Chapter 12
Map and Territory in Physics: The Role of an Analogy in Black Hole Physics

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The generic territory this paper will concern itself with is that of the physical world, and the map is that of theoretical physics: the theories, primarily mathematical, that one generates to describe, to predict new aspects of, that physical world. That such a map is even possible, and furthermore that such a map is such an accurate representation of the physical world is something that amazed and surprised physicists even before the time of Newton. Already Pythagoras astonished both himself and the intellectual world by mathematizing an aspect of the world, that of harmony of musical notes. Two notes produced by different lengths of identical musical strings, such that those lengths bore small whole number ratios with respect to each other, would sound harmonious, while those with arbitrary ratios sounded inharmonious and clashing. Understanding the origin of this mathematization of the physical world formed one of the primary puzzles which exercised the minds of top physicists for 2000 years. That the eventual solution told us as much about the peculiarities of the human mind, as it did about the physical world does not detract from the guiding light that Pythagoras's observation shone in the development of physics (Cohen 1984).

At the same time, analogy has played a guiding role in the rational understanding of the world. In terms of the central metaphor of this book, that of human understanding seen as the interplay in geography between the map and the territory, the question is, "If the map of two regions is the same, how much can we say about the similarity of the territory that the maps describe?"

One of the most astonishing features of modern 20th and 21st century physics has been how similar the mathematical tools are which are used to describe what, on the face of it, are utterly disparate phenomena. Quantum Field theory, developed to describe the quantum mechanics of electromagnetism, and which eventually became the dominant paradigm of elementary particle physics, has also come to dominate the theoretical structure of condensed matter physics.

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In this paper I want to show how one can use the mathematical analogy between two seemingly disparate areas of physics to cast light on both.

Back Holes were one of the most surprising predictions of Einstein's theory of gravity. That theory began with Einstein's insight that gravity, rather than being a force, was actually closely associated with the nature of time. Newton had described the gravity we experience daily as a mysterious force, an "action at a distance", which caused two massive bodies to feel a force of attraction to each other. Einstein (already by 1908) realized that gravity could instead be described as the inequable flow of time from place to place. One often hears that gravity can cause clocks to tick at different rates from place to place. But that is a perversion of the story that his theory tells. Instead it is precisely the ticking of time differently from place to place that is the gravitational field which we usually feel (Unruh 1995). Combining this with Minkowski's description of special relativity as combining distances in time and distances in space into one unified notion of distances in space-time, and with Newton's realization that the motion of matter in the absence of external forces follows straight lines (the shortest distance between two points), Einstein showed how all of Newton's theory of gravity could be subsumed into the law that matter causes time to flow differently from place to place. Of course Einstein's theory, General Relativity, is more complex since if time can flow differently from place to place, then spatial distances can also change from time to time (leading to cosmology, where the distances between each object in the universe can increase or decrease with time without the objects themselves moving, and to the existence of gravitational waves where distance changes can propagate at the speed of light).

Only a few months after Einstein had laid out his theory, Karl Schwarzschild, a German soldier on the Russian front of WWI, found the first exact solution of the equations. He showed that the consequences of the theory were even more dramatic than anyone had expected. It took almost 50 years for physicists to realize that his solution implied that one could have regions of space which could be entirely out of contact with the rest of the universe. Even at the speed of light, anything inside what is now called the horizon in Schwarzschild's solution, cannot communicate or interact with anything outside (unless that outside object also falls into the horizon). In honour of this behaviour, Wheeler popularized the name "Black Hole" for this phenomenon. But almost immediately after these objects had been named, another shock was delivered. Hawking (1977) argued that, if one takes seriously the behaviour of the aforementioned quantum fields near the black hole, it ceased to be black. It radiates, and surprisingly, it radiates as though it were a hot body, with a temperature inversely proportional to the mass. Thus a solar mass black hole has a temperature of about 10^{-6} K, but an earth-mass black hole radiates with a temperature about a million times higher, while the black hole in the centre of our galaxy has a temperature of the order of the coldest temperature ever achieved in terrestrial labs.

