

This chapter presents the simplest model for studying investment. The model has two periods. The current period, denoted as period 1, and the future period, denoted as period 2. Investments are chosen and financed in period 1 and the return to investment is realized in period 2. The model has been used frequently in international macroeconomics, both as an introductory pedagogical device (e.g. Obstfeld and Rogoff 1996) and as a tool for analyzing issues on the research frontier (e.g. D’Erasmus and Mendoza 2015). Here, we use the model to examine government investment decisions. The goal of the chapter is to identify how the level and the allocation of government investment should be determined when using purely economic considerations that are in the national interest. The analysis here provides the benchmark for comparison to the situation with self-interest, election politics, rent seeking, and corruption, as introduced in Chap. 3.

The four principles mentioned in Chap. 1 determine what policies are in the national interest. The first principle is that government should not be a vehicle for redistributing income to public officials or to a relatively small group of their supporters. This principle is formally represented in this chapter by assuming that the government is “benevolent”—seeking to maximize the welfare of the representative private household or the welfare of a group of different private households.

The second principle is efficiency of resource use. Maximizing social welfare requires the level and allocation of government investments to be production-efficient, so that the size of the economic pie is made as large as possible. In our setting this means investments should be made as long as their returns in future income exceed the opportunity costs of the resources used to finance them.

The third principle is that the government should limit large disparities in consumption and equalize opportunities for economic success. This principle applies when there are households with different initial conditions. The utilitarian social welfare function used in chapter weighs each of the country’s households equally. This social welfare function implies that some redistribution of resources maximizes total welfare because of the diminishing marginal value of individual household consumption.

The final principle of good governance is to limit the redistribution of wealth to current generations from unborn generations, consistent with the commonly observed legal restriction that children are not obligated to repay parental debt. In accordance with the second principle of good governance, some redistribution may be called for when future generations are richer than current generations. However, the full costs of this type of redistribution should be made transparent to the current generation so that the extent of the redistribution does not become excessive.

2.1 The Life-Cycle Model of Consumption and Saving

We begin by thinking about how households make their consumption and saving decisions. To think about saving, there must be a future. A two period setting is the simplest way to introduce the future considerations that motivate saving.

Imagine a household that lives for two periods. In each period income is generated by supplying one unit of work to productive activity. The income in period 1 is denoted by y_1 and the income in period 2 by y_2 . A household's lifetime satisfaction or utility is determined by consumption over the two periods of life, c_1 and c_2 . For simplicity, and to allow for quantitative analysis, throughout the book we assume the *lifetime* utility function takes the form

$$U = \ln c_1 + \beta \ln c_2,$$

where $\beta < 1$ is a time discount factor that indicates the relative weight a household places on receiving utility in the future rather than today.

The *single period* utility function, $\ln c$, has the familiar characteristic, one you may recall from introductory economics, of *diminishing marginal utility*. In other words, greater consumption increases satisfaction but at a diminishing rate. All increasing concave functions have this property because their slopes get smaller as the argument of the function increases. The natural log function we are using as our single-period utility function is simply a convenient increasing concave function.

The lifetime utility function includes the satisfaction the household expects to receive from a particular plan for both current and future consumption, combining the utility gained in each period of life. The time discount factor, β , is generally regarded to be less than one because people are impatient; they value satisfaction now over the same satisfaction experienced in the future.

The household's task is to choose a path for consumption that makes U as large as possible. The household can't just choose any consumption path because it is constrained by two considerations. First, it is limited by its income. Second, it may or may not be able to borrow and lend. Being able to borrow and lend is a crucial tool in choosing the best consumption path because, in general, households do not want their consumption to exactly match their income.

For example, suppose y_1 is very low and y_2 is very high. The household would prefer *not* to have their current consumption be very low and their future consumption be very high. This is because of the diminishing marginal utility of consuming

in any one period—if consumption is constrained to exactly match income, the marginal value of consumption would be much higher in the first period than in the second period. Instead, households would prefer to *smooth* consumption over time—make consumption over time more similar than their income over time by raising c_1 above y_1 and lowering c_2 below y_2 .

To manage this consumption smoothing, the household must be able to *borrow* in the first period, when income is low, and pay back the debt in the second period, when income is high. Borrowing allows the consumption path to deviate from the income path in a way that makes the household better off. In the opposite scenario, where y_1 is very high and y_2 is very low, the household wants to lower current consumption and *save* some current income, lend it, and receive repayment on the loan in the future to increase the financing of future consumption.

If the household cannot borrow and lend, then there is actually nothing to decide, it must be the case that $c_1 = y_1$ and $c_2 = y_2$. The more interesting situation allows for borrowing and lending. In this case, we assume a *complete and perfectly competitive credit market*. A complete credit market means there is both a financial asset and a financial liability that the household can acquire. Let the variable a_2 do “double duty” in capturing both household borrowing and lending. If a_2 is positive, it denotes an asset purchased by a household when it uses current income to save. If a_2 is negative, it denotes a liability or debt acquired by the household when it chooses to borrow. A perfectly competitive market means individual households cannot dictate the terms of the borrowing and lending agreement—the terms are instead determined by the market forces of demand and supply for credit. In particular, individual households take the market interest rate as beyond their control,

A word on a potential confusion associated with our notation. We think of the decision to borrow or lend, i.e., the choice of a_2 , as taking place in period 1. So it would be perfectly reasonable to denote this choice as “ a_1 ” instead of as a_2 . The justification for using a_2 is that the repayment of the debt, or the receipt of the principle that was lent out, will occur in period 2. In addition, there will also generally be an interest payment or receipt in period 2, with the rate of interest denoted by r_2 . One can go either way with the notation; the approach we have chosen is the most common convention.

With the possibility of borrowing and lending, the household’s *single period budget constraints* become

$$c_1 + a_2 = y_1$$

and

$$c_2 = y_2 + (1 + r_2)a_2.$$

These budget constraints can be combined, when the household is fully free to choose the value of a_2 , by solving for a_2 using the first period constraint and then substituting the solution into the second period constraint. After some algebraic rearranging, we can write the resulting equation as a *lifetime budget constraint*,

$$c_1 + \frac{c_2}{1 + r_2} = y_1 + \frac{y_2}{1 + r_2}.$$

The lifetime budget constraint, made possible by the ability to borrow and lend, says that, while consumption and income do not have to match period by period, the present value of lifetime consumption spending must equal the present value of lifetime income.

The task of the household is to choose a consumption path that makes them as happy as possible, while satisfying the lifetime budget constraint. Formally, this means choosing c_1 and c_2 to maximize U , taking as given y_1 , y_2 and the perfectly competitive market interest rate, r_2 . The solutions to these types of maximization problems are discussed in the Technical Appendix. Here we simply state and discuss the optimal solutions without derivation. The household's utility maximizing demand for consumption goods and assets are

$$\begin{aligned} c_1 &= \frac{1}{1 + \beta} \left(y_1 + \frac{y_2}{1 + r_2} \right) \\ c_2 &= \frac{\beta(1 + r_2)}{1 + \beta} \left(y_1 + \frac{y_2}{1 + r_2} \right) \\ a_2 &= y_1 - c_1 = \frac{\beta}{1 + \beta} y_1 - \frac{1}{1 + \beta} \frac{y_2}{1 + r_2}. \end{aligned}$$

Consumption in each period is positively affected by the household's lifetime income or *wealth*. With the ability to borrow and lend, current income is not the key factor in explaining consumption—the consumption a household can afford is instead dictated by its wealth. Looking at the solution for a_2 , we see that the household may save and lend ($a_2 > 0$) or may borrow ($a_2 < 0$), depending on the circumstances. Households with relatively high values of y_1 will be savers/lenders and those with relatively high values of y_2 will be borrowers. Patient households, with high values for β , will tend to lend and impatient households, with low values of β , will tend to borrow. Finally, the higher the market interest rate, r_2 , the more likely the household is to save and lend current income. This last result means the “supply of market funds,” provided by household saving, is an upward sloping function of the interest rate, as is typically assumed in elementary economics.

2.1.1 Borrowing Constraints

We have discussed the extreme situations when no credit market exists and when a full complete and perfectly competitive credit market exists. There is an important intermediate case, where the market is *incomplete*. Households are free to save but face restrictions on how much they can borrow. In the case where they cannot

borrow at all, there is what is referred to as a *non-negativity constraint* on a_2 , i.e. household choices must be consistent with the condition $a_2 \geq 0$.

One strategy for identifying when the non-negativity constraint is binding, is to first solve the household problem with no constraints on borrowing, as we have above. Next, use the unconstrained solution for a_2 to see that the condition $a_2 \geq 0$ is equivalent to

$$y_1 \geq \frac{1}{\beta} \frac{y_2}{1 + r_2}.$$

Low values for y_1 , β , and r_2 , and high values for y_2 , increase the likelihood that the household would like to borrow and the condition above will *not* be satisfied. Impatient households with relatively low current income, who also face low market interest rates, will tend to be credit-constrained. In this situation, the best the household can do is choose consumption to match income in each period—just as if there is no credit market at all.

2.2 Introducing the Government

In this section we move from the discussion of an individual household to the economy as a whole. In addition, we start thinking about fiscal policy and the government's role in the economy.

The private sector is made up of representative households that are both consumers and producers who operate just as the household did in Sect. 2.1. In the simplest two-period model, there are N of these private households but they are all *identical* and thus can be represented by a single household. The *representative household* begins period 1 with an exogenous income flow, y_1 , from supplying one unit of labor. The household also supplies one unit of labor to production in period 2. The new twist is we now assume that the output and income in period 2 is affected by the government's provision of public capital (e.g. public education, roads, or public utilities infrastructure). Public capital per household in period 2 is denoted by g_2 (y_1 can also be viewed as a function of the available public capital, g_1 , but that stock is given in the analysis). Period 2 output and income per household is determined by the following production function,

$$y_2 = Ag_2^\mu, \tag{2.1}$$

where A is a productivity parameter, frequently referred to as total factor productivity (TFP), and $0 < \mu < 1$ gauges the impact of public capital on output. The assumption that $\mu < 1$ captures the diminishing marginal productivity of public capital on output. The rationale for diminishing marginal productivity is that as the level of *one* productive input increases, relative to *other* inputs used in production, each additional unit of the input will not be used as intensely in producing goods. We think of y_2 as being produced using not only public capital but also using the time of a worker of given abilities and, perhaps, a fixed amount other inputs such as

land. The productivity of the worker increases with public capital. For example, the more public education received when young and the more roads available to move products as an adult, the more productive the worker. However, the effect of additional public capital on worker productivity diminishes as the level of capital becomes larger—additional expenditures on public education or roads have a diminishing effect.

The fact that output depends on public capital *per household*, rather than the total stock of public capital (G_2), means that there is crowding of the public capital. If the population of workers were to increase, with a fixed G_2 , the productivity of an individual worker would fall. In Sect. 2.6 of this chapter, we discuss what happens if public capital is a *pure public good*, with productive services that are not affected by the population size, or an *impure public good*, with partial crowding as the population increases. The bottom line will be that the alternative assumptions about public capital primarily alter the interpretation of A .

