Chapter 2
Investigating the Relationship Between Prospective Elementary Teachers' Math-Specific Knowledge Domains

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Abstract Notwithstanding long term efforts to differentiate between domains of mathematics-related teacher knowledge, there is no doubt that different forms and aspects of teacher knowledge are interrelated and mutually influence each other. However, the nature of this relation is still open to scholarly debate. First, we give an overview of empirical studies that investigated the relation between different domains of mathematics teachers' knowledge, notably, the domains of "content knowledge" and "pedagogical content knowledge". We demonstrate that the research on the relationship turns out to be multifaceted and we point to the need of cognitive orientated research on the integration of knowledge domains. Second, we present our own ongoing research on the integration of prospective elementary teachers' math-specific knowledge domains by describing our use and analysis of task-based interviews. Preliminary findings indicate that our approach can help to identify mental processes that illuminate the integration of math-specific knowledge domains.

Keywords Elementary school mathematics · Pedagogical content knowledge
Pre-service teachers · Integration · Cognitive processes

2.1 Introduction

One of the most influential ideas in describing professional knowledge for teaching is the distinction between mathematical *content knowledge* (CK) and mathematics-related *pedagogical content knowledge* (PCK) (Shulman 1986; Bromme 1992; Ball et al. 2008). But what is the relation between these knowledge

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domains? To answer this question it is necessary to understand what is meant by "relation". Unfortunately, the term is interpreted and used in many different ways. Before we present our own research, we thus attempt to identify and outline different approaches of research on the relation of mathematics-related CK and PCK. Hereby, we seek to provide a basis in order to classify our own ongoing research and its implications with respect to the field of research.

Among these approaches we identify two main strands: (1) Relations are expressed by theoretically discussing conceptualizations of PCK and referring either to pedagogical or to content-related aspects; and (2) Existing conceptualizations of content and pedagogic content are used to investigate teacher knowledge and the relation between its components empirically. According to the first strand, all conceptualizations of PCK and its components express a relation with CK in a specific way. For example, Shulman defined PCK as the "particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman 1986, p. 9). Thus, from his point of view PCK can be considered as a special form of CK. However, he also referred to it as that "special amalgam of content and pedagogy" (Shulman 1987, p. 8), a metaphor that expresses a more complicated quality of the relation (an amalgam is a rather tight mixture of elements, which results in a material with new qualities). Ball et al. (2008) presented a more empirically grounded conceptualization of mathematical knowledge for teaching. As above, their definitions of the components of PCK express their view about the relation between both. For example their definition of knowledge of content and teaching combines knowing about teaching and knowing about mathematics. By such conceptualizations, however, relations are often expressed implicitly or remain vague (e.g. PCK as an amalgam). Moreover, Depaepe et al. (2013) pointed out in their review article that there is no consensus about the components that PCK covers. However, in all recently discussed theoretical models PCK comprises at least knowledge on students' cognitions, instructional strategies, and representations (ibid.).

According to the second strand, relations are investigated empirically and based on predefined conceptualizations of CK and PCK. In the following section, different lines of empirical research within this strand are discussed in detail.

2.2 Overview of Empirical Studies on the Relation Between Content Knowledge and Pedagogical Content Knowledge

We differentiate between four lines of research where relations are investigated empirically: (a) Relation as correlation between CK and PCK measured by some form of test; (b) Relation as parallel development of CK and PCK, where one is described as a condition for the other; (c) Relation as a form of integration of CK and PCK; and (d) Relation as a form of association of CK and PCK. Studies that address the quality of instruction or the student progress (dependent on CK and PCK) are not included in our overview. We are aware of the fact that it is not always easy to distinguish between the lines of research mentioned above, especially since they are not fully disjoint. Nonetheless, we consider the differentiation useful to give a systematic overview. Next, we discuss each of these four lines of research separately.

