0$. Visual inspection of kinematic chains in Fig. 1(b) and (d) shows that they contain at least one subset isomorphic to Fig. 1(a), therefore the chains in Fig. 1(a and c) compose the *minimal set* of kinematic chains for the case $\lambda = 3$, $M = 0$ and $\nu = \{1, 2\}$.

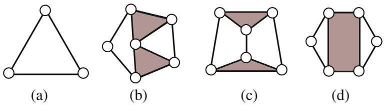

Fig. 1. Atlas of kinematic chains with $\lambda = 3$, $M = 0$ and $\nu \leq 2$

Chains with mobility $M > 0$ also have minimal sets, e.g., the elements of the minimal set of kinematic chains with $\lambda = 3$, $M = 1$ and $\nu \leq 3$ are presented in Fig. 2, again emphasizing that all are variety $V = 0$. There are no chains for the case $\nu = 2$.

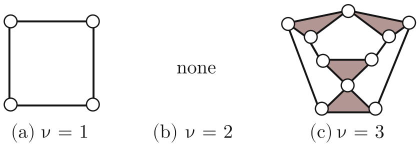

Fig. 2. Minimal set for the case $M = 1$, $\lambda = 3$ and $\nu \leq 3$

The relationship between minimal sets and variety is clear and unidirectional: it must be stressed that whilst all members of the minimal set have variety $V = 0$, not all kinematic chains with $V = 0$ belong to the minimal sets [6,11]. An example is the Watt's chain, shown in Fig. 3: it is variety $V = 0$ and it has mobility $M = 1$, but as it contains two subchains identical to the kinematic chain of Fig. 2(a), it cannot belong to the minimal set presented in Fig. 2.

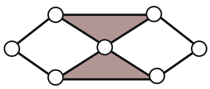

Fig. 3. Watt's chain ($\lambda = 3$, $M = 1$ and $\nu = 2$): despite of being variety $V = 0$, it does not belong to the minimal set for the case shown in Fig. 2

The variety of a kinematic chain is determined by searching for at least one subset which is member of the minimal set for the case of the chain's structural characteristics. The subset which presents the smallest mobility, M', may be used to calculate the variety of the kinematic chain with M DoF, through the difference $V = M - M'$ [11].

5 Kinematic Chains with Zero Variety for Epicyclic Gear Trains

In an *epicyclic gear train* (EGT), some gears not only rotate about their own axes, but also revolve around some other gears. Depending on whether it is an internal or external gear, a gear that rotates about a central stationary axis is called *sun* or *ring* gear, and those whose axes revolve about the central axis are called *planet* gears. The *carrier* or *arm*, a supporting link, keeps the center distance between two meshing gears constant. EGTs can have multiple applications, such as vehicle transmissions, machine tool gear boxes, robot manipulators, and so on.

One of the first methods for enumeration of kinematic chains for EGTs was developed by Buchsbaum and Freudenstein [2]. Tsai [13] presented an atlas of bicolored graphs containing one to four independent loops for EGTs, considering the order of screw system $\lambda = 2$, as proposed by Davies [3,4].

The objective of this section is to enumerate the variety $V = 0$ kinematic chains for EGTs with one and two degrees of freedom, by employing the Eq. 4 to determine the number of joints, i.e., the number of edges of the graphs to be generated, and then the Eq. 2 to calculate the number of links/vertices, considering the cases $\lambda = 2$, $M = \{1, 2\}$ and $\nu = \{1, 2, 3, 4\}$. The values obtained are used into the McKay's graph generator [7,8] and it is demanded to disconsider graphs with independent loops with mobility equivalent to one or two when necessary.

Chains with $M \neq \{1, 2\}$ and higher number of independent loops are not included in the scope of this paper, neither are fractionated chains. The amount of $V = 0$ kinematic chains found is compared to the status of enumeration of non-fractionated chains for $\lambda = 2$ and $M = \{1, 2\}$ presented by Simoni et al. [10], shown in Table 1, to verify their proportion among the chains with the structural characteristics analysed.

Table 1. Current status of enumeration of non-fractionated chains for $\lambda = 2$, $M = \{1, 2\}$ and $\nu \leq 4$ (adapted from [10])

ν	$M = 1$	$M = 2$
1	1	1
2	1	2
3	3	9
4	13	49

5.1 Epicyclic Gear Trains with $M = 1$

In total, eighteen kinematic chains with $\lambda = 2$, $M = 1$ and $\nu \leq 4$ have variety $V = 0$, depicted in Figs. 4 and 5. By comparing the quantity of results that have been found to the values of Table 1, it can be concluded that all kinematic chains with $\nu \leq 4$ representing EGTs with one DoF have $V = 0$.