As stated, that temperature is a function of the mass of the black hole. The mass of the black hole is, via $E = mc^2$, expressible in terms of the energy of the black hole—the total energy which has fallen in to make the black hole. This suggests that the black hole is a thermodynamic object, with an entropy. Using the expression Hawking found (in units in which $\mathrm{G} = \mathrm{c} = \hbar$)

$$T = \frac{1}{8\pi M} \tag{12.1}$$

one finds that the entropy is just one quarter of the area of the black hole, with the area expressed in terms of the Planck area (the area expressed in purely in terms of $\hbar$, G, c). As with any thermodynamic object, this entropy limits the efficiency with which one can use the black hole to convert heat into work (Unruh and Wald 1982). The understanding of Hawking's result has been a driving force in theoretical physics in the past 45 years.

Bekenstein (1973) had suggested that black holes should have an entropy by following Wheeler's unsupported suggestion that black holes, as absorbers of entropy, should also act as thermodynamic objects and had an entropy. This idea ran into the road block that black holes are black. They do not have any temperature except 0. While black holes had formal features which suggested the laws of thermodynamics, at best everyone took these as formal unphysical analogies. Hawking's unexpected result shocked and inspired the theoretical community. Black holes are thermodynamic objects. This result was too surprising to be false, but in the past 40 years, the understanding of the source of thermodynamic aspect of black holes has remained largely a mystery. One of the greatest mysteries is the entropy. Entropy was introduced in the mid 19th century to explain the operation of heat engines. Later in the 19th century, Maxwell, Gibbs, Boltzmann and others explained entropy in terms of statistical mechanics. For them, the entropy is related to the number of different states the system could have at a given energy or temperature consistent with the macroscopic parameters one could access in using the system in a heat engine. But what is the entropy of a black hole? How does it relate to statistical properties of a black hole, and what are those microscopic degrees of freedom needed to give it a statistical interpretation?

However, Hawking's temperature rested on a strange aspect of quantum field theory in the vicinity of the black hole. The evolution of quantum fields is deterministic. The thermal emission must arise from some aspect of the initial state of the field, which was assumed to be the vacuum state. What aspects of that vacuum state result in the thermal emission after the black hole has formed? Hawking essentially operated backwards. Given the final state, of the field, what aspect of the initial state could have produced it? To find it, one can evolve the final state backward in time. And because of the linearity of the field which he used to calculate, one can do this mode by mode. Given any mode of the field (some distribution of the field obeying the classical equations of motion) one must see where it came from in the initial state. It cannot come from inside the black hole (nothing can get out of the black hole by definition of what a black hole is). But it comes from the direction of the black hole. It must therefore come from a vicinity closer and closer to the horizon of the black hole. In fact, the equations of motion say it comes from a region exponentially closer with a scale of the radius of the black hole, and a time scale of the light-travel time across a distance of the order the size of the black hole. It continues to get closer and closer to the horizon until one gets to a time when the black hole forms, when that mode can escape out toward infinity. By that time its wavelength is tiny, and frequency extremely high. For example, for a solar mass black hole, a thermal mode, of frequency near the maximum of the thermal spectrum, emitted one second after the formation of the black hole via collapse, must have originated from a quantum vacuum fluctuation in the initial vacuum state with a frequency of order e^{10^5}. This would have an energy of the order of e^{10^5} times the mass of the whole universe. Clearly, for modes of this energy, the assumptions that the field is a simple linear, non-interacting field is extremely suspect. Does this mean the prediction of thermal radiation is wrong?

This problem with Hawking's derivation was clear very soon after his discovery. It has also misled many researchers throughout the years into believing that Hawking's result depends on high energy Planck scale physics. There certainly seems no way of avoiding this conclusion if one takes his derivation seriously.

In 1972 I was asked by Denis Sciama to give a colloquium at Oxford on black holes. Desperately trying to think of some way of making some of the properties of black holes approachable by the audience who had never heard of such objects, I thought of an analogy, that of a waterfall. If one imagines a waterfall so high that the velocity of the water somewhere exceeds the velocity of sound at some surface, then that surface acts very much like a black hole horizon as far as sound is concerned. Sound cannot escape out of that surface, since the sound there is swept back over the waterfall at the same rate as it is trying to escape. Furthermore, any sound trying to escape from just outside that surface takes a long time to get out. The closer it is to that surface that the sound is emitted, the longer it takes to escape. Both of these features are similar to what happens to light near a black hole. No light can escape from behind the horizon, and the time it takes for the light emitted just outside the horizon gets longer and longer the closer the emission is to the horizon. The sound waves emitted nearer and nearer the horizon are bass-shifted, just as light emitted nearer and nearer the horizon is red-shifted.

