

---

# Contents

<b>1</b>	<b>Weighted Residuals and Galerkin's Method for a Generic 1D Problem</b>	<b>1</b>
1.1	Introduction: Weighted Residual Methods	1
1.2	Galerkin's Method	2
1.3	An Overall Framework	4
<b>2</b>	<b>A Model Problem: 1D Elastostatics</b>	<b>5</b>
2.1	Introduction: A Model Problem	5
2.2	Weak Formulations in One Dimension	6
2.3	An Example	8
2.4	Some Restrictions	9
2.5	Remarks on Nonlinear Problems	10
	References	11
<b>3</b>	<b>A Finite Element Implementation in One Dimension</b>	<b>13</b>
3.1	Introduction	13
3.2	Weak Formulation	14
3.3	FEM Approximation	14
3.4	Construction of FEM Basis Functions	15
3.5	Integration and Gaussian Quadrature	16
3.5.1	An Example	17
3.6	Global/Local Transformations	19
3.7	Differential Properties of Shape Functions	20
3.8	Post-Processing	22
3.9	A Detailed Example	22
3.9.1	Weak Form	22
3.9.2	Formation of the Discrete System	23
3.9.3	Applying Boundary Conditions	24
3.9.4	Massive Data Storage Reduction	25
3.10	Quadratic Elements	25
	Reference	28

<b>4</b>	<b>Accuracy of the Finite Element Method in One Dimension . . . . .</b>	<b>29</b>
4.1	Introduction . . . . .	29
4.2	The “Best Approximation” Theorem . . . . .	30
4.3	The Principle of Minimum Potential Energy. . . . .	31
4.4	Simple Estimates for Adequate FEM Meshes. . . . .	33
4.5	Local Mesh Refinement . . . . .	34
	References. . . . .	35
<b>5</b>	<b>Iterative Solutions Schemes . . . . .</b>	<b>37</b>
5.1	Introduction: Minimum Principles and Krylov Methods. . . . .	37
5.1.1	Numerical Linear Algebra . . . . .	37
5.1.2	Krylov Searches and Minimum Principles. . . . .	39
	Reference . . . . .	43
<b>6</b>	<b>Weak Formulations in Three Dimensions . . . . .</b>	<b>45</b>
6.1	Introduction . . . . .	45
6.2	Hilbertian Sobolev Spaces . . . . .	47
6.3	The Principle of Minimum Potential Energy. . . . .	48
6.4	Complementary Principles . . . . .	49
<b>7</b>	<b>A Finite Element Implementation in Three Dimensions . . . . .</b>	<b>51</b>
7.1	Introduction . . . . .	51
7.2	FEM Approximation. . . . .	52
7.3	Global/Local Transformations. . . . .	54
7.4	Mesh Generation and Connectivity Functions. . . . .	55
7.5	Warning: Restrictions on Elements. . . . .	56
7.5.1	Good and Bad Elements: Examples. . . . .	56
7.6	Three-Dimensional Shape Functions. . . . .	58
7.7	Differential Properties of Shape Functions . . . . .	59
7.8	Differentiation in the Referential Coordinates . . . . .	61
7.8.1	Implementation Issues . . . . .	64
7.8.2	An Example of the Storage Scaling. . . . .	65
7.9	Surface Jacobians and Nanson’s Formula. . . . .	66
7.10	Post-Processing. . . . .	67
	References. . . . .	68
<b>8</b>	<b>Accuracy of the Finite Element Method in Three Dimensions. . . . .</b>	<b>69</b>
8.1	Introduction . . . . .	69
8.2	The “Best Approximation” Theorem . . . . .	70
8.3	Simple Estimates for Adequate FEM Meshes Revisited for Three Dimensions. . . . .	71
8.4	Local Error Estimation and Adaptive Mesh Refinement. . . . .	72
8.4.1	A Posteriori Recovery Methods. . . . .	72
8.4.2	A Posteriori Residual Methods . . . . .	73
	References. . . . .	74

---

<b>9</b>	<b>Time-Dependent Problems</b> . . . . .	75
9.1	Introduction . . . . .	75
9.2	Generic Time Stepping . . . . .	75
9.3	Application to the Continuum Formulation. . . . .	77
	References. . . . .	79
<b>10</b>	<b>Summary and Advanced Topics.</b> . . . .	81
	References. . . . .	84
	<b>Appendix A: Elementary Mathematical Concepts</b> . . . . .	85
	<b>Appendix B: Basic Continuum Mechanics</b> . . . . .	93
	<b>Appendix C: Convergence of Recursive Iterative Schemes</b> . . . . .	107
	<b>Appendix D: Selected in-Class Exam Problems</b> . . . . .	109
	<b>Appendix E: Selected Computer Projects</b> . . . . .	125



<http://www.springer.com/978-3-319-70427-2>

A Finite Element Primer for Beginners

The Basics

Zohdi, T.I.

2018, XIII, 135 p. 41 illus., Softcover

ISBN: 978-3-319-70427-2