

# Texts for Quantitative Critical Thinking

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# Mathematics for Sustainability

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## Foreword

I'm excited to introduce this book to you, because it may be different from any math text you've read before. It will change the way you look at the world and enlarge the way you think about mathematics. No longer will you be just a spectator when people give you quantitative information—you will become an active participant who can engage and contribute new insights to any discussion. Just look at the verbs that underlie the chapter titles: measure, flow, connect, change, risk, decide!

Here's what stands out to me when I read this book: there are many math books that will feed you knowledge, but it is rare to see a book like this one that will help you cultivate wisdom.

There is a deep difference between knowledge and wisdom. A knowledgeable person may be armed with facts, but a wise person considers how to act in light of those facts. A knowledgeable person may think an answer is the end of an investigation, whereas a wise person considers the new questions that result. And a knowledgeable person might ignore the human element of a problem that a wise person deems essential to understand. As the authors illustrate, mathematics that pays attention to human considerations can help you look at the world with a new lens, help you frame important questions, and help you make wise decisions.

Sustainability asks: how can we be wise stewards of Earth's resources? One way or another this question will impinge on some aspect of your life, if it hasn't already. Sustainability is an economic concern because resources are limited. Sustainability is a moral concern, because any scarcity of Earth's resources will harm the weak and vulnerable first. And sustainability is a scientific concern, because we may have the power to improve the lives of those who will be affected.

I know that each of the authors shares a deep vocational commitment in bringing this book to you, and as evidence, I'll speak personally of the author I have the privilege to know as a friend: John Roe, a man of deep grace and humility who made this book his highest professional priority while battling a difficult illness. For him, this project grew out of a conviction and prayerful reflection that his knowledge as a mathematician and an educator could be channeled into wise action on matters that will impact us all.

The authors have poured their hearts into this remarkably important and timely book, and I hope you will engage it with the same fervor. Because it concerns the world you live in, how you will need to live in it, and the problems that you—yes YOU—can solve so that all of us can live in it well.

Francis Edward Su  
Benediktsson-Karwa Professor of Mathematics, Harvey Mudd College  
Past President, Mathematical Association of America

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# Before We Begin...

## 0.1 To the Student

### A Letter from the Authors

Dear Student,

The world that you are inheriting is full of bright possibilities—and also of big problems. Many of the problems center on *sustainability questions* like “can this (key part of our social or economic system) last?” or to put it in a way that has a little more math in it, “*how long can this last?*” For example, modern society is a profligate consumer of energy, most of which is supplied by fossil fuels: coal, oil, and natural gas. Fossil fuel supplies, though of vast and unknown size, are limited. *How long can they last?* What’s more, there is a strong scientific consensus that the carbon dioxide gas (also known as  $\text{CO}_2$ ) released by burning fossil fuels is affecting Earth’s climate, making it more unstable. *How much more  $\text{CO}_2$  can we emit before climate instability becomes dangerous?* These are big problems. Bright possibilities for addressing them include renewable energy sources like wind and solar. No doubt you have heard all of these things before—as well as many other news stories about “sustainability,” both positive and negative.

We started developing the “Mathematics for Sustainability” course, and writing this book, because of three convictions:

- Many of the key choices that humans will have to make in the twenty-first century are rooted in sustainability questions. These include choices that we must make together, as citizens, as well as choices related to individual lifestyles.
- In a democracy, as many people as possible need to participate in well-informed discussion of these sustainability questions. They are too important to be left to “experts.”
- We may engage with sustainability questions from a wide variety of perspectives, including scientific, technological, political, ethical, and religious. For many of these discussions, we need some knowledge of mathematics in order to participate in a well-informed way.

The aim of this book is to help you, the student, gain that mathematical knowledge and the ability to apply it to sustainability questions.

You may not consider yourself a “math person.” Your studies may center on music or English or art or education or architecture or agriculture.<sup>1</sup> But if you want to find out *for yourself* what “the numbers say”—not just to choose which “expert” you prefer to listen to—then this book is for you. Together, we will find out how to model sustainability on local, regional, and global scales. We will learn about

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<sup>1</sup>Students in all these majors have succeeded in the course on which this book is based.



measurement, flow, connectivity, change, risk, and decision-making. Some of the topics we discuss will probably be challenging, perhaps even unsettling. Whatever conclusions you reach, this book will prepare you to think critically about your own and other people’s arguments and to support them with careful mathematical reasoning.