2.2.1 Relation as Correlation Between CK and PCK

The studies in this line of research investigate relations quantitatively by means of written tests and score correlations. For example, Blömeke et al. (2010a, b), Hill et al. (2004) and Krauss et al. (2008) showed that it is possible to distinguish CK and PCK by statistical analysis of test behavior. In that sense, the term "relation" focuses on the question whether it is reasonable to distinguish domains of knowledge at an interindividual level (through factor analysis or by considering the discriminant validity of tests). Their findings also reveal a strong correlation between the constructs, indicating that both constructs are at least highly connected. However, the picture of correlations for prospective elementary teachers is not as clear as it is for prospective secondary teachers (Blömeke et al. 2010a). Depending on the design of research, additional forms of relations between CK and PCK on the one hand and further characteristics on the other hand were addressed. For instance, Krauss et al. (2008) investigated CK and PCK in dependence of school levels, teaching experience or personal theories about mathematics.

Although correlations between CK, PCK and further variables cannot be interpreted in a causal way, the correlative findings are often considered to support certain assumptions, e.g. about the development of PCK and CK, where one is seen as a possible condition for the other. We will refer to these interpretations in the next sub-section.

2.2.2 Relation as Co-development of CK and PCK

The abovementioned studies by Hill et al. (2004) and Krauss et al. (2008) supported the view on CK as a necessary or at least a facilitating condition for PCK. For example, Krauss et al. (2008) reported that secondary teachers in academic tracks scored better in pedagogic content items to the extent that they scored better in content items. Moreover, Even (1993) for example contends that many of the tested prospective secondary teachers did not have an appropriate mathematical concept of function (CK) which, as a consequence, may have led to inappropriate knowledge of the "vertical line test" as a rule for students to check for the univalence aspect of functions (PCK). The importance and necessity of content knowledge has also been investigated by a number of qualitative studies (cf. Ball et al. 2001): for instance, findings reveal the relevance of conceptual understanding (CK) for teaching methods as well as for the generation and analysis of representations and explanations (PCK).

However, Krauss et al. (2008) reported that some secondary teachers in non-academic tracks reached high scores in PCK measures even though they scored low in items associated with mathematics content. This indicates that CK rather facilitates the development of PCK instead of being a necessary condition in a strict logical sense. For teachers in non-academic tracks low scores in PCK always went along with low scores in CK. Additionally this result was interpreted in a way that "strong content knowledge quasi'protects' against a low level of pedagogical content knowledge" (p. 244, translation by the author).

Can CK be considered as a sufficient condition for PCK? Krauss et al. (2008) showed that secondary teachers in academic tracks scored higher in PCK measures compared to teachers in non-academic tracks although the teachers in academic tracks received less education in PCK. This may support the view that CK is even sufficient for PCK. However, referring to Capraro et al. (2005) the condition is not sufficient for pre-service elementary teachers: "having profound mathematical understanding does not ensure pre-service teachers [to] develop pedagogical content knowledge" (p. 108).

How do prospective teachers acquire knowledge about content and pedagogic content? Kleickmann et al. (2017) conducted an experimental study in order to test the assumptions that (a) teachers construct PCK from CK and PK in a process of amalgamation, (b) CK is a necessary condition and facilitates PCK development, and (c) CK is sufficient for teachers' PCK development. The study indicates "that there are different pathways to PCK development" (p. 17). Lin and Hsu (this volume) refer to the question in their discussion of the use of mathematics-pedagogy tasks to facilitate the development of mathematics-related knowledge domains. They propose to implement tasks in teacher preparation courses which offer opportunities to address knowledge about content, student cognition, curriculum and textbook design concurrently. For instance, they present tasks which may not only help to promote prospective teachers' mathematical understanding but also facilitate them to make analogies to student learning.