This analogy was just that, an analogy whose only purpose was to try to clarify some features of a black hole. It indicates a similarity between sound and light, but as it stands it does not indicate that the two territories share a map, a detailed mathematical similarity. In 1980, I was assigned a course on Fluid Mechanics to teach. One evening, while preparing my lecture for the next morning, my mind wandered back to that analogy and I decided to try to see how well the analogy actually worked. Was it more than a pretty picture? To do so I wrote the equations of motion of an irrotational fluid, separating them into some time-independent background flow and a small linear perturbation around that flow. Those perturbations were to represent sound waves. Introducing the velocity potential (possible because the flow was irrotational), and eliminating the fluctuations in the density between the resultant two differential equations, I got an equation for that velocity potential which looked just like the equation for a scalar field in a background spacetime. In this case that effective spacetime is determined by the background flow and density of the fluid, not by the relation between spacetime and gravity as in Einstein's theory.

Furthermore, one could imagine quantizing those linear perturbations, the sound waves. Such a quantization of sound waves is standard practice in condensed matter physics, where the quantized sound excitations are called phonons. One could then follow Hawking's derivation of the thermal emission by a black hole, step by step, for this quantum field (the velocity potential) in this effective spacetime metric. If the fluid flow was such that at some place the velocity of the fluid exceeded the velocity of sound, that effective metric looked in many ways like that of a black hole, with a Killing horizon (i.e., a horizon defined by the condition that the vector denoting the time displacement symmetry becomes null in the effective metric). A straightforward calculation shows that this quantum field should also produce a thermal flux of phonons, just as the black hole produces a thermal flux of photons. In the latter case the temperature is proportional to the inverse mass of the black hole. In this case the temperature is equal to

$$T = \frac{1}{4\pi c}\frac{d(c^2 - v^2)}{dx} \tag{12.2}$$

where x is the distance along the flow lines of the fluid which go into the horizon where $v^2 = c^2$ (Unruh 1981).

One thus has the same map—the propagation of the field in a spacetime—and the same conclusion—the quantized small fluctuations of that field result in thermal emission from the horizon, with the temperature of that emission determined by properties of that background spacetime. The same map of the two diverse territories implies that unexpected features of the territories also seem to be the same. This conclusion that sonic horizons would also produce a thermal quantum spectrum is also a surprising conclusion, but in both the black hole and the dumb hole cases, the problem of ultra high frequencies in the initial states is the same. If maps are identical then the territories, at least to the extent that the maps are accurate, must also be identical.

But this conclusion in the case of dumb holes (the name given to such sonic analogs to black holes) is clearly wrong. In the case of the sound waves, one can understand the emission of the thermal radiation in the same way. Tracing back the modes of the sound which are thermally excited in the future, one finds again that the horizon is a one-way membrane, at least in the simple model of sound derived from the Navier-Stokes equations. Those modes cannot come from inside the horizon, and must therefor be squeezed more and more against the horizon as one goes into the past. The bass-shift of the outgoing waves near the horizon implies an exponential squeezing of the modes against the sonic horizon, just as the light in the black hole case is squeezed against the horizon because of the red shift of the radiation emitted by a source falling into the black hole. But in the case of sound waves, we understand that the hydrodynamic equations are an approximation. At short wavelengths the fluid cannot be described by a continuous density with some velocity, but rather must be described as a conglomeration of distinct, spatially separated atoms. Sound waves ultimately are a description of the average motion of those atoms around some background equilibrium flow. And sound waves cannot have a wavelength shorter than the average distance between the atoms.

The equivalence of the maps in the sonic and the black hole case breaks down. Or does it? After all one has the gut feeling, which goes all the way back to Planck, that at some scale, quantum gravity effects should come into play in the case of the black hole. This can be seen to be in analogy to atomic effects coming into play in the case of the dumb hole.