Just as in Sect. 2.1, we assume a household's lifetime satisfaction or utility is determined by consumption over the two periods of life, c_1 and c_2 , with the lifetime utility function taking the form

$$U = \ln c_1 + \beta \ln c_2, \quad (2.2)$$

where recall $\beta < 1$ is a time discount factor that measures the household's willingness to postpone receiving utility into the future.

Let's begin with the case where a credit market does *not* exist. This is a natural starting point when discussing a closed economy with identical households. If households are identical, then *all* households will want to lend or *all* households will want to borrow. There will be no possibility of credit market transactions because that requires there be a borrower *and* a lender. We initially assume that the government will finance its spending with tax rate, τ , levied on household income. With no credit market, consumption in each period is determined by the period's income and the period's income tax rate; $c_1 = (1 - \tau_1)y_1$ and $c_2 = (1 - \tau_2)y_2$.

2.2.1 Taxes and Government Investment

In this chapter, we assume that government policy is set by a benevolent social planner who chooses tax rates and government investment to maximize the welfare of the representative household. We also assume that public capital fully depreciates in one period, so the public investment decision in period 1 is equivalent to choosing the period 2 public capital stock.

Note that with only two periods and no debt financing of government investment, there is actually no need for period 2 taxes. With $\tau_2 = 0$, we can focus on the optimal choice of first period taxes and government investment. The first period government budget constraint is $G_2 = N\tau_1y_1$ or $g_2 = \tau_1y_1$, with $g_2 \equiv G_2/N$.

Using the government and private household budget constraints, the private household's welfare can be written as a function of government investment,

$$U = \ln(y_1 - g_2) + \beta \ln(Ag_2^\mu). \quad (2.3)$$

Maximizing (2.3) with respect to g_2 , yields the optimal fiscal policy set by the benevolent government,

$$g_2 = \frac{\beta\mu}{1 + \beta\mu} y_1 \quad (2.4a)$$

$$\tau_1 = \frac{\beta\mu}{1 + \beta\mu}. \quad (2.4b)$$

There are two reasons why government capital is valued in this setting. First, government investment is productive and thereby increases lifetime resources of the representative household. The higher the value of μ , the more productive is investment and the greater is the optimal public capital. Second, because there are no other assets available, government capital can help smooth consumption across time. This second reason explains why the time preference parameter, β , affects the optimal level of government capital in (2.4a). If households are more patient, placing a relatively large weight on future utility, then they prefer higher current period taxes and more government capital as a form of indirect saving.

2.2.2 Public Debt and Government Investment

Now let's introduce public debt. With public debt, the government has two ways of financing first-period investment—taxes or borrowing. The presence of public debt gives households a second asset that may help in smoothing their consumption over time in a way that better suits their preferences. Of course, for households to be interested in government debt as an asset, they have to be willing to save. So, we assume that is the case. If the representative household wants to borrow, they would not purchase government debt and serve as a lender to the government. However, remember from Sect. 2.1, that the government could convince households to lend by offering a sufficiently high interest rate on public debt.

Denote the total public debt issued by the government, and purchased by private households in period 1, as B_2 . In period 2, the principle and interest paid to the bond holders is $(1 + r_2)B_2$, where r_2 is the interest rate on government bonds. The debt repayment obligation creates a need for the government to raise revenue in period 2, so we re-introduce the tax rate, τ_2 . If we define $b_2 \equiv B_2/N$, then the two household budget constraints can be written as

$$c_1 + b_2 = (1 - \tau_1)y_1 \quad (2.5a)$$

$$c_2 = (1 - \tau_2)y_2 + (1 + r_2)b_2, \quad (2.5b)$$

and the two government budget constraints as

$$g_2 = \tau_1 y_1 + b_2 \quad (2.6a)$$

$$(1 + r_2)b_2 = \tau_2 y_2. \quad (2.6b)$$

Notice that if we combine the household and government budget constraints we can rewrite the economy's *consolidated* constraints as

$$c_1 = y_1 - g_2 \quad (2.7a)$$

$$c_2 = y_2. \quad (2.7b)$$

Surprisingly, these consolidated constraints imply that public debt neither affects the lifetime resources of the household nor the ability to alter the timing of household consumption. The optimal choice of government investment is the same as in the setting where government borrowing was prohibited.

Why does adding public debt fail to alter the government's policy and household consumption? Even if households like the idea of being able to save by purchasing government bonds, it fails to increase second period consumption opportunities because government borrowing requires that households pay higher second period taxes. The second period taxes completely offset the value of the government bonds purchased and the associated interest payments. For this reason, bonds are not a store of household wealth. Furthermore, households view period 1 taxes and period 1 bonds as equivalent means of financing government investment because both reduce first period consumption in the same way and both fail to directly increase household consumption in the second period. The result that *government bonds are not net wealth, and tax and bond financing are equivalent*, is a fundamental starting point in the conceptual understanding of fiscal policy (Barro 1974). However, as we shall see later in this chapter and in the next chapter, the result fails to hold for empirically important and policy relevant reasons (Kotlikoff 2003, Chapter VII).

2.3 The Small-Open Economy Model

Borrowing and lending across households is not possible when households are identical in all ways, as is the case in the representative agent model, because some households must choose to be lenders and some borrowers. In a representative agent model, either *all* households want to lend, or *all* households want to borrow. One way that the representative household could acquire assets or liabilities is to lend or borrow in an *international* market for funds. Implicit in this idea is the assumption that households in other countries have *different* income paths or time preferences.

The simplest way to introduce an international market for funds is to assume a sufficiently large number of countries are trading with each other. When many countries are engaged in trade, it may be reasonable to assume that the international market for funds is perfectly competitive at the level of an entire country. A single country is so small relative to the entire market, that they take the international interest rate as an exogenous variable that is beyond its influence. This assumption is most accurate for smaller economies, so an open economy model with an exogenous international interest rate for funds is called the *small open economy* model.

In our model, the market for funds is one where households borrow and lend for the purpose of financing consumption. Of course markets for private consumption loans don't just appear. Consumption loan markets are limited in today's most developed economies. It takes a great deal of financial and legal institutional structure to extend, monitor, and enforce domestic, not to mention international, loan contracts for household consumption. Without the underlying financial and legal structure, the costs and risk associated with such lending would be too great for the market to exist. The laws and regulations associated with financial markets are forms of *intangible* public capital that are an important component of a country's infrastructure

In contrast, loans extended from one country's government, or group of governments, to another country's government may be feasible even when private loan markets fail to exist. Governments typically have at least some rudiments of a formal accounting and payment system that allow for funds to be transferred across borders. In addition, political or economic pressure can be used by lending governments to help enforce loan repayment. Thus, it is important to consider the situation where private households do not have access to international loan markets, and yet governments can extend loans to each other.

2.3.1 Private and Public Credit

Let's begin with the simplest case where *both* households and governments can borrow and lend in perfectly competitive international loan markets. We introduce the new notation a_2^* to represent the representative household's holdings of an international asset, $a_2^* > 0$, or an international liability, $a_2^* < 0$. We also need to adjust notation to allow for government lending as well as government borrowing. Toward this end, think of $b_2 > 0$ as government debt associated with borrowing and $b_2 < 0$ as government assets associated with lending. Furthermore, let household *saving*, s , be the accumulation of international assets and domestic government debt. Household saving could be negative, meaning that households of one country could be borrowing from other countries.

The household budget constraints can now be written as

$$c_1 + s = (1 - \tau_1)y_1 \quad (2.8a)$$

$$c_2 = (1 - \tau_2)y_2 + (1 + r^*)s, \quad (2.8b)$$

where r^* is the exogenous interest rate determined by the international market for funds. Treating r^* as an exogenous variable is where we use the assumption of a small open economy operating in a perfectly competitive international market for funds.

The government budget constraints take the same form as (2.6), but now, because we allow for the possibility that the government is a net lender, we must allow for the possibility that $\tau_2 < 0$. The second period tax rate must be interpreted as a *net* tax rate, that can possibly be negative. When the government is a net lender it is able to *transfer* income to second period households, financed by interest and loan repayments from abroad, rather than tax them.

Consolidating (2.6) and (2.8) to form the household's *lifetime budget constraint*, gives us

$$c_1 + \frac{c_2}{1 + r^*} = y_1 + \frac{y_2}{1 + r^*} - \left(\tau_1 y_1 + \frac{\tau_2 y_2}{1 + r^*} \right) = y_1 + \frac{y_2}{1 + r^*} - g_2. \quad (2.9)$$

As in the closed economy setting, government bonds are equivalent to first period taxes and are not net wealth. If the government borrows internationally to avoid using current taxes, domestic households will have to be taxed in the future period to repay the foreign debt and interest. Government lending requires that first period income be taxed away from households. However, the first period taxes paid to a government for the purpose of lending them internationally, are returned with interest to the household in the form of second period transfers when the international loans are repaid.

Furthermore, the household could generate this same outcome for itself by saving and lending privately. If the government taxes more in the first period and lends the revenue, the household would simply lend less privately. The only aspect of government policy that influences the representative household's lifetime consumption possibilities is government investment, regardless of how it is financed.

The benevolent government chooses investment to maximize the lifetime wealth of the household. Maximizing the right-hand side of (2.9) by choosing g_2 generates the following *efficient* investment rule

$$\mu A g_2^{\mu-1} = 1 + r^*. \quad (2.10)$$

The efficient investment rule says that government investment should equate the marginal product of public capital to the opportunity cost of funds, as determined by the interest rate on international loan markets. Unlike (2.4a), the household preference parameter (β) that influences the optimal timing of consumption plays no role in the efficiency condition. Household can now use international consumption loans

to determine the preferred time path of consumption. Government capital no longer needs to do the double duty of increasing future income and optimally smoothing consumption over time.

Given the maximum lifetime wealth that results from efficient public investment, the representative household chooses consumption across the two periods to maximize utility. Using the resulting optimal conditions for maximizing utility, the optimal consumption path satisfies

$$\frac{c_2}{c_1} = \beta(1 + r^*). \quad (2.11)$$

This expression, known as the *Euler equation*, says that consumption rises faster over the life-cycle, the higher is the interest rate. A higher interest rate raises the cost of current consumption relative to future consumption, which causes the household to choose a steeper consumption profile over time. Note that if you combine the solutions for c_1 and c_2 from Sect. 2.1, those solutions will also satisfy (2.11). The Euler equation will always hold whenever households can freely borrow and lend.