2.2.3 Relation as a Form of Integration of CK and PCK

With the term "integration" we refer to the ways of how (prospective) teachers' CK and PCK come together and inform teachers' behavior in teaching-specific contexts or in the course of teaching. Hence, investigating the integration requires analyzing and differentiating teachers' behavior and the knowledge that becomes evident in the process. Escudero and Sanchez (2007), for example, analyzed videotaped lessons of two experienced secondary teachers and described the teachers' behavior and the integrated knowledge. Similarly, Speer and Wagner (2009) identified "component practices of analytic scaffolding" and analyzed the knowledge "needed to enact these practices" (p. 557). For instance, they worked out that the examined undergraduate mathematics teacher (who had good mathematical knowledge) was unable to use the students' contributions as an opportunity for analytic scaffolding due to his lack of PCK.

Rowland et al. (2005) analyzed videotaped lessons of pre-service elementary teachers with the aim "to locate ways in which they drew on their knowledge of mathematics and mathematics pedagogy in their teaching" (p. 255). They identified four dimensions, called the "knowledge quartet", which can be used to observe prospective teachers' knowledge in practice. The knowledge quartet can be a useful tool for the discussion of knowledge domains between prospective teachers' and their mathematics teacher educators.

2.2.4 Relation as a Form of Association of CK and PCK

One form of association can be named "verbal association". Hereby, we denote a relationship operationalized as a connection between CK and PCK (or its components) that results from the proximity of teachers' utterances (which refer to elements of CK and PCK) in time. In this way, it is possible to count knowledge domains (or components of knowledge domains) per utterance in an interview or per episode in a lesson (enumerative approach). To our knowledge, so far no study was conducted in order to map out the relation of mathematics-related knowledge this way. However, Park and Chen (2012) used a model of PCK for teaching science (called the Pentagon model) and applied the enumerative approach to the teaching of four high school biology teachers. They worked out that the occurrence of "the components was idiosyncratic and topic-specific [and that] Knowledge of Student Understanding (KSU) and Knowledge of Instructional Strategies and Representations (KISR) were central" in the episodes (p. 930). In a similar vein, the so called Epistemic Network Analysis is applied by Weiland et al. (2015) in order to investigate "connections" between elements of mathematics content knowledge. This exploratory approach seems to enable further insights in teachers' organization of CK. However, there are no final results available yet.

A different approach to describe the association of CK and PCK is presented by Lehrer and Franke (1992). To derive "conditional relationships" among CK and PCK they applied personal construct psychology and the logic of fuzzy sets to the study of two experienced elementary teachers: after central knowledge "constructs" were identified (and later assigned to CK, PCK, or general pedagogical knowledge) the teachers were asked to rate the constructs to be more or less important (true) to each of the presented fraction problems. Fuzzy logic was applied to receive the strengths of implications (associations) between the constructs. The further analysis of the teaching revealed that teachers' individual implications correspond to their actions in the context of the classroom.

2.2.5 *Summary*

The overview demonstrates that the research with focus on relations between (mathematics-related) CK and PCK turns out to be multifaceted. Most of the previous work seems to focus on correlations or the co-development of knowledge domains. However, findings do not reveal a clear picture and there is still an ongoing debate of whether the findings are influenced by the respective methodology, such as by the types of tasks used in the measurement. According to Buchholtz et al. (2014) strong empirical correlations between CK and PCK reported in the literature can be ascribed to operationalizations of PCK that closely relate to CK.

As reported by Depaepe et al. (2013) only few studies address the integration or association of knowledge domains (Escudero and Sanchez 2007; Speer and Wagner 2009; Lehrer and Franke 1992). Thus, if we seek a deeper understanding of *how* aspects of content and pedagogic content play together when (prospective) teachers are faced with the demands of teaching, it is necessary to further investigate the integration of CK and PCK. The quantity of coincidences of knowledge domains (or components) per utterance or episode does not really help us to understand how CK and PCK come together. Rather, it would be necessary to concentrate both on mental processes *and* the respective knowledge domains or components that relate to those processes. However, existing studies did not fully recognize integration at a cognitive level. For this reason, we think it is of particular interest to further investigate the integration of knowledge domains and their components that way.