One of the worries about the black hole is that perhaps those quantum gravity effects could destroy the thermodynamic edifice erected around black holes via Hawking's discovery. If Hawking radiation really depends on those exponentially large frequencies and exponentially tiny distances which his derivation requires, then the necessary alteration of the theory at those scales by the effects of quantum gravity might destroy the effect he discovered.

It is precisely here that the sonic model might come to the rescue. We understand precisely how the hydrodynamic equations break down, and we understand, at least in theory, what a truer description of a fluid is. It is the collective motion of a bunch of atoms. The calculations of how the fluid behaves in terms of the individual atoms might be horrendously complicated but, unlike the case for quantum gravity, we have a strong faith that the essentials of the theory of fluids are known. So we can ask, "Does the thermal radiation emission by a dumb hole survive the generalization of hydrodynamics to a fully atomic description of the fluid?" If it does not, then one has no faith that the Hawking effect would survive a fully quantum treatment of gravity. If the prediction of dumb-hole thermal radiation does survive, then it may give us clues as to how the black hole thermal radiation might also survive the effects of small scale quantum gravity.

When I wrote the paper which resulted from my evening's distraction from lesson preparation, I realized the potential usefulness of the dumb-hole model in deepening our understanding of black holes. But I had no idea how to actually carry out a calculation treating the atoms of the fluid as fully quantum objects. I tried to imagine how I would even start to carry out a fully non-linear quantum treatment of 10^{25} interacting atoms. Fortunately I gave a seminar at the University of Texas where Ted Jacobson was in the audience. About 10 years later, he realized that one of the key effects of the atomic nature of the fluid was to change the dispersion relation of sound waves, i.e., instead of the velocity of sound, whether phase or group velocities, being a constant, independent of frequency or wavelength, the atomic nature of matter caused the velocity of sound to change at short wavelengths. How it changes depended on the particular nature of the fluid. For liquid helium, for example, both the group and phase velocities would, at short enough wavelengths, decrease from their values at long wavelengths. For a Bose Einstein condensate fluid on the other hand, the velocity of sound would increase as the wavelength became shorter and shorter. It was this realization which allowed people to begin to answer the question as to what the effect of the atomic nature of the fluid on the analog to Hawking radiation could be.

In the above description of how the horizon affects the modes which eventually come away from the horizon in a thermally excited state, the key was that the modes got squeezed up more and more against the horizon as one propagated those modes backward in time, until one got those absurdly high frequencies and wavelengths. The change in the dispersion relation, the change in the velocity of sound with frequency, means that, while initially those modes are again squeezed against the horizon, eventually their wavelength becomes sufficiently short that their velocity is no longer the same as the velocity of the fluid. If their velocity decreases with frequency, those waves must have been swept in from outside the horizon. If the velocity increases with frequency, those waves must have travelled from inside the horizon out to the horizon. In either case, that squeezing of wavelength ends once the wavelength reaches the value where the velocity of the waves changes from the velocity of sound at long wavelengths.

What Jacobson's observation meant was that the modes of propagation of the sound waves always remained in a regime in which they acted like linear sound-waves, with wavelengths much longer than the inter-atomic spacing. One did not have to worry that the highly non-linear regimes of the inter-atomic interactions would destroy ones ability to do calculations. In general the equations can still not be solved analytically, but they can be solved numerically. Soon after Jacobson's observation, both I (Unruh 1995), and then Corley and Jacobson (1996) did just that and found that the change in the dispersion relation at high frequency had essentially no effect on the thermal emission at low enough frequencies the quantum sound emission behaved just as in the hydrodynamic approximation. Although at high frequencies, radiation begins to deviate from thermal, at lower frequencies thermal spectrum is a very good approximation. The thermal spectrum is insensitive to the behaviour of the equations of the field at short spatial or temporal scales. The thermal behaviour of the emission from horizons is a robust phenomenon. This suggests strongly that the concern, that Hawking's derivation requires a specific behaviour of the fields at arbitrarily high frequencies or arbitrarily short spatial scales, is misplaced. Hawking radiation is a low frequency, large (relatively) distance phenomenon. It is not a magic road to Planck scale physics.