2.3.2 Only Public Credit

Suppose now that the market for *private* international loans does *not* exist. Governments can borrow and lend internationally, but not households. The consolidated budget constraints of the *credit-constrained* representative household are

$$c_1 = (1 - \tau_1)y_1 = y_1 - g_2 + b_2 \quad (2.12a)$$

$$c_2 = (1 - \tau_2)y_2 = y_2 - (1 + r^*)b_2. \quad (2.12b)$$

Note these budget constraints now depend on public debt because the government can do something the household cannot do—borrow. The situation differs from the closed economy case because domestic households do not have to purchase government debt, instead the debt can be sold to foreigners. So, debt financing is now possible even if domestic households are credit constrained and do not want to lend.

The government chooses g_2 and b_2 to maximize

$U = \ln(y_1 - g_2 + b_2) + \beta \ln(y_2 - (1 + r^*)b_2)$, producing the following optimal conditions,

$$\mu A g_2^{\mu-1} = 1 + r^* \quad (2.13a)$$

$$\frac{c_2}{c_1} = \beta(1 + r^*). \quad (2.13b)$$

Equation (2.13a) reproduces the efficiency condition for investment given in (2.10) and Eq. (2.13b) generates the same condition for the optimal timing of household consumption over the life cycle given by (2.11). The first-best outcome, where households can also borrow and lend internationally, is reproduced because the government serves as a financial intermediary for private households. The government mimics the borrowing and lending the household prefers in order to generate the same first-best outcome that the household would have chosen if it could have directly participated in perfectly competitive international loan markets.

To understand this result in more detail, consider the perfectly closed economy we started the chapter with, where no international borrowing and lending is possible. Suppose the government, and the representative household it serves, would prefer to borrow internationally but cannot. Under this scenario, the *credit-constrained* solution for government investment in the closed economy, that satisfies (2.4a), implies $\mu Ag_2^{\mu-1} > 1 + r^*$. Government investment is inefficiently low because the marginal product of public capital is greater than the cost of borrowing. The household does not prefer the efficient investment level because the required sacrifice of *first* period consumption would lower its welfare more than if they were able to borrow the funds in the international loan market and instead sacrifice the corresponding *second* period consumption. The credit constraint means that *both* first period consumption and government investment are too low.

If the government can borrow the required funds abroad to achieve the efficient investment level *and* raise first period consumption, the household can be made better off. Here, government bonds are not equivalent to first period taxes as a way of funding investment because, with international borrowing, the domestic household need not purchase the debt and sacrifice current consumption. Thus, issuing government debt raises the private household's lifetime wealth and welfare. The general lesson is that when private households are credit-constrained, and the government has access to an international market for credit, then government debt can be a welfare-enhancing fiscal tool.

To complete our discussion, we need to introduce an important caveat—the conclusion that government debt can be used as a welfare-enhancing fiscal tool must be interpreted carefully. We have assumed that the two periods in the model represent two periods in the life of a single household. Under this interpretation, the lifetime welfare of a credit-constrained household can unambiguously be raised by issuing public debt in international loan markets.

However, as will be discussed below, there is also a *generational* interpretation of the two-period model, with each period representing a distinct generation of the same family. In this case, the credit-constraint takes the form of a *bequest-constraint*. The bequest-constraint means that the current generation is not legally permitted to impose a debt-obligation on their children, the next generation. Positive bequests of assets are fine, but negative bequests, the bequeathing of parental debts, are not allowed. This legal restriction is reflected in the laws of most countries. The government, however, can indirectly relieve the non-negative bequest-constraint by issuing public debt on behalf of the current generation and

then using taxes on future generations to force repayment (Drazen 1978). In this way, the government can circumvent the legal restriction it imposes on individual households, creating a fundamental tension in how fiscal policy affects the welfare of *different* generations.

2.4 Human Capital, Inequality, and Public Debt

In their survey of the theories of why public debt is used, Alesina and Passalaqua (2015) view the credit constraint-motivation as particularly convincing. As poorer segments of the population become more engaged in a country's politics, there would naturally be more pressure to issue public debt to serve as a substitute for the inability to borrow privately. Increasing political voice for the poor, offers a possible explanation for the rise in public debt observed in maturing democracies.

This explanation can be further articulated if one takes the *generational* interpretation of the two period model—with the first period representing the parent's adult lifetime and the second period the adult lifetime of their children. The utility function in our model, given by (2.2), is now interpreted as being comprised of the utility the current generation receives from its own lifetime consumption and the utility the current generation receives from the lifetime consumption of its adult children, a form of *intergenerational altruism*. The reason the generational interpretation creates a more compelling framework for analyzing public debt is that, while *life-cycle* credit markets *may be complete*, the market for *intergenerational* credit transactions are *clearly incomplete*. Parents are allowed to lend and create an asset that could then be bequeathed to children. However, parents are not legally allowed to borrow and then leave the debt for their children to repay.

In a life-cycle credit transaction, the person who borrows is the same person who repays the debt at a later date—everything is settled within an individual's lifetime. A market for life-cycle credit transactions is close to complete in developed countries. Intergenerational credit transactions would include contracts with the parents doing the borrowing and the children repaying the debt in the future. Private intergenerational credit transactions are limited because children are not legally bound to repay the debt taken on by their parents. However, the government, by borrowing today and postponing debt repayment sufficiently far into the future, can create a credit transaction that extends across generations.

To emphasize why these considerations are important, let's go a step further. Interpret g_2 as human capital investments in children, similar to Drazen (1978), that either parents choose directly or that are determined by local governments responding to household preferences in particular communities. These human capital investments include all educational investment that occur at each stage of the child's life—from pre-school investments, to primary and secondary schooling, to parental subsidy of college expenses. We can extend the model a bit and think of p as representing the relative price of educational inputs. Introducing the price of education inputs, changes both the credit constrained choice of g_2 , given by (2.4a),

and the unconstrained or efficient choice of g_2 given by (2.13a). When education has a distinct relative price from other goods, these two equations take the form

“Poor” Household (Bequest-Constrained)

$$pg_2 = \frac{\beta\mu}{1 + \beta\mu}y_1, \text{ with } \mu Ag_2^{\mu-1} > p(1 + r^*), \quad (2.4a')$$

and

“Rich” Households (Unconstrained)

$$\mu Ag_2^{\mu-1} = p(1 + r^*) \quad (2.13a')$$

Finally, to capture what Alesina and Passalaqua (2015) have in mind, let’s add some relevant heterogeneity into the mix by thinking of two household types that differ by their level of first period, or parental, income. “Poor” households have little parental income. If they cannot impose debt repayment obligations on their children (i.e. $a^* < 0$ is not allowed), their preferred investments would be represented by (2.4a’). “Rich” households, on the other hand, have high parental income. They can afford a level of g_2 that satisfies the efficiency condition in (2.13a’), even with no intergenerational credit transactions. In fact, rich parents also leave their children a positive bequest of financial assets ($a^* > 0$).

This re-interpretation and extension of the two-period model allows us to relate several important features of advanced economies that began developing over the last quarter of the twentieth century. First, we have seen a rise in skill-biased technological change and a change in sectoral composition that increased the return to schooling, but at the same time created a rise in wage inequality across households with different levels of schooling (Autor 2014). Second, despite the growing return to education, there has been a slowdown in the growth of years of schooling and in economic growth (Gordon 2016; OECD 2015). Third, there has been a rise in globalization since the 1970s and expanded access to international credit (Azzimonti et al. 2014). Further, we have seen an unprecedented rise in government budget deficits and public debt (Hallerberg et al. 2009; Steuerle 2014). Finally, the relative prices of important investments in education and health have dramatically increased. In relating these five developments, we argue that the credit-constrained story for the rise in public debt can be made even more convincing.

Important Developments in Rich Countries Since 1975

1. Increased return to college and increased wage inequality
2. Slowdown in the growth of schooling obtained by average worker
3. Expansion in international credit markets and a low international cost of funds

4. Rising public debt as a fraction of GDP
5. Increasing relative price of education and health care

Start with the rise in the returns to education, which in our model is captured by a rise in A and $\mu A g_2^{\mu-1}/p$. Rich households would have no trouble responding to the increased return by raising their preferred level of g_2 until (2.13a') was once again satisfied—increasing investment at all stages of their child's life to ensure they can get into the best college possible or even go on to graduate school. However, poor households that are constrained by low levels of y_1 would not alter their levels of g_2 . Notice that A does not enter (2.4a').¹ Thus, a rise in A leads to a rise in wage inequality in the next generation because rich households respond to and benefit the most from a rise in A .

The wage inequality would worsen if the rising demand for educational inputs by rich households drives p up. An increase in the relative price of family investments in education and health care is the fifth important development mentioned above. A rising price of education would lower the actual investments of constrained households, as indicated by (2.4a'). Rich households increase years of schooling, although by not as much as when the relative price remains fixed, but poor households reduce years of schooling for their children. This implies there may not be a strong economy-wide increase in educational attainment despite the growing return to schooling, explaining the second important development on the list.

The rising gap between the return to educational investments and the return to financial or physical assets would increase the poor household's demand for public debt to alleviate their intergenerational credit constraints. At the same time, growing access to international loan markets would lower the cost of funding the demand for public debt, the third development since 1975. Azzimonti et al. (2014) explain the rise in debt-to-GDP ratios across OECD countries as, in part, due to increasing financial liberalization across borders and lower interest rates charged to countries with high public debt.

An increase in public debt unambiguously raises the welfare of the current generation of poor households by allowing both more current consumption and more human capital investment in the next generation. However, nothing guarantees that the extra investment will raise future income enough to cover the debt costs—i.e. the consumption of the future generation could fall. Specifically, public debt raises U and c_1 , but c_2 may rise or fall. This is because the rate of return on investment has fallen from $\mu A g_2^{\mu-1} - 1$ to the international loan rate r^* . This

¹The fact that A does not enter (2.4a') can be explained by offsetting income and substitution effects that are analogous to those associated with a rise in interest rates in the standard life-cycle theory of saving. A higher value of A increases family resources that parents can access by investing less in their children. On the other hand, the opportunity cost of not investing has gone up. For more on the conflicting income and substitution effects associated with saving and investment, see Sect. 2.10 and Chapter 4.

implies that the ratio, c_2/c_1 , must fall, which includes the possibility of an *absolute* fall in c_2 .

The possibility that the next generation from a bequest-constrained household is made worse-off by government debt is discussed in detail by Lord and Rangazas (1993). They find that deficit policies that are supported by the majority of altruistic households currently alive are likely to reduce the consumption opportunities of future generations. This is an important consideration. Most societies create laws that protect future generations from the excesses of the current generation by making it illegal for parents to shift their debt obligations to their children. If these laws are generally supported, then it should not be possible for the government to circumvent them with fiscal policy. Instead, the government should make any intergenerational redistribution clear to the public and help impose the same discipline on the country as a whole that the country's laws place on individual households.

