Preface

Lotfi Zadeh introduced the concept of a fuzzy subset of a set in 1965 as a way to represent uncertainty. His ideas have motivated the interest of researchers worldwide. One such researcher was Azriel Rosenfeld. He was one of the fathers of fuzzy graph theory. His development of the concept of a fuzzy graph provides the motivation of this book and the research it contains.

The book deals with current ideas in fuzzy graphs. It is not an attempt to provide an exhaustive study. There are individual topics in fuzzy graphs that would provide enough material for an entire book in them. Still it covers most of the major developments in fuzzy graph theory during the period 1975–2017. The book should be of interest to research mathematicians, computer scientists, and social scientists. It is the first volume of a two volume set. The second volume focuses on the application of fuzzy graph theory to the problem of human trafficking.

Some of the material in this book has appeared in [127]. We include it here since the development of the book rests on it.

We provide in Chap. 1 only the very basics of fuzzy set theory needed to understand the book. We assume the reader is familiar with basic notions of mathematics including set theory. Since this book is designed primarily for researchers with a knowledge of fuzzy set theory, we only provide a few concepts from fuzzy sets and relations mainly to set forth our notation to be used in the book.

In Chap. 2, we present basic concepts of fuzzy graphs which are needed later in the chapter and in the remainder of the book. For example, we introduce and present basic results on paths, connectedness, forests, trees, and fuzzy cutsets. Other basic concepts include bridges, cutsets, and blocks. We examine the connection between cycles and fuzzy trees. We present deeper results on blocks and in fact give a characterization of blocks in fuzzy graphs. We examine special types of cycles such as strong cycles and locamin cycles. We then present results on important types of fuzzy graphs such as fuzzy line graphs and fuzzy interval graphs. The study of fuzzy interval graphs includes the fuzzy analog of Marczewski's theorem, the Gilmore and Hoffman characterization, and the Fulkerson and Gross characterization. The chapter is concluded with the development of certain operations on fuzzy graphs such as the Cartesian cross product, the composition, union, and join of two fuzzy graphs.

In Chap. 3, we focus on the connectivity of fuzzy graphs. We describe various types of edges in fuzzy graphs with respect to connectivity properties and characterize different fuzzy graph structures. We then consider vertex connectivity and edge connectivity. We provide generalized versions of connectivity parameters introduced by Yeh and Bang in 1975. Menger's theorem is very celebrated result in graph theory. We present a version of Menger's theorem for fuzzy graphs.

Chapter 4 develops further results involving blocks. An application involving undirected network of roads is given. Attention is then turned to critical blocks and block graphs. Connectivity-transitive and cyclically transitive fuzzy graphs are examined next.

Chapter 5 starts by considering connectedness and acyclic levels. A new measure of connectivity of fuzzy graphs, called cycle connectivity, and two different types of bridges, called bonds and cutbonds, are discussed. Various metrics are also examined. Attention is also given to detour distance in fuzzy graphs.

In Chap. 6, the notion of a sequence in fuzzy graphs is introduced. Most of the fuzzy graph structures are characterized using different types of sequences. The notion of saturation in fuzzy graphs is also introduced. The chapter concludes with a study of strong intervals and strong gates in fuzzy graphs.

In Chap. 7, we present the work on interval-valued fuzzy graphs that is mostly due to Akram. It includes results concerning the operations, Cartesian product, composition, union, and join of fuzzy interval graphs. Other results deal with isomorphisms, complete, and self-complementary interval-valued fuzzy graphs. The important work of Craine on fuzzy interval graphs appears in Chap. 2.

In Chap. 8, we present the work on bipolar fuzzy graphs that is mostly due to Akram. This work includes results on operations of bipolar fuzzy graphs similar to the results in Chaps. 2 and 7. Results concerning isomorphisms of bipolar fuzzy graphs as well as results concerning strong and regular bipolar fuzzy graphs are provided. The chapter concludes with the work by Mathew and others on connectivity concepts of bipolar fuzzy graphs.

The authors are thankful to everyone who supported this project. It is our hope that this book will help students and researchers all over the globe to learn and to apply fuzzy graph theory. We welcome all suggestions and comments from everyone so that we can improve this book as a useful resource for students, teachers, scientists, and engineers for addressing the challenges of today's world.

Acknowledgements We thank Dr. George Haddix and his wife Sue for their generous support of our endeavors involving mathematics of uncertainty. We also thank the journal New Mathematics and Natural Computation for allowing us to reuse some of our work.

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http://www.springer.com/978-3-319-71406-6

Fuzzy Graph Theory
Mathew, S.; Mordeson, J.N.; Malik, D.S.
2018, XVII, 320 p. 171 illus., Hardcover
ISBN: 978-3-319-71406-6