As citizens in a democracy, you will ultimately be the ones whose decisions will guide your world toward a sustainable future. We wish you the very best in your studies, and as you participate in building the sustainable society of the future.

John Roe, Russ deForest, Sara Jamshidi  
August 2017

## Sustainability—The Key Idea

In spring 2017, our Earth’s human population surpassed  $7\frac{1}{2}$  billion. Here’s a question. What do you imagine was the population of Earth two thousand years ago, at the beginning of the Common Era?

*Demographers* (scientists and historians who study population) obviously don’t know an exact answer to this question. But they are able to make some good estimates, which are in the range of 200 to 300 million people (Section 8.2). That is to say, the number of people on the whole planet twenty centuries ago was roughly the same as the number in the United States (U.S.) today. Or, to put it differently, the population of our Earth has increased by twenty-five times over that period.

That population increase has not been a steady one. Most of the growth has occurred in the last century. And many other measures of human activity follow a similar pattern. Take a look at the graphs in Figure 1, which are taken from a book by Will Steffen [308]. These graphs show a pattern of accelerating increase that mathematicians call *exponential growth*.<sup>2</sup> This is important news: some good (most societies have regarded large families and increased wealth as good, for example) and some less so (Steffen’s book includes similar curves about pollution and overuse of resources, which most would regard as bad). Both “goods” and “bads” have been growing exponentially, especially over the past two hundred years, since the Industrial Revolution got into gear. *Can this pattern continue?*

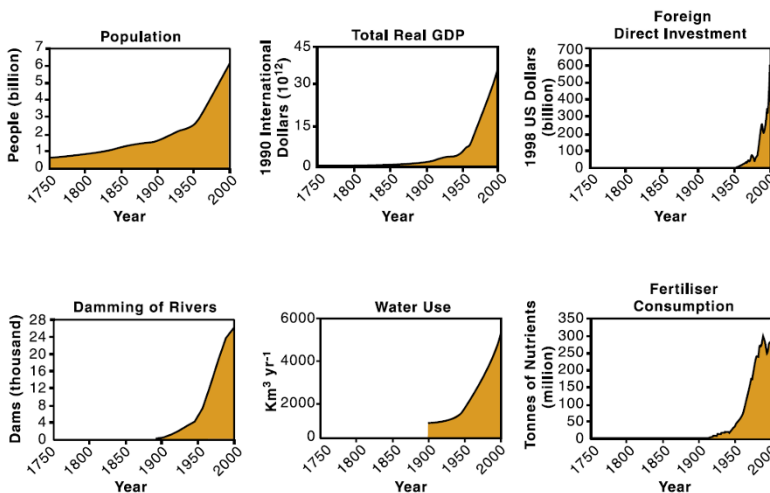


Figure 1: Some measures of the “size” of humanity (from [308]).

Some think so. In July 2015, one presidential candidate declared that his objective for the United States was “4 percent annual growth as far as the eye can see.” That is about the growth rate for the

<sup>2</sup>See Section 3.4 for more about this concept.

curves in Figure 1. Others, though, look at similar data and see it differently. “[These] remarkable charts. . .,” writes Gus Speth, former dean of the Yale School of Forestry and Environmental Studies, “reveal the story of humanity’s impact on the natural earth. The pattern is clear: if we could speed up time, it would seem as if the global economy is crashing against the earth—the Great Collision. And like the crash of an asteroid, the damage is enormous.” [305]. “Societies are now traveling together in the midst of unfolding calamity down a path that links two worlds,” he continues. “Behind is the world we have lost, ahead is the world we are making. . . . The old world, nature’s world, continues, of course; but we are steadily closing it down, roping it off. It flourishes in our art and literature and in our imaginations. But it is disappearing.”

The “old world” that Speth describes is a world in which Earth appears to be huge, teeming with life, abundant, exhilarating and dangerous. Humanity exists on the margin. In the “old world,” to ask about humanity’s impact on nature might seem absurd: much more important to worry about nature’s impact on human beings (diseases? predators? food shortages?) By contrast, the iconic image of the “new world” is the Apollo astronaut’s view of Earth: the “blue marble” (Figure 2), floating in space, gemlike and beautiful, yet cut down to finite size by human achievement. In this finite world it makes sense to ask: How long can we keep growing? Have we already become too big? Can our complex society remain diverse, active, and productive for an extended period of time? Or could we overshoot the limits of our resources and then decline, as many earlier civilizations have done [96]?