The more households that are “poor,” i.e. face constraints on intergenerational borrowing arrangements, the more relevant the model is for explaining the rise in public debt. Three factors suggest that a growing majority of households face intergenerational borrowing constraints. First, econometric studies consistently find that parent's income is positively correlated with educational attainment of children, even when measures of child ability are controlled for statistically (Heckman and Krueger 2005). If households are unconstrained, then marginal variations in parental income would *not* affect the efficient investment in education. Second, most countries have a strong “college-or-bust” mentality among the majority of households (Murray 2008; Bennet and Wilezol 2013). The real cost of college, including educating a child well enough that they can realistically enter college and complete a 4 year degree, is quite expensive. The *relative* burden of financing education has increased over time because there has been little or no increase in real income since the 1970s for the vast majority of households (Autor (2014) and OECD (2015, Table 5)), while the real costs of education has increased over the same period (Gordon 2016). Combine this with the rising relative price of health care and it is easy to see that both the consumption and human capital investments of the majority of households have been increasingly squeezed by economic trends. Finally, statistical studies show that educational investments, at all ages, continue to exceed the return on financial and physical assets (at least for the *average* student). Thus, it is not hard to see why the majority of households might be willing to accept the expansion in public debt—especially if the full extent of the intergenerational transfers associated with current policies is not transparent.

2.5 Public Capital and Productivity

We have been assuming that public capital raises worker productivity, i.e. that $\mu > 0$. There is an empirical literature that attempts to test this assumption. The concept of public capital is quite broad and can include physical infrastructure, the stock of basic research knowledge, human capital acquired via public schooling,

and even the intangible capital reflected in a country's laws and regulations—including the rules and procedures for implementing them. Empirical studies typically use national income accounting measures of public capital that are limited to physical infrastructure. Although there is some debate over the exact estimate of μ , most studies find a positive and statistically significant effect of public infrastructure on output.

The classic empirical study of the productivity effects of public infrastructure was conducted by David Ashauer (1989). His approach allowed for a direct measure of μ , the output elasticity of public capital, which he estimated to be as high as 0.40. Subsequent research that attempted to verify his findings, using different data sets and econometric approaches, found a somewhat lower elasticity. Glomm and Ravikumar (1997) survey the empirical work in the decade following Ashauer's study and conclude that a more reasonable estimate might be 0.20. In an update of his earlier study, Ashauer (2000) found estimates close to 0.30. Several more recent studies also find estimates that cluster around 0.30 (see the survey in Bivens 2012).

It would be useful to have estimates of the effects that extend beyond public physical infrastructure. Less tangible types of public capital may have output elasticities that differ from physical infrastructure. Ideally one would decompose public capital into its different components. For example, a recent study has estimated a parameter very similar to μ that measures the human capital elasticity of public school spending. Interestingly, Manuelli and Seshadri (2014) find a public school spending elasticity estimate of about 0.30. Their estimate is based on an assumption that public school spending has a rate of return similar to that of private physical capital, about 7%. Heckman and others argue that, at the levels of school spending seen in developed countries, the marginal rate of return to public school spending in the average community is much lower than 7% (Heckman and Krueger 2005). This is consistent with the historical analysis of Rangazas (2000, 2002) who finds a public spending elasticity of less than 0.20.

Another measurement issue in empirical studies is related to the quality of public capital and government corruption (Chakraborty and Dabla-Norris 2011). As discussed in the introduction, large portions of the funds officially budgeted for public investment are never actually invested but instead are siphoned off for consumption by public officials and private contractors. In addition, the effectiveness of the public capital that does exist is influenced by how it is maintained and operated by government bureaucrats. This issue not only applies to infrastructure, power plants, and water and sewage facilities, but also to public schools where teacher absenteeism is a problem. The inability to control for these measurement issues will create a downward bias in the estimates of output effects from public capital.

2.6 Pure and Impure Public Capital

Thus far we have assumed that public capital is a private good, similar to private capital. With private capital, if one worker drives a tractor or operates a computer, then it is not possible for another worker to use the same equipment to produce

output. For some types of public capital, the analogy to private capital is not accurate. If a producer is using a public road, this does not inhibit another producer from using the same road, at the same time, in any significant way.

If the transportation services provided by the road are not affected by the total number of producers using the road, then the road would be a *pure public good*—no “crowding” or reduction of services occurs as the number of producers served increases. Roads, while not pure private goods, are not pure public goods either because when the road becomes sufficiently busy with traffic, the total number of producers using the road *does reduce* the transportation services provided per producer. Roads, and many other types of public capital, are best viewed as *impure public goods* where crowding can occur.

This discussion affects the modelling of the production function that relates public capital to output. If public capital were a pure public good, then instead of writing the production function as in (2.1), we would write the production function as

$$y_2 = AG_2^\mu \quad (2.14)$$

where now the *total* public capital stock determines the productivity of an *individual* producer, independent of how many producers there are in the economy.

A more general way of writing the production function, that includes (2.1) and (2.14) as special cases and that introduces impure public goods, is

$$y_2 = A(G_2/N^\xi)^\mu, \quad (2.15)$$

with $0 \leq \xi \leq 1$. The parameter ξ gauges the public goods nature of public capital. If $\xi = 1$, then public capital is a private good, as in the case of private capital. If $\xi = 0$, then public capital is a pure public good. For $0 < \xi < 1$, we have an impure public good, where some crowding occurs.

Now we need to think about how taking the simple route of modeling public capital as a private good, when in fact it is more accurate to model it as a impure public good, affects the analysis. Toward this end, note that we can write (2.15) as

$$y_2 = A \left(\frac{G_2 N}{N^\xi N} \right)^\mu = A (g_2 N^{1-\xi})^\mu \equiv \bar{A} g_2^\mu, \quad (2.16)$$

where $\bar{A} \equiv AN^{(1-\xi)\mu}$. The general production function in (2.16) has the same form as (2.1), but with an adjusted TFP term. This means, even if public capital is an impure public good, we can continue to model it as a private good. However, the TFP associated with a production function of the form in (2.1), i.e. expressed in terms of public capital per producer (as is done with private capital), will increase with population size. For a given ratio of public capital per producer, a larger economy will generate more output *per producer*. This is because the producers, at least to some extent, can share the total public capital, and with more producers there is a greater total public capital stock for any given value of g_2 . Note that the *sharing effect*, that raises TFP, diminishes with population size because $(1 - \xi)\mu < 1$.

So, for large populations, variations in population size do not affect worker productivity very much, when g_2 is held constant.

The lesson here is that we can model public capital as a private good and use (2.1), but we have to remember that the TFP associated with (2.1) is a function of population size if public capital has public good characteristics. For most of our analysis, this consideration will not be important. However, as we will see in the very next section of the chapter, there are instances where the adjusted interpretation of TFP should be kept in mind.

2.7 The Allocation of Public Capital

Now we turn to the *allocation* of public capital. This is important because, as discussed in the introduction, politics will not only affect the size of government budgets but also how a given budget is allocated across regions or neighborhoods of a country. For example, societies tend to have dramatically unequal allocations of infrastructure and educational spending across rich and poor neighborhoods. To examine the possible distortionary influence of politics, we need to start with a benchmark analysis of investment allocation based solely on economic considerations.

Suppose there are two regions P and R. Each region has a representative household with an associated initial income flow and a production function relating local public capital to future output and income. Income flows over the two periods, are y_{1P} and $y_{2P} = A_P(g_{2P})^\mu$, for region P, and y_{1R} and $y_{2R} = A_R(g_{2R})^\mu$, for region R.

To focus on allocation, we simplify the financing decision by assuming that the government does not issue debt. In period 1, the national government levies an income tax on all households equal to τ_1 . The government budget constraint is

$$N_P g_{2P} + N_R g_{2R} = \tau_1 (N_P y_{1P} + N_R y_{1R}). \quad (2.17)$$

Furthermore, we assume households can borrow and lend in a perfectly competitive loan market, so public capital investment decisions are not affected by concerns over intertemporal consumption smoothing.

The household budget constraints in each region take the form,

$$c_1 + s = (1 - \tau_1)y_1 \quad (2.18a)$$

$$c_2 = y_2 + (1 + r^*)s, \quad (2.18b)$$

where we drop the regional notation when it is not necessary for clarity. Household preferences in each region take the same log form as before, see (2.2). Households choose consumption and saving to maximize utility subject to the budget constraints given by (2.18). The resulting optimal consumption choices are

$$c_1 = \frac{W}{1 + \beta} \quad (2.19a)$$

$$c_2 = \frac{\beta W}{1 + \beta}, \quad (2.19b)$$

where $W \equiv (1 - \tau_1)y_1 + \frac{y_2}{1+r^*}$, lifetime after-tax *wealth*. Substituting the optimal consumption choices back into (2.2) yields a *value function* or an *indirect utility function*, giving the maximum lifetime utility associated with a particular value of wealth,

$$V(W) = (1 + \beta) \ln W. \quad (2.20)$$

We assume that the benevolent government chooses fiscal policy to maximize the sum of the utility of its citizens, a measure of aggregate welfare that weighs each individual household equally.² Subject to the budget constraint given in (2.17), the government then chooses the common income tax rate and public capital in each region to maximize

$$N_P V(W_P) + N_R V(W_R). \quad (2.21)$$

The government's problem generates the following rules for the optimal fiscal policy,

$$\frac{N_P y_{1P}}{W_P} + \frac{N_R y_{1R}}{W_R} = \lambda (N_P y_{1P} + N_R y_{1R}) \quad (2.22a)$$

$$\frac{1}{W_P} \frac{\mu A_P g_{2P}^{\mu-1}}{1 + r^*} = \lambda \quad (2.22b)$$

$$\frac{1}{W_R} \frac{\mu A_R g_{2R}^{\mu-1}}{1 + r^*} = \lambda. \quad (2.22c)$$

where λ is the Lagrange multiplier associated with the government budget constraint, which can be interpreted as the marginal value of government revenue. Equation (2.22a) says the tax rate should be chosen to equate the marginal social cost, associated with the drop in current consumption, to the marginal benefit of additional government revenue collected. Eqs. (2.22b) and (2.22c) say that the marginal benefit of investing in each region should be equated to the marginal cost of collecting the government revenue needed to finance the investments.

²Saez and Stantcheva (2016) develop ways to generalize the traditional utilitarian social welfare function used here in order to reflect considerations that may be important for policy formation. For example, society may want policy makers to place greater weights on households that have a greater willingness to work or that have come from disadvantaged family backgrounds.

Equating (2.22b) and (2.22c) gives an allocation rule for government investment,

$$\frac{1}{W_P} \frac{\mu_{AP} g_{2P}^{\mu-1}}{1+r^*} = \frac{1}{W_R} \frac{\mu_{AR} g_{2R}^{\mu-1}}{1+r^*}, \quad (2.23)$$

i.e. the marginal *value* of investment should be equated across regions.

In general, the allocation rule does not indicate equal government investment across regions. The government should invest more in the region with low consumption and high marginal productivity of public capital. A region with lower first period income will receive higher marginal value from greater consumption associated with higher second period income. The rise in second period income will be greater the higher is the region's TFP. Remember from our discussion of impure public goods that regional TFP could differ because of differences in population size. TFP could also differ based on differences in local natural resources or other geographic characteristics such as access to the sea or to the borders of foreign countries.

