

2 ELECTRIC STRESSES

Electric stresses must always be considered if electric field strengths are in the range of the electric strengths of the insulating materials, i.e. calculation of electric field intensities is of fundamental importance.

The most important equations for the description of electric fields in high voltage engineering are summarized in *Section 2.1* [2], [3]. *Section 2.2* describes how different stresses (e.g. by AC, DC and impulse voltages) result in the development of different fields. Basic arrangements can be treated by analytical calculations (*Section 2.3* and *2.4*), but complex insulation systems normally require numerical calculation [4] (*Section 2.5*). Traveling waves need special consideration because of their character as fast changing electromagnetic processes [5] (*Section 2.6*).

2.1 Basic Field Theory

Electric fields cannot be sensed by human beings directly. Fields can only be noticed indirectly by their physical effects. *The electric field describes a physical condition of space.* The electric field strength is defined by its mechanical *force* on electric charges (or charged test bodies).

There are two sources of electric fields:

- Positive and negative electric *charges* are sources and sinks (i.e. starting and ending points) of the field (“*electrostatic field*”), Figure 2.1-1.
- Furthermore, there are electric fields which are induced by *time-varying magnetic fields*. Magnetic field lines can be regarded as *curl lines* of closed field line loops (“*curl field*”), Figure 2.1-2.

Electric charges cannot be divided infinitely. The smallest quantity of charge is the *elementary charge* $e = 1.6022 \cdot 10^{-19}$ As.

Figure 2.1-1:

Electric source field

(irrotational electrostatic field with charges as sources and sinks of the field).

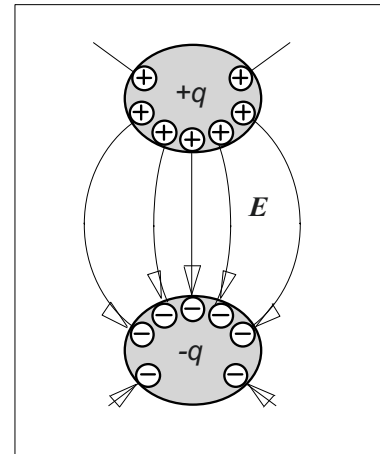
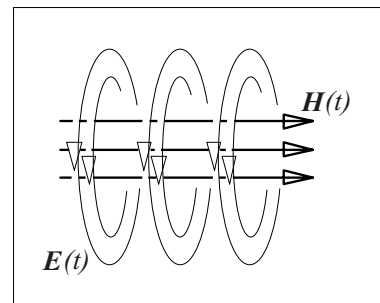


Figure 2.1-2:

Electric curl field

(induced electric field, electromagnetic field).



Charges can be distributed differently,

- as a *single charge* or *point charge* (e.g. electrons with the charge $q = -e$ or protons with $q = +e$),
- as a *line charge* (e.g. on a wire with a negligibly small diameter),
- as a *surface charge* (e.g. on the surface of a conductive electrode, Figure 2.1-1) and
- as a *space charge* (e.g. as a “space charge cloud” in a gas discharge).

Point charges and line charges are idealizations which are useful for field calculations.

The quantities *potential*, *voltage*, *current* and *capacitance* are integral quantities, which have to be derived from the actual field quantities (Section 2.1.2). Although these integral quantities are used very often, it must not be forgotten that they only reflect partial aspects of the field properties. Therefore, exact knowledge and calculation of the electric field is very important for high voltage engineering.

2.1.1 Field Quantities

The **electric field strength** E is defined by means of the mechanical force F on a positive test charge q^+ , Figure 2.1-3:

$$E = F/q^+ \quad \text{or} \quad F = q^+ \cdot E \quad (2.1-1)$$

Electric field strength E and force F are **vector quantities**, which are printed here in ***bold and italic*** letters. The direction of the field vector E is identical with the direction of force F on a positive test charge. These directions are visualized by **field lines** in a field plot. In the case of a negative test charge, the directions of field and force vectors are anti-parallel.

According to Eq. (2.1-1) the magnitude $E = F/q^+$ of the field strength E is calculated from the magnitude F of force F . The density of field lines corresponds to the field strength.

Analogy to the gravitation field: The force on charges in the electric field is analogous to the force on masses in the gravity field g . In the field of the Earth's acceleration g , the force on a test body of mass m is $F = m \cdot g$, parallel to the direction of the gravity field.

In an electrostatic field positive and negative charges are the origin (source) of the electric field. Therefore a field quantity D is defined which is directly related to the field-generating charges: The **dielectric displacement density** (*electric flux density*) D is proportional to the electric field strength:

$$D = \epsilon_0 \epsilon_r E \quad (2.1-2)$$

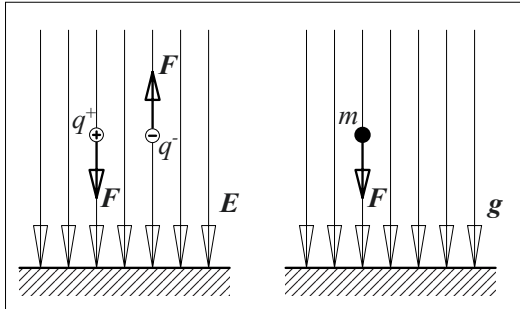


Figure 2.1-3: Forces on test bodies in the electric field (left) and in the gravity field (right).

The magnitude of D corresponds to the surface charge density σ on an ideal conductive electrode, see also Figure 2.1-1:

$$D = \sigma = dq/dA \quad (2.1-3)$$

Therefore the dimension of D is the dimension of a charge divided by the area, i.e. $[D] = \text{As/m}^2 = \text{C/m}^2$.

According to Eq. (2.1-2) the relation between D and E is given by a fundamental physical constant, the **electric constant**, i.e. the (absolute) **permittivity of vacuum**,

$$\epsilon_0 = 8.8542 \text{ pF/m.}$$

Additionally, Eq. (2.1-2) contains the **relative permittivity** ϵ_r , which has no dimension and which depends on the properties of the dielectric material.

The relative permittivity ϵ_r is always greater than 1 because the electric field polarizes available charges within the dielectric material and thereby generates a reverse field. This means that the field strength E for a given charge density (or for a given electric flux density D) is lower than in vacuum with $\epsilon_r = 1$. This effect of *polarization* is described by a factor $\epsilon_r > 1$, Section 4.2.

Technical insulating materials always have a small (residual) **conductivity** κ . The forces of the electric field can therefore accelerate mobile charge carriers, and a drift current proportional to the field strength arises. The **current density** is

$$J = \kappa \cdot E \quad (2.1-4)$$

The field of the current density J is called an **electrical conduction field**; in high voltage engineering it is especially important for DC stresses. The field of the electric flux density D , the so called **dielectric displacement field** is usually dominant in case of AC stresses and always dominant in case of impulse voltages. For relatively slowly changing (quasi-static) processes, induced field components can often be neglected. Rapidly changing fields, e.g. in the case of the skin effect, eddy currents and electromagnetic wave propagation, must be described as **curl fields** or **electromagnetic fields**.

2.1.2 Equipotential Lines, Potential, Voltage and Capacitance

A charge q , which is moved against the force F of the electric field E , has a potential energy W_{pot} , analogous to the potential energy of a mass m in the gravitation field g , Figure 2.1-4. We refer to potential φ based on the value of the charge:

$$\varphi = W_{\text{pot}}/q \quad (2.1-5)$$

A surface of equal potential or equal potential energy is described as an **equipotential surface** (or an *equipotential line* in a two-dimensional sectional view. Both are *orthogonal* to the field lines, i.e. a movement of charges on the equipotential plane (*orthogonal* to the field direction) is possible without any force or energy.

Note: The *equipotential surface* is not the surface of a body or the interface between different materials. Here the term *surface* is used in a general *mathematical* sense in order to describe an area.

Electric field plots in high voltage engineering are very often *visualized* by means of equipotential lines. Their course can be approximated analytically in some simple cases. Numerical solutions also normally consist of potential values which are used for the interpolation of equipotential lines.

Potential and potential energy must always be related to a surface area with $W_{\text{pot}} = 0$ and $\varphi = 0$. This *reference surface* can be chosen freely.

Therefore potential is not an absolute quantity; only potential differences $\Delta\varphi$ and differences of potential energy ΔW_{pot} can be defined. A potential difference is very often referred to as a **voltage** V or *voltage difference* ΔV :

$$\Delta\varphi_{21} = V_{21} = \Delta V_{21} \quad (2.1-6)$$

For a given electric field E the voltage or the potential difference between point 2 and 1 can be calculated by integration. Thereby the difference of the potential energies is described as a line integral of the field force along the integration path. According to Eq. (2.1-1) F is replaced by $q \cdot E$:

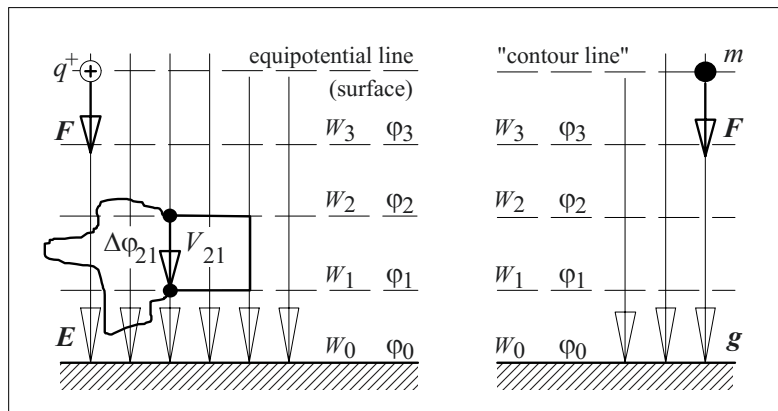
$$\begin{aligned} V_{21} = \Delta\varphi_{21} = \varphi_2 - \varphi_1 &= \frac{1}{q} \Delta W_{21} \\ &= \frac{1}{q} \int_2^1 F \, dx \\ &= \frac{1}{q} \int_2^1 q E \, dx \end{aligned}$$

$$V_{21} = \Delta\varphi_{21} = \varphi_2 - \varphi_1 = \int_2^1 E \, dx \quad (2.1-7)$$

Thus, the voltage or the potential difference between two points 2 and 1 is given by the line integral of the electric field strength E along the path x .

In an *electrostatic field* the result of the integration is independent of the integration path. If different paths are chosen, the result is always the same; it depends only on the potential difference between the starting and

Figure 2.1-4: Potential energy, potential and voltage in the electric field (left) and in the gravitation field (right).



end points, Figure 2.1-4. Therefore it is also referred to as a so-called *potential field*. According to Eq. (2.1-7) and (2.1-8) the field strength is the negative gradient of the potential $E = -\text{grad } \varphi$, it is therefore referred to as a *gradient field*. Sometimes the field is called *irrotational*, because there are *no* time-varying magnetic field lines which can be regarded as curl lines of an induced electric field.

For a *curl field* or an *electromagnetic field*, which is not irrotational, the result of the integration according to Eq. (2.1-7) would depend on the definition of the integration path. For example, the integration along a field line in Figure 2.1-2 would give a finite value, different from zero, even if starting and end point were identical. The definition of a *scalar potential* φ , a potential difference $\Delta\varphi$ and a voltage is **no longer possible** for the curl field or the electromagnetic field. The definition of a *vector potential* shall not be considered further here [2], [3].

If the spatial distribution of a static electric field $E(x) = E(x,y,z)$ is known, the potential distribution $\varphi(x,y,z)$ can be determined from Eq. (2.1-7). Conversely, if the potential distribution is given (*scalar field*), the *electric field strength* can be determined by forming the *gradient*, i.e. by differentiation

$$E = -\text{grad } \varphi. \quad (2.1-8)$$

For Cartesian coordinates x, y, z we find

$$\begin{aligned} E(x,y,z) &= \{E_x, E_y, E_z\} \\ &= -\text{grad } \varphi \\ &= -\{\partial\varphi/\partial x, \partial\varphi/\partial y, \partial\varphi/\partial z\}. \end{aligned} \quad (2.1-8a)$$

For *cylindrical coordinates* r, α, z , the field strength vector E can be described as

$$\begin{aligned} E(r,\alpha,z) &= \{E_r, E_\alpha, E_z\} \\ &= -\text{grad } \varphi \\ &= -\{\partial\varphi/\partial r, r^{-1} \cdot \partial\varphi/\partial \alpha, \partial\varphi/\partial z\}. \end{aligned} \quad (2.1-8b)$$

For *spherical coordinates* r, α, ϑ we find

$$\begin{aligned} E(r,\alpha,\vartheta) &= \{E_r, E_\alpha, E_\vartheta\} \\ &= -\text{grad } \varphi \\ &= -\{\partial\varphi/\partial r, (r \cdot \sin \vartheta)^{-1} \cdot \partial\varphi/\partial \alpha, \\ &\quad r^{-1} \cdot \partial\varphi/\partial \vartheta\}. \end{aligned} \quad (2.1-8c)$$

By specifying a voltage (or a potential difference) the energy that is accumulated during

acceleration of a charged particle in the *electric field* can be directly calculated. This is important for the description of ionization and discharge processes. The kinetic energy results from the difference of potential energies according to Eq. (2.1-5):

$$\begin{aligned} W_{\text{kin}} &= W_{\text{pot2}} - W_{\text{pot1}} \\ &= q(\varphi_2 - \varphi_1) = q \Delta\varphi_{21} \\ &= q V_{21}. \end{aligned} \quad (2.1-9)$$

The electrostatic field is caused by charges on the electrode surfaces in accordance with Figure 2.1-1, i.e. the electrode configuration stores a distinct amount of charge at a given potential difference (voltage). The charge storage capacity is defined by the quotient of the charge and the voltage, which is called “**capacitance**” C :

$$C = q / V = q / \Delta\varphi \quad (2.1-10)$$

In many cases a wide-ranging field configuration can be replaced by a lumped circuit element (i.e. by a capacitance C), Figure 2.1-5.

The use of lumped capacitances as equivalent circuits of distributed fields enables their inclusion in network calculations. This is especially important for the estimation of (parasitic) *stray capacitances* (*air capacitances*) in high voltage measuring circuits or in complex insulation systems.

Furthermore, the whole *capacitively stored energy* W in the field volume can be calculated as a function of the voltage by means of the capacitance C :

$$W = \frac{1}{2} C V^2 \quad (2.1-11)$$

This relation can be understood from the example of a *parallel-plate capacitor* (see Figure 2.1-5 right). For a surface area A , an electrode distance x and a homogeneous electric field strength $E = V/x$, the capacitance is

$$\begin{aligned} C &= q / V \\ &= (D \cdot A) / (E \cdot x) \\ &= (\varepsilon_0 \varepsilon_r E \cdot A) / (E \cdot x) \end{aligned}$$

$$C = \varepsilon_0 \varepsilon_r A / x.$$

I.e.

$$C = \varepsilon A / x \quad (2.1-12)$$

The *stored energy* W is calculated, if the build-up of field strength E is described by the transport of infinitesimal amounts of charge dq against the field force $dF = E dq$. The energy $dW = x dF = x E dq = V dq$, which is necessary for that purpose, is stored in the electric field (as *potential energy* of the charge dq). The integration of all charges gives the total amount of energy:

$$\begin{aligned} W &= \int dW = \int V(q) dq \\ &= \int_0^q (q/C) dq \\ &= \frac{1}{2} q^2 / C \\ &= \frac{1}{2} C V^2 \quad \text{q.e.d.} \end{aligned}$$

The *volume density of the energy* w in the homogeneous (uniform) field of the parallel-plate capacitor is derived by the division of energy W by the volume $v = A \cdot x$. For the homogeneous field w is independent of the position:

$$\begin{aligned} w &= W/v \\ &= [\frac{1}{2} (E x)^2 \varepsilon A/x] / (A x) \\ &= \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 \end{aligned}$$

Within an infinitesimal small volume element ΔV the field can be regarded as homogeneous for any field configuration. Therefore the **volume density of energy** is always given by

$$\begin{aligned} w &= \partial W / \partial V \\ &= \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 \\ &= \frac{1}{2} E D. \end{aligned} \quad (2.1-13)$$

This means that the energy density increases quadratically with the field strength. Therefore, E has to be as high as possible in *energy storage capacitors*, and the maximum volume density of energy is essentially determined by the electrical breakdown strength of the dielectric material.

2.1.3 Maxwell's Equations

Classic problems of high voltage engineering are mostly limited to static, stationary and quasi-stationary electric fields, e.g. for DC, power frequency AC and impulse stresses.

Nevertheless, high electrical stresses can occur for all kinds of fields, as described in Chapter 1, and high voltage engineering has to deal not only with conventional DC, AC and impulse fields, but also with very rapidly changing fields.

Therefore, the **Maxwell's Equations** for *stationary* (not moving) *bodies* are the starting point of this description, from which the respective valid simplifications are deduced. For clarity, this description is limited to the *integral form* of the field equations, Figures 2.1-6 ff.

Maxwell's Equations are subdivided into three categories,

- the main **Field Equations** (the first and second circular law, i.e. Ampere's and Faraday's law resp.), which describe the relation between time-varying electric and magnetic field quantities (Figure 2.1-6),
- the **Continuity Equations**, which describe the sources or the lack of sources of the field quantities (i.e. the continuity of magnetic flux and current, Figure 2.1-7) and

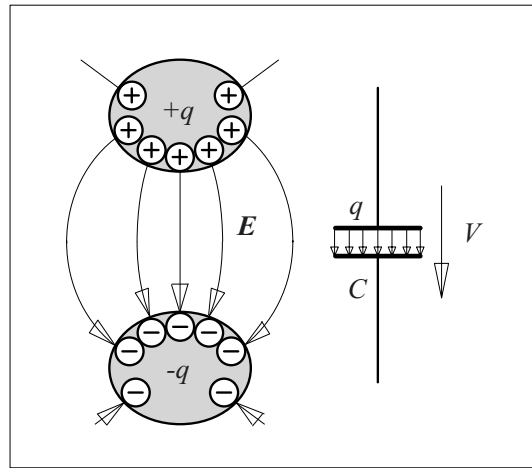


Figure 2.1-5: Allocation of an ideal capacitance to the electrostatic field between two electrodes.

- the **Material Equations** (constitutive relations), which describe the relation between field quantities under the influence of different material properties (Figure 2.1-8).

These equations can be evaluated analytically for defined special cases. For this purpose, it is necessary to find simplifications, which result from spatial symmetries (e.g. fields with plane, cylindrical or spherical symmetry) and particular time dependencies (e.g. static DC fields or harmonic fields with sinusoidal time dependence).

2.1.3.1 Maxwell's Main Field Equations

The *physical meaning* of the main field equations consists in the insight that a time-varying magnetic flux $\iint \mathbf{B} \, d\mathbf{A}$ induces an electric curl field $\mathbf{E}$ (**law of induction, Faraday's law, second circular law**), Eq. (2.1-14). The induced voltage (electromotive force) is the integral of $\mathbf{E}$ around a closed loop. It is equivalent to the derivative of the magnetic flux (through the loop) with respect to time. Furthermore, a magnetic curl field $\mathbf{H}$ is caused by an electric current, i.e. by a "flux of a current density" (**law of the magnetomotive force, Ampere's law, first circular law**), Eq. (2.1-15). The current or the magnetomotive force is calculated both from the conduction current density $\mathbf{J}$ (moving charge carriers) and/or from the displacement current density $\partial\mathbf{D}/\partial t$ (time-varying electric field).

Finally, Maxwell's Main Field Equations describe the generation of an electric (or magnetic) curl field from a time-varying magnetic (or electric) field. This mutual interdependence is the reason for the propagation of electromagnetic waves, which occur, for example, as line-bound waves on transmission lines or measuring cables in high voltage engineering.

2.1.3.2 Maxwell's Continuity Equations

The *physical content* of the Continuity Equations consists of a statement about the continuity and the sources of magnetic and electric fields, Figure 2.1-7.

Magnetic fields are solenoidal, i.e. free of sources. If a closed surface area A is considered, there are no sources or sinks of magnetic field lines within the enclosed volume, i.e. the magnetic flux $\iint \mathbf{B} \, d\mathbf{A}$, which enters the volume on one side, must leave it on the other side, Figure 2.1-7 (left). A mathematical description of the solenoidal character of the magnetic field is given, if the closed surface integral $\oint \mathbf{B} \, d\mathbf{A}$ is set equal to zero, because of the compensation of incoming and outgoing magnetic fluxes, Eq. (2.1-16).

The continuity equation for the **field of the displacement density** Eq. (2.1-17a) states that the displacement density field $\mathbf{D}$ is free of sources, if there are no charges within an closed surface A ($Q = 0$). These charges would act as sources and drains of the field and the closed-surface integral would give a magnitude different from zero. Calculating the dielectric displacement flux $\iint \mathbf{D} \, d\mathbf{A}$ over a closed surface A , the enclosed charge Q must be introduced a term which is different from zero.

Additionally, the **electric current density** is free of sources, if the sum of conduction current density $\mathbf{J}$ and displacement current density $\partial\mathbf{D}/\partial t$ is considered, Eq. (2.1-17b). This equation can directly derived from Eq. (2.1-15).

Note: A relation between Eq. (2.1-17b) and (2.1-17a) can be established in the following way: A time-varying conduction current $i(t) = \iint \mathbf{J} \, d\mathbf{A}$, which is flowing via a conductor towards an electrode, will be continued as a displacement current $\iint \partial\mathbf{D}/\partial t \, d\mathbf{A}$ within the non-conductive dielectric material adjacent to the electrode, Figure 2.1-7 (right). The conduction current is associated with a displacement and an accumulation of charge carriers close to the surface of the electrode, i.e. at the interface between conductive and insulating materials.

The integration of Eq. (2.1-17b) over time provides a relation between the displacement density $\mathbf{D}$ and the charge, which is enclosed by the closed surface A if it is assumed that there is no charge on the electrode at the beginning of the integration process:

$$\oint_A \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \, d\mathbf{A} = 0$$

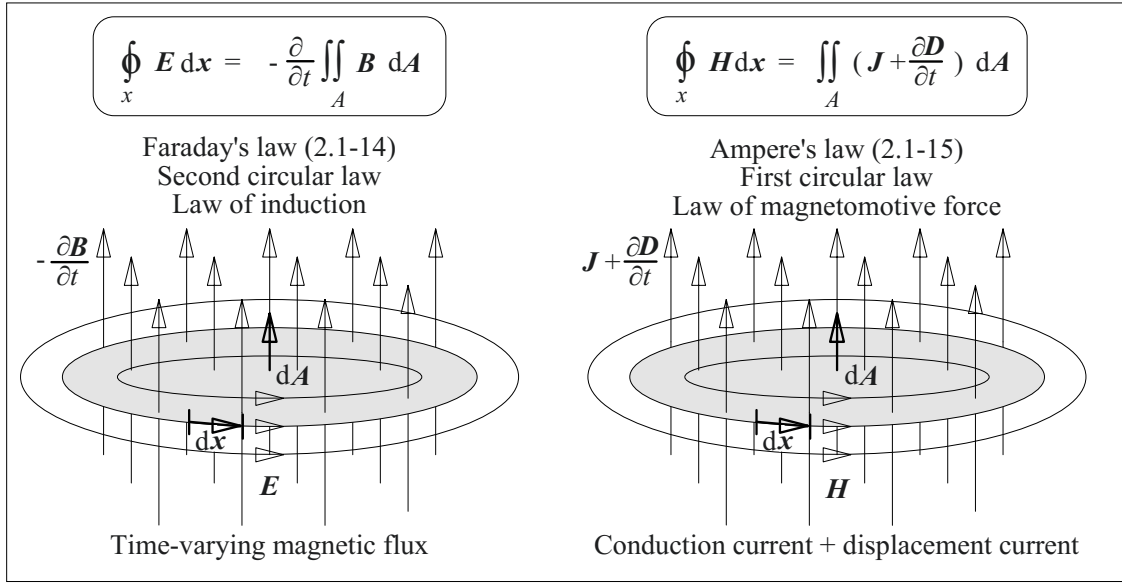


Figure 2.1-6: Integral form of Maxwell's main field equations for stationary (not moving) bodies. Coupling of electric and magnetic field quantities by means of Faraday's law of induction (left) and by Ampere's law of magnetomotive force (right).

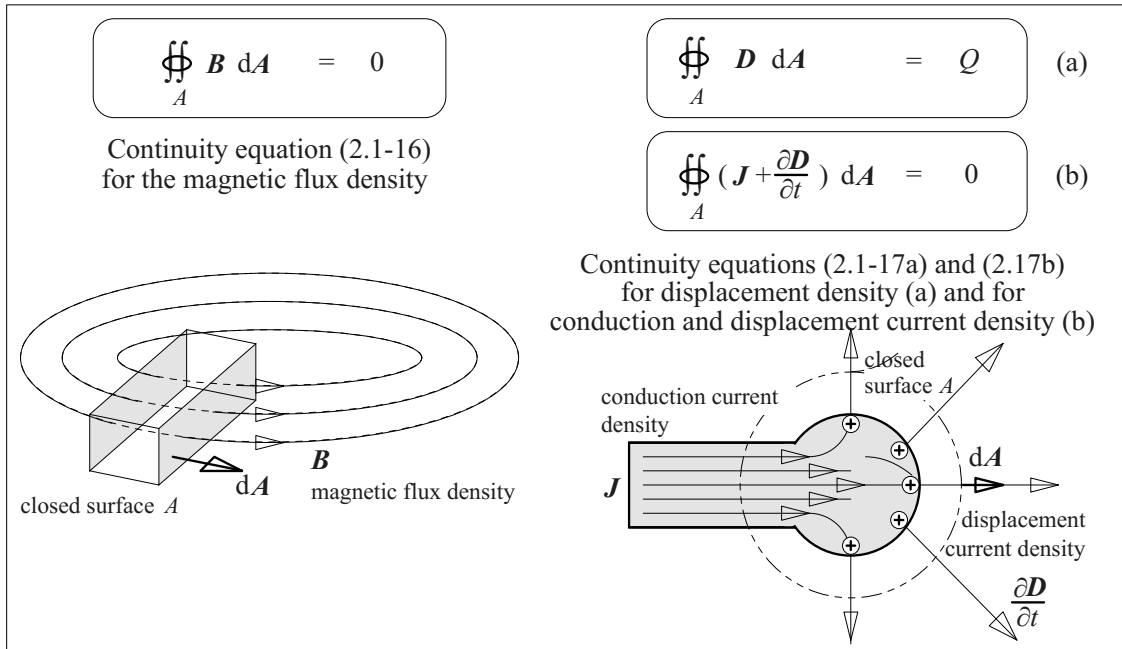


Figure 2.1-7: Integral form of Maxwell's continuity equations for the magnetic flux density (left, spatial view of a closed surface area A) and for conduction and displacement current density (right, sectional view).

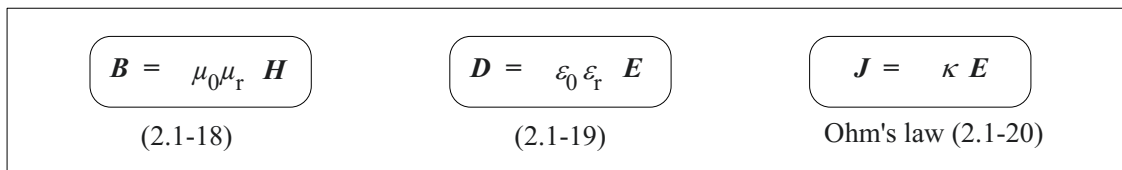


Figure 2.1-8: Material equations (constitutive relations) for the magnetic and electric field quantities.

$$\oint_A \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{A} = - \oint_A \mathbf{J} \cdot d\mathbf{A} = i(t)$$

The integration over the time gives Eq. (2.1-17a):

$$\oint_A \mathbf{D} \cdot d\mathbf{A} = \int i(t) dt = Q.$$

Nevertheless, it must be noted that this derivation required additional assumptions. Therefore the two equations are not equivalent.