Our overview of empirical studies classifies existing studies aiming at the relationship between mathematics-related CK and PCK. In consequence, in the next section we are able to present our focus of research with respect to the gap of research outlined above and in contrast to the other lines of research. As pointed out already, the lines of research are not entirely independent. For instance, investigating the integration of knowledge domains may also lead to new insights according to the association of knowledge domains (because integration goes beyond association) or the development of PCK (because the identification of mental processes may give hints on how PCK can be acquired). Thus, the overview also offers an opportunity to refer back to existing studies when we interpret our preliminary and future findings with respect to the other lines of research in the discussion section.

2.3 Focus of Research

Referring to the gap of research outlined above, our aim is to investigate the integration of mathematics-related CK and PCK. As we mentioned earlier, by "integration" we understand the ways of how CK and PCK inform (prospective) teachers' mental processes when they deal with typical demands in teaching-specific contexts or in the course of teaching. To us, it is particularly important to identify and describe mental processes that help to make clear how domains of knowledge come together. Of course, it is not possible to observe mental processes directly, such as to remember, understand, apply, analyze, evaluate, or create (Stern 2017). However, it is possible to analyze observable behavior in order to generate hypotheses about mental processes in which different mathematics-related knowledge domains are activated. Describing the integration of mathematics-related knowledge domains that way is a key aspect if we seek to improve our understanding of (professional) knowledge structures of (prospective) teachers.

It is not possible to investigate the integration without investigating the mathematics-related knowledge that (prospective) teachers use. According to Liljedahl et al. (2009), knowledge domains may become more and more integrated or even unified with time. At the same time, the integration of knowledge domains may depend on the amount of professional training and the experience of (prospective) teachers. Our aim is to shed more light on mathematics-related knowledge of prospective elementary teachers and its integration in the first phase of mathematics teacher education (i.e., during their bachelor degree courses). However, it is not our aim to investigate the development of knowledge domains or the development of its integration.

In the first phase of teacher education the knowledge domains initially emerge and often cover incomplete or incorrect knowledge. Thus, we are not only interested in knowledge that can be considered as correct from a normative point of view (professional knowledge), but we also consider incomplete, subjective, or experience-based knowledge. For this reason, it is not appropriate to use an existing conceptualization of mathematics-related knowledge. Rather it seems necessary to develop an empirically-grounded and adapted conceptualization which applies to our prospective teachers and which can be used to further investigate the integration.

We pose the following research questions:

(1) Which domains of prospective elementary teachers' mathematics-related knowledge can be distinguished in their reasoning while solving pedagogical tasks in mathematics teaching (as posed in bachelor-degree courses)?; and
(2) Which are the mental processes of prospective elementary teachers' that are based on different domains of knowledge?

2.4 Task-Based Interviews

We assume that the knowledge domains can best be investigated in situation-specific contexts. Thus, we considered the use of tests or questionnaires inappropriate. The method of lesson observation seem to fit better but does not offer the possibility to address further questions in relation to mathematics and the teaching of mathematics. Considering teaching in vivo also is not appropriate since the participants of the study are prospective elementary teachers (in their second to fourth semester) who typically have very little teaching experience and issues of classroom management may be predominant.

In previous studies the use of task-based interviews was applied successfully with respect to the investigation of teacher knowledge and beliefs (e.g. Ball 1988; Biza et al. 2007; Ma 1999). We apply this method including the task structure proposed by Biza et al. (2007) because the tasks are common to the training of prospective teachers. In the semi-structured interviews excerpts from textbooks and other curriculum materials are used as a basis for questions concerning tasks of teachers which are typical for arithmetic teaching in second to sixth grade. The list of core tasks presented by Bass and Ball (2004) served as a guiding framework. It comprises the following tasks where mathematical work is involved:

> [S]etting and clarifying goals, evaluating a textbook's approach to a topic, selecting and designing a task, re-scaling tests, choosing and using representations, analyzing and evaluating student responses, analyzing and responding to student errors, managing productive discussions, figuring out what students are learning […] (Ball and Bass 2004, p. 296).