The fact that (2.23) is not a pure efficiency rule that would simply determine the allocation of investment by equating the marginal product of public capital across regions, captures the possible conflict between the government efficiency principle and the principle of narrowing economic disparities. Larger investments in a poor region may be justified, even if the return on investment is relatively low, because any gain in income has a strong effect on household welfare when household wealth is low.

The possible conflict between the two principles when deciding on the allocation of investment depends on the fiscal tools available to the government. We are not allowing for any fiscal variables that *directly* address differences in first period income across the regions. Region P could be interpreted as “poor” and region R as “rich,” if $y_{1P} < y_{1R}$. In principle, rich households could be targeted with higher tax rates that finance transfers of income to poor households. In this case, the investment allocation could be made strictly on efficiency grounds. However, unless the tax-transfer scheme completely equated first period incomes, then optimal government investment will be affected by income inequality. Here, public investment must again do “double duty,” trying to satisfy equity and efficiency considerations.

Even if $y_{1P} = y_{1R}$, differences in regional TFP could affect lifetime income, which in turn would prevent a equalization of marginal products across regions. The only situation where (2.23) implies an investment rule that equates the marginal products across sector is where *both* first period incomes *and* TFP are equal across regions. In this special case, public capital should be equal across regions. Furthermore, because $W_P = W_R = W$, then (2.22a) gives $u\lambda = 1/W$. This implies, using (2.22b) and (2.22c), that the marginal product of public investment equals the international opportunity cost of funds. The assumptions of this special case essentially take us back to the representative agent model.

2.8 Fiscal Federalism

Fiscal federalism relates to the economics of the public sector when policies are conducted by different levels of government, i.e. national as well as regional governments, such as state and local governments. Here we study fiscal federalism by extending our analysis of the regional allocation of public capital to the situation where both national and regional governments invest.

Begin by noting that, in principle, our analysis of national governments from sections 2 through 7 applies equally as well to regional governments. In particular, if the regional households, or the regional government, can borrow and lend in international credit markets, then regional investment would be efficient. The national government could also invest public capital in the region, but this would not affect the efficient level of investment. Regional governments would simply reduce their funding for investment one-for-one with the national government's investment. This would free up income, equal in value to the national governments investment, for the regional government and its households to use as they wish. Thus, national investment in a region would be equivalent to income transfers to the region. Furthermore, the income transfers would be used to finance consumption and saving in financial assets. None of the newly available income would be used for public investment.

If the national government is to have a role in determining regional investment in public capital, it must be when the regional households and governments are unable to borrow and lend in international markets. For this reason, we study the national government's allocation of public capital when regional governments and their households are credit constrained.

2.8.1 Tax Financing of Regional Investment

Let's start with the situation where the national government is also unable to borrow internationally. In this case, the government can still impact outcomes by redistributing income across regions in a way that raises aggregate welfare.

We first need to establish how the regional government sets its policy, taking the national policy as given. The regional government chooses a first period tax that is used to finance a local public capital investment, denoted by g_2^l , a perfect substitute for the public capital that is provided by the national government, as before, denoted by g_2 . The consumption of households in the region is given by

$$c_1 = (1 - \tau_1)y_1 - g_2^l \quad (2.24a)$$

$$c_2 = y_2 = A(g_2^l + g_2)^\mu, \quad (2.24b)$$

where τ_1 is the national income tax rate.

Taking the national policy as given, the regional government chooses g_2^l to maximize the representative household's utility, $\ln c_1 + \beta \ln c_2$, subject to (2.24). The resulting optimality conditions can be used to solve for regional government investment and household consumption,

$$g_2^l = \frac{\beta\mu}{1 + \beta\mu} (1 - \tau_1)y_1 - \frac{1}{1 + \beta\mu} g_2 \quad (2.25a)$$

$$c_1 = \frac{(1 - \tau_1)y_1 + g_2}{1 + \beta\mu} \quad (2.25b)$$

$$c_2 = A \left(\frac{\beta\mu}{1 + \beta\mu} [(1 - \tau_1)y_1 + g_2] \right)^\mu. \quad (2.25c)$$

There is an important feature of these solutions. Regional households are treating g_2 as a source of disposable income; note how $(1 - \tau_1)y_1$ and g_2 appear together in (2.25b) and (2.25c). This is because of two assumptions. First, we are assuming that locally and nationally provided public capital are perfect substitutes. Second, we assume that the nationally provided public capital is not so large as to drive local public capital to zero. These two assumptions imply that national investment works like a cash transfer because a dollar of national investment will free up a dollar of local funds previously used to finance local investment. As with household income generally, the funds freed by national investment are partly consumed, as in (2.25b), and partly invested in local capital, as in (2.25c). This is why (2.25a) says that the reduction in g_2^l is not one-for-one with the rise in g_2 . Despite the fact that national investment is equivalent to a cash transfer, it will raise total investment in the region to some extent.

Now let's turn to national policy. A household's maximum welfare, in either region, is found by substituting (2.25b) and (2.25c) back into the utility function to get the indirect utility or value function

$$V(\tau_1, g_2) = E + (1 + \beta\mu) \ln [(1 - \tau_1)y_1 + g_2] \quad (2.26)$$

where E is an expression involving terms that are independent of policy. Note, in particular, that the local TFP associated with public capital does not affect the marginal value of investment by the national government. This is because while a higher TFP *raises* the marginal return on investment it also *lowers* the marginal value of additional income. Under our assumption about preferences, these two effects exactly cancel. Thus, efficiency considerations related to the level of TFP and the return to investment do not enter to the government's decision making.

The national government chooses τ_1 , g_{2P} , and g_{2R} to maximize the sum of household value functions,

$$N_P V_P(\tau_1, g_{2P}) + N_R V_R(\tau_1, g_{2R}), \quad (2.27)$$

subject to the government budget constraint, (2.17). The first order conditions from the national government's problem can be used to derive the following allocation rule

$$y_{1R} - \frac{N_P}{N_R} (g_{2P} - \tau_1 y_{1P}) = y_{1P} + (g_{2P} - \tau_1 y_{1P}). \quad (2.28)$$

In the absence of efficiency considerations, for the reasons stated above, the allocation rule requires an equalization of disposable income across regions.

Think of R as the rich region and P as the poor region, in the sense that $y_{1P} < y_{1R}$. To equalize disposable income, fiscal policy must create a net transfer to the household of the poor region, $(g_{2P} - \tau_1 y_{1P}) > 0$. The rich household's burden in making the transfer is $\frac{N_P}{N_R} (g_{2P} - \tau_1 y_{1P})$.

In general, there is an unintended consequence of the income transfer on investment efficiency and total output that depends on the relative size of the returns to investment in each region, $\mu A_R (g_{2R}^l)^{\mu-1}$ and $\mu A_P (g_{2P}^l)^{\mu-1}$. The fact that the poor region has lower income and therefore lower levels of local investment, means it is quite possible that its marginal return on investment is higher. In this case the optimal policy would not only equalize disposable income, but would also raise the economy's total output in period 2. This is an example of where the usual trade-off between equity and efficiency goals does not exist.

The effect of national policy on the investment in the poor region becomes stronger if local investment is so low that national investment drives local investment to zero. Notice from (2.25a), that there is a sufficiently large value for g_{2P} that would make g_{2P}^l zero. Any national investment beyond this value for g_{2P} would increase investment in the poor region one-for-one. Thus, if the goal is to raise investment and *future output*, there is a strong case for the national government focusing public investment on the poor region. However, given the social welfare function in (2.27), the best policy to raise *utility* in the poor region would be to use income transfers rather than in-kind transfers of public capital. Remember, when households are credit-constrained, both consumption and investment are too low. With income transfers, the household could optimally divide the transfers across consumption and investment, according to their time preference. This is an example of the policy tension between in-kind and cash transfers.

2.8.2 Bond Financing of Regional Investment

Now suppose the national government can borrow on international credit markets and uses bond financing for national public investment. In period 1, government bonds, $b_2 \equiv B_2/N$, are issued to fund investment in the two regions,

$$N_P g_{2P} + N_R g_{2R} = b_2 N, \quad (2.29)$$

where $N \equiv N_P + N_R$.

In period 2, taxes must be raised to repay the debt and interest,

$$b_2 N(1 + r^*) = \tau_2 (N_P y_{2P} + N_R y_{2R}). \quad (2.30)$$

The household budget constraints are then

$$c_1 = y_1 - g_2^l \quad (2.31a)$$

$$c_2 = (1 - \tau_2) y_2 = (1 - \tau_2) A (g_2^l + g_2)^\mu. \quad (2.31b)$$

As before, we begin by deriving local government policy to get

$$g_2^l = \frac{\beta\mu}{1 + \beta\mu} y_1 - \frac{1}{1 + \beta\mu} g_2 \quad (2.32a)$$

$$c_1 = \frac{y_1 + g_2}{1 + \beta\mu} \quad (2.32b)$$

$$c_2 = (1 - \tau_2) A \left(\frac{\beta\mu}{1 + \beta\mu} [y_1 + g_2] \right)^\mu. \quad (2.32c)$$

Using (2.32), the value function for a household is now,

$$V(\tau_2, g_2) = E + (1 + \beta\mu) \ln [y_1 + g_2] + \beta \ln (1 - \tau_2).$$

Given (2.29), (2.30), and the local government response function given by (2.32a), the national government chooses τ_2 , g_{2P} , and g_{2R} to maximize $N_P V_P(\tau_2, g_{2P}) + N_R V_R(\tau_2, g_{2R})$. Using the optimality conditions for the national government's problem, we derive the following equations that determine the allocation of investment.

$$\frac{\mu(1 - \tau_2)\bar{y}_2}{g_{2P}^l + g_{2P}} + \frac{\mu\tau_2 y_{2P}}{y_{1P} + g_{2P}} = 1 + r^* \quad (2.33a)$$

$$\frac{\mu(1 - \tau_2)\bar{y}_2}{g_{2R}^l + g_{2R}} + \frac{\mu\tau_2 y_{2R}}{y_{1R} + g_{2R}} = 1 + r^*, \quad (2.33b)$$

where $\bar{y}_2 \equiv (N_P y_{2P} + N_R y_{2R})/N$, the average income in period 2. The allocation rule is found by combining (2.33a) and (2.33b). The allocation rule is now more complicated because of the second expression found on the left-hand-side of (2.33a) and (2.33b). These *tax terms* bring in a particular efficiency consideration. They give the value of the marginal tax revenue captured by the national

government due to the marginal return on public capital investment in the region. No such effect was present under first period tax financing because the first period tax base is exogenous.