One can understand this in a hand-waving way by the following argument. Consider a mode of the field which begins life far from the location of the future black hole, and which has a very high frequency. Our assumption is that a mode begins in its ground, or vacuum, state. Because of its high frequency, it sees the surrounding metric change on scales which are of much lower frequency and longer spatial scales than its own. By the quantum adiabatic theorem, a quantum system which is perturbed on time scales much longer than its own does not change its state. If it begins in its ground state, it remains in its ground state. As the mode propagates near the horizon of the black hole, this adiabatic behaviour of the surrounding spacetime continues until the frequency has been red-shifted by its propagation along the horizon to a value which is the same order as the rate of change of the surrounding metric (the time scale and spatial scale of the curvature of the black hole). It is only at this point that that the time-dependence of the surrounding spacetime begins to change the state of that mode of the field, creating particles (excitations away from the ground state of that mode). If this argument is correct (and no rigorous derivation exists which demonstrates that this argument is correct), then the Hawking radiation is truly a low energy, long wavelength process.

Entropy

As part of his thesis project under John Wheeler, Jacob Bekenstein argued that black holes should have an entropy. Wheeler had argued that because black holes could absorb the entropy of the matter falling into the black hole, it should also have entropy itself. Otherwise one could get rid of an arbitrary amount of entropy from the external universe, and perhaps violating the second law of thermodynamics. Since a classical black hole has a zero temperature, and since a zero temperature heat bath can (barring the third law) absorb and arbitrary amount of entropy, Wheeler's argument was somewhat shaky. Bekenstein however ran with the idea. Hawking had just shown that the laws of classical General Relativity, together with the requirement that matter always have positive energy, implied that the surface area of a black hole must always increase. Since entropy (by the second law) must also always increase, it was very suggestive to Bekenstein that perhaps there was some relation between the area of a black hole and its entropy. He generated a number of arguments that this identification of entropy and area was more than an analogy. However, this analogy foundered on the problem that if the black hole had an entropy, and since it certainly had energy, it must also have a temperature. Classical black holes have at best a zero temperature. Geroch pointed out that if one regarded the area as the entropy one could violate the second law of thermodynamics if the black hole temperature was zero.

It was Hawking's discovery that quantum field theory implied that black holes did have a temperature, a temperature moreover which was a function of the mass of the black hole that gave a way out of this impasse. Using the standard thermodynamic relation, $dE = TdS$, one found that the entropy must be equal to 1/4 of the surface area of the black hole, as measured in Planck units. Various arguments showed that this entropy was more than just a fluke. In particular, if one operated a heat engine with the gravitational field of the black hole being used to convert heat energy to work, then such a heat engine obeyed the standard Carnot efficiency if the surface area of the black hole was the entropy required in the Carnot argument. The entropy of the black hole is a real thermodynamic entropy.

The big question then was whether or not the arguments of Maxwell, Gibbs and Boltzmann, that entropy is related to the uncertainty of microstate of the system under the constraint that the few degrees of freedom used by the heat engine be fixed, were correct. What are these internal degrees of freedom of a black hole? Or, alternatively, is the entropy of a black hole not of any statistical origin, but is a "pure" entropy, unrelated to a counting of the microscopic degrees of freedom?

It is these questions which the sonic model can perhaps also shed light on. For the sonic analog, there is no relation between the energy in the waterfall, and the temperature. There is then also no entropy associate with a dumb hole. Yet, in both the black and dumb hole cases, one finds that the fields living on this spacetime (e.g. photons in black holes, and phonons in the dumb hole case) are emitted with a thermal spectrum. That thermal spectrum is not the result of the dynamics of any hidden degrees of freedom of the spacetime, but is a direct consequence only of the smooth metric structure which determines the equations of motion of the quantum fields.

In ordinary statistical mechanics, there is an intimate relation between the microscopic degrees of freedom of the thermal object and the thermal radiation emitted by that object. It is precisely those microscopic degrees of freedom which create the radiation which escapes from the body. It is because those micro degrees of freedom move and change that the radiation is created. In the case of both the black holes and the dumb holes this is not the case. The background metric does not change. It is not due to its alterations, due to its thermal excitation, that the radiation is created. Rather it is because of the quantum field's motion over the smooth surface of the spacetime that the radiation is created. To me this suggests that the entropy (which, as I said, is a genuine thermodynamic entropy in the case of black holes) is not the result of microscopic degrees of freedom, but is fundamentally thermodynamic entropy, unrelated to any microscopic degrees of freedom.