For instance, we use the so-called Multiplication Poster by Wittmann (1998), shown in Fig. 2.1, as a representation of a holistic approach to the multiplication table and link it to the following scenario: "Suppose you plan to use the Multiplication Poster with your second graders. Reflect on your ideas and reasoning. How do you proceed?" Depending on the responses the interviewer poses questions such as: "What do you think about?" "What is the Multiplication Poster about?" "What about it could be easy or difficult for your students?" and "What are the goals you want them to reach?" Following the task structure mentioned above and in order to cover as many tasks from the list as possible, we further asked prospective teachers to examine and respond to fictional student solutions and errors.

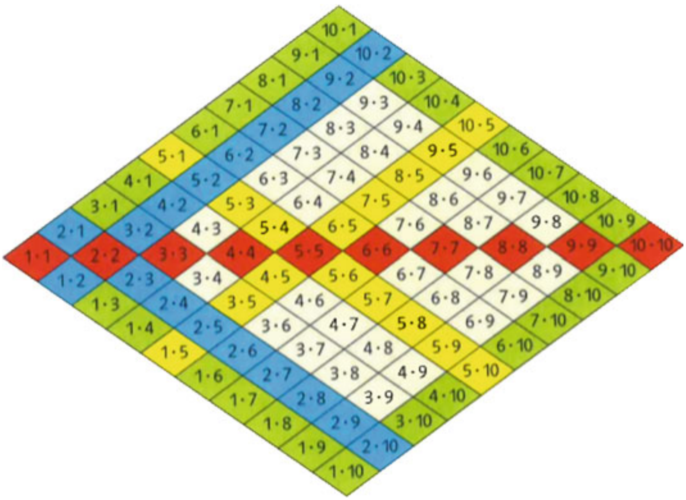

Fig. 2.1 Multiplication Poster © Klett und Balmer AG, Verlag, Zug 2007

2.5 Sampling and Analysis

Acquisition (theoretical sampling) and analysis of data (constant comparison) is based on the approach of Grounded Theory (Strauss and Corbin 1996) which is adequate for the purpose of developing theory from empirical interview data. The analysis of data is carried out using the software MAXQDA 11.

2.5.1 Participants

Our qualitative design with a small sample risk overemphasizing individual cases since prospective teachers' abilities and backgrounds are very diverse. To avoid this problem, we drew the sample following the maximum variance principle (Patton 1990). Six prospective teachers participated in our interviews which lasted between forty and sixty minutes each. Three of them were enrolled in the second semester (and attended the mathematics-related courses "Elementary Algebra and Arithmetic" and "Mathematical Thinking of Children"), one of them in the third semester (who additionally attended the course "Geometry and Applied Mathematics"), and two of them in the fourth semester (who additionally attended the course "Planning and Implementation of Mathematics Instruction"). All of them were bachelor students in an elementary education program in Switzerland.

2.5.2 Analysis

All six interviews were transcribed in full. Open and axial coding were taken out simultaneously (because they go hand in hand). Nonetheless, for reasons of readability we will outline both steps separately. Both steps of analysis are still in progress.

Open coding involves a constant comparison of data and asking questions like "On what knowledge does the interviewed person draw back in the situation?" These are core elements of this step which very closely sticks to the data. The goal is to explore the knowledge that the participants refer to in the interviews. We assign codes to segments of data if a certain concept is reflected (a piece of knowledge) according to our interpretation. The meaning of concepts represented by the codes is established by comparing data and writing memos. Memos also include discussions about the dimensions of concepts (such as frequency, depth, or duration).

The step of open coding is taken out highly inductive. This means that the names of codes are mainly developed in vivo (names are derived directly or with little variation from the data) or from the data in the sense that concepts are grounded in the data and named by the researcher during the analytical process (Strauss and Corbin 1996, p. 49). In order to remain open to the data a sentence-by-sentence coding is applied to the transcripts. Codes are not required to be disjoint.