To begin the interpretation of allocation rule associated with (2.33), suppose the tax terms are zero. Then (2.33) tells us that total investment should be equalized across regions, $g_{2P}^I + g_{2P} = g_{2R}^I + g_{2R} \equiv g$. This also would imply that $\bar{y}_2 = \bar{A}(g)^\mu$, where $\bar{A} \equiv (N_P A_P + N_R A_R)/N$, the average TFP across regions. Finally, the common investment in each sector would be privately efficient on average because the after-tax rate of return to investment would equal the opportunity cost of funds, $\mu(1 - \tau_2)\bar{y}_2/g = 1 + r^*$.

Now re-introduce the tax terms. Suppose we continue to keep total investment in each region equal. From (2.32a), this would also mean that $y_{1P} + g_{2P} = y_{1R} + g_{2R}$ because $g_2^I + g_2 = \frac{\beta\mu}{1 + \beta\mu}(y_1 + g_2)$ in each sector. However, the left-hand-sides of (2.33a) and (2.33b) would only be equal if second period income is equalized across sectors. This can only be true if $A_P = A_R$. Differences in TFP across sectors now create a reason to deviate from equalizing investment across regions. The presence of the tax terms mean, if the rich region has superior TFP, then total investment there must be greater there than in the poor region. The intuition for this result is that the national government collects more tax revenue by deviating from the equalization of total investment across regions and investing more in the rich region. The need to collect taxes in the future to finance debt financing creates an added incentive for the government to invest in the high TFP region.

2.9 A Note on Migration

An important extension to Sects. 2.7 and 2.8 is to allow for population migration from one region to another. For example, if economic opportunities are greater in region R than in region P, because of superior production technologies and greater local public capital provision, then households from poor regions would tend to move to rich regions.

We do observe long-term migration flows from poor to rich regions, but the pace of the migration is typically slow.³ Urban areas tend to be richer than rural areas in developing countries. Nevertheless, history shows that it takes decades for the rural–urban migration in developing economies to be completed (even in the absence of explicit government policies that restrict migration). Evidence suggests that migration is quite costly for households in poor regions. The costs are, in part, due to incomplete markets for land and insurance that bind households to rural areas in order to protect land claims and to receive informal insurance from local social networks. Moving to the city can also be costly due to cultural and language

³See Das et al. (2015 Chapters 5–8) for a complete discussion of the economics of internal migration across regions and sectors.

differences, as well as incomplete social security and social safety net arrangements for new migrants. Due to the gradual and incomplete nature of internal migration across regions, it might be a reasonable approximation in the short-run to assume no migration as we have done.

However, policy with a longer term perspective must account for migration flows from poorer to richer areas. An important consideration is that migration from poor to rich areas is in the national interest of a developing economy. Workers are more productive in the rich regions because of the fundamentals that made the region rich to begin with—superior technologies or a more concentrated population that creates a larger sharing effect from public capital (see Sect. 2.6). Movement of workers away from poor regions will tend to raise national productivity and welfare, as well as equalize living standards across regions. In this sense, the national government should encourage the natural migration flow by “favoring” the rich region with its public capital allocation. When conditions in the poor region are desperate, the migration flow can become too rapid, causing a crowding of public services in rich regions. For this reason, investment in the poor region cannot be ignored. The optimal policy is a mix of public investment across regions, but one that favors the rich region on efficiency grounds (Mourmouras and Rangazas 2013; Das et al. 2015, Chapter 8).

The inclusion of migration can reverse the logic of our analysis in Sects. 2.7 and 2.8. The logic without migration says that it is in the national interest to favor the poor region because the value of nationally funded investment is higher there due to low levels of local investment. However, one way of making the poor-region households better off is to encourage migration to the richer regions by favoring rich regions with national policy. The difference in policy recommendations is based on the precise source of differences in the return to investment in public capital.

The argument for favoring rich regions in the presence of migration predominately applies to *developing* countries. In developing countries, it is much more likely for the absence of land and insurance markets to bind workers to backward regions that have both inferior technologies and a smaller sharing effect due to less concentrated populations. In this situation $A_R > A_P$ and workers should be encouraged to migrate to richer urban areas.

In *developed* countries, with complete markets and modern technologies found in all regions, the logic for favoring rich areas has much less force. Regional differences in developed economies are more likely due to under-investment in local public capital, particularly public education, in poor areas. The national government can raise national welfare by redistributing investment, or income transfers, to the poor regions of *developed countries* as indicated in Sects. 2.7 and 2.8.

2.10 A Dynamic Generational Model

In this section we alter the interpretation of the investment model in a manner that will allow a more complete dynamic analysis that stretches beyond two periods. As suggested earlier, we can think of each period as representing a generation. The

current generation has to choose how much to consume and how much to invest in the productivity of the next generation. For this set-up to make sense, parents must have some concern about the economic welfare of their children. Some aspect of children's economic situation must then enter the utility function of the parent. One could continue to assume the world ends after two periods, now representing two generations, but we will instead extend the future out indefinitely and allow for a truly dynamic analysis.

We take this interpretation not only to build a bridge from a simple investment model to a more complete growth model, but also to make a particular point. A major concern, addressed in some detail in Chap. 4, is that the saving and investment shares of total income are declining in the U.S. and other developed countries (Dobrescu et al. 2012 and Kotlikoff 2015). As discussed in Chap. 4, one explanation for this trend is that policies have become increasingly biased toward current older generations at the expense of younger and unborn generations. This policy bias can be explained by the formation of interest groups that trade political support for government transfer payments and subsidies. In various ways, the expansion in transfers to current older generations reduces saving and investment in the future.

While politics plays an important role in explaining the decline in saving and investment shares, we also want to point out that such a decline can occur for more fundamental economic reasons. In particular, even in a world where the current generation has concern for the future generations, investment shares can fall over time in the absence of politics.

2.10.1 The Growth Model

Let's build a generational model from the basic elements of the closed economy, investment model discussed in Sect. 2.2. Assume that the government taxes the current generation to finance public investment that raises the productivity of the next generation. To create a generational model, we also need to change the interpretation of household preferences. We assume that the current generation gains utility from the future productivity of their children. The form of the utility function is basically the same as in earlier sections

$$U_t = \ln c_t + \beta \ln y_{t+1}, \quad (2.34)$$

but now lifetime utility is a function of parent's consumption and the adult income of their children.

The consumption of generation- t is determined by the budget constraint,

$$c_t = y_t - \tau_t y_t = y_t - g_{t+1}, \quad (2.35)$$

where the second equality comes from the assumption that the government taxes the current generation to finance investments in the future generation. Substituting (2.35) into (2.34), defines the objective function that the government maximizes when choosing its public investment. The solution for public investment from the

government's maximization problem, can be used to derive the following *transition equation* for public capital,

$$g_{t+1} = \frac{\beta\mu}{1 + \beta\mu} A g_t^\mu. \quad (2.36)$$

First, notice how similar (2.36) is to the optimal choice of g_2 from the two-period investment model in Sect. 2.2. As before, the tax rate on current income to finance public investment is $\beta\mu/(1 + \beta\mu)$. Now, however, current income is explicitly linked in past investment in every period, $y_t = A g_t^\mu$.

The basic logic for the investment rule is also essentially the same as before. The added element in (2.36) is that it makes a connection between public capital *over time*. Eq. (2.36) is called a *transition equation*, in mathematics a *difference equation*, because it describes *changes* in public capital from period to period. Given some initial value for public capital, (2.36) determines the public capital in the next period. The new value of public capital then becomes the initial value, from the perspective of the next period, determining yet another value in the dynamic sequence.

The dynamic path for government capital given by (2.36) can be traced using Fig. 2.1, with g_t plotted on the horizontal axis and g_{t+1} plotted on the vertical axis. Imagine that the economy begins in period 1 with $g_t = g_1$. To find out what the capital stock will be in period 2, move vertically up to the plot of the transition equation to find g_2 . In period 2, g_2 will now be the initial capital stock. To see this, move horizontally from the transition equation to the 45-degree line and then back down vertically to the horizontal axis. The process then repeats itself until one reaches $g_t = \bar{g}$, where the transition equation crosses the 45-degree line. At this point, the capital stock remains constant from period to period and the economy is

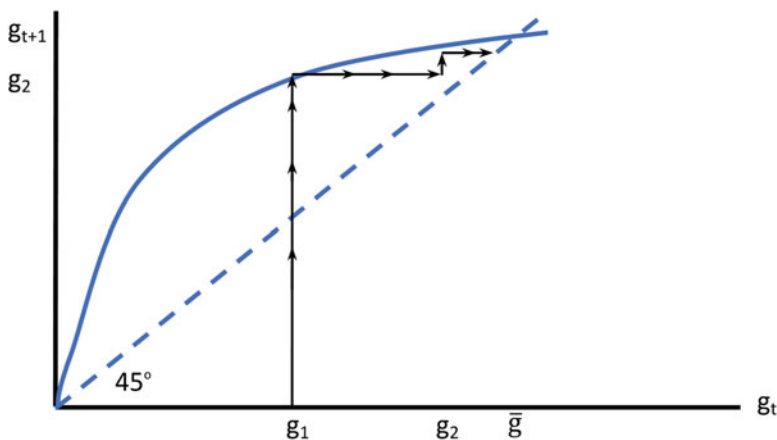


Fig. 2.1 The transition equation for government capital

said to have reached a *steady state* equilibrium.⁴ An algebraic solution for the steady state is found by setting $g_{t+1} = g_t = \bar{g}$ in (2.36) and then solving the equation for $\bar{g} = \left(\frac{\beta\mu A}{1 + \beta\mu} \right)^{\frac{1}{1-\mu}}$.

2.10.2 The Investment Share

In (2.36), the *fraction* of current output and income that is invested is a constant, also equal to the income tax rate. The *investment share* in the model is constant as the economy grows. To examine how investment shares may change over the course of development, we need to leave the simple log preferences in favor of a more general and flexible class of preferences represented by a *constant elasticity of substitution* (CES) utility function.

With a CES utility function it remains true that a generation- t household derives satisfaction from its own lifetime consumption, c_t , and the future lifetime income of its child, y_{t+1} . However, the CES utility function in c_t and y_{t+1} takes the form

$$U_t = \frac{(c_t^{1-1/\sigma} - 1) + \beta(y_{t+1}^{1-1/\sigma} - 1)}{(1 - 1/\sigma)}. \quad (2.37)$$

This utility function has the standard property that the marginal utility of each of its arguments is positive but diminishing. The two parameters of the function are, the now familiar, pure time discount factor (β) and a new parameter, the *intertemporal elasticity of substitution* ($\infty > \sigma > 0$). The intertemporal elasticity of substitution is a measure of the willingness to substitute current consumption for future income when the relative price of future income falls, but this won't be made clear for a while. Subtracting 1 from each argument is done for a purely technical reason. It allows the logarithmic utility function, $U_t = \ln c_t + \beta \ln y_{t+1}$, to appear as a special case when $\sigma = 1$ (see the Technical Appendix and Problem 24).