Eq. (2.1-17a) plays an important role for field calculation purposes:

$$\oint_A \mathbf{D} \cdot d\mathbf{A} = Q \quad (2.1-21)$$

The charge can also be seen as the integral of the space charge density η over the volume V , which is enclosed by the closed surface area A :

$$\oint_A \mathbf{D} \cdot d\mathbf{A} = \iiint_V \eta \cdot dV \quad (2.1-22)$$

Eq. (2.1-21) or (2.1-22) is called “**Gauss’s law**”. It states that the integral of the flux density $\mathbf{D}$ over any closed surface equals the charge enclosed [481]. It allows the analytical calculation of electrostatic fields in some important cases, Figure 2.1-9.

2.1.3.3 Material Equations

The material equations describe the interaction of electric and magnetic fields with materials, Figure 2.1-8. They constitute the relations between $\mathbf{E}$ and $\mathbf{D}$, $\mathbf{B}$ and $\mathbf{H}$, and $\mathbf{E}$ and $\mathbf{J}$, therefore they are also called **constitutive relations**.

A magnetic field $\mathbf{B}$ can cause an orientation of magnetic dipoles (“elementary magnets”) within a material. This effect of *magnetic polarization* causes an additional field, which enhances or reduces the resulting field strength. Eq. (2.1-18) accounts for the magnetic polarization by the factor μ_r (relative permeability).

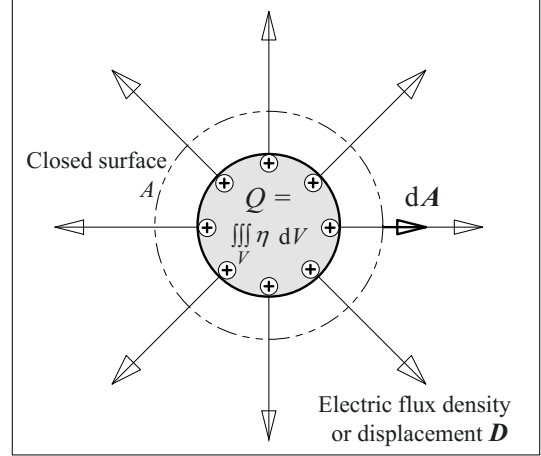


Figure 2.1-9: Charges as sources of the electric flux density (displacement $\mathbf{D}$), i.e. as reason of the static electric field (“Gauss’s law”).

An electric field $\mathbf{E}$ can cause a displacement of charges or an orientation of electric dipoles within a material. This effect of this *electric polarization* causes an additional field, see Section 4.2. If the amount of charge on the electrodes is constant, the resulting electric field strength is reduced. If the field strength is imposed by an impressed voltage or field strength, additional charges are accumulated on the electrodes by the polarization effect. Eq. (2.1-19) accounts for the increase in electric displacement density (proportional to the charge) via the dielectric polarization by the factor ϵ_r (relative permittivity). ϵ_r is always greater than 1, because each type of matter can be polarized to a greater or lesser extent, e.g. by displacement of lattice atoms, by orientation of polar molecules or molecule groups, or by displacement of atomic nuclei relative to their electron shells.

Electric polarization is of high importance for high voltage engineering, since it determines *relative permittivities* and *capacitances*. Additionally there are *polarization losses* (*dissipation losses*) depending on temperature and frequency.

Eq. (2.1-19) is the basis of all high voltage field calculations for insulating systems which consist of more than one insulating material

and which are stressed with *time-varying electric fields*.

Moreover, an electric field E can accelerate free and mobile charge carriers within a material. Because of collision processes there is a mean drift velocity of the charge carriers in the direction of the electric field, described by a current density J proportional to the electric field strength E . Eq. (2.1-20) correlates the electric current density with the electric field strength by the factor κ (electric conductivity); it is equivalent to Ohm's law.

Eq. (2.1-20) is the basis of all high voltage field calculations for insulating systems, which are stressed with *direct voltage (DC voltage)*.

2.1.4 Classification of Fields

Depending on the gradient of field changes $\partial \dots / \partial t$ (i.e. the time rate of change), Maxwell's Equations can be simplified. Three categories have proved to be useful [394]:

1. Static and stationary (steady-state) fields (Section 2.1.4.1): For **static fields** there are no changes of the field quantities E , H and J , i.e. the derivatives with respect to time are zero, $\partial \dots / \partial t = 0$.

Concerning the time rate of change, field quantities are time-invariant, constant or "**static**", i.e. they do not change in time. There is absolutely no coupling of magnetic and electric field quantities by Faraday's law of induction (2.1-14) or by Ampere's law (2.1-15) regarding the displacement current density $\partial D / \partial t$. Consequently there is no propagation of electromagnetic waves. *Concerning the propagation of waves*, static fields are "**stationary**", i.e. they are fixed to a location and do not propagate.

Note: Traditionally the time-invariant E - and H -fields are referred to as **electrostatic** and **magnetostatic fields**. The time-invariant electrical conduction field J is mostly referred to as a **steady-state (stationary) conduction field**. This is motivated by the conduction current,

which is related to energy transport, the *power* as derivative of the energy with respect to time is therefore not equal to zero [395]. The field of the conduction current density J is caused by the *motion* of charges, and it is no longer *static* in terms of being *immobile*. Furthermore, there is coupling of J with the magnetic field H as described in Ampere's law of electromotive force (2.1-15).

Nevertheless, *concerning the field quantities*, all three kinds of fields (E , H and J) are both time-invariant (*static*) and fixed to a location (*stationary*) [394]. Usually the terms "**static fields**" and "**steady-state conditions**" are used.

2. Quasi-static (quasi-stationary) fields (Section 2.1.4.2 and 2.1.4.3): For **quasi-static** (quasi-stationary) fields there are (slowly) time-varying field quantities, but their coupling is so weak that the *electric curl field induced by the magnetic field of the displacement current* and the **wave character** of the fields can be **neglected**.

Note: If this is interpreted in the **time domain**, it means that the *wave propagation time* τ within a field volume with the dimension x has to be small in comparison with the rise time T_r of the time-varying field, see Eq. (2.1-36ff): $\tau \ll T_r$. In a *limited volume* with dimension x the change of field strength with time has almost no influence on field distribution; the field can therefore be regarded as "**quasi-static**" (similar to the static field distribution).

In the **frequency domain** the *dimension* x of the field volume must be small in comparison with a *quarter-wavelength* $\lambda/4$, see Eq. (2.1-36) and (-37): $x \ll \lambda/4$. In a *limited volume* with dimension x the change of field strength with time does not result in remarkable wave propagation effects, field changes can be regarded as quasi-simultaneous and the field can therefore be regarded as "**quasi-stationary**" (approx. fixed to the location).

Typically, quasi-static (quasi-stationary) fields are **slowly changing fields** (or low frequency fields) (Section 2.1.4.2 and 3)

- either *in conductors*, where the displacement current is negligible in comparison with a conduction current (*inductive field*),
- or *in insulating materials*, where the induced curl field is negligible in comparison with an electrostatic field (*capacitive field*).

Additionally, there are fast **changing quasi-stationary fields in conductors**, if $x \ll \lambda/4$. As long as the displacement current is negligible, the electromagnetic wave propagation orthogonal to the conductor surface is negligible because of extreme damping. Nevertheless, these fields are characterized by the superposition of the impressed electric conduction field and an induced electric curl field. This results in *eddy currents* that are superimposed to the impressed conduction current. Thereby they cause *current displacement (skin effect, Heaviside effect)*.

3. Non-stationary fields (Electromagnetic Waves, Section 2.1.4.4): Generally, for rapidly varying fields the mutual coupling of electric and magnetic fields must not be neglected, because of a high *rate of change* and/or the *dimensions* of the field volume. Then the characteristics of **electromagnetic wave propagation**, the spatial dimensions and propagation time effects must be considered.

2.1.4.1 Static and Stationary Fields

a) Electrostatic and magnetostatic fields

For static fields the right side of Eq. (2.1-14) and (2.1-15) is equal to zero, since there is neither a time-varying magnetic flux nor a dis-

placement current, nor a conduction current. Thus, there is no coupling between electric and magnetic field quantities.

Strictly speaking, static fields only exist as *magnetic* fields of permanent magnets. Static *electric* fields are a *theoretical fiction*, assuming that immobile charges are sources of a static electric field without causing conduction current, charge transport or energy transport. This would only be possible for a *perfect* dielectric ($\kappa = 0$) that does not exist in reality.

Nevertheless, a **static electric field** is often assumed in order to approximate the field and to *simplify field calculations*: A static field is only dependent on the permittivities of dielectric materials. It should be noted that the calculation results are not valid for static electric fields and stationary conduction fields, for which the conductivities would be significant, but only for quasi-static (quasi-stationary) cases, Section 2.1.4.3.

b) Stationary (steady-state) conduction fields

Conduction currents are accompanied by energy transport and by a stationary (steady-state) power flow different from zero, Eq. 2.1-24. In comparison with electrostatic and magnetostatic fields, **steady-state conduction fields** are characterized by a time-invariant conduction current density $\mathbf{J}$ and a proportional time-invariant electric field strength $\mathbf{E}$, Figure 2.1-10.

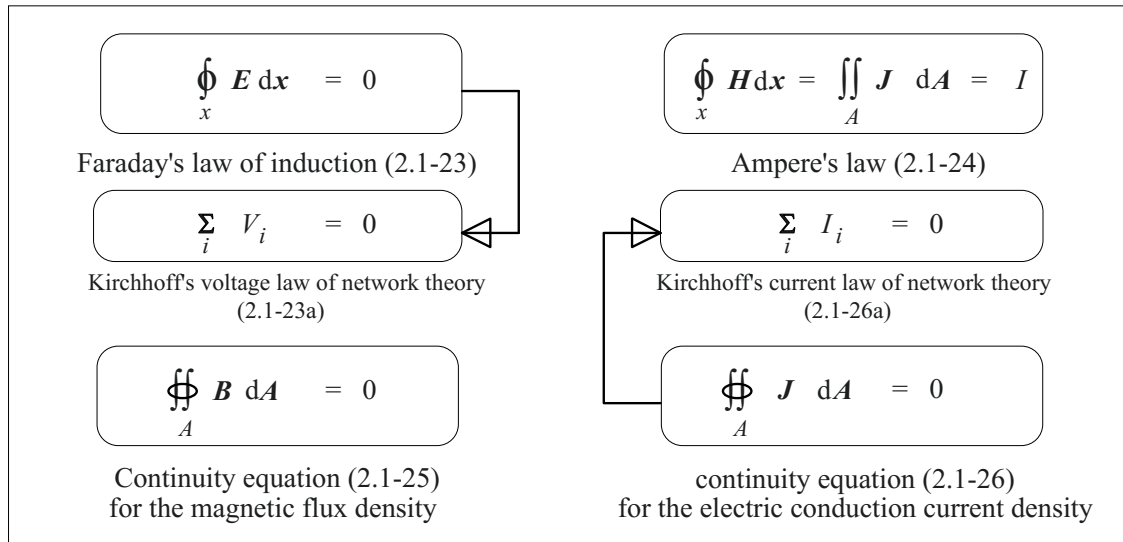
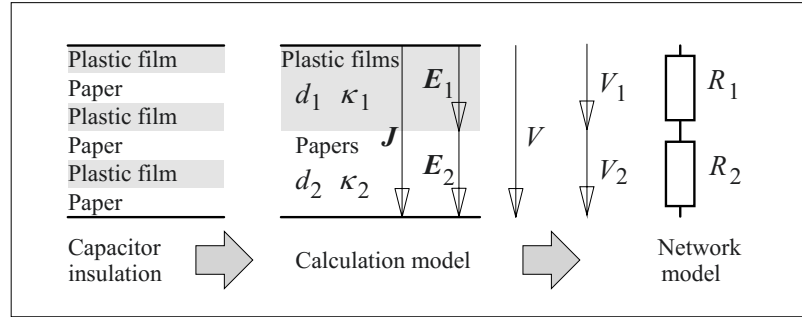


Figure 2.1-10: Simplification of Maxwell's equations for stationary fields (all derivatives of field quantities with respect to time are equal to zero).

Figure 2.1-11:

Stationary (steady-state) conduction field in a capacitor insulation stressed with constant DC voltage.



According to Eq. (2.1-23) the ring integral of $E \, dx$ around a closed loop and the sum of the voltages V_i in a closed loop are zero, therefore it is an irrotational field. This means that the calculation of voltages and potential differences between two points with Eq. (2.1-7) is independent of the integration path, Figure 2.1-4. Therefore, it is a so called “*potential field*” with unequivocally defined voltages and potential differences.

The electric field under electrical stress with constant **direct voltage** (DC voltage) is always a *stationary (steady-state) conduction field*. The field distribution does not change with time, but owing to the (residual) *conductivity* κ according to Eq. (2.1-20) there is a conduction current $J = \kappa \cdot E$, which determines the field distribution. The *permittivities* and Eq. (2.1-19) do *not* have *any significance* for the formation of a steady conduction field.

Example: Capacitor dielectric

A constant DC voltage ($U = 3 \, \text{kV}$) is applied to the dielectric of a capacitor for a very long time. The insulation is arranged in layers of polymeric films ($d_1 = 30 \, \mu\text{m}$, $\kappa_1 = 10^{-16} \, \text{S/m}$) and oil-impregnated paper ($d_2 = 30 \, \mu\text{m}$, $\kappa_2 = 10^{-14} \, \text{S/m}$). The field stress, i.e. the field strength, shall be calculated for the materials, Figure 2.1-11.

In a calculation model, two equivalent layers replace all films and papers. The total voltage is calculated with Eq. (2.1-7) from the sum of the two partial voltages on the polymeric films and the impregnated papers:

$$V = d_1 E_1 + d_2 E_2.$$

Furthermore, the current density is equal in both materials according to the Continuity Equation (2.1-26):

$$J = \kappa_1 E_1 = \kappa_2 E_2$$

Field strengths are calculated as

$$E_1 = V / (d_1 + d_2 \cdot \kappa_1 / \kappa_2) = 99 \, \text{kV/mm}$$

and

$$E_2 = E_1 \cdot \kappa_1 / \kappa_2 = 1 \, \text{kV/mm}.$$

Result: $E_1 = 99 \, \text{kV/mm}$ (polymeric films)

$$E_2 = 1 \, \text{kV/mm} \quad (\text{papers})$$

In spite of the same thickness, the papers are stressed with $V_2 = E_2 \cdot d_2 = 0.03 \, \text{kV}$ only, i.e. with approx. 1 % of the total voltage. Because of their low conductivity (their high specific resistance $\rho = 1/\kappa$) the polymeric films are stressed with 99 % of the total voltage.

Note: Paper layers in capacitor dielectrics are used in order to impregnate the gaps between the films with insulating oil, i.e. the paper acts as wick for the impregnation. The insulation strength must be guaranteed by the polymeric films, which normally have a significantly higher breakdown strength than oil-impregnated paper.

2.1.4.2 Quasi-stationary (Inductive) Fields in Conductors

In materials with **high conductivities** (*conductors*), it is possible to neglect displacement current density $\partial D / \partial t$ in comparison to conduction current density J up to the GHz range. Therefore, the field can be regarded as **quasi-stationary** and the electromagnetic wave character is negligible, Eq. (2.1-29) and (-31), Figure 2.1-12. The condition is

$$\partial D / \partial t = \epsilon_0 \epsilon_r \partial E / \partial t \ll J = \kappa E. \quad (2.1-27)$$

The *spatial and temporal limits* for the quasi-stationary regime, which are defined in Eq. (2.1-36) and (-37), are also valid here.

$\oint_x \mathbf{E} \, d\mathbf{x} = - \frac{\partial}{\partial t} \iint_A \mathbf{B} \, d\mathbf{A}$ Faraday's law of induction (2.1-28)	$\oint_x \mathbf{H} \, d\mathbf{x} \cong \iint_A \mathbf{J} \, d\mathbf{A} = I$ Ampere's law (2.1-29)
$\oiint_A \mathbf{B} \, d\mathbf{A} = 0$ Continuity equation (2.1-30) for the magnetic flux density	$\oiint_A \mathbf{J} \, d\mathbf{A} \cong 0$ Continuity equation (2.1-31) for the conduction current density

Figure 2.1-12: Maxwell's equations for quasi-stationary inductive fields in conductors (Displacement current density in conductors is neglected).

Owing to the high electric conductivity κ , even low electric field strengths $\mathbf{E}$ cause high current densities $\mathbf{J}$. Therefore, even low induced field strengths according to Eq. (2.1-28) must be considered. This means that there is a coupling between the electric conduction current field and the magnetic field by Faraday's law of induction.

The electric field is no longer irrotational, i.e. an unequivocal *definition of scalar potentials or voltage differences* according to Eq. (2.1-7) is *not possible*, the line integral $\int \mathbf{E} \cdot d\mathbf{x}$ would depend on the integration path.

Example: Eddy currents and current displacement

The effect of “current displacement” and “eddy currents” in conductors can be explained by the electric curl field $\mathbf{E}(t)$, which is induced by the time-varying magnetic field, causing time-varying current density $\mathbf{J}(t)$. The induced field $\mathbf{E}(t)$ causes an eddy current density $\mathbf{J}_e(t) = \kappa \mathbf{E}(t)$, which enhances the current density at the conductor's surface and reduces it within the conductor (current displacement, skin effect).

Even for AC at power frequency 50 or 60 Hz, current displacement causes an increase in resistance in the range of a few percent for common conductor materials. In the magnetic cores of electrical machines and transformers, there are so-called eddy current losses, which are reduced by thin, mutually insulated core laminations. Since these subjects are not directly related to high voltage engineering, we refer you to more information given in the literature [2].

Even for very **fast changing currents in conductors**, the fields are **quasi-stationary** in the direction orthogo-

nal to the conductor's surface. The displacement current can be neglected in comparison with the conduction current and there is almost no or an extremely strongly damped and spatially limited electromagnetic wave propagation orthogonal to the conductor's surface.

The quasi-stationary inductive field is important for high voltage engineering only in *materials with high conductivities*, e.g. in conducting electrodes, conducting connection lines and transformer windings. In *dielectric materials with low conductivities*, the displacement current density must normally not be neglected in comparison with the very small conduction current density, even at slowly changing fields, Section 2.1.4.3.

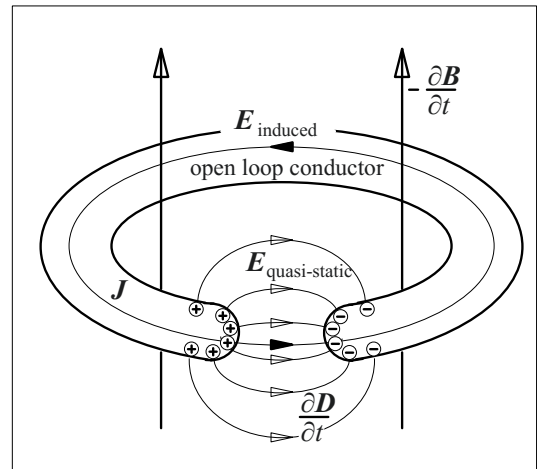


Figure 2.1-13: Quasi-stationary fields within and outside the conductor material (inductive and capacitive fields).

$\oint_x E \, dx \quad \cong \quad 0$ Faraday's law of induction (2.1-32)	$\oint_x H \, dx = \iint_A \left(J + \frac{\partial D}{\partial t} \right) dA$ Ampere's law (2.1-33)
$\oiint_A B \, dA = 0$ Continuity equation (2.1-34) for the magnetic flux density	$\oiint_A \left(J + \frac{\partial D}{\partial t} \right) dA = 0$ Continuity equation (2.1-35) for the conduction and displacement current density

Figure 2.1-14: Maxwell's equations for quasi-stationary/ quasi-static (capacitive) displacement fields in insulating materials (Induction of electric curl fields by magnetic fields is neglected).

Example: Transformer winding

The fields in a transformer winding are determined by the time-varying magnetic field, which induces an electric curl field *within the conductor*. Thereby charges are displaced to the conductor's surface, Figure 2.1-13. The conduction current is much greater than the displacement current, therefore it is a **quasi-stationary inductive field**.

On the *outside of the conductor*, the conduction current is continued in the dielectric (mainly) as a displacement current. The surface charges cause a quasi-static electric field. *Within the dielectric*, the source field is much stronger than the (induced) curl field, which is normally negligible. Then it is a **quasi-static** (quasi-stationary) **capacitive field**.

2.1.4.3 Quasi-stationary/ Quasi-static (Capacitive) Displacement Fields in Dielectrics

High voltage engineering predominantly considers electric fields in *dielectrics*, i.e. in *insulating materials* (so-called “non-conductors”) with a comparatively low (residual) conductivity κ . Eq. (2.1-27) is not fulfilled and displacement currents normally exceed small conduction currents, even at low frequencies. In Ampere's law (2.1-33) and in the Continuity Equation (2.1-35) both current components must be considered, Figure 2.1-14.

The electric field is a source field and exceeds the induced electric field strength if the rate of change of the magnetic field remains low, e.g. see Figure 2.1-13. Then *potential and voltage*

differences can *approximately* be defined according to Eq. (2.1-7) because the induced curl field is neglected relative to the source field, see Eq. (2.1-32). This means that the electromagnetic wave character is neglected and the field is both a **quasi-stationary** and a **quasi-static** field. Sometimes it is referred to (imprecisely) as a “**static electric field**” or an “**electrostatic field**” only.

Power frequency (AC) voltage, switching impulse (SI) voltage and lightning impulse (LI) voltage are the most important voltage stresses of high voltage insulating materials. In most cases, they can be described as quasi-static (quasi-stationary) capacitive fields.

Within a field volume under consideration, a quasi-static (quasi-stationary) description is allowed, if all changes of the fields are *nearly synchronous*, i.e. without any significant delay. This means that traveling wave processes, which have to be considered for fast changing fields, can be neglected. Thereby **limits for the validity of the quasi-static description** are defined **in space and time** as follows:

The *propagation time* $\tau = x/u$ of an electromagnetic wave through the volume under consideration (length x , traveling wave velocity u) must be *negligibly small* in comparison with the time over which the field changes.

This means for *sinusoidally* changing fields with the period T and the frequency f :

$$\tau = x/u \ll T/4 = (4f)^{-1} \quad (2.1-36a)$$

and with the wavelength $\lambda = T \cdot u$

$$x \ll \lambda/4. \quad (2.1-36b)$$

For a *transient* process with a rise time T_r we find accordingly

$$\tau = x/u \ll T_r \quad (2.1-37)$$

With a limit of 0.5 % for the voltage error, the length x of long lines should be

$$x < \lambda/60. \quad (2.1-38)$$

For overhead lines with air insulation, which shall be regarded as quasi-static (quasi-stationary), at $f = 50$ Hz (60 Hz) x must remain < 100 km (< 80 km). For cables with a reduced traveling wave velocity, this limit is further reduced. For a lightning impulse voltage with $T_r \cong 1 \mu s$, the length in air is only $x < 22$ m, if the rise with T_r is approximated by a quarter-period of an AC voltage with $T = 4 \mu s$.

In the common insulating materials displacement currents normally exceed conduction currents for impulse and AC voltages:

$$\partial D / \partial t = \epsilon_0 \epsilon_r \partial E / \partial t \gg J = \kappa E \quad (2.1-39)$$

This is the case of the quasi-static (quasi-stationary) “dielectric **displacement field**”, which is often called “electrostatic field”. Field distributions are determined by the relative permittivities ϵ_r of the different materials, Figure 2.1-15 (top). Simple geometries can be described by a network model with capacitances.

In case of predominant conduction current, the field is a (quasi-stationary) electrical **conduction field**. Field distributions are determined by the conductivities κ , Figure 2.1-15 (bottom). Simple geometries can be described by a network model with resistances.

Example: Capacitor dielectric (continued)

A power frequency AC voltage ($f = 50$ Hz, root-mean-square/ r.m.s. voltage $V = 3$ kV) is applied to a capacitor insulation according to Figure 2.1-11 ($d_1 = d_2 = 30 \mu m$, $\epsilon_{r1} = 2.2$, $\epsilon_{r2} = 4.4$). The electric field strength in the different materials shall be determined, Figure 2.1-15.

The total voltage is calculated from the sum of the two partial voltages on the polymeric films and the impregnated papers according to Eq. (2.1-7):

$$v(t) = d_1 E_1(t) + d_2 E_2(t)$$

From this, the r.m.s. values for V and D follow:

$$V = d_1 E_1 + d_2 E_2.$$

Figure 2.1-15:

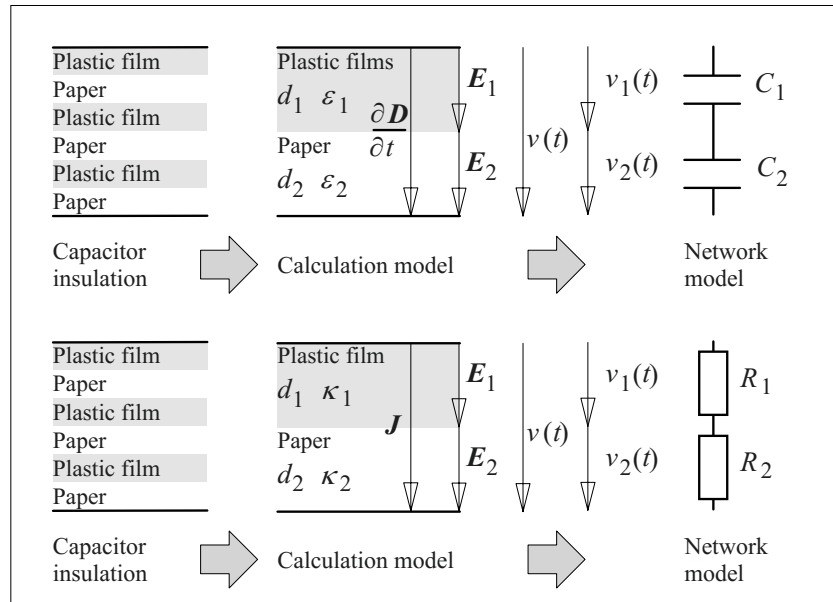
Quasi-static (quasi-stationary) fields in a capacitor insulation stressed with a time-varying voltage $v(t)$.

Top: Displacement current dominates (dielectric displacement field, often denominated as “electrostatic field” or “capacitive field”).

$$\partial D / \partial t \gg J$$

Bottom: Conduction current dominates (Electrical conduction field).

$$J \gg \partial D / \partial t$$



Moreover, the displacement current density according to Eq. (2.1-35) is equal in both materials:

$$\partial D / \partial t = \varepsilon_1 \cdot \partial E_1 / \partial t = \varepsilon_2 \cdot \partial E_2 / \partial t$$

The conduction current density J is neglected. The time integral gives the r.m.s. values

$$D = \varepsilon_1 \cdot E_1 = \varepsilon_2 \cdot E_2.$$

From this, field strengths are calculated from the equation for the r.m.s. voltage V ,

$$E_1 = V / (d_1 + d_2 \cdot \varepsilon_1 / \varepsilon_2) = 67 \text{ kV/mm}$$

and

$$E_2 = E_1 \cdot \varepsilon_1 / \varepsilon_2 = 33 \text{ kV/mm.}$$

Note: Obviously the plastic dielectric with the lower permittivity $\varepsilon_{r1} = 2.2$ is stressed with $E_1 = 67 \text{ kV/mm}$, i.e. twice as strongly as the oil-impregnated papers, the field is displaced into the material with the lower permittivity. This effect of “**field displacement**” is of high importance for high voltage engineering: In the given case, the material with the lower breakdown strength (the paper) is fortunately stressed with the lower field strength. However, very often there are cases where the electric field is “displaced” into the dielectric with the lower strength (e.g. in air bubbles with $\varepsilon_r = 1$).