Regarding the minimal set, by visual inspection, it inferred that it is composed by exclusively two kinematic chains, those shown in Fig. 4(a) and in Fig. 5(n), as the latter is the only one that does not contain one subset isomorphic

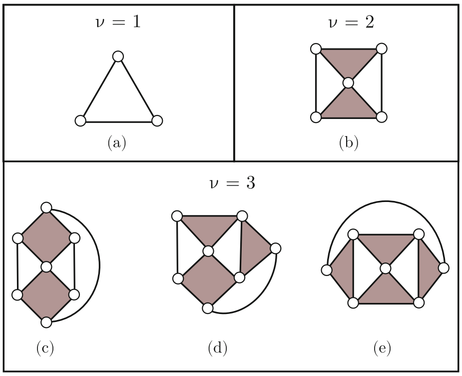

Fig. 4. Variety $V = 0$ kinematic chains with $\lambda = 2$, $M = 1$ and $\nu \leq 3$

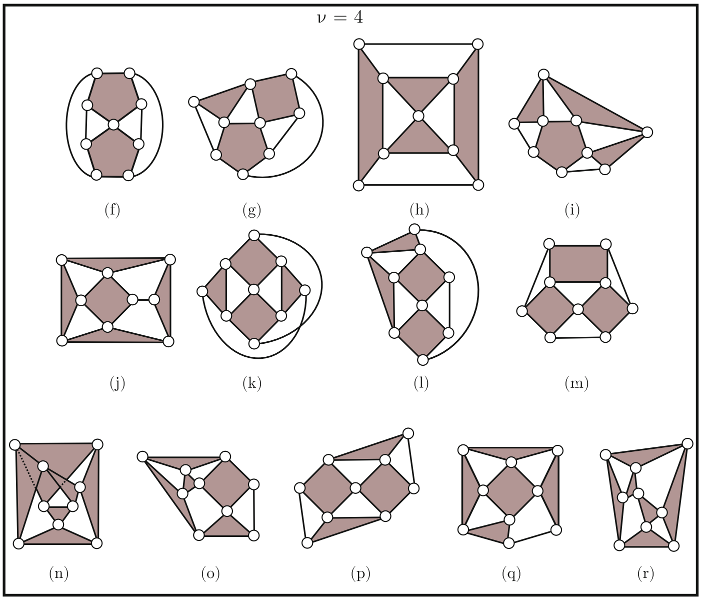

Fig. 5. Variety $V = 0$ kinematic chains with $\lambda = 2$, $M = 1$ and $\nu = 4$

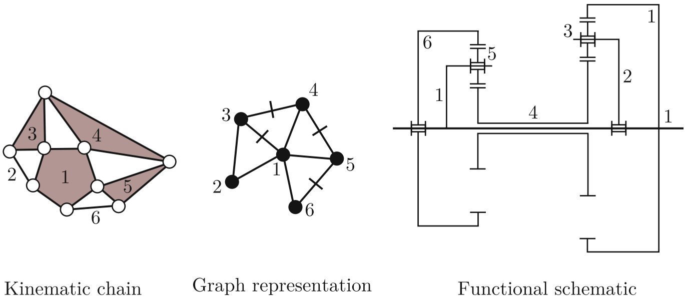

Fig. 6. Kinematic chain of Fig. 4(i), its graph representation (where the edges crossed by a perpendicular line stand for gear joints and the remaining ones, revolute joints) and the functional schematic of the Simpson gear set

to the former. It must be remarked that in the kinematic chain of Fig. 5(n), there are three ternary links that cross each other, what is represented by dashed lines.

Further development for a specific case is seen in Fig. 6, where the kinematic chain in Fig. 5(i) is shown with its graph representation, whose edges crossed by a perpendicular line stand for gear joints and the remaining ones, revolute joints. The links are enumerated from 1 to 6 in order to generate a functional schematic of a possible EGT configuration, known as Simpson gear set. It is a compound EGT consisting of two basic EGTs, each having a sun gear (link 4), a ring gear (links 1 and 6), a carrier (links 1 and 2) and four planets (links 3 and 5). The two sun gears are connected to each other by a common shaft (link 4), whereas the carrier of one EGT is connected to the ring gear of the other EGT by a spline shaft (link 1). The Simpson gear set is most used in three-speed automotive automatic transmissions [13]. As its respective kinematic chain is variety $V = 0$, any joint can be chosen for actuation without restriction, i.e., any link can be the system's input, respecting the EGT's configuration.

5.2 Epicyclic Gear Trains with $M = 2$

There are eight variety $V = 0$ kinematic chains with $\lambda = 2$, $M = 2$ and $\nu \leq 4$, as shown in Fig. 7, fewer than the number of chains enumerated in Table 1. In regards to the previous section, the number of results found for $M = 2$ are also lesser than the ones for $M = 1$. The minimal set is reduced by half with the increase of mobility as well, on this circumstance being composed exclusively by the chain represented by Fig. 7(a), as all the kinematic chains with $V = 0$ present an identical subset to this chain. Again, in Fig. 7(h), the crossing of some links is represented by dashed lines.