This is **the** sustainability question from which all other such questions derive. We can put it another way by thinking of the successive generations of humanity on this planet. Each generation inherits Earth’s resources from its predecessors and passes them on to its successors. In a lasting or sustainable society, each generation would leave the world system in as good a shape as it found it; my generation’s enjoyment of Earth’s bounty would not deprive the next generation (yours) of the opportunity for similar enjoyment. Thus we arrive at the famous definition<sup>3</sup> given in the Brundtland Report [255] as early as 1987:



Figure 2: The “blue marble.”

### Definition 1

**Sustainability** is the ability of a social, economic, or ecological system to meet the needs of the present generation without compromising the ability of future generations to meet their own needs. A process or practice is **sustainable** to the extent that it contributes to the sustainability of the social, economic, or ecological systems in that it is embedded.

It is important to recognize that working for sustainability *does not* mean just trying to keep things as they are. “Things as they are” include patterns of privilege and inequality that deprive many members of the *present* generation of their basic needs. A sustainable world system must continue to extend the provision of these needs to a growing share of its population—ultimately, to all—even

<sup>3</sup>The Brundtland Report in fact defines “sustainable development”; we have slightly modified its language to arrive at a definition of “sustainability” itself.

as it also works to ensure that these needs can be provided in a way that doesn't ask future generations to pay the bill. This double challenge, we believe, will define the century that you live in.

We should also recognize that questions about sustainability and about the needs of the present and future generations are inherently value-laden. Particularly when we approach these questions on a global scale, we should expect to encounter different value judgments concerning how human well-being is defined and what are the basic needs whose provision should be sustained.

### *Think about it...*

The Brundtland definition of sustainability, Definition 1, was written in 1987. Yet such ideas appear many years earlier in the thought of indigenous peoples around the world. For instance, the Iroquois Confederacy's "Great Law of Peace," which is older than the U.S. Constitution, contains a clause that is often paraphrased as, "In every deliberation, we must consider the impact on the seventh generation. . . even if it requires having skin as thick as the bark of a pine." Thus, Westerners' recent "discovery" of the notion of sustainability might be more properly described as a "rediscovery" of ideas that are rooted in many traditions (including some of our own). How do you feel about this? Why might Western society have lost touch with the sustainability idea for part of its history?

## Sustainability and Resilience



Figure 3: Marine rescue operation in Galveston, Texas, following Hurricane Harvey, August 31, 2017.

The time scale on which we ask sustainability questions is a long one—many generations. Some changes to our world are not perceptible unless we take such a long view. The rate at which Earth's climate is changing, measured by global average surface temperature, is no more than a couple of hundredths of a degree per year: of little consequence from one year to the next but (as we will see) hugely significant on longer time scales. But change does not always come in tiny steps: our world also experiences sudden, extreme events. (Think of Hurricane Harvey, which flooded Houston in August 2017, doing damage initially estimated at \$60 billion.) Extreme events are unavoidable: the question is not how to avoid them, but how quickly we can bounce back from them. This "self-healing" capacity is **resilience**.

### **Definition 2**

**Resilience** is the ability of a social or economic system to absorb and respond to a sudden shock without damage to its core functioning.

Sustainability and resilience are not the same, but they are closely related: a system that is already near the limits of sustainability may be pushed to collapse by a shock that a more resilient system would easily have survived. In the same way, an infection that a healthy person would shrug off may be fatal for someone whose immune system is already compromised by malnutrition. Many historical examples can be found in [96].

One way to think about the relationship between sustainability and resilience is through the idea of *control*. It is tempting to believe that the more elements of our socio-ecological system we can bring

under human control, the better we will be able to steer it on a safe path and so the more sustainable it will be. For example, wildfire management in the Western U.S., and elsewhere, focused for many years on fire suppression: bringing fires under human control. This seemingly allowed development in wildfire-threatened areas to be lasting, or “sustainable.” Only recently has it become clear that these measures, which indeed halted many local fires, also set the stage for huge buildups of fuel that make the forests less *resilient* to the possibility of a catastrophic burn. Similar ideas apply to flood control measures like overflow dams and levees. In Section 4.4 we will develop a mathematical perspective on *tipping points*—situations in which a system lacks resilience because a small shock can produce an overwhelming response.