Axial coding denotes the process of relating concepts and categories back together. Similar concepts are grouped and summarized under categories (knowledge domains) according to shared properties (Strauss and Corbin 1996, p. 47). At first, assigning these codes to categories and thus to (components of) knowledge domains is based on existing models of teachers' mathematics-related knowledge (theoretical sensitivity) (e.g. Ball et al. 2008; Krauss et al. 2008; Rowland et al. 2005; Shulman 1986). We use this generic structure as an initial point to further apply the method of constant comparison. Moreover, the assignation of codes to categories takes place irrespectively of content-related correctness. Hence, data are not evaluated as right or wrong, but as being of a certain kind of knowledge.

In the following, we give some examples illustrating codes, codings, and the assigning of codes to categories. Some of these illustrating codings are presented again as parts of interview sequences when we describe how codes (and categories) relate to each other in mental processes (see preliminary findings section).

2.5.2.1 Knowledge About Students' Cognitions

The following two codes are developed in vivo and irrespectively of being appropriate or not with respect to the represented concept (usually 7×3 is not considered to be most difficult). They are both assigned to knowledge about students' cognitions because they deal with the question what students will find hard or confusing (cf. Ball et al. 2008).

Code	Example (coding)
7×8 and 7×3 are most difficult	"I think that 7×8 and 7×3 are most difficult."
Multiplication Poster is confusing at first	"All at once [there are all multiplications up to 10 in the Multiplication Poster]. That is confusing. I mean the students cannot easily assess it."

2.5.2.2 Curricular and Teaching Related Knowledge

The notation of "core multiplications" concerns a certain approach to the multiplication table which is popular in Germany and Switzerland. It denotes the strategy of reducing an arbitrary multiplication to multiplications with factors 1, 2, 5 or 10 (core multiplications), e.g. $7 \times 8 = 5 \times 8 + 2 \times 8$, by applying the distributive property. The core multiplications are examined in detail from second grade on. This strategy is contained by many textbooks and standards for second grade.

Code	Example (coding)
Core multiplications	"For example, second graders work intensively with the 1, 2 and 5 series. In classes you normally start with them and build on them."

The utterances "second graders work intensively with" and "you normally start with them and build on them" were taken as indicators for teaching-related knowledge [cf. curricular knowledge about textbooks and standards, Shulman (1986)]. Alternatively, one may interpret this approach as a common way of making the multiplication table comprehensible. As in this case, it is not always easy to distinguish between curricular and teaching-related knowledge [cf. "knowledge of content and teaching", Ball et al. (2008)]. One may also think about assigning the code additionally to content knowledge. However, the focus on curricular aspects and the absence of mathematical analysis is an argument against it.

2.5.2.3 Content Knowledge

When codes refer to mathematical facts, concepts, procedures or other more syntactical forms of knowledge, they were assigned to content knowledge.

Code	Example (coding)
Breaking apart	"One can use the 2 and 5 series as a basis. When you know the 5 series you can go ahead from here. For 9×8 you don't have to count 9 so often. You can take 5×8 and then proceed."
Associative law	"It doesn't matter where you start from. $3 \times 3 \times 3$ is the same as 3×9 or 9×3."

The first code example was assigned to content knowledge because the concept of core multiplications (see above) is addressed here from a mathematical point of view (the strategy of "breaking apart" is explained with an example without referring to the distributive property explicitly). The second code example again refers to a fundamental mathematical concept, the associative law.

According to the second research question we seek to investigate the integration of concepts by detecting the participants' mental processes. We ask questions such as "How does the interviewed person proceed in this situation; how does the person make a connection between different domains of knowledge?" and go through all the interviews again. We tried to find answers by writing memos and analytic stories considering the paradigm, the dimensions and the questions that enhance theoretical sensitivity proposed by Strauss and Corbin (1996).