Using the more general CES utility function changes the solution for optimal investment. The new solution for public investment can be used to derive the following *transition equation* for public capital,

$$g_{t+1} \left(\Gamma + g_{t+1}^{(\sigma-1)(1-\mu)} \right) = \Gamma y_t = \Gamma A g_t^\mu, \quad (2.38)$$

where $\Gamma \equiv (\beta\mu)^\sigma A^{\sigma-1}$. Just as in the more special case given by (2.36), the transition equation in (2.38) can be sketched with g_t plotted on the horizontal axis and g_{t+1} plotted on the vertical axis. The plot will look like that in Fig. 2.1. The transition equation is increasing and concave, with a unique steady state where the transition equation crosses the 45-degree line.

⁴The economy never literally reaches the steady state, although it will get arbitrarily close.

In general, there is not a closed form solution for g_{t+1} in (2.38). In addition to the case where $\sigma = 1$, there is a second special case where we can get an explicit closed-form solution for the transition equation. If $\sigma = (2 - \mu)/(1 - \mu) > 2$, then (2.38) becomes a quadratic equation in g_{t+1} .⁵ Solving for the only positive root gives us the following transition equation,

$$g_{t+1} = \frac{\Gamma}{2} \left(\sqrt{1 + \frac{4Ag_t^\mu}{\Gamma}} - 1 \right). \quad (2.39)$$

As mentioned, the sketch of (2.39) is of the same concave shape as displayed in Fig. 2.1. However, (2.39) has a different implication for the investment share than (2.36). Using (2.38) we can derive an expression for the economy's investment share (\tilde{g}_t). Divide both sides of (2.38) by y_t and by the expression $\Gamma + g_{t+1}^{(\sigma-1)(1-\mu)}$ to find

$$\tilde{g}_t \equiv \frac{g_{t+1}}{y_t} = \frac{\Gamma}{\left(\Gamma + g_{t+1}^{(\sigma-1)(1-\mu)} \right)}. \quad (2.40)$$

If $\sigma = 1$, the investment share is a constant throughout the entire dynamic path to the steady state. However, if $\sigma > 1$, as in (2.39), the investment share declines as government capital grows. Thus, the economy experiences an increasing consumption rate for the current generation over time—as we observe in the data for the U.S. and other developed countries.

The intuition as to why the behavior of \tilde{g}_t depends crucially on σ is as follows. As government capital grows, the return to government capital investment falls (because $\mu < 1$). The decrease in the return lowers the opportunity cost of consumption by the current generation, which creates an incentive for the current generation to consume more and investment less (a *substitution effect*). However, the lower return also lowers the income of the future generation, for any level of investment, and creates an incentive for the current generation to compensate by investing more (an *income effect*). Which of these two effects dominates depends on how willingly the current generation trades off current consumption for future income. The willingness to carry out intertemporal substitution of consumption at different dates is governed by σ . The higher is σ , the more likely that the substitution effect dominates and \tilde{g}_t falls over time in a growing economy. The critical value is $\sigma = 1$, where the two effects exactly offset and \tilde{g}_t remains constant.⁶

⁵Note that $\sigma = (2 - \mu)/(1 - \mu)$ is greater than 2 because it is increasing in μ , so its smallest value is when $\mu = 0$.

⁶There are other models where the investment rate in the future generation can decline as a fraction of family resources even in the case where $\sigma = 1$. See Mourmouras and Rangazas (2007) and Das et al. (2015, Chapter 3).

2.11 Principles for Tax Collection

There is a large literature that extends the principles of good governance by looking at the issue of how best to collect taxes—a topic we have ignored. A fundamental issue in this literature is to find ways of minimizing the *distortionary* effects of taxation on economic behavior that lead to *excess burdens*. Excess burdens are costs that go beyond the loss in income associated with paying taxes. The excess burden of taxation includes the efficiency losses in welfare and output that occur when behaviors, such as work effort and saving choices, are altered by taxation. A complete discussion of *optimal taxation*, that examines the balance between equity and efficiency objectives, goes well beyond the scope of this book. A good serious introduction to this topic is Salanie (2011).

One defense of ignoring the distortionary effects of taxation is to argue that the behavior we focus on is not strongly responsive to taxation. There is certainly empirical evidence that suggests this may be a reasonable approximation with respect to labor supply and saving behavior—where the evidence for significant distortionary effects is quite inconclusive. Tax issues are raised again in the policy discussion of Chap. 7, when we discuss a variety of considerations that should influence the design of a tax system such as simplicity and transparency, sin taxes and corrective taxes that have *beneficial* effects on behavior, and tax evasion.

2.12 Conclusion

Our final section of the chapter gives a quick summary of the lessons from the two-period model of government investment. These lessons for government policy are based solely on economic logic in the absence of politics that may be in conflict with the national interest. The lessons provide a useful benchmark for comparison as we extend the analysis to include rent seeking and corruption in Chap. 3.

2.12.1 Basic Principles

1. Government capital is valued primarily because it raises future production and lifetime resources. When households lack access to a complete market for financial assets, government capital also provides a physical asset that can smooth consumption over time.
2. Government capital can be modeled as a private input to the production function, but if the capital is a pure or impure public good, then the associated TFP will be an increasing function of population size.
3. Public debt is equivalent to first period taxes and provides no net wealth to the nation when either (i) the economy is closed and domestic household must purchase the debt or (ii) the economy is open and private households can borrow and lend internationally. However, in an open economy where private

households have no access to international credit markets, but the country's government does, public debt can be a welfare improving fiscal tool.

4. One implication of (2.3) is that financial liberalization across borders may be a reason for the rise in public debt over the last quarter of the twentieth century. International lenders have been willing to purchase the public debt of developed countries, such as the United States, at low interest rates. Receiving funds from foreign sources lowers interest rates and reduces the cost of borrowing. A variety of trends in the developed world are creating incentives to allow government borrowing with little public resistance.
5. A caveat concerning (2.3) is that while debt can raise the welfare of current generations, it may nevertheless lower the welfare of future generations (even when current generations possess intergenerational altruism). Given that most societies have laws prohibiting individual households from leaving debt for their children to repay, fiscal policies should tend to exhibit the same discipline.

2.12.2 Regional Issues and Inequality

1. Unless there are policy tools that can completely eliminate regional income differences, without creating distortions, the optimal allocation of government capital across regions will be determined by equity, as well as efficiency, considerations. Both equity and efficiency considerations will *tend to*, but *not necessarily*, bias government capital allocation toward poor regions. The tendency will be strongest in developed economies where TFP is similar across regions.
2. When regional governments (i) can provide the same capital inputs as national governments and (ii) have access to credit markets, there is no role for the national government in public investment. However, in *developed* economies, where regional governments are credit-constrained, the national government should generally redistribute wealth from rich to persistently poor regions by biasing public capital allocation in that direction.
3. In *developing* economies that are undergoing major structural transformations, with gradual migration from poor to rich regions based on a superior technology in the rich region, the government should bias its funding support toward richer regions to help speed migration flows. However, poorer regions cannot be completely ignored or migration to the rich regions could become too rapid.

2.12.3 Identifying the Influence of Politics

While the social welfare function we assume gives equal weight to all households, the analysis does not imply equal treatment of all households under national fiscal policy. Both efficiency and equity considerations can cause different regions to be treated differently by national policy. This means that one must take care in

interpreting differential treatment as stemming from a bias based on differences in political influence across regions.

A similar warning applies to explaining the observed decline in investment in future generations by current generations or the increased use of intergenerational redistribution. Political explanations based on interest groups and selfish concerns of politicians who seek re-election may not be necessary. Economic fundamentals can cause the current generation to choose a declining investment rate in future generations, or vote for the accumulation of public debt, even when they value the economic welfare of their children and there are no special-interest political motivations present.

2.13 Exercises

Questions

Questions 1–4 should be answered using the model of Sect. 2.1.

1. What is the household's lifetime budget constraint? When does it represent a meaningful constraint on household choices?
2. Which of the following are *choice variables* of the household?
 - (a) y_1
 - (b) y_2
 - (c) c_1
 - (d) c_2
 - (e) a_2
 - (f) r_2
3. If a household is able to borrow and lend, how does an increase in each of the following affect c_1 , c_2 , and a_2 ? Repeat the exercise when households are not able to borrow and lend.
 - (a) y_1
 - (b) y_2
 - (c) r_2
 - (d) β
4. What does it mean to be credit-constrained? What factors increase the likelihood of being credit-constrained?
5. Suppose there is a closed economy made up of identical households. Why can there be no private credit market where borrowing and lending actually takes place? What considerations determine the optimal government investment in this setting, assuming the government finances investment exclusively using first period taxes?
6. In a closed economy made up of identical households, explain the meaning of the following statements.
 - (a) government debt provides no net wealth to private households

- (b) financing government investment with taxes is equivalent to using bond finance

What considerations, not captured by the representative agent model of Sect. 2.2, might cause households to prefer bond financing over tax financing in period 1?

7. Answer the following questions, assuming there is a small-open economy where private households can borrow and lend in a perfectly competitive international credit market.
 - (a) Do households prefer that the government finances public capital investment using taxes or bonds?
 - (b) What is the optimal rule for government investment? What is the economic intuition behind the rule?
 - (c) What is the optimal consumption path for private households? How does the international interest rate and the household's time discount factor affect the optimal path?
8. Answer the following questions, assuming there is a small-open economy where the government can borrow and lend in a perfectly competitive international credit market, but private households cannot.
 - (a) Do households prefer that the government finances public capital investment using taxes or bonds?
 - (b) What is the optimal rule for government investment? What is the economic intuition behind the rule?
 - (c) What is the optimal consumption path for private households? How does the international interest rate and the household's time discount factor affect the optimal path?
9. When does government borrowing have the potential to raise household welfare?
10. Explain the generational interpretation of the two-period model. Why are private credit constraints more likely under the generational interpretation than under the *life-cycle* interpretation?
11. Under the generational interpretation of the two-period model, explain how the preferred investment of a "rich" household (one that makes positive bequests) is affected by the following events.
 - (a) an increase in A
 - (b) a decrease in r^*
 - (c) a proportional increase in A and p
12. Repeat question 11 for the case of a "poor" household (one that is bequest-constrained).
13. Use the generational interpretation of the two-period model to explain why the majority of households may be in favor of government debt-financing since the 1970s.
14. What is a public good? What is an impure public good? In Eq. (2.1) is government capital assumed to be a private or a public good? How can one generalize (2.1) to allow for the possibility that government capital is a pure or impure public good?

15. State and explain intuitively the *utilitarian social welfare function* given in (2.21).
16. Why does the rule for allocating government investment across two different regions not necessarily imply equal investment levels across regions?
17. When does regional investment by a national government have an impact on the welfare of households living under a regional government?
18. When does regional investment by a national government raise total government investment in the region? Does regional TFP affect the level of national investment in the region? Explain
19. Suppose that regional governments cannot borrow in international markets. Intuitively explain the regional investment rules of a national government with and without the ability to borrow in international markets.
20. How does internal population migration across regions affect the national government's regional investment strategy?
21. Explain the concept of a transition equation using Fig. 2.1.
22. Use the model from Sect. 2.10 to explain what happens to the following variables as the economy approaches the steady state from below.
 - (a) government investment
 - (b) worker productivity
 - (c) consumption
 - (d) return to government investment
 - (e) growth rate of worker productivity
23. Use a transition equation to explain why a government might find it optimal to lower the rate of investment as an economy develops.