In Fig. 8, the chain of Fig. 7(b) has its links enumerated from 1 to 5 and it is shown with its graph representation, and a feasible EGT configuration, where the link 2 is chosen to be the ground link. EGTs with mobility $M = 2$, associated with a continuous variable unit (CVU), may compose an *infinitely variable transmission* (IVT), a transmission system which allows for step-less

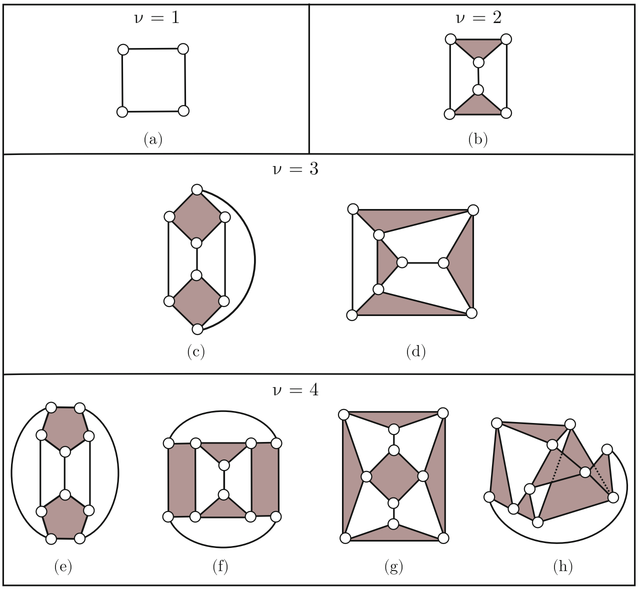

Fig. 7. Variety $V = 0$ kinematic chains with $\lambda = 2$, $M = 2$ and $\nu \leq 4$

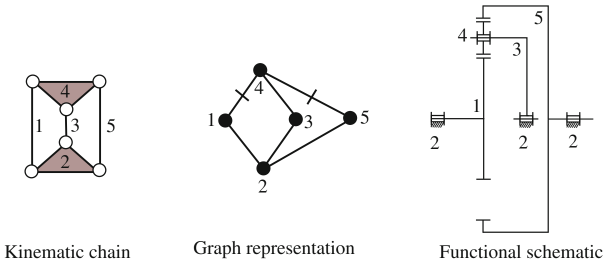

Fig. 8. The kinematic chain in Fig. 7(b), its graph representation (where the edges crossed by a perpendicular line stand for gear joints and the remaining ones, revolute joints) and a feasible EGT configuration

variability of speed ratio, including zero velocity ratio. It is used as the power train for exercising machines and agricultural tractors [1,14].

As the kinematic chain in Fig. 7(b) is variety $V = 0$, the choice of the two actuated joints can be randomly made, i.e., any links can be the inputs, although the EGT's configuration must be taken in consideration. By analysing the EGT's functional schematic in Fig. 8, it can be inferred that the two inputs may be chosen among three links: the sun gear (link 1), the ring gear (link 5) and the carrier (link 2), the one not selected as input being defined as the system's output. There are three alternatives for the inputs' choice: *sun gear/carrier*, *ring gear/carrier* and *sun gear/ring gear*. The last one is represented in Fig. 9,

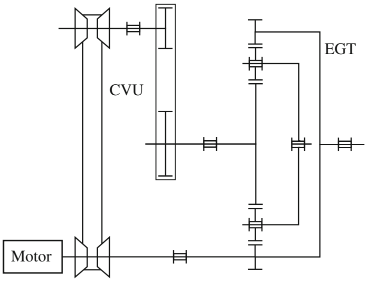

Fig. 9. Possible IVT configuration with the EGT of Fig. 8

where the links 1 and 5 are the two inputs of the two DoF EGT and the link 2 is the output.

6 Conclusion

The concept of variety is a good criterion to reduce the options' range of kinematic chains for a defined purpose. A kinematic chain with variety $V = 0$ erases a constraint in the selection of the actuated joints, allowing to choose them at random, what can be interesting depending on the aimed utilization, thus the importance of enumerating the chains which have this characteristic.

This paper presented the variety $V = 0$ kinematic chains for epicyclic gear trains with mobilities $M = 1$ and $M = 2$. A comparison between the number of results found and the current status of enumeration of non-fractionated chains for $\lambda = 2$, $M = \{1, 2\}$ and $\nu \leq 4$ is made to verify the proportion of chains among them whose actuated joints can be chosen randomly. It was corroborated that, as stated by Tischler et al. [12], the amount of kinematic chains decreases when lesser the variety's value: while all the 18 kinematic chains with $\lambda = 2$, $M = 1$ and $\nu \leq 4$ are variety $V = 0$, only 8 of the 61 kinematic chains with $\lambda = 2$, $M = 2$ and $\nu \leq 4$ have zero variety. It was also verified that the minimal sets for both cases are not large, the minimal set for the case $M = 1$ consisting of two chains and the one for the case $M = 2$ being composed only by one chain.

In spite of the variety $V = 0$ kinematic chains allowing to actuate any joint without restriction, the EGT's configuration must be taken into account when determining the system's inputs/outputs, as seen in Sect. 5.2. Nonetheless, by using only chains with zero variety, this selection can be made without hesitation.

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