2.6 Preliminary Findings

As mentioned above, the analysis is still in progress. For the moment, we are able to present preliminary findings. Despite its preliminary character, these findings are meaningful in order to illustrate that our analysis is appropriate to reconstruct mental processes which appear to be informed by (math-specific) knowledge domains. First, we refer to our empirically grounded differentiation of knowledge domains. Second, we refer to the integration of knowledge domains by presenting to examples of mental processes.

2.6.1 Prospective Elementary Teachers' (Math-Specific) Knowledge Domains

So far, the analysis revealed four different domains of knowledge which we briefly describe in the following (for examples of codes relating to the respective knowledge domain see analysis section).

- *Knowledge about students' cognitions*: This domain of knowledge combines knowledge of mathematics and the learners of mathematics. It covers mathematics-related knowledge about (task) difficulties, conceptions and misconceptions, behavior patterns and "thinking paths" of elementary students. However, it does neither refer to "pure" mathematical knowledge even though it is used to identify student errors nor to knowledge about students which is not math-specific.
- *Curricular and teaching-related knowledge*: Curricular knowledge is knowledge about teaching standards, teaching programs, textbooks and other associated materials. It deals with the question which demands or representations are included and how they are sequenced in the standards, textbooks etc. Teaching-related knowledge is knowledge about the use of valuable representations of mathematics in terms of temporal sequences in the context of classroom teaching.
- *Content knowledge*: This domain includes substantial and syntactical as well as conceptual and procedural knowledge about the subject content which underlies the teaching of mathematics in elementary classes (in Switzerland first to sixth grade). Ideally, content knowledge is profound according to Ma (1999). However, we avoid making further claims about its quality in order to make it possible to capture any kind of content knowledge (since our focus of research is on the integration of knowledge domains).
- *Didactical knowledge*: This is the only domain of knowledge which is not content-related. It comprises general pedagogical and psychological knowledge such as knowledge about learning theories, forms of teaching and learning, teaching strategies, didactical principles or methods of assessment.

Knowledge about students' cognitions, curricular knowledge, and teaching-related knowledge can be considered as dimensions of our empirically grounded conceptualization of prospective teachers' PCK which closely sticks to types of knowledge known from the literature. However, differences remain. For example, so far we were not able to distinguish between curricular and teaching related knowledge (see above) or between "specialized content knowledge" and "common content knowledge" as proposed by Ball et al. (2008). In addition and in contrast to conceptualizations of professional knowledge, we also assigned incomplete, subjective, or experience-based knowledge to the respective domains. The findings presented above more serve the purpose of creating the necessary foundation to investigate the integration of prospective teachers' knowledge instead of representing a new contribution to the research of (professional) knowledge.

2.6.2 *Mental Processes in Which Different Mathematics-Related Knowledge Domains are Activated*

In the following we present two examples of prospective elementary teachers' mental processes which involve the activation of different mathematics-related knowledge domains. The mental processes are illustrated by sequences of codes and codings which appear in the original sequence as it has been found in the interview. Partially, the same codes and codings were discussed separately in the analysis section in order to illustrate the development of codes and its assigning to categories.

2.6.2.1 Evaluating Typical Task Difficulties from a Mathematical Point of View

This type of process involves mathematical analysis and thus the activation of content knowledge to evaluate the difficulty of tasks for children. In the example below, the prospective teacher first relies on his content knowledge when evaluating which tasks may be easy or hard. He identifies prime numbers. Probably he assumes multiplications with prime numbers to be more difficult because prime numbers have no factors apart from 1 and itself (and thus cannot be computed as easy as others by applying strategies such as breaking apart). In a second step, he argues that multiplications with the prime numbers 2 and 5 are easy. According to the multiplication with 5, he again relies on content knowledge by referring to the special role of 5 in the decimal system. According to the doubling, he knows that this is easy for second graders. Finally, this leads to the judgment that the 3 and the 7 series are most difficult (knowledge about students' cognitions).

Code sequence	Example (coding)
Prime numbers role of 5 in the decimal system	"Well, 7 is a prime number. And 5 of course is also a prime number. But it is the half of 10. Thus you can handle 5 more easily. 3 also is a prime number.
Doubling is easy	And 2 and 1 series are easy for the students at that level.
3 and 7 series are most difficult	So, I think the 3 and 7 series are most difficult for the pupils."