Problems

Use the model of Sect. 2.1 to answer Problems 1–4.

1. Sketch the lifetime budget constraint with c_1 on the horizontal axis and c_2 on the vertical axis. Label each of the following features of the sketch.
 - (a) x-intercept
 - (b) y-intercept
 - (c) slope
2. Suppose a household can borrow and lend in a perfectly competitive credit market. Assume $\beta = 0.2$ and $r_2 = 0.10$. Compute the value for the optimal choice of a_2 when
 - (a) $y_1 = 10$ and $y_2 = 0$
 - (b) $y_1 = 0$ and $y_2 = 10$
 - (c) $y_1 = 10$ and $y_2 = 10$
3. Assume $\beta = 0.2$, $r_2 = 0.10$, and $y_1 = y_2 = 10$. Compute U when the household cannot borrow and lend and when it can. In which case is U higher? Explain.
4. Repeat Problem 1 for a household that is free to lend but not to borrow, i.e., the choice of a_2 must satisfy the non-negativity constraint, $a_2 \geq 0$.
5. Set up and solve the optimization problem needed to derive (2.4a, 2.4b).

6. If $\mu = 1/3$, $\beta = 0.5$, $A = 6$, and $y_1 = 8$, compute the values of the following variables in the closed economy model of government investment in Sect. 2.2.
 - (a) g_2
 - (b) τ_1
 - (c) c_1
 - (d) c_2
 - (e) U
7. Use the closed economy model of government investment to sketch the lifetime consumption possibilities of the representative household. Begin by noting that consumption in the two periods can be related by the following equation, $c_2 = A(y_1 - c_1)^\mu$. Continue by placing current consumption on the horizontal-axis and future consumption on the vertical-axis. What is the horizontal intercept? The vertical intercept? If you know calculus, what is the slope? If you don't know calculus, you should plot a few points by using the parameter assumptions from Problem 6.
8. Derive the lifetime budget constraint, given by (2.9), of a household trading in a perfectly competitive open economy.
9. Derive (2.10 and 2.11) and (2.13a, 2.13b). Be sure to state the underlying assumptions made in the two different cases.
10. Place g_2 on the horizontal axis and then separately plot the left-hand-side and the right-hand-side of (2.10) as functions of g_2 . Use the diagram to locate the productively efficient level of g_2 . Use the figure to determine what happens to the productively efficient g_2 when there is an increase in A . Repeat for an increase in r^* .
11. Derive the adjusted first order conditions that replace (2.13a and 2.13b) when the government faces a binding borrowing constraint. Let s^g denote government saving that could finance loans to the international credit market, if positive. The borrowing constraint means $s^g \geq 0$.
12. *Preferences in Two Dimensions*

A common way of sketching preferences involves the concept of an *indifference curve*. In our model an indifference curve associated with the household's lifetime utility function gives *all combinations* of c_1 and c_2 that generate a *given level* of satisfaction. For a given level of satisfaction or utility, \bar{U} , the combinations of c_1 and c_2 that are used to construct an indifference curve are defined by the condition, $\bar{U} = \ln c_1 + \beta \ln c_2 = \ln(c_1 c_2^\beta)$. Recall that the natural exponential function is the inverse of the natural log function, $e^{\ln x} = x$. If we take the exponential of both sides of the condition defining an indifference curve, we get $e^{\bar{U}} = c_1 c_2^\beta$ or $c_1 = e^{\bar{U}} / c_2^\beta$. The last expression gives the value of c_1 that generates the same satisfaction level \bar{U} for different possible values of c_2 , forming consumption pairs that the household is indifferent to because they all yield the same utility.

Assume $\beta = 0.50$ and consider 5 different values of c_2 : 1, 4, 9, 16, 25.

- (a) If $\bar{U} = 1$, what are the values for c_1 that correspond to each of the 5 values of c_2 ?
- (b) If $\bar{U} = 2$, what are the values for c_1 that correspond to each of the 5 values of c_2 ?
- (c) Use the five c_1 – c_2 pairs to sketch the two indifference curves from (a) and (b) on a diagram with c_1 on the horizontal axis and c_2 on the vertical axis.
- (d) Why are the indifference curves downward sloping? Give an economic interpretation of the slope. Why do you think the slope becomes flatter as you move along the horizontal axis by considering higher values of c_1 ?

Sketches of indifference curves are important in the analysis of Problems 13 and 14.

13. *Credit-Constrained Investment in Pictures* (an Extension of Problem 7)

Let's sketch the solution associated with maximizing (2.3), assuming it is consistent with a closed credit-constrained economy (i.e. g_2 is less than the efficient level). Our sketch will display the consumption possibilities over the two periods. Plot c_1 on the horizontal axis and c_2 on the vertical axis. Note that the consumption possibility frontier (CPF) is generated by choosing different values of g_2 that serve to generate different values for c_1 and c_2 .

- (a) State the maximum possible values of c_1 and c_2 in general form (variables not numbers). Label them on the sketch.
- (b) Show $dc_2/dc_1 = -\mu Ag_2^{\mu-1}$ and $d^2c_2/dc_1^2 = \mu(\mu-1)Ag_2^{\mu-2}$. Use these results to determine the shape of the sketch. If you don't know calculus, use your numerical plot from Problem 7 to guide the sketch of the curve's shape.
- (c) Display the consumption solution associated with (2.4) by depicting a tangency between the CPF and the indifference curve corresponding to the maximum value of (2.3). Note: the indifference curve generated by the log utility function will have the standard convex shape, so just assume that to be true. What is the value of c_1 at the point of tangency? The value of c_2 ? Again in general form.

14. *A Portrait of Investment with International Borrowing*

Following up on Problem 13, suppose now that the country can borrow abroad at the international interest rate, r^* .

- (a) Label the credit-constrained solution from Problem 13 on the CPF with the letter A. Is the absolute value of the slope of the CPF at A greater than, equal to, or less than $1 + r^*$?
- (b) Starting at point A, in which direction must you move along the CPF to reach the point associated with a productively efficient level of investment? Go in this direction, choose a point associated with efficient investment, and label the point B.
- (c) We know that B does not represent an optimal consumption combination. Why?

- (d) The economy can achieve the efficient investment level and at the same time increase the value of c_1 by borrowing internationally. Draw a tangent line with the slope $-(1+r^*)$ through the point B. The economy can increase c_1 by moving along this tangent line, away from point B in the south easterly direction. Sketch a tangency between an indifference curve and this tangent line at a point labelled C, where c_1 is greater than its value at point A but c_2 is less than its value at point A.
- (e) If the model represents a single generation that lives for two periods, is the representative household better off at C than at A? What welfare implication do you draw if we interpret the model as representing two distinct generations that live for one period?
15. Explicitly incorporate a price, p , for investment goods and derive (2.4a')
16. If $\mu = 1/3$, $p = 1$, $A = 6$, and $r^* = 0.10$, find the productively efficient value of g_2 . What is the preferred value of g_2 for a bequest-constrained household, if $\beta = 0.5$ and $y_1 = 8$? Suppose that rich and poor households live in distinct communities and the level of g_2 is determined at the community level to match the household preferences. If rich households plan to make positive bequests and poor households are bequest-constrained, what is the resulting income gap for the children from rich and poor communities when they become adult workers?
17. Repeat Problem 16 in the following two new scenarios:
- the value of A rises to 12 and
 - the value of A rises to 12 *and* the value of p rises to 1.5.
18. Suppose that, in contrast to the assumption of Problem 16, the value of g_2 is determined at the national level and is common across all households. Further assume that when g_2 is determined at the national level that it reflects the preferences of rich households and is productively efficient. Using the parameter assumptions of Problem 16, compute the utility of a poor household when g_2 is determined at the community level and when it is determined at the national level. Explain your results.
19. Suppose the governments in two locations (countries, cities, regions) provide the same value of g_2 . The two locations, A and B, are otherwise identical except the population size in location B is twice that of location A. If $\xi = \mu = 1/3$, what is the ratio of y_2 in location B relative to location A?
20. Use the Lagrangian method for constrained optimization to derive (2.22) and (2.23).
21. In the model used to allocate public capital across regions or communities, assume $N_P = N_R$. From (2.17), we then have $g_{2P} + g_{2R} = \tau_1 y_1$, where $y_1 \equiv (y_{1P} + y_{2R})/2$. This implies $g_{2R} = \tau_1 y_1 - g_{2P}$. Now sketch both sides of the equality in (2.23) as functions of g_{2P} , i.e. plot the left and right hand sides of (2.23) with g_{2P} on the horizontal axis. Locate the welfare maximizing value of g_{2P} using the diagram. Use the diagram to show what happens to the optimal g_{2P} if each of the following increase:
- A_R ,
 - A_P ,

(c) r^* .

What happens if y_{1P} decreases and y_{1R} increases, leaving the value of y_1 unchanged?

22. Derive the behavior of a local regional government operating in a federal system as given by (2.25) and (2.26).
23. In a federal system, total investment in a particular region is $g_2^l + g_2$. Use (2.25a) to derive an equation for total investment in the region. If $\beta = 0.5$ and $\mu = 0.4$, compute the effect of an increase in g_2 on total investment, i.e. compute $d(g_2^l + g_2)/dg_2$. For what values of g_2 is your computation valid? What is the value of $d(g_2^l + g_2)/dg_2$ when g_2 become sufficiently high to render your first computation invalid?
24. Show that the general CES utility function, given in (2.37), includes the log utility function as a special case.
25. Maximize (2.34) with respect to g_2 and derive the transition equation given in (2.36).
26. Use (2.38) to derive the two explicit transition equation given by (2.36) and (2.39). Use calculus to show that these two transition equations are concave functions.
27. Let's study the dynamic transition of the model using (2.36) from Sect. 2.10, while making the following parameter assumptions: $A = 1$ and $\beta = \mu = 0.5$.
 - (a) What is the steady state value for g ?
 - (b) Trace the transition path out for five periods if the initial public capital stock is 0.01. Do the same if the initial public capital stock is 0.08.
28. Let's study the dynamic transition of the model using (2.39) from Sect. 2.10, while making the following parameter assumptions: $\sigma = 3$, $A = 1$ and $\beta = \mu = 0.5$. Compared to Problem 27, we are now focusing on a situation where σ differs from 1; in this case, a value greater than 1.
 - (a) What is the value for Γ ?
 - (b) Trace the transition path out for five periods if the initial public capital stock is 0.01.
 - (c) Based on your transition path calculations, what would be a good approximate value for the study state g ?
29. Use (2.40) to compute the investment rates associated with the transition path you calculated in Problem 28.

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