The question, whether a slowly changing field must be regarded as an electrostatic field or as an electrical conduction field, can be answered by considering the **transition process** between

the initial displacement field and the stationary conduction field, Figure 2.1-16 gives an example. If the time, which is relevant for field variations, is significantly shorter than a time constant τ describing the transient behavior of the insulation system, an **electrostatic field** (dielectric **displacement field**) can be assumed:

$$\text{AC quarter-period } T/4 \ll \tau \quad (2.1-40)$$

$$\text{or rise time } T_r \ll \tau$$

Fields, which vary very slowly in comparison with the transition time constant τ of the dielectric system, can be regarded as **electrical conduction fields**, which are determined by conductivities:

$$\text{AC quarter-period } T/4 \gg \tau \quad (2.1-41)$$

$$\text{or rise time } T_r \gg \tau$$

Example: Capacitor dielectric (continued)

Following the application of a DC voltage, the system needs several hours to approach a steady-state condition, i.e. to charge the capacitance C_1 of the highly resistive polymeric films via the insulating resistance R_2 of the comparatively conductive oil-impregnated papers.

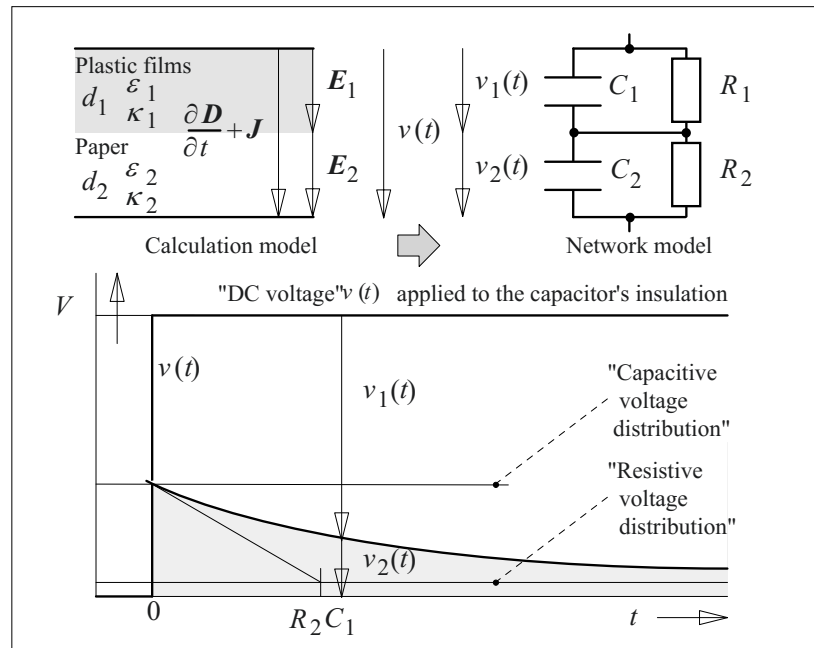


Figure 2.1-16:

Transition from the displacement field (electrostatic field, quasi-static field) with a “capacitive voltage distribution” to a stationary (steady-state) conduction field with a “resistive voltage distribution” during the application of a so-called “DC voltage” $v(t)$.

The relevant time-constant is approximately $R_2C_1 \cong 0.5$ h, Figure 2.1-16. This time-constant is significantly longer than $T/4 = 5$ ms for an AC voltage at $f = 50$ Hz. Therefore, the assumption of an electrostatic field (dielectric displacement field) in the above-mentioned example was justified. Immediately after the application of the DC voltage, a “capacitive voltage distribution” is caused, according to the electrostatic field (displacement field). It marks the beginning of a **transition process**, approaching a “resistive voltage distribution”, according to the electrical conduction field.

Example: Self-discharging of a Dielectric

The self-discharging of a capacitor or a homogeneous dielectric is also an exponential transition process. The time-constant $\tau_d = R \cdot C = \varepsilon/\kappa$ results from a calculation in a network model, which describes the exponential discharging of a capacitance C via the parallel insulating resistance R . The geometric quantities cancel each other out, so that the time constant is $\tau_d = \varepsilon/\kappa$.

If field theory is considered, the result is the same: In a self-discharging dielectric material the conduction current $J \cdot dA$ is fed from the change of the electric field, i.e. the “electric circuit” is closed by the anti-parallel displacement current $-\partial D/\partial t \cdot dA$, which is equal in magnitude. By means of the constitutive (material) Equations (2.1-19) and (-20) a differential equation for the decrease of the electric field strength $E(t)$ is derived:

$$\kappa \cdot E(t) = -\varepsilon \partial E / \partial t$$

The solution is an exponentially decreasing field strength with the time constant $\tau_d = \varepsilon/\kappa$; see Eq. (2.4-3).

In the previously discussed example of a capacitor dielectric, the time constants for the self-discharging of the polymeric films are $\tau_{d1} = \varepsilon_1/\kappa_1 = \varepsilon_0\varepsilon_{r1}/\kappa_1 \cong 50$ h and for the oil-impregnated papers $\tau_{d2} = \varepsilon_2/\kappa_2 = \varepsilon_0\varepsilon_{r2}/\kappa_2 \cong 1$ h.

Transition processes in complex insulation systems have to be determined by network analysis. Especially for **DC and polarity reversal (PR) stresses** it is necessary to base the analysis on the actually given field distribution (e.g. on a given steady-state conduction field). According to the field changes (application or polarity reversal of a “direct voltage”) a displacement field (electrostatic, quasi-static or AC field resp.) must be superimposed. During a more or less complex *transition* process, a new steady-state condition in the form of a (stationary) conduction field is approached. Thereby it is possible that the insulating sys-

tem is stressed for a short time completely differently from as one would assume from a pure AC or DC voltage distribution at the beginning and at the end of the transition [7].

2.1.4.4 Non-stationary, Time-varying Fields (Electromagnetic Waves)

Fast changing fields, which no longer fulfill the conditions for a quasi-static (quasi-stationary) description (eq. (2.1-36 to (-38))), have to be described by *Maxwell’s Equations* (2.1-14) to (-17) in their universal form. Therefore, the mutual coupling between electric and magnetic field quantities must especially be considered. This results in **electromagnetic waves** with finite phase velocity u .

Maxwell’s Main Field Equations (2.1-14) and (-15), i.e. Faraday’s and Ampere’s law, transferred into their differential form, can be differentiated with respect to time and can be mutually inserted into each other. The results are two independent partial differential equations for the electric and magnetic field quantities $E(x,t)$ and $H(x,t)$. It can be shown that the solution approaches

$$f(z - ut) \quad \text{and} \quad g(z + ut)$$

both satisfy the differential equations. They can be interpreted as wave processes that are propagating in the $+z$ - and $-z$ -directions. Boundary conditions and material properties determine the particular distribution of the electromagnetic wave-fields $E(x,t)$ and $H(x,t)$.

Example: Uniform plane wave

For example, a uniform plane wave in $+z$ -direction within a non-conductive insulating material ($\kappa = 0$) shall be considered: From Maxwell’s Main Field Equations, i.e. from Faraday’s and Ampere’s law it is concluded that the field vectors E and H are orthogonal to the direction of propagation and orthogonal to each other, Figure 2.1-6. Such a field is called *transverse electric and magnetic field*, *transverse electromagnetic wave* or *TEM wave*, Figure 2.1-17.

The so-called *phase front* is spanned by the vectors E and H orthogonally relative to the direction of propagation. It is a surface or plane of constant phase (e.g. the

wavefront), and it propagates with the *phase velocity* u in $+z$ -direction, Figure 2.1-17 with Eq. (2.1-42). The magnitudes of the related vectors $\mathbf{E}$ and $\mathbf{H}$ have a constant ratio $E/H = Z$, the so called *wave impedance*, Figure 2.1-17 with Eq. (2.1-43).

The *phase velocity* in vacuum and in gases is equal to the *speed of light*

$$u = u_0 = 300 \cdot 10^6 \text{ m/s}, \quad (2.1-44)$$

the *wave impedance* is equal to the *intrinsic impedance of the free space*

$$Z = Z_0 = 377 \Omega. \quad (2.1-45)$$

In *insulating materials* with $\mu_r \cong 1$ the quantities u_0 and Z_0 are to be divided by the square root of ϵ_r :

$$\begin{aligned} u &= u_0 / \sqrt{\epsilon_r} \\ Z &= Z_0 / \sqrt{\epsilon_r} \end{aligned} \quad (2.1-46)$$

High voltage engineering must always consider the traveling wave character of the fields, if the quasi-static description causes significant errors. According to Eq. (2.1-36) to (-38), this limit is reached for materials with $\epsilon_r = 1$ (e.g. for air)

- for a length of approx. 100 km (80 km) in the case of AC voltage $f = 50 \text{ Hz}$ (60 Hz),
- for a length of approx. 5 km in the case of switching impulse (SI) voltage (time to crest $T_{cr} = 250 \mu\text{s}$),
- for a length of approx. 25 m in the case of lightning impulse (LI) voltage (front time $T_1 = 1.2 \mu\text{s}$) and
- for lengths of less than 0.2 m in the case of “fast transients” (rise time $T_r < 10 \text{ ns}$) in gas insulated switchgear (GIS) or gas insulated lines (GIL).

For AC and SI voltages we can assume *quasi-static (quasi-stationary) conditions* in many cases, for LI voltages these conditions can be assumed if the circuit dimensions are limited to a few tens of meters. Overvoltages with rise times in the μs range are *always* to be regarded as *traveling waves* in the distributed systems of energy or data transmission. The same applies to the very steep fast transients, which

excite significant traveling waves even in systems with lengths of a few meters only. It is concluded that high voltage engineering mostly has to deal with *guided traveling waves* on transmission lines, see Section 2.6. Important practical examples are

- lightning strikes into an overhead transmission line,
- „fast transients“ (FT) in pressurized gas-insulated switchgear (GIS),
- impulse generators for the so-called “pulsed power technology” and
- measurement signals on long measuring cables.

2.2 Electrical Stresses in High Voltage Engineering

Insulating systems in high voltage engineering are based on insulating materials, which have to satisfy very different requirements. Of course, the *electric strength* plays a dominant role, but it is only a single property among

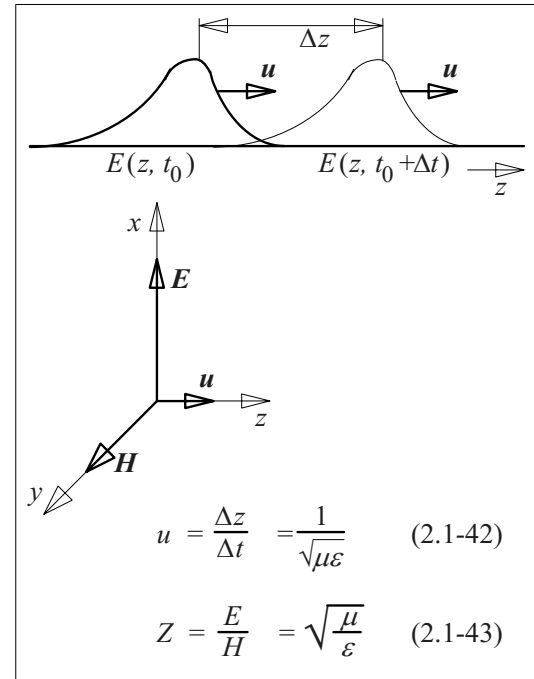


Figure 2.1-17: Uniform plane wave with transverse $\mathbf{E}$ - and $\mathbf{H}$ -field vectors.

many others. Each insulating material has a specific *profile* of properties that determines whether the material is suitable for a special application or not. In the following some important elements of such profiles are summarized:

1. Electric strength (dielectric strength),

e.g. short term strengths and life-time characteristics for AC, DC and impulse voltage stresses, also under the influence of pollution.

2. Dielectric properties,

e.g. permittivity, dissipation factor, conductivity and surface resistance.

3. Thermal properties,

e.g. permissible holding and maximum temperatures, thermal conductivity, coefficient of thermal expansion, thermal capacitance, flammability, tracking resistance, temperature dependences of material parameters.

4. Mechanical properties,

e.g. tensile and flexural strength, modulus of elasticity, degree of hardness.

5. Resistance to environmental influences,

e.g. weather-resistance, resistance to ultraviolet rays and resistance against chemical impacts.

6. Processing possibilities,

e.g. casting, extruding, welding, bonding and mechanical machining.

7. Availability and costs,

for procurement and processing.

8. Possibilities for recycling and disposal.

The design of an insulating system has must ensure that the property profiles of the dielectric materials satisfy all test and service stresses. These stresses can be of an electrical, thermal, mechanical and chemical/physical nature.

In the following, only the various **electrical stresses** will be discussed, such as DC voltages (Section 2.2.1), AC voltages (Section 2.2.2), switching and lightning impulse voltages (Sections 2.2.3 and 2.2.4), fast rising impulses (e.g. fast transients, Section 2.2.5) and mixed stresses (Section 2.2.6). During the calculation of electric field strengths, it is necessary to consider the different characters of the electric fields. Figure 2.2-4 gives an overview at the end of this chapter.

The wide range of *non-electric stresses* in high voltage engineering will be addressed together with the description of special insulating material properties (Chapter 5) and the different technical applications (Chapter 7).

2.2.1 DC Voltage Stress

Many technical applications are associated with high DC voltage stresses:

1.) In devices for the DC supply of X-ray tubes, in monitors, charging devices, copiers, impulse circuits, lacquering and coating devices and in test equipment there are high electric DC fields, especially in the barrier junction of the rectifier components and in the dielectrics of smoothing and energy storage capacitors, Figure 2.2-1.

These stresses are *frequently* not a matter of *pure steady-state DC voltage*, often there are *mixed voltage stresses*: Grading capacitors and rectifier components are additionally stressed by a superimposed AC voltage. The voltage of smoothing capacitors contains a certain “ripple” and energy storage capacitors (“surge capacitors”) are discharged abruptly. Furthermore, in many cases the DC voltage is not applied long enough for the formation of a steady-state (stationary) conduction field, Fig-

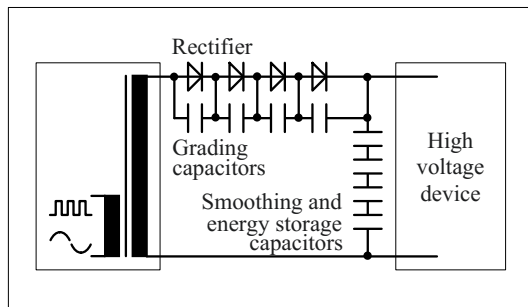


Figure 2.2-1: Half-wave high-voltage rectifier circuit with grading and smoothing capacitors.

ure 2.1-16. The individual voltage stress of single components is derived only from the analysis of voltages in a given circuit. Different circuits for the generation of high DC voltages are described in Section 6.2.

2.) In high-voltage direct-current (HVDC) transmission systems AC-transformers feed converter circuits, which are arranged in series on the DC side, Figure 2.2-2. Thereby, there are *mixed AC and DC* stresses on the insulation.

The insulation also has to withstand *transient processes* occurring after switching on and off or polarity reversal of the DC voltage. Therefore, special DC test procedures with polarity reversals and voltage profiles determine the DC insulation design, see Figure 2.2-4.

For the *calculation* of electric field strengths after voltage changes or reversals it is necessary to superimpose a dielectric *displacement field* (quasi-static field), which describes the amplitude of the voltage change, and the initially given *stationary conduction field* (initial steady-state condition), Section 2.1.4.4.

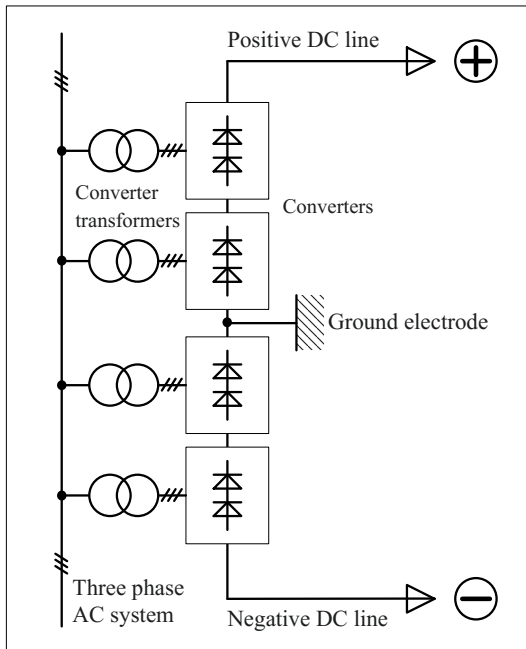


Figure 2.2-2: Converter station of a high-voltage direct-current (HVDC) transmission system.

During steady-state DC voltage application, there are problems with very high line-to-ground voltages (approx. above 500 kV). Pollution and wetting of insulator surfaces cause unpredictable distortions of the electric fields because of comparatively high and non-uniform conductivities of the wetted deposits on the surfaces [7] ... [10].

2.2.2 AC Voltage Stress

Energy transmission with three phase AC systems requires the application of high voltages in order to limit the losses. Therefore insulation systems are stressed by high AC voltages with frequencies of $f = 50$ Hz or $f = 60$ Hz. According to the considerations of Sections 2.1.4.3 and 2.1.4.4 *quasi-static* (quasi-stationary) *displacement fields* can normally be assumed in the insulation of power apparatus. This means that the *permittivities* determine the distribution of the electric fields for the common insulating materials with low conductivities.

The transmission voltage of a three-phase AC system is defined by the “highest voltage for equipment” V_m , which must not be exceeded in any part of the electrical grid [11]. This voltage is the highest r.m.s. value of the phase-to-phase voltage for which the equipment and its insulation is designed. Standardized values are

in the “medium voltage (MV) range”

$$(1 \text{ kV} < V_m < 52 \text{ kV})$$

$$V_m = \begin{array}{ll} 3.6 \text{ kV}, & \\ 7.2 \text{ kV}, & \\ \mathbf{12 \text{ kV}} & \text{(common in Germany),} \\ 17.5 \text{ kV}, & \\ \mathbf{24 \text{ kV}} & \text{(common in Germany),} \\ 36 \text{ kV}, & \end{array}$$

in the “high voltage (HV) range”

$$(52 \text{ kV} \leq V_m < 300 \text{ kV})$$

$$V_m = \begin{array}{l} 52 \text{ kV}, \\ 72.5 \text{ kV}, \end{array}$$

123 kV	(common in Germany),
145 kV,	
170 kV,	
245 kV	(common in Germany),

and in the “**extra high voltage (EHV) range**” ($V_m \geq 300$ kV)

$V_m = 300$ kV,	
362 kV,	
420 kV	(common in Germany),
525 kV,	
765 kV.	

Note: In a very general sense, “*high voltage*” is any voltage level above *low voltage* (1 kV). The boundaries between *medium* and *high voltage* depend on local circumstances, history or common usage. Nevertheless, the band 30 kV to 100 kV frequently contains the accepted boundary. The above-mentioned classification is used in German transmission and distribution systems for instance.

Note: Sometimes in Germany the standardized voltage levels $V_m = 12, 24, 123, 245$ and 420 kV are still denominated with the old “nominal voltages” 10, 20, 110, 220 and 380 kV (400 kV).

Insulation breakdown is normally determined by the highest occurring value of the AC voltage, i.e. by the **peak value**. For sinusoidal voltages and under normal service conditions, the insulation between phases (line-to-line, index “LL”) is stressed with the peak value

$$\hat{V}_{LL} = \sqrt{2} \cdot V_m \quad (2.2-1)$$

and between phase and ground (line-to-ground, index “LG”) with the peak value

$$\hat{V}_{LG} = \sqrt{2} \cdot V_m / \sqrt{3} \quad (2.2-2)$$

Electrical equipment is designed and rated to withstand this *continuous voltage stress* for many decades.

For a short time, power-frequency *overvoltages* can occur, e.g. during a sudden load reduction. In grids up to $V_m = 123$ kV the neutral point (star point) does not always have a solid grounding. In the case of a single phase-to-

ground fault, the neutral-point potential is therefore shifted and the insulations between the unaffected phases and ground are stressed with the phase-to-phase voltage according to Eq. (2.2-1). Resonant overvoltages at power frequency should be excluded by the grid topology, but they can occur together with harmonics.

The strength of insulation against power-frequency overvoltages has to be proven by means of a specified AC voltage withstand test with a duration of 1 minute (“*rated short-duration power-frequency withstand voltage test*”). The r.m.s. value of the test voltage is always specified in relation to the highest voltage for equipment V_m [11]. This reference of test voltages to the maximum voltage stresses in service is called “*insulation coordination*”, Section 6.1.4.

The test voltage value is nearly $3 \cdot V_m$ for the lower voltage levels and approx. between $2 \cdot V_m$ to $1.5 \cdot V_m$ for the higher levels. This short-duration test voltage is an important design parameter for insulation systems.

For the higher voltage levels a successful AC voltage withstand test is not sufficient. Depending on the kind of equipment, *insulation quality* must be guaranteed by the proof of *partial discharge* (PD) intensity limits at different AC test voltage levels (see Section 3.6, 6.4.2 and Chapter 7).

AC voltage tests on **cables** with high capacitances are performed with *very low frequency* (VLF) $f = 0.1$ Hz in order to reduce the capacitive reactive power. Alternatively, tests with higher frequencies can be performed with *resonance test circuits instead of the less meaningful DC tests*, Figure 2.2-4, Section 6.2.1.

Transformers must be tested with *increased frequencies* (e.g. with $f = 100$ Hz for 50 Hz transformers and $f = 120$ Hz for 60 Hz transformers) in order to avoid saturation of the magnetic core after exceeding the design voltage and at the start of the test voltage. If the

frequency is doubled, the induced voltage $V_i \sim \partial B / \partial t \sim \omega B$ is also increased by a factor of two without any increase in magnetic flux density B , Figure 2.2-4.

Significantly higher frequencies occur if the line voltage contains *harmonics*. This can result in a distortion of the voltage curve, so that peak values differ significantly from the peak values of a sinusoidal voltage with the same r.m.s. value. Furthermore, harmonics can cause enhanced capacitive currents and enhanced dielectric *losses*. Lossy and thick insulation systems (e.g. in old power factor correction capacitors with oil-impregnated paper insulation) are subject to higher thermal stresses.

2.2.3 Switching Impulse Voltage Stress (“Internal Overvoltages”)

Pulse-shaped overvoltages can be caused by switching operations in the electrical grid, e.g. by interruption of a currents during the opening of inductive circuits. As the origin is in the grid itself, the *switching impulses* (SI) are called “*internal overvoltages*”.

For equipment with $V_m > 300$ kV the electric strength against SI overvoltages is normally proven during a type test. The peak value of the standardized “*rated switching impulse withstand voltage*” V_{rs} is defined in relation to V_m as part of the insulation coordination [11], Section 6.2.3.1. Normally the peak time T_p (time to crest) is 250 μ s and the time to half-value (tail time) is 2500 μ s.

During the calculation of electric fields in common insulating systems, *quasi-static* (*quasi-stationary*) conditions in the form of *dielectric displacement fields* can be assumed, which are determined by *permittivities*, cf. Section 2.1.4.4. The errors of a quasi-stationary analysis are negligible up to lengths of approx. 5 km, if $T_p = 250$ μ s, Eq. (2.1-37), Figure 2.2-4.

2.2.4 Lightning Impulse Voltage Stress (“External Overvoltages”)

Direct lightning strikes into **power apparatus** cause travelling waves in the widely distributed overhead lines and cables. These waves lead to very high short-duration overvoltages. Also lightning strikes into line towers, overhead ground wires or into other structures nearby can lead to rapidly changing fields and traveling wave processes being coupled in. As the surges are generated atmospherically, i.e. they are caused by external lightning impulses (LI), we refer to “*external overvoltages*”.

Amplitudes and time responses of external overvoltages are subject to strong variations. Nevertheless, characteristic properties are a fast rising voltage in the μ s range and a significantly slower decline of the overvoltage impulse (Section 6.2.4). For of electrical power equipment a standardized lightning impulse voltage with a so-called front time $T_1 = 1.2$ μ s and a time to half-value (tail time) $T_2 = 50$ μ s is defined. As part of the insulation coordination, the different service voltage levels V_m are each associated with a peak value of the “*rated lightning impulse withstand voltage*” V_{rL} [11]. They are more than twice the so-called short-duration AC withstand voltage.

For the calculation of electric fields, *quasi-static* (quasi-stationary) conditions and *dielectric displacement fields* can be assumed in relatively small systems only, i.e. for dimensions limited to approx. 25 m, Figure 2.2-4. In systems with distributed parameters (distributed systems), e.g. in cables and overhead lines, the *wave-character* of the fields must be considered. This is especially the case if the trailing part of the impulse voltage is chopped. Depending on the inductance of the circuit, chopping times far below 100 ns can occur. In circuits with low losses (e.g. if a non-damped capacitive voltage-divider is used), the chopping can be an excitation of significant transient traveling wave oscillations.

Another example for pulsed electric stresses is the *discharging of energy storage capacitors*, which are often referred to as *impulse capacitors*, Figure 2.2-4. Typically the discharge periods or the time-constants of the discharge processes are in the μs range. Therefore in smaller systems quasi-static fields can be assumed.

(Impulse) discharge circuits of high power pulse technology (pulsed power) are used in many technical applications, Section 7.4.2:

- In *medical engineering* acoustic shock-waves are generated by igniting a spark gap in water or by an electroacoustic transducer. The energy is supplied by the discharging of a high voltage capacitor. For example, in a *lithotripter* the resulting shock wave is focused on a kidney stone (nephrolith) or a gallstone in order to pound it to tiny pieces.
- In production technology focused acoustic shock waves can be used for *high-speed forming* of metallic materials.
- Shock waves are used for *electrodynamic fragmentation*, i.e. for the fragmentation

and grinding of inhomogeneous or composite materials, e.g. for *recycling* applications [12].

- *Electroporation* can be applied for the opening and disruption of biological cells at room temperatures with low energy consumption.
- High energy impulses can be used to generate very fast temperature rises for the production of *nanometric particles* by melting and condensing.
- Pulse discharge circuits are necessary for the power supply of *impulse lasers* and for other *impulse (flash) light sources*.

2.2.5 Fast-rising Impulse Stresses (“Fast Transients”)

There are many examples of fast rising impulses in different technical applications:

- 1.) In **gas-insulated switchgear (GIS)** discharge processes are caused for example by flashovers or by switching of disconnectors.

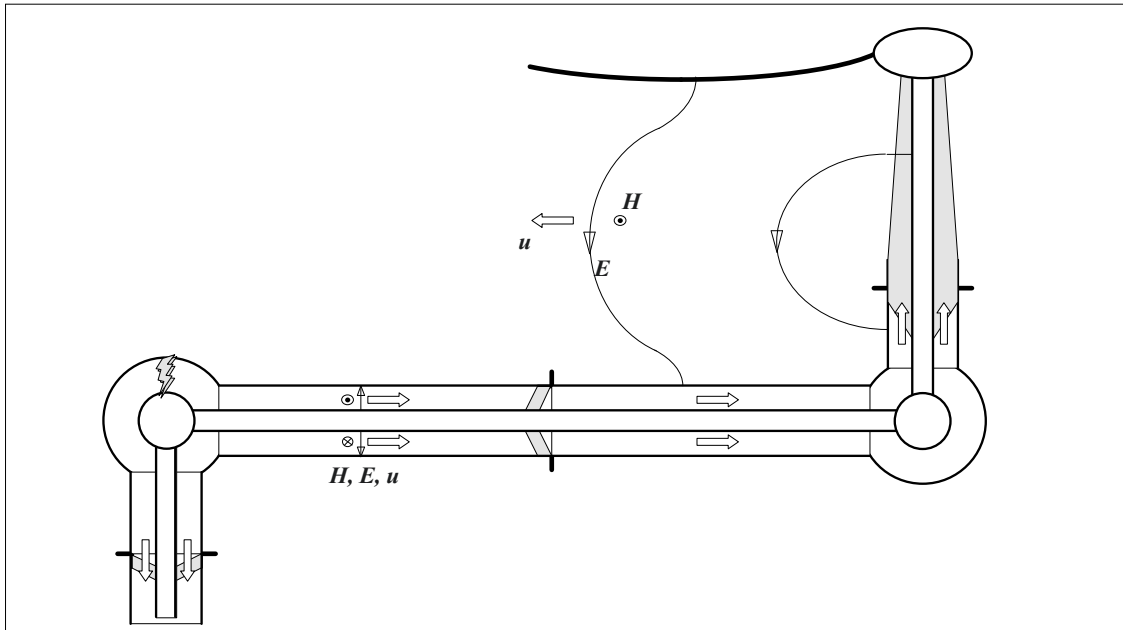


Figure 2.2-3: Propagation of traveling waves within and on the outside of a gas-insulated switchgear (GIS) after a breakdown in gas (schematic illustration without respect to reflections).