Experiment

My original paper on the sonic analog was entitled "Experimental Black Hole evaporation?" What excited me was the possibility that one could, in a terrestrial laboratory, carry out experiments which were directly related to the thermal emission by black holes. No matter what the approximations used to solve the theory, they are approximations and one is never sure how accurate they are. Furthermore there can be additional physical effects which are not included. One example is that the viscosity of a fluid might affect the thermal radiation. Or turbulence in the fluid, or a host of other effects. In the presence of quantum and classical fluctuations, the exact location of the horizon is uncertain. Do those fluctuations in the position of the horizon affect the thermal radiation? If the high frequency behaviour of the field (e.g. its squeezing against the horizon) changes the horizon then one might expect that the location of the horizon could be important. The waves could be squeezed up against the position of the horizon at one time, only to have the horizon shift so that those squeezed waves are now either inside or outside the horizon. If the claim above is true, that the thermal emission is not a high frequency phenomenon, but represents the reaction of the field at low frequencies and long wavelengths to the changes in the metric field, then one would not expect the exact location of the horizon to be important. This is a question that, potentially, experiments could resolve.

There have now been a number of experiments to look for the thermal nature of the radiation (Daniele Faccio et al. 2013). One set of experiments, initiated by Germaine Rousseaux, and carried to completion by a group at the University of BC (Weinfurtner et al. 2010), used water as the medium for creating a dumb hole, and used the surface gravity waves as the field which carries the thermal emission. Of course the quantum emission would be impossible to see. Its temperature (of the order of 10^{-12K}) is far colder than the temperature of liquid water, but a stimulated emission experiment could be carried out. As Einstein, with his A and B coefficient analysis, showed, knowledge of stimulated emission is sufficient to also understand the spontaneous emission in a system. In these experiments the alteration of the dispersion relation was created by the transition from shallow water waves to deep water waves. The experiment showed that the spectrum of the quantum emission, assuming that Einstein's analysis is correct, would be thermal, with a temperature of the order of 10^{-12} K.

Another recent experiment was by Jeff Steinhauer (2016) using BECs. He looked for fluctuations in the density of the BEC as the measurable quantity of the created quantum phonons. In his case the experiment was too noisy to be able to see a thermal spectrum, but there was a suggestion that there was entanglement between the waves travelling in opposite directions, away from the horizon. Such entanglement would be expected for the creation of Hawking radiation by a horizon, and would be a signature that the process creating those fluctuations was quantum, and not simply the amplification of some classical noise source.

An additional path has been the attempt to use light in a medium to form a black hole type horizon by altering the index of refraction in the medium (see for example Belgiorno et al. 2010). Since the media are solids one cannot have the medium flowing with different velocities. Instead one must have a region in which the velocity of the light is changed, with that region travelling at almost the velocity of light. In most of the experiments of this nature this is done by using an intense region of light whose non-linear interaction with the medium changes its refractive index. So far this promising approach has not yet exhibited quantum emission.

Conclusion

All maps are approximations to the territory they describe, including the mathematical maps which physics use to describe their territory, the world. That the maps which describe different territories can be similar at a certain level of approximation allows us to gain understanding of a poorly understood territory by applying the lessons from the better understood territory. This is the role that analogy has played throughout history. What we see in the example which this article has looked at it that that understanding can come from the differences as much as, or perhaps even more so, than from the similarities.

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For an introduction to both the theory and experiments in Analog gravity, see the volumes **Analogue Gravity Phenomenology: Analogues Spacetime and Horizons, from Theory to Experiment** ed. by Daniele Faccio et al (Springer, 2013) or **Analogue spacetimes: The first 30 years** ed. by V.M.S. Cardoso, L.C.B. Crispino, S. Liberati, E.S. Oliveira, M. Visser (Editora Livraria da Fisica, Sao Paulo, 2013)

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