2.6.2.2 Remembering Content Knowledge in the Function of Illustrating Curricular or Teaching-Related Knowledge

In most of the interviews it appeared that content knowledge was mainly activated in a context of expressed curricular or teaching-related knowledge. In this case, the participants first relied on their curricular or teaching-related knowledge. For instance, they talked about the sequencing of tasks or topics in class, about teaching materials and so on. In the following, content knowledge was activated in the function of illustrating these statements by explicitly connecting the sequencing of tasks or the use of materials in class with mathematical concepts. In the example below, the student indicated the mathematical concept of “breaking apart” with an example in order to illustrate the prior statements. Interestingly, content knowledge was activated very often in the context of articulated curricular or teaching-related knowledge. Conversely, participants never relied on content knowledge primarily (i.e., in order to fully analyze the mathematical potential of the Multiplication Poster) or activated pedagogical content knowledge in the context of articulated content knowledge.

Code sequence	Example (coding)
Core multiplications	“For example second graders work intensively with the 1, 2 and 5 series. In classes you normally start with them and build on them.
Breaking apart	One can use the 2 and 5 series as a basis. When you know the 5 series you can go ahead from here. For 9×8 you don’t have to count 9 so often. You can take 5×8 and then proceed.”

2.7 Discussion

Although the analysis is still in progress, we find hints that it is possible to investigate the mathematics-related knowledge and its integration by applying the described method. We presented examples of our preliminary findings which illustrate the reconstruction of mental processes.

In their bachelor degree courses the participants of our study attended courses which have a strong emphasis on either content knowledge (such as “Elementary Algebra and Arithmetic” or “Geometry and Applied Mathematics”) or pedagogical content knowledge (such as “Mathematical Thinking of Children” or “Planning and Implementation of Mathematics Instruction”). Moreover, the participants had no or limited teaching experiences. Therefore, it can be said that (to a certain extent) the initial teacher education program dealt with the dimensions of subject matter and pedagogic content discretely. Nonetheless, it becomes apparent that the reconstructed mental processes demonstrate how prospective elementary teachers integrate mathematics-related knowledge even at an early stage of preparation. Moreover, the participants never primarily and predominantly relied on content knowledge in these processes so far. We interpret these preliminary findings to oppose the widespread view that domains of knowledge develop independently at first and become more integrated with time.

As reported in the overview section, some studies focused on the conditions of the development of PCK. According to Krauss et al. (2008), for instance, CK facilitates the development of PCK. However, statements like this are formulated in a fairly general manner and do not really help us to understand how cognitive processes look like. Thus, the future findings of our qualitative analysis may help to complement previous studies. For instance, the example we discussed earlier with respect to the process of "Evaluating typical task difficulties from a mathematical point of view" can be interpreted to demonstrate qualitatively the possibility of developing knowledge of students' cognitions by relying on content knowledge.

Even though we did not analyze the (content) knowledge in terms of correctness or profoundness, we interpreted the reconstructed mental processes afterwards with respect to these qualities. For example, in many processes the full mathematical potential of the Multiplication Poster remains unrecognized which is probably due to limited content knowledge. We conclude that it may be valuable to connect mathematical concepts with pedagogy and pedagogical content at an early level of teacher preparation. The use of mathematics-pedagogy tasks proposed by Lin and Hsu (this volume) can be considered as a possible approach to meet this objective. Admittedly, it is necessary to be careful with generally formulated implications, among other things because teacher preparation programs in other countries or at other school levels may have different requirements.

After completing our analysis it may be necessary to stabilize, specify or complement findings by refining the sample. Moreover, we plan to compare our findings for prospective elementary teachers with the integration of knowledge domains of mathematics teacher educators. By interviewing teacher educators we hope to make professional knowledge structures visible in terms of their mental processes.

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