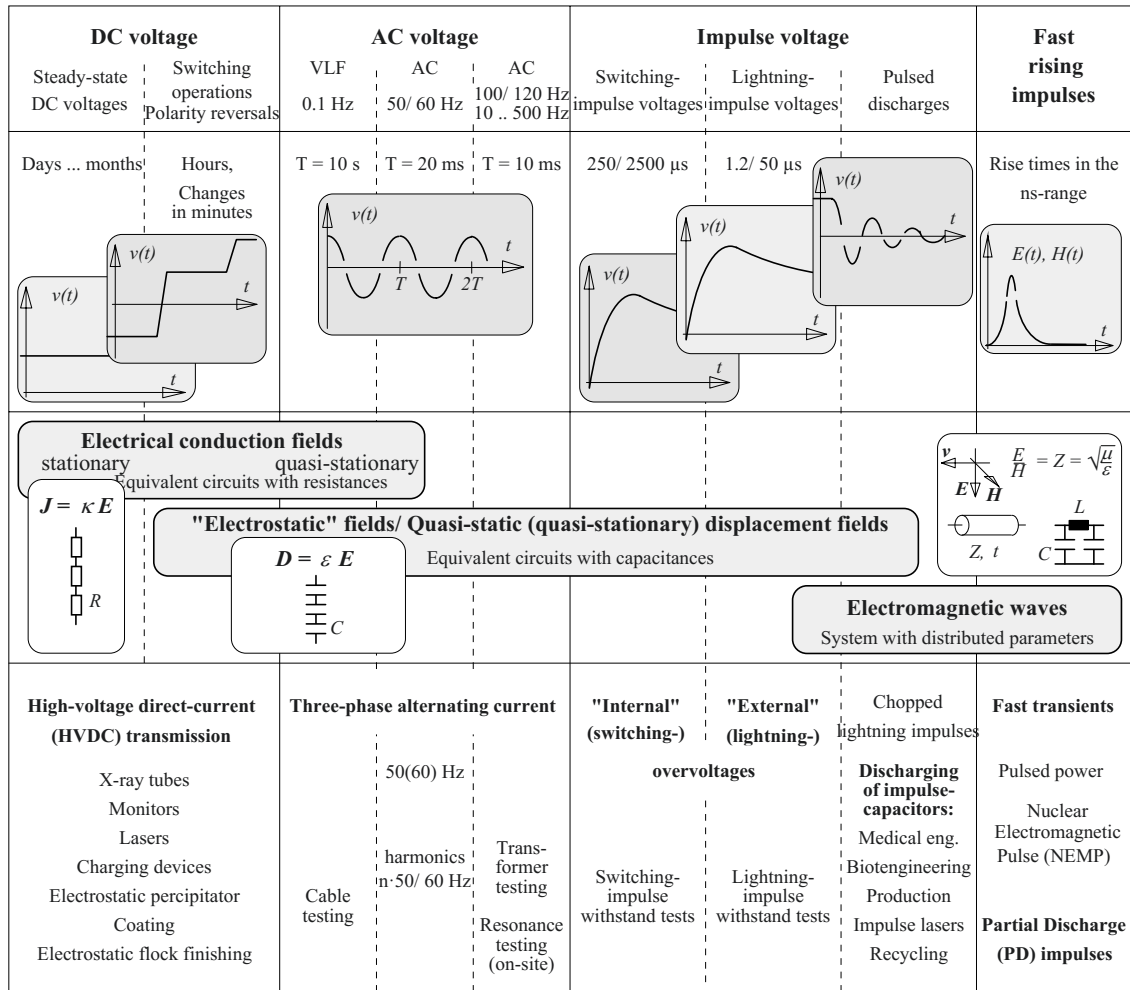


Figure 2.2-4: Overview of important technical voltage stresses in high voltage engineering: Typical time curves (top), kinds of fields and equivalent circuits (middle) and typical applications (bottom).

Because of the high gas pressures and the low insulation distances, discharges have *rise times* in the ns range, therefore they can excite traveling waves. Owing to the length (frequently many meters) of the coaxial tubular conductors and shielding, *traveling waves* can often propagate without significant damping. At discontinuities of the line impedance *reflections* occur, and different waves are superimposed. Normally they propagate within the tubular shielding, but via the bushings they can also propagate outside, Figure 2.2-3 and -4 [13].

Therefore, highly stressed insulations (e.g. in transformers and bushings) are endangered by *significant transient overvoltages*. In unfavor-

able situations, the excitation of *self-resonances* (e.g. in the transformer windings) can result in further voltage overshoots. Wave propagation on the *outside* of the shielding can cause unwanted electromagnetic interferences in the *secondary systems* of the plant. Specific measures have to be taken in order to guarantee "*electromagnetic compatibility*" (EMC).

2.) **Testing of power equipment** with respect to very fast voltage transients is normally performed together with a lightning impulse test by a fast "*chopping*" of the voltage by means of a chopping spark gap. The time to chopping is 4 to 6 μs (chopped lightning impulse, chopped-wave lightning impulse).

For the calculation of fast transients, the fast-changing character of the processes must be considered. Mostly the transients can be described as guided TEM-waves on coaxial lines (*traveling waves*, Section 2.6). On a surface with constant phase (wavefront) the closed-loop integral about $\mathbf{E} \cdot d\mathbf{x}$ is zero (the vectors $\mathbf{H}$ and $\mathbf{B}$ do not penetrate the wavefronts!), therefore voltages between the inner and the outer conductor can be defined according to Eq. (2.1-7). *Attention:* The definition of voltages with components parallel to the direction of wave propagation is **no longer** possible!

3.) Other *examples* for very fast rising high-voltage impulses can be found in **pulsed-power technology** for the generation of extremely short impulses with extremely high power ratings. These pulses are generated by means of traveling-wave lines and they are used to feed particle-beam accelerators in science for the investigation of materials in extreme conditions and for the ignition of controlled nuclear fusion processes. The rise times and the half-value widths of these impulses are in the range of some ns and some 10 ns resp., peak power and peak voltage reach the TW and the MV range [14], [15].

4.) In the case of a nuclear explosion in the space outside of the Earth's atmosphere, it is expected that the action of the resulting radiation in the atmosphere will separate positive and negative charge carriers in the vertical direction. Separation and recombination of charges will cause a pulsed electromagnetic field, the so-called **nuclear electromagnetic pulse** (NEMP). It is expected that high overvoltages will be induced in the widely distributed systems of telecommunications, information technology and energy transmission and distribution.

5.) If a high AC field strength is applied to an insulation defect, **partial-discharge (PD) impulses** occur, normally without causing an immediate breakdown. These impulses also have very short rise times in the ns range. For a single partial discharge the dissipation of energy and the charge magnitude are very small.

Nevertheless, the discharge impulses are a dangerous phenomenon of AC voltage stress, owing to their erosive effects in sensitive, mainly organic insulating materials. The fast changing electromagnetic field of partial discharges is important for partial discharge measurements.

6.) The transmission characteristics of **high-voltage measuring systems** are determined by means of step-generators providing rectangular pulses with rise times in the ns range. Because of the large spatial dimensions, traveling wave oscillations on measuring cables and direct coupling of free electromagnetic waves must be considered [18], [19].

2.2.6 Mixed-field Stresses

In many cases electric stresses are a combination of the cases described above. Then it is often difficult to determine the electric field strengths and the relevant electric strengths. Important examples:

1. Superposition of DC and AC voltages in the converter transformers of HVDC systems.

2. DC voltage and polarity reversal tests: A stationary conduction field and a quasi-stationary displacement field are superimposed, if the amplitude or the polarity of the voltage is changed. Depending on conductivity κ and self-discharge time-constant ε/κ , it takes a very long time to approach a new steady-state condition. During such a transient process, significant stresses can occur on materials in a layered insulation, which are much less highly stressed initially and in the steady state.

3. Rectifier and converter circuits: Many electronic devices are stressed with a superposition of DC, AC and impulse voltages.

4. Energy-storage and impulse capacitors: During the charging process, the dielectrics are stressed with an increasing voltage. Depending on the time of storage, the field after voltage application approaches a steady-state conduc-

tion field. During the discharging process there is a pulsed stress, often in the form of a damped oscillation.

5. Fast transients: Rapidly changing traveling waves are superimposed on the actual field condition given by the quasi-static power frequency state. Thereby high overvoltages can occur, for which the equipment is not insulated sufficiently.

In *field calculations* for the determination of electrical stresses, the different kinds of fields are normally calculated separately and *superimposed linearly* into *mixed stresses*. In contrast to magnetic materials with non-linear magnetization curves, solid insulating materials behave more or less linearly, as long as discharges do not occur. Liquid dielectrics can have *non-linear conductivities*, however, Section 4.2.2.2.

In situations with mixed stresses, it is often difficult to *evaluate* calculated field strengths. For the example of impulse capacitors, the steady-state DC voltage is absolutely noncritical in comparison with the fast changing stress during the discharging process, although the amplitudes are identical in both cases.

For practical design purposes, *breakdown* and *lifetime tests* are necessary, with conditions close to the *real service conditions*.

2.3 Conduction and Displacement Fields in Homogeneous Dielectrics

For static, stationary and slowly changing (quasi-static, quasi-stationary) fields in insulating materials, the electric field can be regarded as irrotational, i.e. as an electrostatic field. An induced electric curl field does not occur or can be neglected. Therefore, the definition of potential differences and voltages is acceptable.

If the whole field volume that is to be considered consists of a **single homogeneous** (uniform) **insulating material** (dielectric), the field distribution is not determined by material properties (permittivity ϵ , conductivity κ) and the electric field calculations are based on the same relationships. Therefore, the field calculation methods, which are described in the following, can be applied for most of the.

First of all, the *direct analytic evaluation of the field equations* is performed (Section 2.3.1 and 2.3.2). It allows the calculation of basic field configurations with homogeneous, spherically symmetric and cylindrically symmetric fields. Some important high voltage field configurations can be approximated from this.

A *graphical method* (Section 2.3.3) allows a qualitative or semi-quantitative field sketch to be determined using some simple drafting rules. Often this is very valuable for a first qualitative estimation of field conditions.

The *method of conformal mapping* (Section 2.3.4) allows the calculation of some special cases, e.g. the field stress enhancement at the edges of a parallel-plate capacitor (edge field).

By means of the fields of *equivalent charges* (Section 2.3.5) it is also possible to calculate important field configurations, e.g. sphere against sphere or cylinder against plane.

In many cases, only the *maximum field strength* is of interest. For many field configurations, it can be derived from already calculated cases by *similarity relationships and geometry factors* (Schwaiger's field efficiency factor, Section 2.3.6).

Before numerical field calculation methods were available, the only possibility for the determination of field strengths in systems with arbitrarily shaped electrodes was the *measurement* of electrical *conduction fields* in a semi-conductive liquid (*electrolytic tank*) or on a semi-conductive paper (*field plotter*), Section 2.3.7).

2.3.1 Analytic Evaluation of the Continuity Equation (Gauss's Law)

2.3.1.1 General Calculation Method

If Eq. (2.1-17), i.e. the Continuity Equation for conduction and displacement current

$$\oiint_A \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{A} = 0,$$

is integrated over the time, it results in "Gauss's law", Eq. (2.1-21) and (2.3-1). It states that the integral of the flux density $\mathbf{D}$ over any closed surface, i.e. the total displacement flux $\oiint \mathbf{D} \cdot d\mathbf{A}$, equals the charge Q enclosed.

$$\boxed{\oiint_A \mathbf{D} \cdot d\mathbf{A} = Q} \quad (2.3-1)$$

With Eq. (2.3-1), field calculations can be performed for some basic configurations in four steps:

Step 1:

For configurations with symmetrical fields, Eq. (2.3-1) is solved for the magnitude D of displacement vector $\mathbf{D}$, in order to get a relationship between field-generating charge Q and the electric field strength $E = D/\epsilon$.

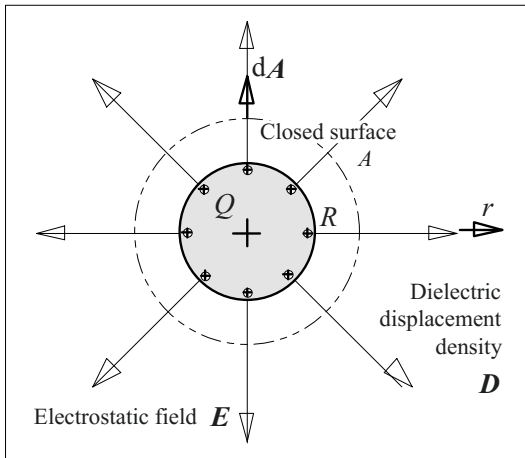


Figure 2.3-1: Spherically symmetrical electrode in free space (Gauss's law).

Example: Spherical electrode in free space

The magnitude of the electric field strength E shall be calculated as function of voltage V and radius r for the spherically symmetric arrangement according to Figure 2.3-1. The counter-electrode with the negative counter-charges is assumed to be infinitely far away.

Step 1:

The surface of a sphere with the radius r is chosen as the closed surface for integration. Thereby the symmetry of the configuration is utilized, because the displacement density has a constant magnitude $D(r)$ over the entire chosen surface. Furthermore, the vectors $\mathbf{D}$ and $d\mathbf{A}$ are parallel to each other, over the entire closed surface. The scalar product $\mathbf{D} \cdot d\mathbf{A}$ equals the product of the magnitudes $D \cdot dA$. The displacement density is constant all over the surface of integration, and it can be brought out from under the integral sign in Eq. (2.3-1):

$$\oiint_A \mathbf{D} \cdot d\mathbf{A} = D(r) \oiint_A dA = Q$$

The remaining integral over the closed surface gives the surface area $A(r) = 4\pi r^2$ itself:

$$D(r) \cdot A(r) = D(r) \cdot 4\pi r^2 = Q$$

This gives

$$D(r) = Q/(4\pi r^2) \quad \text{and} \quad E(r) = Q/(4\pi\epsilon r^2). \quad (2.3-2)$$

The magnitude of the electric field strength decreases proportionally to $1/r^2$, i.e. quadratically with the radius.

Step 2:

In high voltage engineering the electric field strength E normally has to be given as a function of the applied voltage V . This is possible, if the field strength, which is derived from Gauss's law according to Eq. (2.3-1), is integrated according to Eq. (2.1-7):

$$\boxed{V_{21} = \Delta\varphi_{21} = \varphi_2 - \varphi_1 = \int_2^1 \mathbf{E} \cdot d\mathbf{x}} \quad (2.3-3)$$

Thereby a relationship between Q and V is determined, i.e. $Q = f(V)$.

Step 3:

According to Eq. (2.1-10) the ratio of Q and V defines the **capacitance** C of the field configuration:

$$C = Q/V \quad (2.3-4)$$

Step 4:

The desired relationship between **field strength** E and **voltage** V

$$E = f(V)$$

is derived from the first step with $E = f(Q)$ and from the second step with $Q = f(V)$.

Step 5:

In an additional step, maximum values of field strength can be determined and optimization problems can be solved, e.g. the minimization of maximum field strength.

Example: Spherical electrode (continued)**Step 2:**

Continuing with the former example, the voltage between the electrode surface with radius $r = R$ and the counter-electrode (carrying the negative counter-charges) with the radius $r \rightarrow \infty$ is given by

$$\begin{aligned} V_{R\infty} &= \int_R^{\infty} E(r) dr = \frac{Q}{4\pi\epsilon} \int_R^{\infty} \frac{1}{r^2} dr \\ &= \frac{Q}{4\pi\epsilon} \left[\frac{-1}{r} \right]_R^{\infty} \\ &= Q / (4\pi\epsilon R), \end{aligned}$$

if the integration is performed radially, i.e. in parallel with the electric field E . Therefore it gives

$$Q = 4\pi\epsilon R V. \quad (2.3-5)$$

Step 3:

From that, the capacitance is derived:

$$C = Q / V = 4\pi\epsilon R. \quad (2.3-6)$$

Step 4:

According to Eq. (2.3-2) and (2.3-5) from steps 1 and 2, the electric field strength is

$$E(r) = V R / r^2. \quad (2.3-7)$$

Step 5:

Maximum field-strength is found on the electrode surface for the smallest possible radius $r \rightarrow R$, i.e.

$$E_{\max} = E(R) = V / R. \quad (2.3-8)$$

to (2.3-8). Maximum field strength occurs at the electrode surface.

Note: The field of the conducting sphere in free space with a vanishingly small radius $R \rightarrow 0$ approaches the field of a *point charge*. This theoretical case is important for field calculations with the equivalent-charge simulation method, Section 2.3.5.

The smaller the radius of curvature R of an electrode, the higher the electric edge-field strength will be, Eq. (2.3-8).

Generally, **sharp edges** (with small radii R) **must therefore be avoided in high voltage engineering**, in order to avoid electrical overstressing of the adjacent insulating material.

Example: Sharp-edged point electrode

The field in the vicinity of a metallic point electrode shall be calculated approximately as a spherically symmetric field with $R = 1$ mm. What is the expected peak voltage $\hat{V}$ for the inception of discharges in air ($\hat{E}_i = 5$ kV/mm)?

From Eq. (2.3-8) we conclude that $\hat{V} = \hat{E}_{\max} \cdot R = 5$ kV.

Note: High field strengths at sharp-edged point electrodes do only occur in a small volume close to the point. The inception field strength is therefore significantly higher than the commonly used peak value $\hat{E}_i = 3$ kV/mm = 30 kV/cm which is valid for air gaps in the centimeter range, Figure 3.2-15. Discharge inception cannot be described exactly by specifying a constant inception field strength, see Chapter 3.

In high voltage engineering, field stress enhancements at sharp edges are reduced by sufficient radii of curvature. In many cases, sharp-edged parts have to be protected by *shielding electrodes* (e.g. spheres or toroids). The field strengths on the surface can be estimated by approximation from Eq. (2.3-8) if approximately spherically symmetric conditions are assumed with a counter-electrode very far away:

$$E_{\max} = V / R \quad (2.3-8)$$

The capacitance in air can be approximately described according to Eq. (2.3-6) using the rule of thumb

2.3.1.2 Spherically Symmetric Fields

The electric field of a “conducting sphere in free space” was calculated in the former example, see Figure 2.3-1 and Equations (2.3-6)

$$C/\text{pF} \approx R/\text{cm} . \quad (2.3-9)$$

Note: If the counter-electrode (e.g. the grounded wall, ceiling or floor) is located at a finite distance, higher capacitances and field strengths occur. Numerical field calculation methods achieve a significantly better accuracy than the analytic estimation, Section 2.3.5.

Example: Shielding electrodes

The *diameters of shielding electrodes* for use in air ($\hat{E}_{\text{bd}} = 30 \text{ kV/cm}$, $\varepsilon_r = 1$) and insulating oil ($\hat{E}_{\text{bd}} = 150 \text{ kV/cm}$, $\varepsilon_r = 2.2$) shall be sized for the voltage amplitudes $\hat{V} = 10 \text{ kV}$, 100 kV and 1 MV so that the field strengths do not exceed $2/3$ of the breakdown field strength.

The diameters are derived from Eq. (2.3-8):

$$D = 2R = 2\hat{V} / (2/3 \cdot \hat{E}_D) .$$

Peak voltage $\hat{V}$	10 kV	100 kV	1 MV
Air:			
D	1 cm	10 cm	1 m
C	0.5 pF	5 pF	50 pF
Insulating Oil:			
D	2 mm	2 cm	20 cm
C	0.2 pF	2 pF	22 pF

Note: The results show that *high voltage laboratories* need shielding electrodes with diameters in the meter range.

Much *more compact high voltage equipment* can be designed by means of electrically stronger insulating materials (e.g. insulating oil or sulfur hexafluoride SF_6). It has to be considered that E_{bd} of insulating oil is not a constant value, but inter alia is dependent on the width of the oil ducts (“volume effect”, “distance effect” or “size effect”, see Chapters 3 and 5).

A so-called *spherical capacitor* consists of an inner sphere with the radius R_1 and a counter electrode consisting of a concentric outer sphere with the finite radius R_2 , Figure 2.3-2a.

The *calculation of field strength* is performed in five steps, analogous with the calculation steps for the spherical electrode in free space:

The use of Gauss’s law (eq. (2.3-1), Step 1) on the spherically symmetrical configuration according to Figure 2.3-2a gives the same equation (2.3-2) again because of the similar field conditions. E decreases with the radius r proportional to $1/r^2$ (step 1).

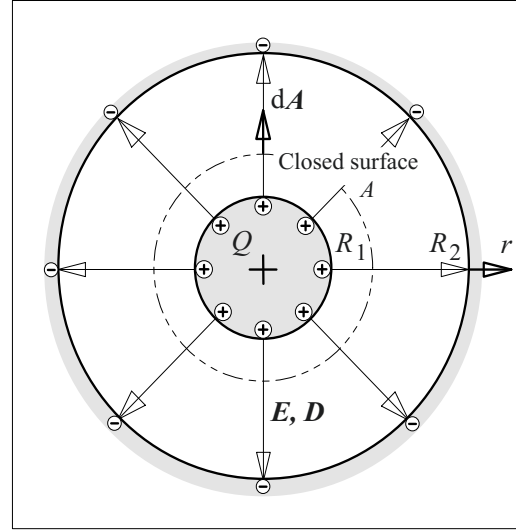


Figure 2.3-2a: Spherical capacitor (or cylindrical capacitor resp.).

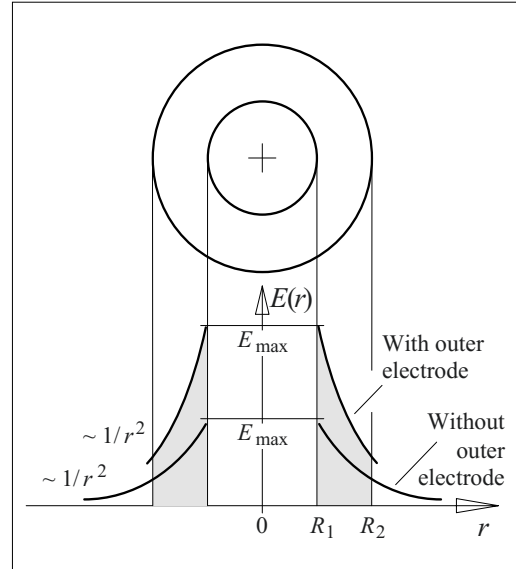


Figure 2.3-2b: Field strength curves $E(r)$ for the spherical capacitor.

The integration of the field strength according to Eq. (2.3-2) (Step 2) must not be performed from radius R to infinity, because the field is limited to the space between the inner and outer electrodes, i.e. integration has to be performed between the radii R_1 and R_2 . Inserting the new integration limits and $V_{12} = V$, we obtain

$$V = \frac{Q}{4\pi\epsilon} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (2.3-10)$$

and

$$Q = \frac{4\pi\epsilon}{\frac{1}{R_1} - \frac{1}{R_2}} V. \quad (2.3-11)$$

The *capacitance* of a spherical capacitor is given by $C = Q/V$ (Step 3):

$$C = \frac{4\pi\epsilon}{\frac{1}{R_1} - \frac{1}{R_2}} \quad (2.3-12)$$

The field strength curve between inner and outer electrodes is derived from Eq. (2.3-2) and (2.3-11) (Step 4):

$$E(r) = \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right) r^2} V \quad (2.3-13)$$

For the extreme value $R_2 \gg R_1$ Eq. (2.3-12) and (2.3-13) approach Eq. (2.3-6) and (2.3-7).

The maximum field strength at the inner sphere with $r = R_1$ (Step 5) is higher in comparison with the sphere in free space, Figure 2.3-2b. The areas under the field strength curves correspond to the integral of the function $E(r)$, i.e. to the applied voltage V .

We obtain the *maximum field strength* from Eq. (2.3-13) for the radius $r = R_1$ on the surface of the inner sphere (*surface or edge field strength*):

$$E_{\max} = E_1 = \frac{R_2}{R_1 R_2 - R_1^2} V \quad (2.3-14)$$

From the calculated field strengths *optimization problems* can be solved by means of *extreme value determination*:

For a given outer radius R_2 the inner radius R_1 shall be chosen so that the *maximum field strength* (*surface/edge field strength*) E_1 will be minimal.

In the extreme cases $R_1 \rightarrow 0$ and $R_1 \rightarrow R_2$, there are infinitely high surface/edge field

strengths E_1 . The *optimum inner radius* R_1 for a minimum surface field strength E_1 is between 0 and R_2 therefore, and it is determined, if the derivate of E_1 with respect to the variable R_1 is set to zero, see Eq. (2.3-14):

$$\frac{\partial E_1}{\partial R_1} = - \frac{R_2 - 2R_1}{(R_1 R_2 - R_1^2)^2} R_2 V \stackrel{!}{=} 0$$

We obtain for R_1 and $E_{1\min}$:

$$R_1 = R_2 / 2 \quad \text{and} \quad E_{1\min} = 4 V / R_2$$

(2.3-15)

Another example for optimization problems is the *maximization* of the *capacitively stored energy* $W = \frac{1}{2} C V^2$ by variation of the inner radius R_1 for a given outer radius R_2 and an permissible field strength E_{bd} . This is particularly important for capacitors in which the greatest quantity of energy is to be stored for given dimensions.

The capacitively stored energy is calculated according to Eq. (2.3-12) and (2.3-14)

$$W = \frac{1}{2} C V^2$$

$$W = \frac{1}{2} [4\pi\epsilon R_1 R_2 (R_2 - R_1)^{-1}] [E_{\text{bd}} (R_2 - R_1) R_1 / R_2]^2$$

$$W = E_{\text{bd}}^2 2\pi\epsilon (R_2 - R_1) R_1^3 / R_2$$

$$W = E_{\text{bd}}^2 2\pi\epsilon (R_2 R_1^3 - R_1^4) / R_2$$

In the extreme cases $R_1 \rightarrow 0$ and $R_1 \rightarrow R_2$ the field energy is minimal, i.e. $W \rightarrow 0$. The radius R_1 for *maximum field energy* is determined, if the derivate of W with respect to the variable R_1 is set to zero:

$$\partial W / \partial R_1 = E_{\text{bd}}^2 2\pi\epsilon (R_2 3R_1^2 - 4R_1^3) / R_2 = 0$$

The result for R_1 is

$$R_1 = R_2 3/4. \quad (2.3-16)$$

For many applications in high voltage engineering the electric field can be regarded as spherically symmetric, either by approximation or in limited field regions, Figure 2.3-3.