### Ecosystem Services

In Speth’s “new world” there is a tight connection between the economy, environmental quality, human well-being, and the well-being of the entire community of living and nonliving things that we call Earth’s *ecosystem*. This requires a significant change in our thinking. From the perspective of the “old world,” Earth was simply a source of abundance: nature provided. Our “new world” perspective requires a deeper understanding of what it is that nature provides, and indeed of the fact that human activity can no longer be so neatly distinguished from “nature” at all. Economic activity and human prosperity are embedded in the natural world, and they depend on and are ultimately constrained by the productive capacity of the whole Earth ecosystem (Figure 4)—which itself is constrained by the rate at which energy arrives from the Sun.

In Figure 4, we should therefore think of the outermost oval, which represents the entire ecosystem of the Earth, as of more or less unchangeable size. That means the inner ones cannot grow too much. In other words, the definition of a sustainable society implicitly involves the uncomfortable idea of *limits*: limits on the “goods” that are available to each generation and also limits on the capacity of Earth’s ecosystem to absorb the “bads” generated by human activities. The recognition that such limits exist sets the stage for the mathematical tools we develop throughout this book. We need to be able to interpret and use quantitative information to assess the size and nature of such limits and so to reach well-justified decisions about sustainability questions. If we lack such tools, we’ll be tempted to believe either that there “are no limits” or that if limits exist, they are so far off that they need be of no concern to us. That may be true in some cases, but in others we may find that the limits are startlingly close.

The notion of **ecosystem services** [342] provides one way to conceptualize this dependence of human activity on the planet’s ecosystems.

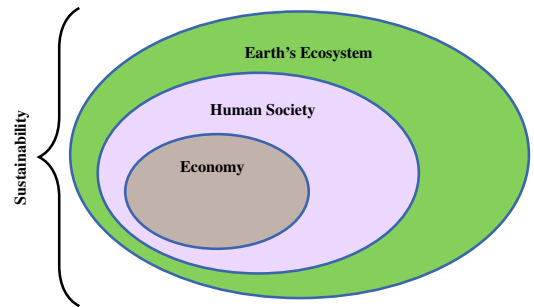


Figure 4: Sustainability and limits. Redrawn from [65].

#### Definition 3

**Ecosystem services** are benefits that human society obtains from Earth’s ecosystems. These services range from pollination of food plants and provisioning of freshwater and other resources to climate regulation, soil production, and recreational opportunities.

Specific ecosystem services are not fixed once and for all: human activity may enhance or degrade the capacity of Earth’s ecosystems to provide these services. Moreover, many ecosystem services

(such as clean air, clean water, climate regulation) are *public goods*—they cannot be “owned” in any practical sense. While some ecosystem services are local or regional (think of the services provided by a community park or a state forest), many are global in nature (like the pollution-absorbing capacity of the atmosphere): effective management and stewardship therefore require international cooperation. In Chapter 6 we develop mathematical tools to study such cooperation. This will allow us to understand some of the pitfalls and difficulties in reaching mutually beneficial agreements.

*Think about it...*

Gaylord Nelson, former U.S. senator for Wisconsin and founder of Earth Day, wrote in 2002:

*The economy is a wholly owned subsidiary of the environment, not the other way around.*

What do you think he means? Do you agree?

## About This Book

A few words about the *structure* of this book. **Part I** is the longest part of the text. In it, we develop mathematical tools and apply them to short examples. It is organized into six chapters corresponding to key concepts that arise in the mathematical study of sustainability: measuring, flowing, connecting, changing, risking, and deciding. Here is an overview of those concepts:

**Chapter 1: Measuring.** In this chapter we discuss how to measure, and how to express *how big* some quantity is. From the point of view of sustainability, the kind of answer that matters is often not some absolute number, but a *comparison* or a *level of importance*, so we’ll talk about how we might judge whether some quantity represents something important, and how we might make a decision on how to respond.

**Chapter 2: Flowing.** It’s common to talk about the *balance of nature*. But this image of a “balance” can suggest something that is static, unchanging. That is not the way natural ecosystems operate. Individual components of the system are constantly changing, even as the system as a whole maintains its equilibrium. For example, consider a mountain lake. The water level in the lake may stay the same, but the actual water in the lake today is not the same as the water that was there yesterday: new water has arrived through rain and snowmelt, old water has left through runoff and evaporation. It’s the balance between these various *flow* processes that keeps the water level—the *stock*—constant. This chapter is devoted to exploring these concepts of flow and stock in detail.