2.3.1.3 Cylindrically Symmetric Fields

The so called “*cylindrical capacitor*” consists of concentric (coaxial) cylinders with the radii R_1 and R_2 , Figure 2.3-4. To begin with, field

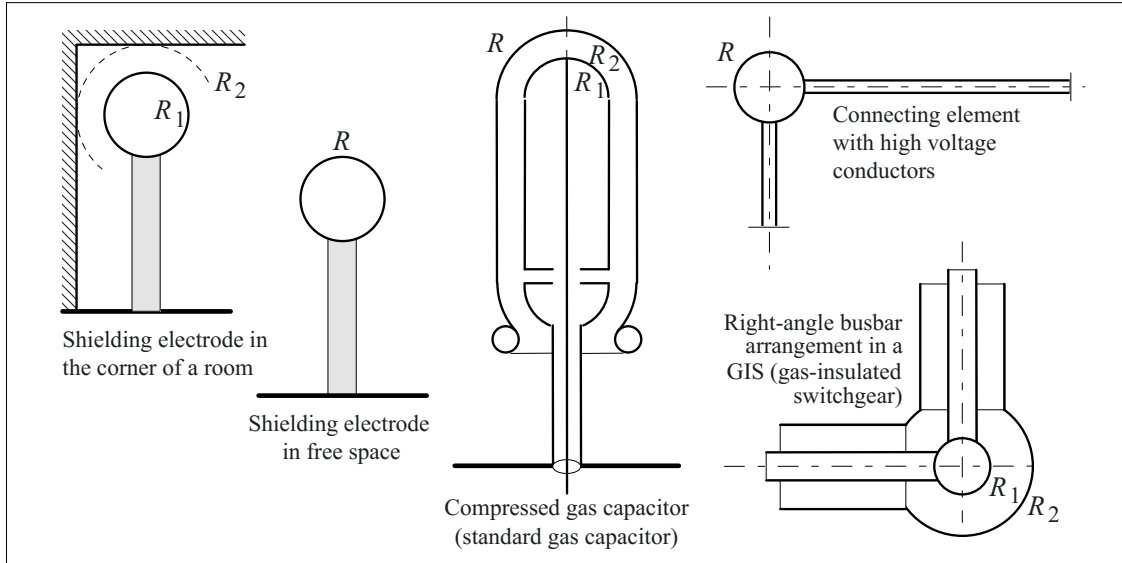


Figure 2.3-3: Examples for spherically and cylindrically symmetric fields in HV engineering (approximations).

distortions at the edges of the cylinders are neglected, i.e. it is assumed that there is a two-dimensional field, which does not change along the axis of the cylinders.

Calculation of *field strengths* is performed in *five steps*, as in the case of the spherically symmetric field:

For the application of Gauss's law, Eq. (2.3-1), *Step 1*, a closed surface is defined, completely

enclosing the inner cylinder. It consists of a cylindrical lateral surface with the radius r and the cylinder length z , and two plane end areas, Figure 2.3-4.

The lines of dielectric displacement density $\mathbf{D}$ are nearly orthogonal to the vectors of the area elements $d\mathbf{A}$ of the end areas. For the integration over the total closed surface, the contribution of the end areas can therefore be neglected. On the lateral surface $\mathbf{D}$ and $d\mathbf{A}$ are in

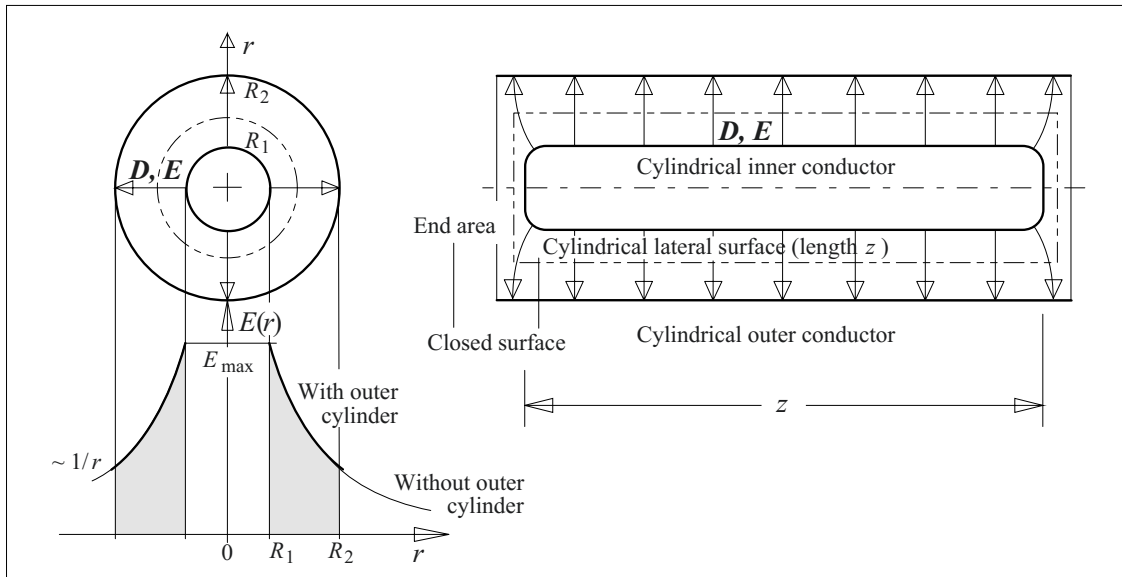


Figure 2.3-4: Cylindrically symmetric electrode configuration (top) with field strength profile (bottom).

parallel, so the vector product can be replaced by the scalar product of the magnitudes. $D(r)$ is nearly constant on the lateral surface and can be brought out from under the integral sign. The remaining integration of dA over the lateral surface gives the surface area value $A = 2\pi r z$

$$Q = D(r) \iint dA = D(r) A = \varepsilon E(r) 2\pi r z$$

The magnitude of the electric field strength decreases with the radius r proportional to $1/r$:

$$E(r) = Q / (2\pi\varepsilon z r) \quad (2.3-17)$$

The integration of field strength $E(r)$ from the inner to the outer cylinder according to *Step 2* and Eq. (2.3-3) with $V_{12} = V$ gives

$$V = \frac{Q}{2\pi\varepsilon z} \ln \frac{R_2}{R_1}, \quad (2.3-18)$$

i.e.

$$Q = \frac{2\pi\varepsilon z}{\ln \frac{R_2}{R_1}} V. \quad (2.3-19)$$

The *capacitance* of the cylindrical capacitor is $C = Q/U$ (*Step 3*):

$$C = \frac{2\pi\varepsilon z}{\ln \frac{R_2}{R_1}} \quad (2.3-20)$$

In *Step 4* the *field strength profile* between inner and outer cylinders is evaluated from Eq. (2.3-17) and (2.3-19):

$$E(r) = \frac{V}{r \ln \frac{R_2}{R_1}} \quad (2.3-21)$$

The area under the field strength curve equals the voltage (potential difference) between the cylinders, which is the integral of $E(r)$ in radial direction, Figure 2.3-4.

In *step 5* the *maximum field strength* is derived from Eq. (2.3-21) for the radius $r = R_1$:

$$E_{\max} = E_1 = \frac{V}{R_1 \ln \frac{R_2}{R_1}} \quad (2.3-22)$$

Also for cylindrically symmetric fields we find: the smaller the radius of curvature R_1 of the inner cylinder the higher the electric field strength at the inner cylinder will be, Eq. (2.3-22). However, the increase of field strength is smaller than in the spherically symmetric field. **Small radii are also to be avoided in cylindrically symmetric fields** in order to stay below the breakdown strength of the adjacent dielectric materials.

Eq. (2.3-22) gives the maximum field strength for the ideal cylindrically symmetric field and not the field strength at the *edges of the cylinders*. Significant *field stress enhancements* and *high local fields* can occur there depending on the electrode profiles at the electrode edges.

Note: It is natural to look at a “cylindrical conductor in free space”, as was done for the “sphere in free space”, i.e. the extreme case with the counter-electrode with the negative counter-charges is assumed to be infinitely far away, i.e. $R_2 \rightarrow \infty$. If the field strength is integrated according to Eq. (2.3-17), the result is an infinite voltage (potential difference), which can directly be seen from Eq. (2.3-18). For a finite potential difference and $R_2 \rightarrow \infty$ the field strength will be zero, see Eq. (2.3-21).

Therefore it is always necessary to consider an *outer cylinder with a finite radius* $R_2 < \infty$ in a cylindrically symmetrical field.

The field between two cylinders approaches the field of an ideal “line charge” for the extreme case $R_1 \rightarrow 0$ and $R_2 \rightarrow \infty$. This theoretical extreme case is important for field calculations with the equivalent charge method together as with “point charges” and other equivalent charges (Section 2.3.5). Nevertheless, the counter-charges are not located infinitely far away, but are also line charges at finite distances.

From the calculated field strengths *optimization problems* can be solved by means of *extreme value determination*:

For a given outer radius R_2 the inner radius R_1 shall be chosen so that the *maximum field strength (surface/edge field strength)* E_1 will be minimal.

In the extreme cases $R_1 \rightarrow 0$ and $R_1 \rightarrow R_2$, there are infinitely high surface/edge field strengths E_1 . The *optimum inner radius* R_1 for

minimum surface field strength E_1 is between 0 and R_2 therefore, and it is determined from Eq. (2.3-22), if the derivate of E_1 with respect to the variable R_1 is set to zero.

During the differentiation, the rules for derivate of a fraction have to be applied to the whole fraction at first. Additionally, the rules for the derivate of a product have to be applied for the differentiation of the denominator [6].

$$\frac{\partial E_{\max}}{\partial R_1} = V \frac{0 - \left[\ln \frac{R_2}{R_1} + R_1 \frac{R_1}{R_2} \frac{-R_2}{R_1^2} \right]}{\left(R_1 \ln \frac{R_2}{R_1} \right)^2}$$

$$\frac{\partial E_{\max}}{\partial R_1} = V \frac{- \left[\ln \frac{R_2}{R_1} - 1 \right]}{\left(R_1 \ln \frac{R_2}{R_1} \right)^2} \stackrel{!}{=} 0$$

R_1 and $E_{1\min}$ are derived from Eq. (2.3-22):

$$R_1 = R_2 / e \quad \text{and} \quad E_{1\min} = e V / R_2$$

(2.3-23)

Note: The irrational number $e = 2.71828...$ is the *base of the natural logarithm* and it is occasionally called “*Euler number*” or “*Napier’s constant*”.

Another example for optimization problems is the *maximization* of the *capacitively stored energy* $W = \frac{1}{2} C V^2$ by variation of the inner radius R_1 for a given outer radius R_2 and a permissible field strength E_{bd} . This is particularly important for capacitors in which the greatest possible quantity of energy is to be stored for given dimensions.

The maximum stored energy is calculated according to Eq. (2.3-20) and (2.3-22)

$$W = \frac{1}{2} C V^2$$

$$W = \frac{1}{2} [2\pi\epsilon z / \ln(R_2/R_1)] [E_{bd} R_1 \ln(R_2/R_1)]^2$$

$$W = E_{bd}^2 \pi\epsilon z R_1^2 \ln(R_2/R_1).$$

In the extreme cases $R_1 \rightarrow 0$ and $R_1 \rightarrow R_2$ the field energy is minimal, i.e. $W \rightarrow 0$. The radius R_1 for *maximum field energy* is determined, if the derivate of W with respect to the variable R_1 is set to zero:

$$\begin{aligned} \partial W / \partial R_1 &= E_{bd}^2 \pi\epsilon z \\ &\cdot [2R_1 \ln(R_2/R_1) + R_1^2 (R_1/R_2)(-R_2/R_1^2)] \\ &= E_{bd}^2 \pi\epsilon z R_1 [2 \ln(R_2/R_1) - 1] \\ &= 0 \end{aligned}$$

The result for R_1 is

$$R_1 = R_2 / e^{1/2}. \quad (2.3-24)$$

For many *applications* in high voltage engineering the electric field can be regarded as cylindrically symmetric, either by approximation or in limited field regions. Some examples are already shown in Figure 2.3-3. Cylindrically symmetric fields also occur in high-voltage cables, in gas insulated switchgears, in bushings and close to cylindrical conductors.

Example: Thin wire

The field in the vicinity of a thin wire shall be approximated by a cylindrically symmetric field with $R_1 = 1$ mm and $R_2 = 1$ m. What is the expected peak voltage $\hat{V}$ for the inception of electrical discharges in air ($\hat{E}_i = 4$ kV/mm)?

From Eq. (2.3-22) it is concluded that

$$\hat{V} = \hat{E}_{\max} R_1 \ln(R_2/R_1) = 27.6 \text{ kV}$$

Note: High field strengths at thin conductors with very small radii only occur in a small volume close to the conductor. Inception field strength is therefore significantly higher than the commonly used value $\hat{E}_i = 3$ kV/mm = 30 kV/cm which is valid for air in the centimeter range, Figure 3.2-15. Discharge inception cannot be described exactly by a constant inception field strength, Chapter 3.

Example: Tubular conductor and cable

The *diameters of high-voltage conductors* with *coaxial outer conductors* shall be sized for use in **air**-insulated tubular conductors ($\hat{E}_{bd} = 30$ kV/cm, $\epsilon_r = 1$), in **oil**-insulated tubular conductors ($\hat{E}_{bd} = 150$ kV/cm, $\epsilon_r = 2.2$) and in **thermoplastic**-insulated cables (polyethylene, $\hat{E}_{bd} = 450$ kV/cm, $\epsilon_r = 2.2$). Peak voltages shall be $\hat{V} = 10$ kV/ 100 kV/ 1 MV; $2/3$ of the breakdown strength must not be exceeded and the outer diameter shall be as small as possible.

The smallest outer diameters are achieved, if the ratio of the radii $R_2/R_1 = e$ is chosen such that the maximum field strength E_1 is minimal. With $\hat{E}_{1\min} = 0.67 \hat{E}_{bd}$ we get

$$D_2 = 2 R_2 = 2 e \hat{U} / (0.67 \hat{E}_{bd}) \quad \text{and}$$

$$D_1 = 2 R_1 = 2 \hat{U} / (0.67 \hat{E}_{bd}).$$

Note: The same result is obtained by extreme value determination, i.e. if Eq. (2.3-22) is solved for R_2 , and if the derivate of R_2 with respect to R_1 is set to zero in order to determine the minimum.

Voltage	$\hat{V}$	10 kV	100 kV	1 MV
Air:	D_2	2.7 cm	27 cm	2.7 m
	D_1	1 cm	10 cm	1 m
	C'	56 pF/m	56 pF/m	56 pF/m
Insulating Oil:	D_2	5.4 mm	5.4 cm	54 cm
	D_1	2 mm	2 cm	20 cm
	C'	122 pF/m	122 pF/m	122 pF/m
Poly-ethylene:	D_2	1.8 mm	1.8 cm	18 cm
	D_1	0.7 mm	0.7 cm	7 cm
	C'	122 pF/m	122 pF/m	122 pF/m

Note: The result shows that the application of electrically strong insulating materials (insulating oil, SF₆, polyethylene) allows very compact designs in compari-

son with air. The assumption of a constant value for $\hat{E}_{bd}$ neglects that the electric strength of insulating oil and polyethylene decreases with increasing insulation thickness ("volume effect", "size effect"). The mentioned voltages and field strengths are short-term strengths, as may be used for designs with respect to *short-term voltage tests*. Permissible *service voltages and field strengths* are significantly lower, especially for oil and polyethylene (Section 2.2.2 to 2.2.4). The *capacitance per unit length C'* only depends on the permittivity ϵ_r , because the ratio of radii is the same in all cases.

2.3.1.4 Uniform (Homogeneous) Fields

Between two parallel plane electrodes at a distance d there is a uniform electric field with the constant field strength $E = V/d$ ("parallel-plate capacitor"). In the first instance, field distortions at the electrode edges shall be neglected. Even in this simple case, the calculation of field strength is performed *in the five steps* mentioned above, just in order to illustrate the method:

For the application of Gauss's law (eq. (2.3-1), *Step 1*) a closed surface is defined, enclosing one of the electrodes completely. It consists of a surface A between the plane parts of the electrodes and of additional faces in the outer parts of the field volume, which extend surface A to form a closed surface, Figure 2.3-5.

The outer faces are only penetrated by a small displacement flux $\iint \mathbf{D} d\mathbf{A}$; its contribution to the total flux over the closed surface is therefore neglected.

Between the electrodes, $\mathbf{D}$ and $d\mathbf{A}$ are in parallel, so the product of the vectors can be replaced by the scalar product of the magnitudes. D is nearly constant on the surface A and can be brought out from under the integral sign. The remaining integration of dA over the surface A gives the area of the surface A itself:

$$Q = D \iint dA = D A = \epsilon E A$$

Therefore, the magnitude of the electric field strength E is constant for all values of x between the electrodes:

$$E(x) = Q/(\epsilon A) = E_0 = \text{const.} \quad (2.3-25)$$

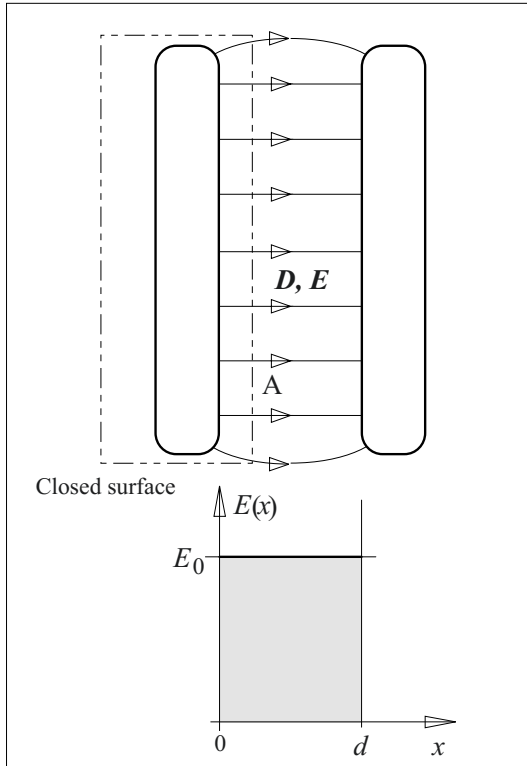


Figure 2.3-5: Uniform electric field in a parallel-plate capacitor (approximation without consideration of field distortions at the electrode edges).

The integration of field strength $E(x)$ according to Eq. (2.3-3) (Step 2) from one electrode to the other gives

$$V = E_0 d = Q d / (\epsilon A).$$

This means

$$Q = \epsilon A V / d. \quad (2.3-26)$$

The *capacitance* of the parallel-plate capacitor follows from $C = Q/V$ (Step 3):

$$C = \epsilon A / d \quad (2.3-27)$$

The *field strength curve* between the elec-

trodes is calculated from Eq. (2.3-25) and (2.3-26), Step 4:

$$E(x) = E_0 = V/d = \text{const.} \quad (2.3-28)$$

The area under the field strength curve equals the voltage (potential difference) between the electrodes which is the integral of $E(x)$ in the x -direction, Figure 2.3-5.

The indication of *maximum field strength* (Step 5) is unnecessary for the uniform field. Nevertheless, field stress enhancements can occur at strongly curved edges of the electrodes (see also Figure 2.3-8 and 2.3-9).

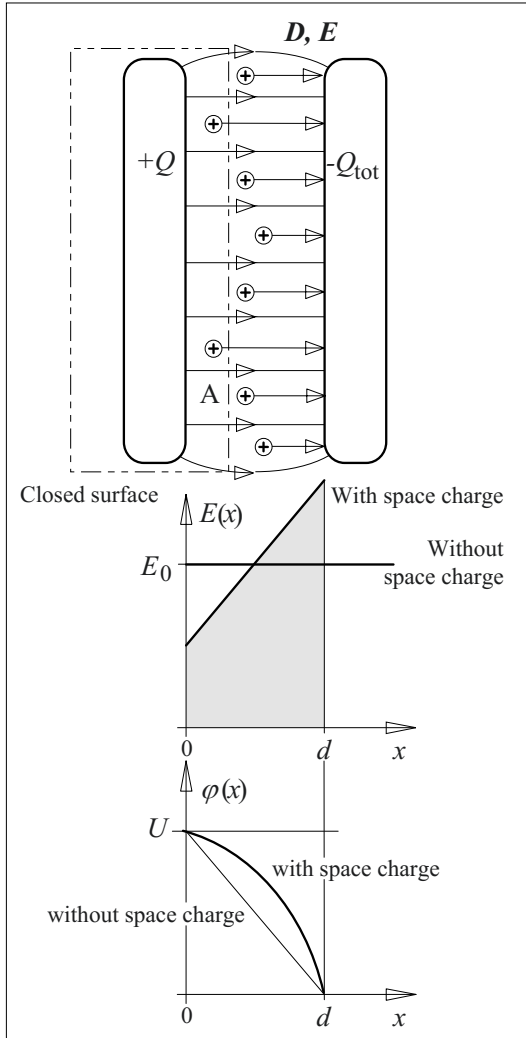


Figure 2.3-6: Space charges in the dielectric of a parallel plate capacitor (see fig. 2.3-5).

2.3.1.5 Field Distortions by Space Charges

Electric discharges in gaseous dielectrics can generate “*space charge clouds*” which strongly modify („distort“) the local electric field. Space charges can also be generated in liquid and solid insulating materials under the influence of electrical stresses.

Example: Space charge in a parallel-plate capacitor

The influence of space charges and the basic calculation method are explained for the example of a uniform field with constant and positive space charge density η (volume density of charge). The counter-charges are assumed to be on the negative electrode, Figure 2.3-6:

During the application of Gauss’s law (eq. (2.3-1), Step 1) it has to be considered that the total enclosed charge within the closed surface is dependent on the actual position of the surface between the electrodes. The total charge consists of the sum of the charges on the enclosed electrode and the (space) charges within the enclosed insulating volume:

$$\begin{aligned} Q(x) &= Q + \iiint \eta \, dV \\ &= Q + \eta A x \\ &\stackrel{!}{=} D \iint dA = D A = \epsilon E A. \end{aligned}$$

This means that the magnitude of the *electric field strength* E between the electrodes is *no longer constant*; it increases linearly with x and it becomes maximal at the counter-electrode at $x = d$, Figure 2.3-6:

$$E(x) = Q(x)/(\epsilon A) = Q/(\epsilon A) + x \eta/\epsilon \quad (2.3-29)$$

The further calculation can be performed in analogy to the steps described above, but it is to be considered that the total stored charge Q_{tot} consists of the positive

charge Q on the electrode and the positive charge $||| \eta$ $dV = dA \eta$ stored in the dielectric.

$$Q_{\text{tot}} = Q + d \cdot A \cdot \eta$$

Thereby the capacitance $C = Q_{\text{tot}}/V$ is increased. The potential no longer decreases linearly with x , but is described by a second order polynomial, which can be derived by the integration of Eq. (2.3-29), Figure 2.3-6.

In **non-uniform fields**, e.g. in spherically or cylindrically symmetric fields, space charges can *increase or decrease the (geometric) inhomogeneity* (non-uniformity) of the field, depending on polarity. Thereby the discharge behavior in non-uniform fields is strongly influenced (*“polarity effect”*, Section 3.2.5.2).

Note: Especially in the insulation of a capacitor consisting of layers with different materials (Section 2.4), the space charges stored in the material and the surface charges at the interfaces can cause a dangerous *“re-charging”* of the electrodes and a *“recovery voltage”*, even after a temporary short circuit of the electrodes (Section 2.4.4.3). Therefore, capacitors must be **short-circuited permanently**.

2.3.2 Analytic Solution of Poisson’s Equation

The continuity of the displacement density is described by Eq. (2.3-1) in *integral* form. The equivalent in differential form is

$$\text{div } \mathbf{D} = \eta. \quad (2.3-30)$$

According to Eq. (2.1-8) the field strength is

$$\mathbf{E} = - \text{grad } \varphi$$

and **Poisson’s Equation** can be derived. It can be used for **electrostatic (irrotational) fields**:

$$\text{div grad } \varphi = \nabla^2 \varphi = \Delta \varphi = - \eta/\varepsilon$$

$$(2.3-31)$$

Note: Poisson’s Equation is called **Laplace’s Equation** for $\eta = 0$.

The *differential operators* div (divergence), grad (gradient), ∇ (nabla, Hamilton’s operator, del operator) and Δ (delta, Laplace’s operator)

are expressed differently depending on the *coordinate system* (Cartesian, cylindrical and spherical coordinates) [2], [3], [6]. Poisson’s Equation is written

in *Cartesian coordinates* (x, y, z)

$$\Delta \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = - \eta/\varepsilon, \quad (2.3-32)$$

in *cylindrical coordinates* (r, α, z)

$$\begin{aligned} \Delta \varphi &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \varphi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \alpha^2} + \frac{\partial^2 \varphi}{\partial z^2} \\ &= - \eta/\varepsilon \end{aligned} \quad (2.3-33)$$

and in *spherical coordinates* (r, α, ϑ)

$$\begin{aligned} \Delta \varphi &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \varphi}{\partial r} \right) + \frac{1}{r^2 \sin \vartheta} \frac{\partial}{\partial \vartheta} \left(\sin \vartheta \frac{\partial \varphi}{\partial \vartheta} \right) \\ &\quad + \frac{1}{r^2 \sin^2 \vartheta} \frac{\partial^2 \varphi}{\partial \alpha^2} \\ &= - \eta/\varepsilon \end{aligned} \quad (2.3-34)$$

Note: The derivation of Eq. (2.3-32) to (-34) is omitted and referred to the literature [2], [3], [6].

The *application of Poisson’s Equation* shall be exemplified for the uniform field of a parallel-plate capacitor without space charges, Figure 2.3-5. Nevertheless, all the other cases in Section 2.3.1 can be calculated.

Example: Uniform field without space charges

Step 1: At first, *Poisson’s Equation* is simplified to *Laplace’s Equation* ($\eta = 0$) which is only dependent on the variable x . Naturally, Cartesian coordinates are used here, with $\varphi(x, y, z) = \varphi(x)$. Now Eq. (2.3-32) is

$$\Delta \varphi = \partial^2 \varphi / \partial x^2 = - \eta/\varepsilon = 0.$$

Step 2: The simplified *differential equation* is solved in *general form*, by two integrations in this case:

$$\partial \varphi / \partial x = k_1 \quad \text{und} \quad \varphi(x) = k_1 x + k_2.$$

Step 3: The integration constants k_1 and k_2 are determined from the *boundary conditions*. From

$$\varphi(x=0) = V \quad \text{we obtain} \quad V = 0 + k_2 \quad \text{and from}$$

$$\varphi(x=d) = 0 \quad \text{we obtain} \quad 0 = k_1 d + k_2.$$

With the solutions $k_2 = V$ and $k_1 = -V/d$ the potential is

$$\varphi(x) = V(1 - x/d)$$

Step 4: For a given potential distribution, the electric field is defined unequivocally. The vector of the electric field strength E can be obtained by the *calculation of the gradient* according to Eq. (2.1-8). In a uniform field with Cartesian (x, y, z)-coordinates the result

$$E = -\text{grad } \varphi = \{-\partial\varphi/\partial x, 0, 0\} = \{U/d, 0, 0\}.$$

gives a constant magnitude of the field strength

$$E = V/d = E_0 = \text{const.} \quad \text{q.e.d.}$$

Note: If Poisson's Equation is evaluated in *cylindrical or spherical coordinates*, the calculation of the gradient (the field vector resp.) must also be performed in cylindrical or spherical coordinates, Eq. (2.1-8), [2], [3], [6]. According to the steps described above, the symmetries of the configuration should be used for the *simplification* of Poisson's Equation. The *general solution of the differential equation* gives a general expression for the *potential* distribution and the constants of integration have to be determined by means of the *boundary conditions*. The electrostatic field strength is derived from the potential distribution by calculation of the *gradient*.

2.3.3 Graphical Field Mapping (for Plane Fields)

Normally, technical field configurations in high voltage engineering differ more or less from the basic configurations discussed in the former chapters. Therefore it is helpful to draw qualitative distributions of *field lines and equipotential lines* graphically, i.e. free-hand, just by approximation and without any complicated calculations.

If some *drawing rules* are regarded, a **field map or field pattern** for a *plane (two-dimensional) configuration* can be created. It gives a qualitative impression of the electrical stress, but with appropriate care it is often also possible to roughly determine field strengths and capacitances.

Graphical field mapping gives a good impression of the distribution of field lines and equipotential lines. Therefore, it can be used to support basic physical understanding and to perform plausibility checks of numerically calculated field distributions, i.e. to exclude coarse calculation mistakes.

The **value of the graphical mapping** lies in the *rapid creation of a qualitative overview map*, which cannot replace a numerical calculation, but which can prepare for and supplement it. Furthermore, graphical mapping requires a thorough analysis of the field geometry. Thereby a *valuable and deep understanding* of the physical character of the electrical stress is created.

The **drawing rules** are deduced from the properties of field lines and equipotential lines. (Frequently just referred to as "potential lines"). At first, a *plane, two-dimensional field* is discussed, which does not change in the third dimension and can be drawn in the drawing plane, Figure 2.3-7:

- 1.) Field lines and equipotential lines are **orthogonal (rectangular)** to each other.
- 2.) **Electrode surfaces** are equipotential surfaces (normally reference and high voltage potentials are 0 % and 100 %).
- 3.) Field lines and electrode surfaces are orthogonal (deduced from 1. and 2.).
- 4.) The distance a between two equipotential lines is always related to the same potential difference ΔV . The distance b between two field lines (or displacement lines) is always related to the same charge ΔQ on the electrodes, i.e. to the same displacement flux. From this it follows that the element capacitance $\Delta C = \Delta Q/\Delta V$, which is related to every "box" or element with a length z , is equal for all "boxes" (elements) on the field map:

$$\Delta C = \Delta Q/\Delta V = \varepsilon z b/a = \text{const.} \quad (2.3-35)$$

This means that the **aspect ratio** b/a is equal for all elements:

$$b/a = \text{const.}$$

(2.3-36)

The best way for field mapping is to draw *square elements*, i.e. if $b/a = 1$ is chosen. The aspect ratio is correct, if the four sides

of the square element touch an **inscribed circle**, Figure 2.3-7.

Traditional tools of graphic field mapping are *blank paper, pencil and eraser (rubber)*; also good choices are simple graphics programs which make it easier to improve the map iteratively.

It is advisable to *begin* with the drawing in an area where the *potential distribution is known*. The *electrode contours* give an orientation for the distribution of the *equipotential lines*.

As a first approximation, *field lines* are drawn orthogonally to the equipotential lines and the electrode contours. The aspect ratios of the resulting boxes have to be constant according to Eq. (2.3-36). *Irregularities* with respect to the *drawing rules* (1. to 4.) indicate how the actual map has to be *improved* by repositioning of field lines and equipotential lines.

In practice there will often need to be a greater number of *iteration steps* in order to achieve a satisfactory result.

The graphical method of field mapping shall be explained for the practically important example of the fields at the edges of a parallel-plate capacitor, Figure 2.3-8 and 2.3-9:

Example: Edge field of a parallel-plate capacitor

Step 1 (Figure 2.3-8a): At first, the known *potential distribution* in the uniform or known part of the field is drawn (1). The further course of the *equipotential lines* is approximately orientated with the given electrode contours (2).

Note: It is advisable, to start with a small number of equipotential lines only (e.g. lines for 0 %, 25 %, 50 %, 75 % and 100 %). The completed map can be further refined by interpolation if necessary.

Step 2 (Figure 2.3-8a): The map is supplemented by *field lines* rectangular to the equipotential lines, with the aspect ratio $b/a = 1$. It is advisable to proceed along an electrode contour (e.g. on the high voltage side).

The *inscription of circles* shows whether the aspect ratios of the boxes differ significantly from the desired value 1 in cases (3).

Step 3 (Figure 2.3-8b): Now the *initial map is improved*. In this example the distance between the 25 %-line and the lower electrode is increased (4). The 75 %-

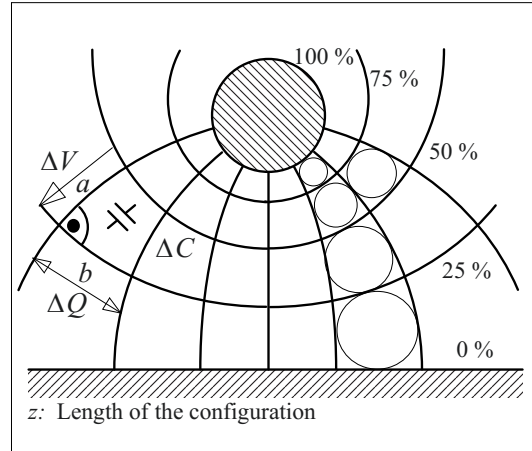


Figure 2.3-7: Graphic mapping of field lines and equipotential lines for two-dimensional fields.

line is shifted closer to the upper electrode (inner side and edge), the distance to the outer side is increased considerably (5). It must be taken in to account that the field strength in the electrode edge region must decrease from the upper electrode towards the lower electrode, i.e. the distance of the equipotential lines increases.

A *check* of the aspect ratios and the angles shows that the field map still needs to be improved.

Step 4 (Figure 2.3-8c): The final field pattern is obtained by *iterative approximations* with respect to the drawing rules.

In the given example it is advisable to start with the inscribing of circles in the region with the uniform field and to proceed towards the regions with non-uniform fields (6). Position and direction of the equipotential lines and the field lines and the diameters of the inscribed circles have to be adjusted iteratively and *step-by-step*.

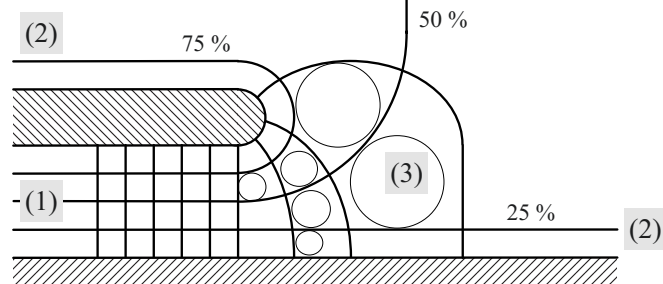
The *evaluation* of a completed field pattern provides approximate information about the *location and magnitude of maximum field strength*, the *field strength profiles* along contours and the *capacitance* to be assigned to the electric field.

The field strength for any element of the field pattern is

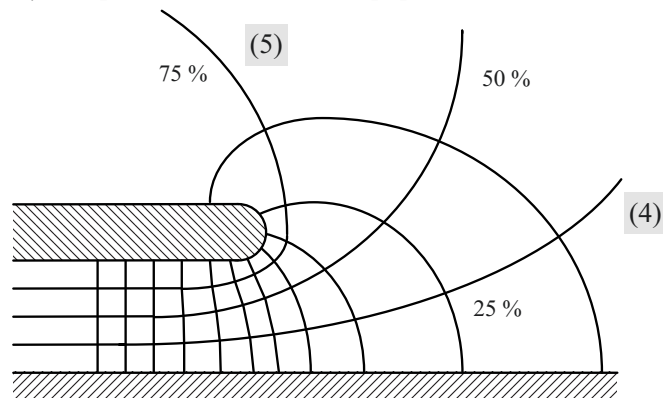
$$E \approx \Delta V/a. \quad (2.3-37)$$

ΔV is the potential difference and a is the distance between two equipotential lines for the considered element ("box"). E is the medium field strength in the element ("box"), its accuracy depends on the accuracy of the drawing.

a) Rough approximation of field lines and equipotential lines



b) Improved field lines and equipotential lines



c) Further improved field lines and equipotential lines

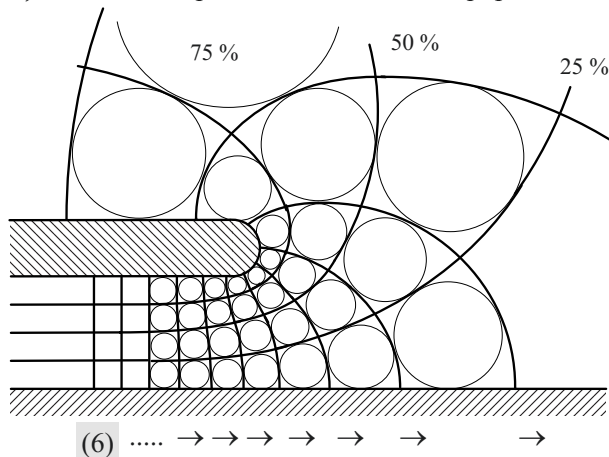


Figure 2.3-8:

Graphical mapping of field lines and equipotential lines for the two-dimensional (plane) edge field of a parallel-plate capacitor in different stages of iteration:

a) A first rough approximation which does not match the drawing rules in many items.

b) Improved map according to the mismatch in a former step.

c) Further improved map generally matching the drawing rules.

For qualitative conclusions, iteration c) is often sufficient.

Field strengths, which are taken from graphic field patterns, have to be *handled with care*. In general, accurate quantitative values require a numerical or an analytical analysis if possible.

The whole field volume can be regarded as a series and parallel connection of equal element capacitances ΔC , Figure 2.3-7. The number of parallel branches n_p and the number of series connections n_s can be counted from the field map. According to Eq. (2.3-35) and with $b/a = 1$ the total capacitance is

$$C_{\text{tot}} = \Delta C n_p / n_s = \varepsilon z n_p / n_s \quad (2.3-38)$$

Often the determination of capacitance is possible with less inaccuracy. Capacitance is an integral quantity and Graphical inaccuracies compensate each other as a result of the integral view of the entire field space.

Example: Edge field of a parallel-plate capacitor (continued)

Step 5 (Figure 2.3-8c and 2.3-9): The point of maximum field strength is at the inner side of the electrode curvature. The magnitude of maximum field strength is

$$E_{\text{max}} \approx \Delta V / a_{\text{min}} = 0.25 V / a_{\text{min}}$$

The minimum distance a_{min} between the 100 % and the 75 % equipotential line is nearly half that in the uniform region of the field. Therefore, the field stress enhancement factor is approximately 2. Indeed, the real maximum field strength will be somewhat higher because the field strength is not constant within the smallest square element, and the measurement according to Eq. (2.3-37) gives a *medium value* for the element only. A field strength profile along the 100 % electrode contour can be determined from the field pattern with Eq. (2.3-37), Figure 2.3-9.

The capacitance of the ideal parallel-plate capacitor $C_0 = \varepsilon A/d$ has to be increased by an additional edge-field capacitance C_{edge} : $C_{\text{tot}} = C_0 + C_{\text{edge}}$. C_{edge} is calculated with Eq. (2.3-38) for $z = 1$ m in air and for the region which is displayed in Figure 2.3-8c (i.e. just the region with the “circles”, but only at the curvature and at the outer side):

$$C_{\text{edge}} \approx \Delta C n_p / n_r = \varepsilon z n_p / n_r = \varepsilon z 5/4 \approx 11 \text{ pF}.$$

The described graphical method can be used for plane two-dimensional fields. Nevertheless, it can also be applied to **rotationally**

symmetric fields, which are two-dimensional as well. For the example in Figure 2.3-7, it is now assumed that there is a horizontal axis of rotation at the lower line in the Figure. Thereby the rod-shaped elements ΔC with the length z are transformed into circular, ring-shaped elements with the circumference $2\pi r$:

$$\Delta C = \varepsilon 2\pi r b/a$$

Because of $\Delta C = \Delta Q / \Delta V = \text{const.}$, the aspect ratio of the elements is

$$b/a = \text{const.}/r. \quad (2.3-39)$$

Therefore, the aspect ratio b/a has to be adjusted *depending on* r , proportional to $1/r$. Thus, the accurate drawing of a field image is much more difficult.

Graphical field mapping can also be applied to **configurations with several dielectrics** (Section 2.4). Additionally to the drawing rules described above, it is necessary to consider the “*refraction laws*” for field lines and equipotential lines at the interfaces between insulating materials, Figure 2.4-10 and 2.4-25.

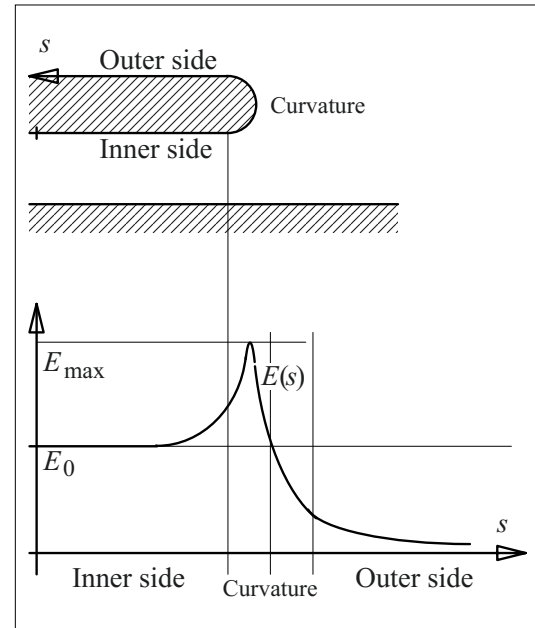


Figure 2.3-9: Qualitative profile of field strength magnitude along the 100 % electrode contour (coordinate s).

For **three-dimensional fields** only rough qualitative drawings are possible without any quantitative information. In general three-dimensional field lines do not lie in a drawing plane, they penetrate it normally. Therefore it is not possible to draw the field lines in a plane. A *two-dimensional field map* has to be restricted to the *equipotential lines*, which are the intersecting lines between the equipotential surface and the drawing plane. Meaningful field patterns can only be calculated with numerical field calculations (Section 2.5).

Nevertheless, *rough sketches* are valuable to support the engineer's imagination and physical understanding, but they must not be overestimated.

2.3.4 Conformal Mapping (for Plane Fields)

Conformal mapping is a method for the *analytic* calculation of some important *two-dimensional high-voltage fields*.

Note: Conformal mapping was of high importance before numerical field calculation methods were available. Today it is of *historical* interest mainly: There are only a few important field configurations still based on conformal mapping.

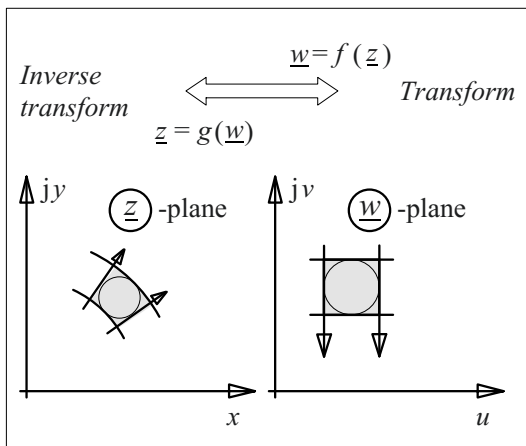


Figure 2.3-10: Conformal mapping of field lines and equipotential lines from the complex z -plane into the w -plane.

The basic idea of conformal mapping is to *transform* the x,y -plane (together with a given complicated electrode configuration) into a u,v -plane, where the electrode configuration can be *calculated easily*. Afterwards the solution is transformed inversely, back into the x,y -plane, Figure 2.3-10.

For this purpose, the geometric x,y -plane is regarded as a complex z -plane ($z = x + jy$) and the geometric u,v -plane as a complex w -plane ($w = u + jv$). Thereby, the two geometric axes are replaced by a real and an imaginary axis. The so called **conformal mapping** is performed by a complex function

$$w = f(z)$$

or

$$u + jv = f(x + jy).$$

It maps the points from the z -plane onto the w -plane. The complex function has two important properties [2], [3], [6], Figure 2.3-10:

- Figures of *finite size* may be subject to deformations by conformal mapping, but the **angles** between curves, and hence the orthogonality of field lines and equipotential lines is preserved, i.e. conformal mapping is **isogonal**.
- Figures** of sufficiently *small size* **preserve their shape**, and infinitesimal elements described by field lines and equipotential lines preserve their aspect ratio during the transformation.

Note: These statements are not valid for the *origin*, which is a *singularity (pole)*.

This means that *potential fields*, which are calculated in the z -plane, preserve their potential field character during the *transformation* onto the w -plane and vice versa, Figure 2.3-10. Nevertheless, the macroscopic field pattern may be deformed.

Note: Mathematically, every regular function of a complex quantity $f(z) = f(x+jy)$ fulfills Laplace's Equation (2.3-22) for the two-dimensional case, i.e. if the space charge η is zero:

$$\begin{aligned}
 \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 &= \\
 f''(\underline{z}) (\partial \underline{z} / \partial x)^2 + f''(\underline{z}) (\partial \underline{z} / \partial y)^2 &= \\
 f''(\underline{z}) 1^2 + f''(\underline{z}) j^2 &= \\
 f''(\underline{z}) - f''(\underline{z}) &= 0
 \end{aligned}$$

q.e.d.

From

$$f(x+jy) = \underline{w} = u(x,y) + j v(x,y)$$

it is further concluded that

$$\begin{aligned}
 \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 &= \\
 (\partial^2 u / \partial x^2 + j \partial^2 v / \partial x^2) + (\partial^2 u / \partial y^2 + j \partial^2 v / \partial y^2) &= \\
 (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) + j (\partial^2 v / \partial x^2 + \partial^2 v / \partial y^2) &= 0.
 \end{aligned}$$

This equation can only be fulfilled, if real and imaginary parts are both zero. This means that the functions $u(x,y)$ and $v(x,y)$ are both *solutions of Laplace's Equation*.

If the curves in the $\underline{w}$ -plane, which are defined by $v = \text{const.} \sim \varphi$, are regarded as *equipotential lines* (Figure 2.3-10 right), the function $\varphi(x,y) \sim v(x,y) = \text{const.}$ defines the potential distribution in the x,y -plane. The orthogonal curves with $u = \text{const.}$ can then be regarded as *field lines*, Figure 2.3-10.

Example: The function $\underline{w} = \underline{z}^2$

The function $\underline{w} = \underline{z}^2 = (z \cdot e^{j\alpha})^2 = z^2 \cdot e^{j2\alpha}$ doubles the angles α of all complex vectors $\underline{z}$ emanating from the null point. The function can therefore be used to transform an **electrode consisting of two orthogonal walls** in the

x,y -plane ($\alpha = 90^\circ$) into a plane electrode in the u,v -plane ($2\alpha = 180^\circ$), Figure 2.3-11.

In the u,v -plane, the field above a plane electrode is a *uniform field* and the potential increases linearly with voltage V and with the distance v (k is a constant):

$$\varphi = v V k$$

This is the equation for equipotential lines in the w -plane. The relation between w - and z -plane is given by

$$\underline{w} = \underline{z}^2 = (x + jy)^2$$

and

$$u + jv = (x^2 - y^2) + jxy.$$

Therefore, the equipotential lines ($v = \text{const.}$) are *hyperbolas* in the x,y -plane, symmetric with respect to the bisecting line between x - and y -axis.:

$$\varphi \sim v = xy = \text{const.}$$

For the field lines ($u = \text{const.}$) there are hyperbolas symmetric with respect to x - and y -axis.:

$$u = x^2 - y^2 = \text{const.}$$

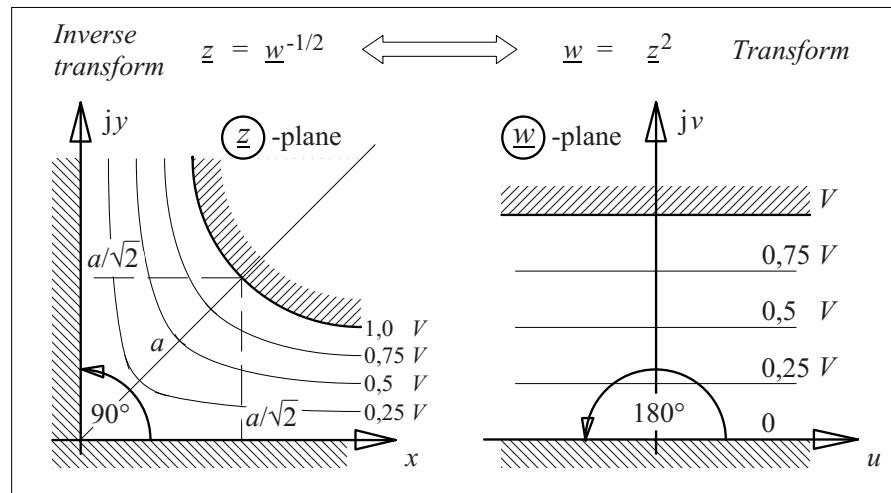
The potential distribution in the x,y -plane is

$$\varphi = v V k = xy V k.$$

In the x,y -plane, the potential of the rectangular electrodes is set to zero (reference electrode). The equipotential line with the diagonal distance a from the origin is selected as a counter-electrode with the potential $\varphi = V$. The constant $k = 2/a^2$ is determined with the boundary condition $\varphi = V$ for $x = y = a/2^{1/2}$:

$$\varphi = xy V 2/a^2$$

Figure 2.3-11:
Conformal mapping
of field lines and
equipotential lines
for a rectangular
electrode: $\underline{w} = \underline{z}^2$



The electric field strength E is the gradient of φ :

$$E = -\text{grad } \varphi = -\{\partial\varphi/\partial x, \partial\varphi/\partial y, \partial\varphi/\partial z\}$$

$$E = -V/2a^2 \{y, x, 0\}$$

The magnitude is

$$E = 2V (\sqrt{x^2 + y^2})/a^2.$$

In the *inner corner* of the reference electrode ($x \rightarrow 0$, $y \rightarrow 0$) there is no longer any field strength, i.e. $E \rightarrow 0$.

At the surface of the *hyperbolic high voltage electrode* in the axis of symmetry ($x = y = a/2^{1/2}$) the field strength is $E = 2V/a$, i.e. twice as high as in a uniform field with the same electrode distance a . However, field strengths increase further outside the axis of symmetry.

The situation close to the axis of symmetry is comparable with a curved conductor (e.g. a tubular conductor) in the corner of a building.

Generally it is difficult to find a function which transforms a *given configuration* into a calculable basic configuration. Therefore a different technique is used: starting from *given functions* $\underline{w} = f(\underline{z})$, one investigates field configurations that arise in the x, y plane.

In this way a large number of technically relevant configurations could be calculated analytically. Meanwhile, any field configuration can be directly calculated numerically (Section 2.5). Therefore, it is not necessary to discuss all of the many special cases that are more or less suitable for conformal mapping; they can be found in the literature [2], [3], [4], [16], [17].

Figure 2.3-12 shows some calculable configurations and the related transforms. Some are discussed in the following.

Example: Conductor bundle

In order to reduce the field strengths at the surfaces of conductors of high voltage overhead lines (for the voltage levels $V_m = 245$ kV and above), single conductors are normally replaced by conductor bundles.

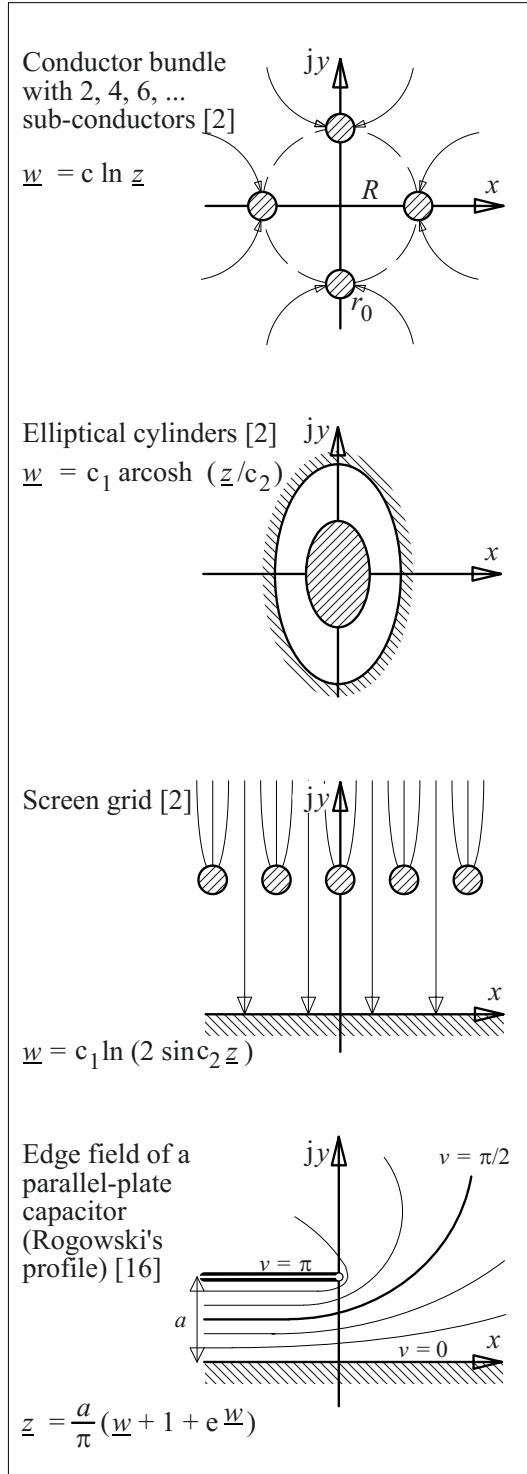


Figure 2.3-12: Examples for two-dimensional fields, which can be calculated by conformal mapping. (see the literature).

A conductor bundle consists of n parallel sub-conductors with the radius r_0 . They are uniformly distributed on a circle with the radius R , and they have the same potential, Figure 2.3-12 top. An *equivalent radius* R' for a single cylindrical conductor with the *same capacitance* against a distant counter-electrode is calculated by means of the function $\underline{w} = \ln \underline{z}$ for small sub-conductor radii $r_0 \ll R$ [2]:

$$R' = R (n r_0 / R)^{1/n} \quad (2.3-40)$$

If two identical conductor bundles are led in parallel with the distance a , the charge can be determined from the capacitance calculations according to Section 2.3.5.3. The maximum field strength at the conductor surface is deduced from this [2]:

$$E = V / \{2 r_0 n \ln(a/R')\} \quad (2.3-41)$$

The reduction ratio of field strength E to field strength E_0 at the surface of a single conductor with the radius r_0 is

$$E/E_0 = \ln(a/r_0) / \{n \ln(a/R')\}. \quad (2.3-42)$$

These equations are also valid for conductor bundles which are led at a distance h above a conducting plane (ground), if a and V are replaced by $a = 2h$ and $V = 2 \Delta V_{L/G}$. This plane is then regarded as a plane of symmetry in a configuration with two parallel conductor bundles, see also Section 2.3.5.3.

Example:

Edge field of a parallel-plate capacitor

The *field at the edge of a parallel-plate capacitor* is especially important for high voltage engineering, Figure 2.3-12 bottom. During the transformation into the $\underline{w}$ -plane a uniform field is achieved by a clockwise rotation of the upper electrode surface by 180° about an axis laying in the edge of the electrode. The equipotential lines in the uniform field of the $\underline{w}$ -plane are defined by the equation $v = \text{const.}$

In the $\underline{z}$ -plane there are equipotential lines, which are bent around the edge of the *upper*

electrode. Close to the edge the field strength is strongly enhanced; it approaches an infinite magnitude at the edge itself. At the *lower electrode* the field strength decreases in the outward direction.

The volume between the electrodes is divided into two parts by the equipotential surface that is represented by the line $v = \pi/2$, the so called **Rogowski profile**: In the *upper volume* the equipotential lines curve around the upper electrode and field strengths along these lines reach local maxima $E > E_0 = V/a$. In the *lower volume* the field strength never exceeds the value of the uniform field $E \leq E_0 = V/a$. Electrodes which are shaped according to the Rogowski's profile (**Rogowski electrodes**) are of high importance in high voltage engineering, because field stress enhancements at the electrode edges can be avoided with them. An important application is the testing of the *breakdown strength* of materials.

Along Rogowski's profile, the field strength continuously decreases in the outward direction. Therefore the curvature of the electrode could be increased further in order to achieve a more compact construction. This is required for higher voltages with very large electrodes. The smallest dimensions are achieved if the field strength is kept constant along the curved electrode: $E = E_0$. The corresponding contour is referred to as "**Borda's profile**", which can also be determined by conformal mapping or by iterative numerical optimization. The **Borda's profile** is a spiral contour with a continuously decreasing radius of curvature. An important application is *field grading in cable entrance fittings and cable joints*, Figure 2.4-36, 7.1.1-4 and -5.