**Chapter 3: Connecting.** Our lives are more interconnected now than at any time in history, and not just through social media. Rather than most of our food being grown close to where we live, for example, we have gotten used to obtaining food from supermarkets that are supplied by a production and transportation network that reaches all over the globe. Energy supply, too, is a vast and complex network of tanker routes, pipelines, electrical grid connections, and truck deliveries, to mention only a few. Human-made networks like these are not the only ones: we have also grown much more aware of the complexity of the *natural* networks that connect the web of life on earth. In this chapter we will study the mathematical language that is used to understand these various kinds of networks.

**Chapter 4: Changing.** In this chapter we will look at examples in which stock-flow systems are out of equilibrium—that is, how they respond to *change*. Although we start by studying a simple model of continuous growth or decay, from the point of view of sustainability the important questions arise when growth is limited by some external factor (such as the fact that we live on a finite planet). We’ll study how the *strength of feedbacks* governs a system’s response to change, and how this leads to the key idea of a *tipping point*—a moment when the system “flips” suddenly to a new state. Some

scientists are concerned that Earth's climate system might be approaching one or more tipping points. We'll ask how it might be possible to tell.

**Chapter 5: Risking.** Any kind of realistic thinking about sustainability must consider likelihoods or *risks*. Nuclear power generation does not produce any greenhouse gas emissions, and it is constant and reliable, but what about the possibility of a catastrophic accident? This question asks us to balance the near-certainty of a steady benefit against a small risk of disaster. It is hard to do so without numbers. How likely are you to die from a nuclear reactor meltdown? The math involved in working with these sorts of questions is called *probability and statistics*. In this chapter, we are going to learn about these techniques and how they can help us make good decisions when faced with limited knowledge and uncertain outcomes.

Finally, **Chapter 6: Deciding** is where the rubber meets the road. In the end, you and your generation are going to have to make some sustainability decisions. These will range from the personal, through the local, to the national and global. Human behavior does not always follow the “rational” principles discussed in Chapter 5. Even if mine always did, I am not the only person involved. Other people's decisions interact with mine in a complicated way—we are in fact a *network* of decision-makers, with no final authority. This decision-making interaction can be studied mathematically, which we will do. The chapter concludes, however, with an extended reflection on how our sustainability decisions can never be *purely* mathematical, but must also engage our fundamental personal and ethical commitments.

In **Part II (Chapter 7)** of the book we provide a collection of Case Studies in which we apply the mathematical tools developed in Part I to answer particular questions related to sustainability and to explore extended examples. We believe that it is important that you, the student, learn to write extended pieces of this sort, and a student writing requirement has been a major part of the course on which this book is based. Why? This is how you build the ability to assess, develop, and present quantitative evidence in support of your own ideas. These skills are vital to you as a future leader, as an engaged citizen, and as an effective advocate for the things you care about. We don't want you just to learn mathematical techniques but also to be able to incorporate them in extended and persuasive written pieces—pieces that might be published in a course blog, in a student or hometown newspaper, or even on a national platform.

Finally, **Part III (Chapter 8)** of the book contains reference material: suggestions for further reading, tables of useful data, and the list of figures, bibliography, and index. We suggest checking Part III regularly, especially if you need some numerical information (such as the heat capacity of water, or the amount of sunlight that falls on Earth, or the planet's estimated coal reserves) to help you answer one of the exercises or formulate an argument in one of your more extended written pieces.



## Online Resources

This book has a *website*, <http://math-for-sustainability.com>, as well as an email address for comments and suggestions, [comments@math-for-sustainability.com](mailto:comments@math-for-sustainability.com). On the website you will find many different kinds of supporting materials:

- Hints and solutions to selected exercises.
- Online calculators and examples. Especially later in the book we will describe many models for sustainability-related processes that *change over time* (such models are called *dynamic*). The best way to visualize such a model is also dynamic, like a video clip rather than a collection of snapshots. That can't be done on paper, but it can be done online, and the website uses several different modeling tools to help you see the way our dynamic models evolve over time.
- Extended discussions of particular topics (refreshing the case studies in part II of the book, some of which may become outdated as technology advances or the environment changes). If *you* publish a written piece in a newspaper somewhere, please write to us (using the email address above) and tell us! We'll be happy to link to your writing from the book's website.
- Corrections or updates. We've tried hard to make this the best book possible. But there are bound to be some mistakes left. What's more, some information will simply become outdated. We'll keep a list of corrections or updates on the website. Again, if *you* notice an apparent mistake, please email us. You'll be helping many future students by doing so.