Note: Electrodes with a contour shaped according to *Borda's* profile are frequently but *imprecisely* referred to as *Rogowski electrodes*.

In many practical cases it is often sufficient to use a cylindrical electrode edge with a *constant radius of curvature* R greater than the electrode distance d :

$$R > d \quad (2.3-43)$$

The distance d is the distance between two identical rounded plate electrodes, see Figure 2.3-5. Therefore, the distance in the Figures 2.3-8 and 2.3-9 is $d/2$.

Note: The **field stress enhancement** is estimated for this case, if a cylindrically symmetric field with $R_1 = R$, $R_2 = R + d/2$ and $R = d$ is assumed according to Eq. (2.3-22) and (2.3-28). The result $E_1/E_0 = 2.47$ strongly overestimates the field stress enhancement, which is significantly lower. A better result is achieved, if an increased “effective outer radius” $R_2 = 2R + d/2$ is assumed at the edge of the electrode, Figure 2.3-8c). The result $E_1/E_0 = 1.1$ is consistent with the real field stress enhancement which can also be estimated by *Schwaiger's field efficiency factor*, Section 2.3-6 [22].

These estimations show that results are strongly dependent on the simplifying assumptions. *Estimations* cannot generally replace exact analytical or accurate numerical calculations. Similar to the graphical field mapping (Section 2.3.3), however, field strength estimations are a good tool to get a feeling for the orders of magnitude.

2.3.5 Charge Simulation Method

The Charge Simulation Method (CSM) is another traditional tool for the analytical calculation of specific high voltage engineering field configurations. It can also be used for the numerical calculation of any electrostatic field (Section 2.5).

The Charge Simulation Method is directly based on the physics of electrostatic fields, whereby source fields are generated by **superposition of the fields of many single charges**. However, these *equivalent charges* are not positioned on the electrode surfaces where they should be in reality. Instead, one proceeds from simplified charge distributions (point charges, line charges, ring charges etc.) and selects the *electrode contours* from the resulting equipotential surfaces afterwards. Therefore, the method is sometimes called the **“indirect method”** [67]. The charges represent only a limited number of *equivalent charges*, giving the same electrostatic field between the electrodes as an infinite number of

real charges. In English the term **„charge simulation method”** (CSM) is common, but the method is also called **“equivalent charge method”**.

In the following chapters important high voltage engineering fields are discussed, i.e. fields in the vicinity of two point charges (Section 2.3.5.1), between two conducting spheres (Section 2.3.5.2), in the vicinity of two line charges in parallel (Section 2.3.5.3) and between conducting cylinders (Section 2.3.5.4). Because of their symmetries, these cases also include the fields between spheres or cylinders and conducting planes [2].

This allows some important field configurations to be calculated analytically, e.g. spherical screening electrodes beside flat walls, spherical spark gaps (measuring spark gaps), cylindrical conductors beside flat walls and overhead lines.

2.3.5.1 Conducting Spheres (Point Charges)

The fields in the vicinity of conducting spheres result from the *superposition of the fields or potentials* of point charges.

The potential in the vicinity of a **single point charge** Q is derived from the electric field strength according to Eq. (2.3-2) by integration from r to ∞ :

$$\varphi(r) = \frac{Q}{4\pi\epsilon} \cdot \frac{1}{r} \quad (2.3-44)$$

The reference potential is $\varphi(\infty) = 0$. The counter charge $-Q$ is assumed for $r = \infty$. The field of **two point charges** Q_1 and Q_2 results from the superposition of the potentials; r_1 and r_2 are the distances from the point charges Q_1 and Q_2 to a reference point P at which the potential φ is formed by superimposing the potentials $\varphi_1 + \varphi_2 = \varphi$, Figure 2.3-13:

$$\varphi(r) = \frac{1}{4\pi\epsilon} \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right) \quad (2.3-45)$$

Example: “Image charges” with equal magnitude and opposite polarity

If the charges $Q_1 = Q$ and $Q_2 = -Q_1 = -Q$ have the same magnitude but opposite polarity, the condition for equipotential surfaces is

$$\varphi(r) = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \text{const.} \quad (2.3-46)$$

The counter-charges $-Q_1 = -Q$ and $-Q_2 = +Q$ are infinitely far away and compensate each other in this case. Because of the mirror-symmetrical position of the charges $Q_1 = +Q$ and $Q_2 = -Q$ with respect to the plane equipotential surface $\varphi = 0$, the terms “image charges” and “surface of reflection” can be used.

According to Eq. (2.3-46) the potential $\varphi = 0$ results for infinite radii r_1 and r_2 and for finite radii if $r_1 = r_2$, i.e. for the plane of symmetry orthogonal to the connecting line between the two charges, Figure 2.3-14.

For the actual positions of the charges the potentials

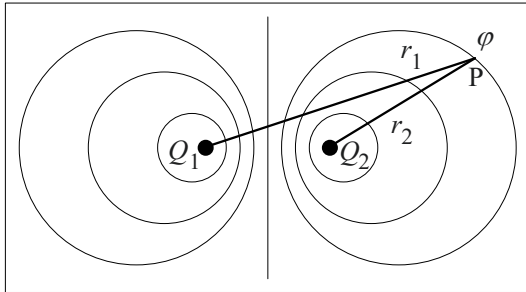


Figure 2.3-13: Superposition of electrostatic fields, which are generated by two point charges.

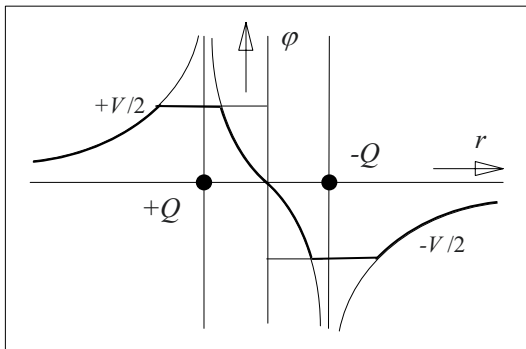


Figure 2.3-14: Potential profile on the connecting line between two point charges $+Q$ and $-Q$ (see also fig. 2.3-13). $+V/2$ and $-V/2$ are the potentials of two equipotential surfaces, which were selected as electrode contours (potential difference V).

theoretically approach infinite positive and negative values ($r_1 = 0$ or $r_2 = 0$). This is the extreme case of the so called *point charge*.

For the calculation of practical electrode configurations two equipotential surfaces (e.g. with $\varphi = +V/2$ and $\varphi = -V/2$) have to be selected, which serve as electrode surfaces. In the given example, however, these surfaces are *not* spherical.

For two charges Q_1 and Q_2 with opposite polarity and *different* charge amounts the equipotential surface $\varphi = 0$ shall be determined. Obviously, this will not be the geometric plane of symmetry (as in the former example). Nevertheless, in a more general sense Q_1 and Q_2 are referred to as “image charges” and the surface $\varphi = 0$ as “surface of reflection”.

With Eq. (2.3-45) the condition for the surface of reflection $\varphi = 0$ is

$$Q_1/r_1 + Q_2/r_2 = 0$$

and

$$r_1/r_2 = Q_1/(-Q_2) = k. \quad (2.3-47)$$

In the case of opposite polarities the ratio of the charges $k = Q_1/(-Q_2)$ is a positive quantity and Eq. (2.3-47) describes a *circle equation*. This will be shown in the following, Figure 2.3-15. The center point (midpoint) M of the circle is chosen as the origin of a Cartesian x,y -coordinate system.

From Eq. (2.3-47) and Figure 2.3-15 we find

$$k^2 = \frac{r_1^2}{r_2^2} = \frac{(a+b+x)^2 + y^2}{(b+x)^2 + y^2}$$

This equation is rearranged in order to constitute the circle equation in the x,y -coordinate system:

$$k^2(b+x)^2 + k^2y^2 = a^2 + 2a(b+x) + (b+x)^2 + y^2$$

$$(k^2-1)[(b+x)^2 + y^2] = a^2 + 2ab + 2ax$$

$$(k^2-1)[x^2 + y^2] = a^2 + 2ab + 2ax - (k^2-1)[b^2 + 2bx]$$

$$x^2 + y^2 = \frac{a^2 + 2ab}{k^2 - 1} - b^2 + \frac{2a \cdot x}{k^2 - 1} - 2b \cdot x = \frac{a^2 k^2}{(k^2 - 1)^2}$$

This expression describes a *circle* around the coordinate origin M, if the term containing x (on the right side of the equation) is zero. The first term, which is independent of x and y , is then equal to the square of the radius r_0 :

$$x^2 + y^2 = r_0^2 + 0$$

Two equations for the *position b of the equivalent charge Q_2* and for the *radius r_0 of the equipotential line* are defined from this. Then b is derived from the second term containing x , and this term is set to zero:

$$b = \frac{a}{k^2 - 1} \quad (2.3-48)$$

The first term provides r_0 by inserting b from Eq. (2.3-48):

$$r_0^2 = \frac{a^2 + 2ab}{k^2 - 1} - b^2$$

According to Eq. (2.3-48) b is inserted:

$$r_0^2 = \frac{a^2(k^2 - 1) + 2a \cdot a}{(k^2 - 1)^2} - \frac{a^2}{(k^2 - 1)^2}$$

With $k > 1$ we find for the radius r_0 of the equipotential line $\varphi = 0$ the relation

$$r_0 = \frac{a k}{k^2 - 1} \quad (2.3-49)$$

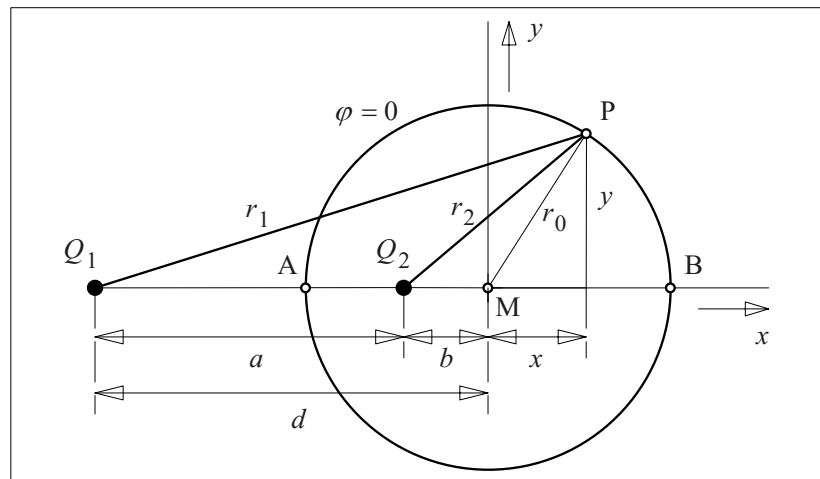
Note: It was shown that the condition $\varphi = 0$ in the x, y -plane corresponds to a *circle* equation according to Figure 2.3-15. This is valid for all planes containing the connecting line between the charges Q_1 and Q_2 . Such planes result from a rotation of the *drawing plane* under consideration about the connecting line (i.e. the x -axis). The *circular equipotential line $\varphi = 0$* is thereby transformed into a *spherical equipotential surface*; the configuration under consideration is spherically symmetric.

The surface of the sphere remains an equipotential surface, if *another charge Q_3* is inserted at the origin M, Figure 2.3-16. The potential at any point P is given by the superposition of the potentials, which are attributed to the three *equivalent charges Q_1, Q_2 and Q_3* :

$$\varphi_P = \varphi_1 + \varphi_2 + \varphi_3 \quad (2.3-50)$$

With Eq. (2.3-44) and $\varphi_1 + \varphi_2 = 0$ we get for all points S on the surface of the sphere

Figure 2.3-15:
Geometric description of the equipotential surface $\varphi = 0$ as circular equation or as spherical surface.



$$\varphi_S = \varphi_3 = \frac{Q_3}{4\pi\epsilon r_0}. \quad (2.3-51)$$

Example: Metallic sphere without charge

An uncharged metallic sphere (radius r_0) is exposed to the field of a point charge Q_1 . The distance d between Q_1 and the center point M of the sphere is given. The following quantities shall be determined:

- (1) Magnitude and position of the equivalent charge,
- (2) Potential of the sphere's surface,
- (3) Potential distribution in the field volume and
- (4) Maximum field strength.

Note: The metallic sphere without charge in an electric field can be regarded as a model of a conducting particle or as an electrode at free potential.

(1) Equivalent charges

A configuration with a spherical equipotential surface can be described by three equivalent charges Q_1 , Q_2 and Q_3 according to Figure 2.3-16. The specification of an uncharged sphere means that the charges Q_2 and Q_3 must be equal in magnitude and opposite in polarity:

$$Q_3 = -Q_2 \quad (2.3-52)$$

Q_2 can be determined with Eqs. (2.3-47) to (-49) from the given quantities Q_1 , r_0 and d :

$$\begin{aligned} d &= a + b \\ (48) \quad &= a + a / (k^2 - 1) \\ &= a k^2 / (k^2 - 1) \\ (49) \quad &= r_0 k \\ (47) \quad &= r_0 (-Q_1/Q_2) \end{aligned}$$

The magnitude of the equivalent charge Q_2 is

$$Q_2 = -Q_1 r_0/d. \quad (2.3-53)$$

The division of Eq. (2.3-48) by Eq. (2.3-49) gives

$$b/r_0 = 1/k = -Q_2/Q_1. \quad (2.3-54)$$

The position b of the equivalent charge Q_2 is calculated with Eq. (2.4-53):

$$b = r_0^2/d \quad (2.3-55)$$

Therefore, the magnitude Q_2 and the position b of the equivalent charge are expressed by the given quantities Q_1 , r_0 and d . Q_2 is the so called "image charge" with respect to Q_1 , the "surface of reflection" is the surface of the sphere in this case. According to Eq. (2.3-52),

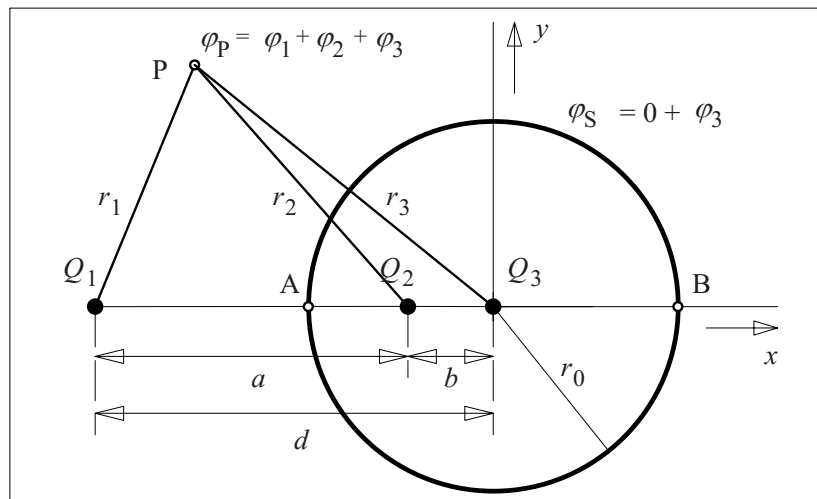


Figure 2.3-16:
Conducting sphere with the
potential φ_S , described by
three equivalent charges.

the equivalent charges Q_3 and Q_2 , have the same magnitude, but opposite polarity.

Note: The considered charges Q_1 , Q_2 and Q_3 cause the *correct field distribution on the outside* of the conducting sphere, but they do not represent the *real charge distribution* on the electrode, therefore they are called **equivalent charges**. On a conducting electrode, real charges are distributed continuously over the surface because of the electrostatic forces. The area charge density equals the local dielectric displacement density, Eq. (2.1-3) and Figure 2.3-17. Only the total charge magnitudes are identical with the equivalent charges.

(2) Potential of the sphere's surface

Prior to the insertion of the charge Q_3 the surface of the sphere has the potential $\varphi = 0$ because of the image charges Q_1 and Q_2 , Figure 2.3-15. With Q_3 and Eqs. (2.3-51) to (-53) the

potential is

$$\begin{aligned}\varphi_S &= 0 + \varphi_3 \\ &= Q_3 / (4\pi\epsilon r_0) \\ &\stackrel{(52)}{=} -Q_2 / (4\pi\epsilon r_0) \\ &\stackrel{(53)}{=} (Q_1 r_0/d) / (4\pi\epsilon r_0) \\ \varphi_S &= Q_1 / (4\pi\epsilon d). \quad (2.3-56)\end{aligned}$$

Therefore, the *potential of the sphere's surface* is *identical* with the potential of the center point M in the field of the point charge Q_1 prior to the insertion of the sphere. This is also valid for any field configuration, which always can be generated from the superposition of single equivalent charges, Figure 2.3-17.

(3) Potential distribution in the field volume

In the field volume on the outside of the conducting sphere, the potential is given by the

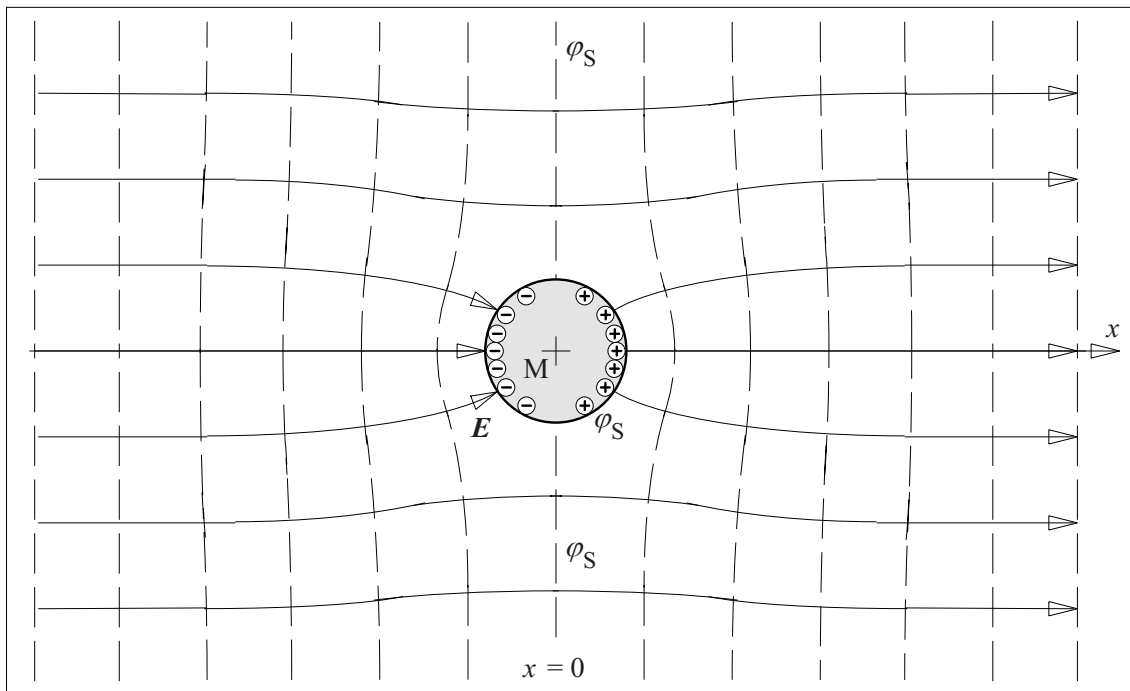


Figure 2.3-17: "Distortion" of a uniform electrostatic field by a conducting sphere without any net charge. The potential of the sphere is equal to the potential of the center point M prior to the insertion of the sphere. The charge distribution in this picture refers to real charge carriers which are distributed continuously over the surface of the sphere. In contrast to this, calculations are performed with a few "equivalent charges", which are located on the x-axis, see fig. 2.3-16.

superposition of the potentials, assigned to the three charges:

$$\varphi_P = \varphi_1 + \varphi_2 + \varphi_3$$

With Eq. (2.3-44) and Figure 2.3-16 we find

$$\varphi_P = \frac{1}{4\pi\epsilon} \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} \right).$$

Q_2 and Q_3 can be replaced by Q_1 according to Eq. (2.3-53) and (-52):

$$\varphi_P = \frac{Q_1}{4\pi\epsilon} \left(\frac{1}{r_1} - \frac{r_0}{dr_2} + \frac{r_0}{dr_3} \right) \quad (2.3-57)$$

In this equation, the distances r_1 , r_2 and r_3 between P and the equivalent charges Q_1 , Q_2 and Q_3 have to be replaced by the coordinates of point P, Figure 2.3-16. Field strengths are calculated from the gradient of the potential according to Eq. (2.1-8).

(4) Maximum field strengths

The maximum field strength occurs on the x -axis for $x = -r_0$ at point A on the surface of the sphere. The electric field on the x -axis has an x -component only; according to Eq. (2.1-8a) it can be determined by the derivate of the potential φ with respect to x .

In the interval $-d < x \leq -r_0$ the potential $\varphi(x)$ is calculated along the x -axis from Eq. (2.3-57) with $r_1 = d + x$, $r_2 = -(b + x)$ and $r_3 = -x$.

The distances r_1 , r_2 and r_3 between P and the equivalent charges are positive quantities (magnitudes) and therefore have to be inserted in the considered interval $-d < x \leq -r_0$ on the negative x -axis according to the preceding definition; x defines the position of the considered point P on the x -axis, Figure 2.3-16:

$$\varphi(x) = \frac{Q_1}{4\pi\epsilon} \left(\frac{1}{d+x} + \frac{r_0}{d} \cdot \frac{1}{b+x} - \frac{r_0}{d} \cdot \frac{1}{x} \right)$$

for $-d < x \leq -r_0$

The x -component of the electric field strength E_x is calculated from the gradient of the potential φ :

$$\begin{aligned} E_x &= -\partial\varphi/\partial x \\ &= \frac{-Q_1}{4\pi\epsilon} \left(\frac{-1}{(d+x)^2} - \frac{r_0}{d(b+x)^2} + \frac{r_0}{dx^2} \right) \end{aligned} \quad (2.3-58)$$

for $-d < x \leq -r_0$

The maximum field strength E is found at $x = -r_0$ at point A. It is equal to the x -component of the field strength E_x at this point:

$$\begin{aligned} E &= E_x = \dots \\ &= \frac{-Q_1}{4\pi\epsilon} \left(\frac{-1}{(d-r_0)^2} - \frac{r_0}{d(r_0^2/d-r_0)^2} + \frac{r_0}{dr_0^2} \right) \\ &= \frac{-Q_1}{4\pi\epsilon} \left(\frac{-1}{(d-r_0)^2} - \frac{d}{r_0(r_0-d)^2} + \frac{1}{dr_0} \right) \\ &= \frac{-Q_1}{4\pi\epsilon} \frac{-dr_0 - d^2 + (d^2 - 2dr_0 + r_0^2)}{dr_0(d-r_0)^2} \\ &= \frac{Q_1}{4\pi\epsilon} \frac{3dr_0 - r_0^2}{dr_0(d-r_0)^2} \end{aligned}$$

After canceling of r_0 , the maximum field strength at point A at $x = -r_0$ is

$$E = E_x = \frac{Q_1}{4\pi\epsilon} \frac{3d - r_0}{d(d-r_0)^2}. \quad (2.3-59)$$

This equation allows one to calculate the *field stress enhancement* caused by the sphere. Prior to the insertion of the sphere, the charge Q_1 causes the electric field strength

$$E_1 = E_{1x} = \frac{Q_1}{4\pi\epsilon} \frac{1}{(d-r_0)^2}. \quad (2.3-60)$$

at point A according to Eq. (2.3-2). With the Eqs. (2.3-59) and (-60), the ratio of the enhanced and the original field strength is

$$\frac{E}{E_1} = 3 - \frac{r_0}{d} \quad (2.3-61)$$

For a nearly *uniform field* E_0 the distance d is $d \gg r_0$ and the field strength is enhanced by a **factor of 3**, Figure 2.3-18:

$$E = 3 E_0 \quad (2.3-62)$$

Note: This result shows that *conductive particles* can cause dangerous field stress enhancements in insulations. Very often real particles differ more or less from the ideal spherical shape and therefore cause *significantly higher field stress enhancements*. Clean processing of insulating materials for the *prevention of con-*

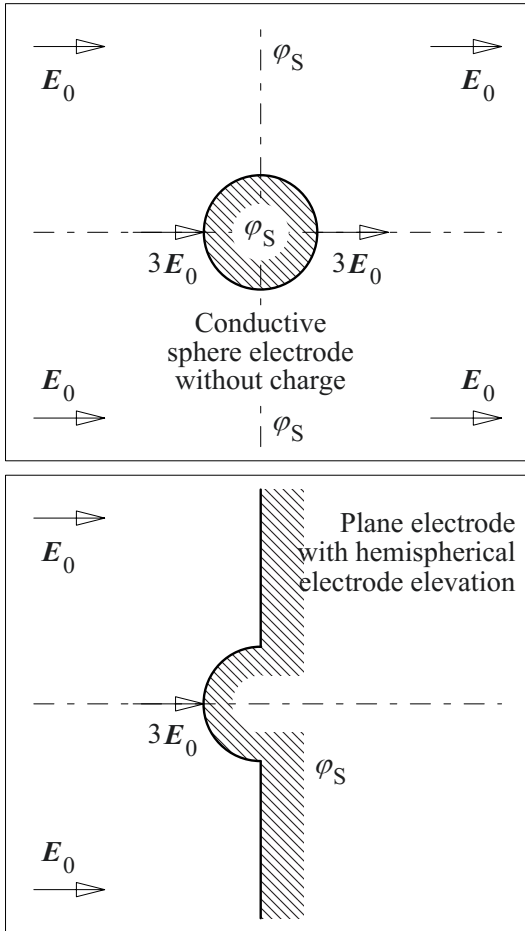


Figure 2.3-18: Field enhancement by a conductive sphere without charge (top) and by a hemispherical electrode elevation on a plane electrode (bottom) in the originally uniform field.

ducting impurities is a basic requirement of high voltage technology manufacturing!

In the special case of a uniform field ($d \gg r_0$) the configuration is symmetric with respect to the equipotential plane ϕ_S , Figure 2.3-17. This plane can then be seen as a plane electrode with a *hemispherical electrode elevation*, Figure 2.3-18. The maximum field strength is three times as high as in the undisturbed uniform field.