## Conclusion



Figure 5: Pope Francis has said: “These ancient stories...bear witness to a conviction which we today share: that everything is interconnected, and that genuine care for our own lives and our relationships to nature is inseparable from ... justice and faithfulness to others” [123].

We hope that through this book you will gain a clearer understanding of the sustainability issues that we humans face and of some choices that we need to make. But let's be clear: mathematics cannot make these choices *for* us. As remarked above, the choices human beings make on such fundamental questions reflect their deepest ethical and personal commitments (compare Figure 5). What mathematics can do, though, is to *inform* our choices by making their likely consequences clearer. It can help us *prioritize* issues by ranking them in terms of the relative size of the risk they pose and the relative severity of their potential outcomes. Understanding mathematics can help us *avoid* falling for some plausible-looking “solutions” that really don't achieve much or are even harmful. Finally, mathematics carries its *own values* also, like communicating clearly, reasoning logically, and considering all possibilities. These values, as well as the specific content of mathematics, can help us all in the decisions that we will all have to make together.

**Summary of Ideas: To the Student**

- **Sustainability** refers to the ability of a social or economic system to keep functioning without degrading its environment—to provide for its own needs and also preserve the ability of future generations to provide for their needs.
- **Resilience** refers to the ability of a social or economic system to “self-heal”—to recover from a disruptive event.
- **Ecosystem services** refer to the benefits that society receives from Earth’s ecosystems.
- Many questions about sustainability and resilience involve *measurement*, *change*, *connection*, and *risk*, all of which can be expressed in the language of mathematics.
- **This book** introduces some of the mathematical ideas that are helpful in making decisions that involve sustainability.



## 0.2 To the Instructor

*If you want to make a course interesting, then you should study something of interest [356].*

This text supports a course that is aimed at college students—many thousands of them, in our large universities—who would not describe themselves as “mathematicians” or “scientists” but who need to take at least one course that supports quantitative literacy as part of their degree requirements. Often such students have found themselves steered into courses in the precalculus sequence: courses that may be excellent preparation for future scientists and engineers, but that fail to catch a student’s attention as their last experience of mathematics. One of us sometimes asks such students, “Would you rather learn the quadratic formula or would you rather save the world?” This book is for those students who would like to save the world, or at least take a step in that direction. Similarly, it is for those instructors who would like to “teach as if life matters” [328], or at least take a step toward using the mathematics classroom to help students think more clearly about some of the issues that are only going to become of increasing importance over the twenty-first century.

Each instructor will, of course, use this book in the way that they see fit. However, one of our primary goals is to advance student skill in quantitative literacy. A **required** student writing component has played an essential role in accomplishing this goal and has accounted for over one-third of the total grade in the course as we have taught it. The Association of American Colleges and Universities provides the following definition for quantitative literacy [249]:

Quantitative Literacy (QL) is a “habit of mind,” competency, and comfort working with numerical data. Individuals with strong QL-skills possess the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats.

Because of its intended audience, the book does not require any mathematics beyond high school algebra; the most complicated idea that appears is a fourth root, which shows up in a few places in Section 2.4. In particular, *no calculus* is required to read and study this book. If the instructor does know calculus, though, they will find that it provides background to the chapters on “Flowing” and “Changing,” and may even find it helpful—for themselves, not for the students—to investigate how our presentation can be translated into the classical language of differential equations.

Nor does the book require (from you, the instructor) a great deal of sustainability-specific mathematical background. Naturally, the more you know, the more you will be able to help your students, but in the end what we’re presenting in Part I of the book is a self-contained set of mathematical techniques, and if you learn about them from the text, that will be fine. Should you wish to pursue the material further, the reading suggested in Section 8.1 gives some possible starting points.

The book’s website is at <http://math-for-sustainability.com>. As well as the student-oriented material described in the previous section, this website contains additional resources and suggestions specifically for instructors. These include a quite specific description of the writing component of the course as we have taught it. You’re welcome to use this model exactly as it is, to adapt it, or to do something entirely different. We do believe, however, that this book will be most effective if it is used in conjunction with a requirement for some student response in the form of extended writing. Our experience suggests that most students are glad to have the opportunity to integrate their mathematical learning with social and environmental concerns and to express themselves in this way.

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