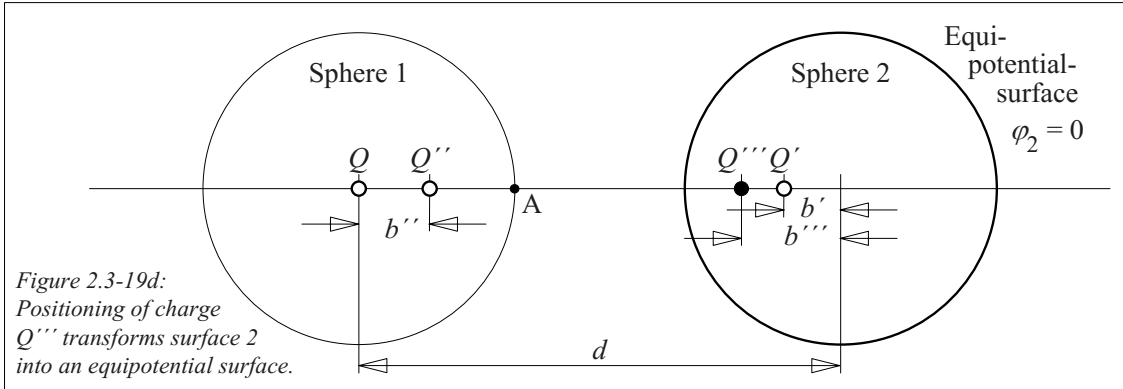
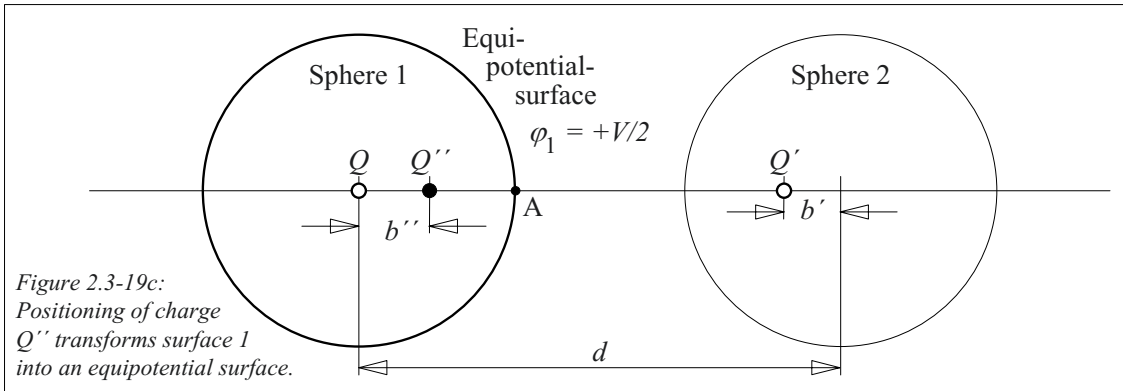
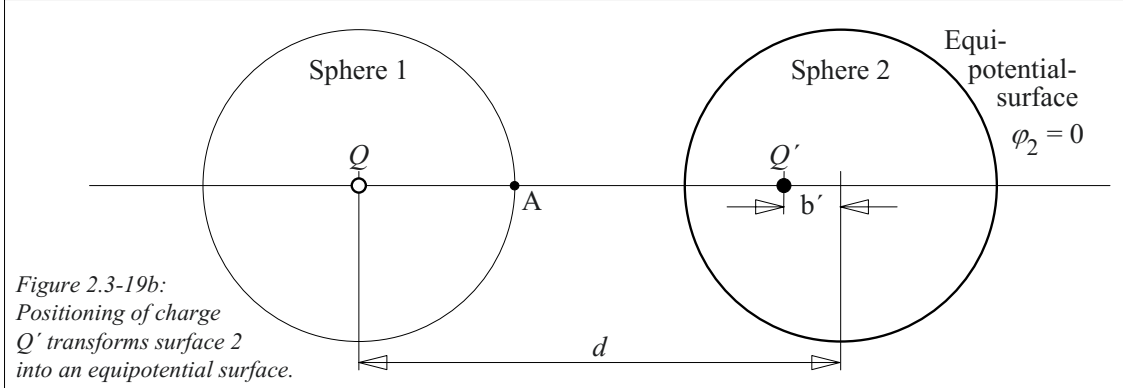
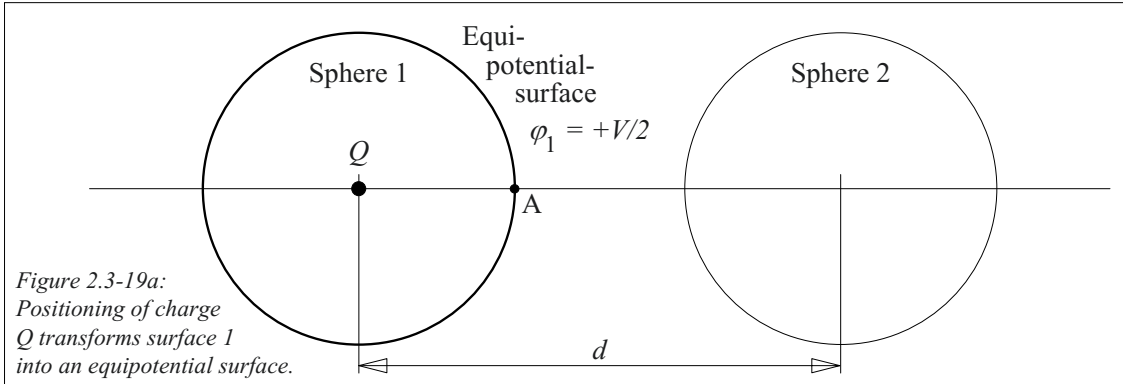
The field stress enhancement caused by a conducting hemisphere on a *plane* electrode can also be regarded as a model for the field stress enhancement on an *uneven electrode surface*. The unevenness of electrode surfaces is responsible for early discharge inception on electrodes at voltages with macroscopic field strengths significantly below anticipated inception field strengths (in air $\hat{E}_D$ is approx. 30 kV/cm).

2.3.5.2 Field between Two Conducting Spheres (Sphere-to-sphere Gap)

The charge simulation method also allows one to calculate the field between two spherical electrodes (**sphere-to-sphere arrangement**) and between a spherical electrode and a plane of symmetry (**sphere-to-plane arrangement**). Important practical applications are *sphere gaps* or *spherical high voltage shielding electrodes* close to a flat wall. Therefore, the comparatively laborious iterative calculation is explained here, Figure 2.3-19:

Two conducting spheres are specified with the radius r_0 , the center point's distance d and the potential difference $\Delta\phi = V$. The calculation is performed in two steps:

- (1) By positioning *equivalent charges*, the specified spherical surfaces are approximated by the calculated equipotential surfaces.
- (2) *Field strengths* are determined from the superposition of equivalent charge fields after the positioning of the charges.



(1) *Equivalent charges*

(a) At first, an equivalent charge Q is positioned at the center point of a virtual sphere 1. Because of the spherically symmetric field the spherical surface 1 is an equipotential surface. The potential $\phi_1 = V/2$ is determined by choosing the magnitude of Q . Now the spherical surface 2 is not an equipotential surface, because of the radially symmetric field of the charge Q in the center point of sphere 1, Figure 2.3-19a.

(b) The spherical surface 2 can be transformed into an equipotential surface with the potential $\phi_2 = 0$ by positioning an *image charge* Q' at a distance b' from its center point, Figure 2.3-19b. The charges Q and Q' are comparable to the charges Q_1 and Q_2 in Figure 2.3-15. After the positioning of Q' , the sphere 1 is no longer an equipotential surface.

(c) Sphere 1 becomes an equipotential surface again by positioning an equivalent charge Q'' as an image charge with respect to Q' , Figure 2.3-19c. The charge Q in the center point 1 causes a spherically symmetric field with concentric equipotential surfaces. The potential is the sum of the potentials related to the charge Q and the charge couple (Q', Q''):

$$\phi_1 = \phi_1(Q) + \phi_1(Q', Q'') = V/2 + 0 = V/2$$

Now sphere 2 is no longer an equipotential surface.

(d) Sphere 2 becomes an equipotential surface again if Q''' is positioned as an image charge with respect to Q'' , Figure 2.3-19d. The potential of sphere 2 is zero again:

$$\phi_2 = \phi_2(Q, Q') + \phi_2(Q'', Q''') = 0 + 0 = 0$$

Now sphere 1 is no longer an equipotential surface.

(e) If further image charges are placed alternately, the state of two spherical equipotential surfaces with the potentials $\phi_1 = V/2$ and $\phi_2 = 0$ can be further *iteratively* approximated.

If a second analogous **row of charges** of opposite polarity (starting with the equivalent charge $-Q$ at the center point of sphere 2) is formed, the potentials of the two spheres 1 and 2 are approximated by the potentials $\phi_1 = 0$ and $\phi_2 = -V/2$. After the superposition of the two rows of charges the intended potential difference between the two spheres $\Delta\phi = V/2 - (-V/2) = V$ is obtained.

In the following, the **equations**, which are necessary for the calculation of magnitude and position of the equivalent charges, are compiled. The listing refers to the *steps (a) to (e)* that were described above:

(a) Q causes the equipotential surface 1 ($\phi_1 = V/2$) according to Eq. (2.3-44):

$$\begin{aligned} b &= 0 \\ Q &= 0.5 V 4\pi\epsilon r_0 \end{aligned} \quad (2.3-63a)$$

(b) Q' and Q cause the equipotential surface 2 ($\phi_2 = 0$) according to Eq. (2.3-55) and (-53):

$$\begin{aligned} b' &= r_0^2/d \\ Q' &= -Q r_0/d \end{aligned} \quad (2.3-63b)$$

(c) Q'' and Q' , together with Q , cause the equipotential surface 1 ($\phi_1 = 0 + V/2 = V/2$). The distance d to the counter charge must be reduced by the distance b' :

$$\begin{aligned} b'' &= r_0^2/(d - b') \\ Q'' &= -Q' r_0/(d - b') \end{aligned} \quad (2.3-63c)$$

(d) Q''' and Q'' , together with Q' and Q , cause the equipotential surface 2 ($\phi_2 = 0 + 0 = 0$). The distance d to the counter charge must be reduced by the distance b'' :

$$\begin{aligned} b''' &= r_0^2/(d - b'') \\ Q''' &= -Q'' r_0/(d - b'') \end{aligned} \quad (2.3-63d)$$

(e) and so on

Note: Because of the *recursive character*, these equations are well-suited to producing a simple numerical iteration program. This example illustrates the basic idea of *numerical field calculation* with the charge simulation method, which iteratively approximates the given electrode contours by placing equivalent charges.

Example: Sphere gap with $r_0 = 0.2 d$

A sphere gap shall be calculated for the special case with $r_0 = 0.2 d$. For this purpose the Equations (2.3-63..) are evaluated numerically, Figure 2.3-20. The first charge row (*white background*) causes the potential $\varphi_1 = V/2$ on sphere 1, the second charge row (*grey background*) causes the potential $\varphi_2 = -V/2$ on sphere 2. Row no. 1 starts with the equivalent charge $+Q$ at the center point of sphere 1, alignment no. 2 with the equivalent charge $-Q$ in the center point of sphere 2. It should be noted that each row contains charges on both spheres alternately. All of the positive charges are aligned within sphere 1, all negative charges within sphere 2. With increasing number of iteration steps the magnitude of the charges and the distances to the adjacent charges

are reduced drastically.

The *capacitance of the sphere gap* can be calculated with Eq. (2.1-10) and (2.3-63a) from the charge sums in the spheres:

$$C = Q_{\text{sum}}/V = 1.25 Q/V = 2.5 \pi \epsilon r_0 \quad (2.3-64)$$

Surprisingly, this capacitance is smaller than the capacitance

$$C = 4 \pi \epsilon r_0$$

between a sphere with the same size and a concentric counter electrode infinitely far away, Eq. (2.3-6).

The plane of symmetry between the spheres is an equipotential plane with the potential $\varphi = 0$. The calculated arrangement also covers the case of a *spherical electrode against a plane electrode* (sphere-to-plane arrangement). In comparison with Eq. (2.3-74) the capacitance is doubled:

$$C = 5 \pi \epsilon r_0 \quad (2.3-65)$$

(2) *Maximum field strengths*

The potentials assigned to the equivalent charges can be superimposed, as in the above Section 2.3.5.1. The field distribution is derived from the potential distribution using gradient generation.

For the determination of *maximum field strength* on the connecting line of the spheres' center points at the spheres' surfaces (point A, Figure 2.3-19) it is possible to *sum the magnitudes of the field vectors directly* because all field vectors related to the equivalent charges are in parallel at this point.

According to Eq. (2.3-2) the field component of a single charge is

$$E_i = Q_i / (4\pi\epsilon r_i^2).$$

The index "i" is the number of a single equivalent charge, r_i it the distance between equivalent charge Q_i and field point A, Figure 2.3-19.

At point A both the positive charges in sphere 1 and the negative charges in sphere 2 cause field vectors in the same direction. Therefore, the summation of all field components E_i has to be performed with the *same* positive sign.

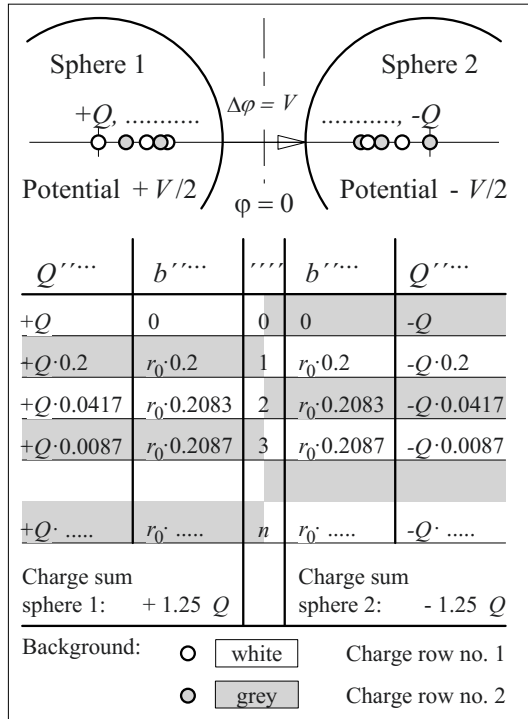


Figure 2.3-20: Position and magnitude of the equivalent charges for the calculation of a sphere gap with $r_0 = 0.2 d$.

According to the alternating polarity of the charges, the signs in Eq. (2.3-66) alternate as well:

$$E_{\max} = \frac{1}{4\pi\epsilon} \left[\frac{+Q}{r_0^2} + \frac{-Q'}{(r_0 - b')^2} + \frac{+Q''}{(r_0 - b'')^2} + \dots \right. \\ \left. - \frac{-Q}{(d - r_0)^2} - \frac{+Q'}{(d - r_0 - b')^2} - \frac{-Q''}{(d - r_0 - b'')^2} - \dots \right] \quad (2.3-66)$$

Note: The correct field magnitude at point A is also achieved, if all *charges* are inserted as *magnitudes* and all *summands* are superimposed with *positive sign*.

The charges belong alternately to the first charge row (started in sphere 1 with $+Q$) and to the second row (started in sphere 2 with $-Q$).

The *first line of the equation* contains the summation of all contributions from the positive charges in sphere 1. The distance between $+Q$ and point A is equal to the sphere's radius r_0 , the following distances are each reduced by $b', b'', b''', \dots$

The *second line of the equation* contains the summation of all contributions from the negative charges in sphere 2. Now the distance between $-Q$ and point A is equal to $(d - r_0)$, the following distances are also each reduced by $b', b'', b''', \dots$

It no longer makes sense to insert Eq. (2.3-63) into Eq. (2.3-66). In fact, the numerical values for the equivalent charges and their positions are used.

Example: Sphere gap (continued)

In the former example a sphere gap with $r_0 = 0.2 d$ was concerned and the equivalent charges Q, Q', Q'', Q''' and their positions b, b', b'', b''' were determined. By inserting the numerical values in Eq. (2.3-66) we find with

$$Q = 0.5 V / 4\pi\epsilon r_0$$

according to Eq. (2.3-63a):

$$E_{\max} = 0.736 V / r_0 = 3.68 V / d = 2.21 V / s$$

The radius of the spheres is $r_0 = 0.2 d$, the distance between the center points of the spheres is d and the "sparking distance" between the spheres is $s = 0.6 d$.

The results can be compared with the field strength in a *parallel-plate capacitor*

$$E = 1 \cdot V / s,$$

with the field strength at the surface of a *single sphere*

$$E = 1 \cdot V / r_0$$

and the maximum field strength in a sphere gap *with a very large gap* ($d \gg r_0$)

$$E = 0.5 \cdot V / r_0.$$

2.3.5.3 Parallel Line Charges

Some important configurations in high voltage engineering can be calculated by means of *line charges* with a uniformly distributed charge Q along the line length L . In the following, the electric field in the vicinity of two parallel line charges with equal magnitude but opposite polarity is discussed, Figure 2.3-21.

The potential distribution in the field volume is determined by the superposition of the two potentials assigned to the two line charges. It is a two-dimensional field so that consideration of a plane orthogonal to the line charges is sufficient, Figure 2.3-22. The counter-charges and the reference potential $\varphi_B = 0$ cannot be considered to be at an infinite distance as for the spherically symmetric field. Here, infinite potential difference would occur, see Section 2.3.1.3. For calculation purposes finite radii r_{B1} and r_{B2} are introduced in order to specify the distances between the charges and the coaxial counter-charges. They can be eliminated later on, if counter charges at large distances are assumed, Figure 2.3-22. The calculation is performed with the assumption and superposition of two cylindrically symmetric fields around the two line charges.

The superposition of the potentials φ_1 and φ_2 , which are assigned to the charges $+Q$ and $-Q$, is performed at point P. With Eq. (2.3-18) we find

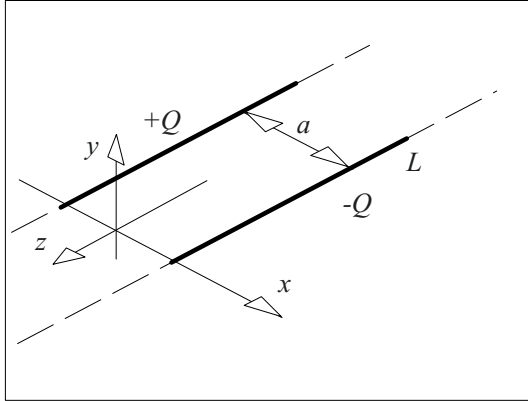


Figure 2.3-21: Infinitely long line charges in parallel (three-dimensional view).

$$\begin{aligned}
 \varphi &= \varphi_1 + \varphi_2 \\
 &= \frac{Q/L}{2\pi\epsilon} \ln \frac{r_{B1}}{r_1} - \frac{Q/L}{2\pi\epsilon} \ln \frac{r_{B2}}{r_2} \\
 &= \frac{Q/L}{2\pi\epsilon} \ln \left(\frac{r_{B1}}{r_1} \cdot \frac{r_2}{r_{B2}} \right).
 \end{aligned}$$

If the reference potential is assumed to be far away, i.e. if

$$r_1, r_2, a \ll r_{B1} \approx r_{B2}$$

and

$$r_{B1}/r_{B2} \approx 1,$$

the distances to the reference potential r_{B1} and r_{B2} can be reduced:

$$\varphi = \frac{Q/L}{2\pi\epsilon} \ln \frac{r_2}{r_1} \quad (2.3-67)$$

According to Eq. (2.3-67) equipotential surfaces $\varphi = \text{const.}$ are described by the condition

$$r_2/r_1 = k = \text{const.} \quad (2.3-68)$$

During the discussion of point charges in Section 2.3.5.1, it was already shown that such a condition is a *circle equation* for the drawing plane under consideration, see Eq. (2.3-47) ff. Therefore, all of the *equipotential surfaces* are *cylindrical surfaces* in the two-dimensional field under consideration of the parallel line charges.

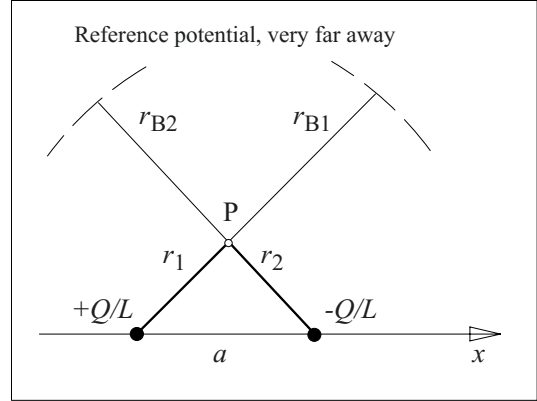


Figure 2.3-22: Infinitely long line charges in parallel (sectional view).

It can be shown that the *field lines are circles* too. They all pass through the intersections of the line charge axes $+Q$ and $-Q$ with the plotting plane [2]. This results in a *graphical plotting method* for a field and equipotential line plot, Figure 2.3-23:

- At first, a circle with the radius $r = a/2$ is plotted through the charge-axis points $+Q$ and $-Q$. The two semicircles describe two field lines.
- At any point $P_1, P_2, \dots$ the “field-line circle” and the “equipotential-line circle” intersect orthogonally.
- Owing to symmetry, all center points $M_1, M_2, \dots$ of the equipotential-line circles are located on the x -axis. Furthermore, the radii $M_1P_1, M_2P_2, \dots$ touch the field-line circle tangentially, and the center points are determined from the intersections of the tangents at $P_1, P_2, \dots$ with the x -axis.
- Additional field lines are plotted as circles with center points on the y -axis.

The potential of the points $P_1, P_2, \dots$ and of the associated equipotential surfaces is given by Eq. (2.3-67). The plane of symmetry between the charges $+Q$ and $-Q$ is an equipotential surface with the reference potential $\varphi = 0$. Field lines and equipotential lines on both sides are given by reflection in the plane of symmetry.

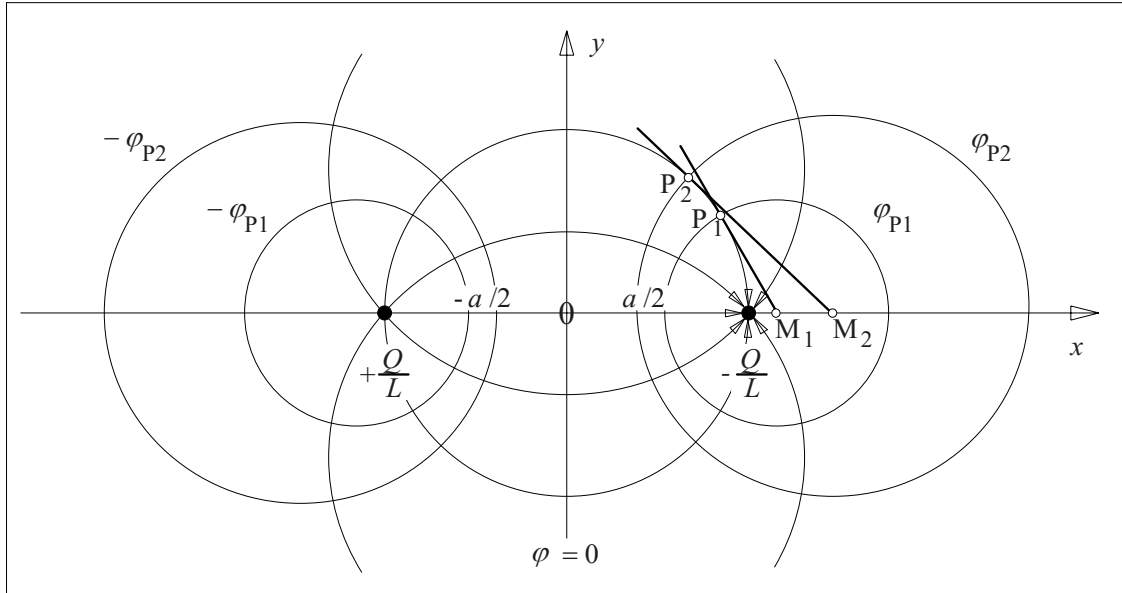


Figure 2.3-23: Graphical design of a pattern with field lines and equipotential lines for parallel line charges.

Potential values have opposite polarities on both sides, Figure 2.3-23.

Any equipotential surfaces caused by the equivalent line charges can be interpreted as electrode contours for field calculations with the charge simulation method. The field of the parallel line charges contains some important high-voltage electrode arrangements that are discussed as *examples* in Section 2.3.5.4. They include parallel cylinders, cylindrical conductors beside plane electrodes, eccentric tubular conductors and overhead line conductors, Figure 2.3-23.

2.3.5.4 Fields in the Vicinity of Cylindrical Conductors

Example 1: Parallel Cylinder Conductors (Cylinder-to-Cylinder, "Two-conductor Line")

If two cylindrical conductors with the radius r_0 and the distance d of the center points is given, the distance a of the equivalent line charges is unknown, i.e. the position b of the equivalent charges has to be determined, Figure 2.3-24.

The geometric relations were already considered in the case of the field between two point

charges. In both cases, the relation $r_2/r_1 = k = \text{const.}$ describes a circle with a radius r_0 and a distance b between the circle's center point and the position of the equivalent charge, Eq. (2.3-47) and (-68).

Note: In the case of point charges, the factor k both has a geometric meaning and describes a ratio of charges. This means that k is given for given charges, and that there is only one circular equipotential line (spherical equipotential surface resp.). In the case of parallel line charges, there is no relation between k and the charge magnitudes. Therefore, there can be any number of circular equipotential lines (or cylindrical equipotential surfaces).

The geometric relations Eq. (2.3-48) and (-49) can therefore be taken from Section 2.3.5.1:

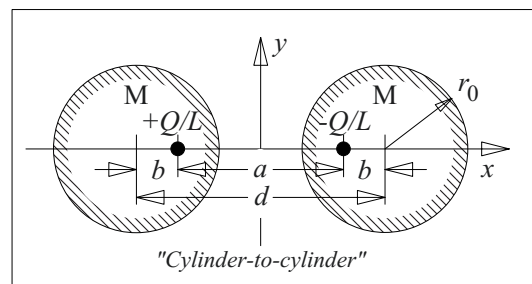


Figure 2.3-24: Parallel cylindrical conductors, calculation with equivalent line charges.

$$\begin{aligned} b &= a / (k^2 - 1) \\ r_0 &= a k / (k^2 - 1) \end{aligned} \quad (2.3-69)$$

The unknown *position* b (or a) of the *equivalent charges* shall now be expressed by the given quantities r_0 and d . the factor k is to be eliminated in the process. With

$$\begin{aligned} d &= a + 2b \\ &= a + 2a / (k^2 - 1) \\ &= a (k^2 + 1) / (k^2 - 1) \end{aligned}$$

$$\begin{aligned} (d/2)^2 - r_0^2 & \quad \text{we form} \\ &= \frac{a^2}{4} \cdot \frac{k^4 + 2k^2 + 1}{(k^2 - 1)^2} - \frac{a^2}{4} \cdot \frac{4k^2}{(k^2 - 1)^2} \\ &= \frac{a^2}{4} \cdot \frac{k^4 - 2k^2 + 1}{k^4 - 2k^2 + 1} = \frac{a^2}{4} \end{aligned}$$

The distance between the equivalent charges is

$$a = 2 \sqrt{(d/2)^2 - r_0^2}.$$

If the conductor's radius r_0 is replaced by the diameter $d_0 = 2r_0$, we get

$$a = \sqrt{d^2 - d_0^2}. \quad (2.3-70)$$

According to Figure 2.3-24 the distance $b = (d - a)/2$ between the conductor's axis and the line charge is determined from this.

The *potential distribution* along the x -axis, where the highest electric field strengths occur, is derived from Eq. (2.3-67):

$$\phi(x) = \frac{Q/L}{2\pi\epsilon} \ln \frac{r_2(x)}{r_1(x)}$$

The distances $r_1(x)$ and $r_2(x)$ have to be defined in intervals, so that the distances are positive. In the interval of interest $-a/2 < x < +a/2$ (between the line charges) the potential is

$$\phi(x) = \frac{Q/L}{2\pi\epsilon} \ln \frac{a/2 - x}{a/2 + x}. \quad (2.3-71)$$

The *field strength profile* $E(x) = E_x(x)$ along the x -axis is derived from Eq. (2.3-71):

$$\begin{aligned} E(x) &= -\partial\phi/\partial x \\ &= -\frac{Q/L}{2\pi\epsilon} \cdot \frac{\partial}{\partial x} [\ln(a/2 - x) - \ln(a/2 + x)] \\ &= -\frac{Q/L}{2\pi\epsilon} \left[\frac{-1}{(a/2 - x)} - \frac{1}{(a/2 + x)} \right] \\ &= \frac{Q/L}{2\pi\epsilon} \left[\frac{1}{(a/2 - x)} + \frac{1}{(a/2 + x)} \right] \end{aligned} \quad (2.3-72)$$

The same result is obtained by direct *superposition of the field strengths*, Eq. (2.3-17).

Figure 2.3-25 shows the profiles of potential and field strength along the x -axis between the conductors according to Eqs. (2.3-71), (-72).

Within the conductors themselves, the equations of the charge simulation method give false results. The potential within an ideal conductor is constant, the electric field strength tends towards zero.

Outside of the conductors, for $x > d/2 + r_0$ and for $x < -d/2 - r_0$, potential and field strength magnitudes decrease in the outward direction. The field strengths at the outside of the conductors are significantly lower than the field strengths at the inner sides where the conductors are facing each other.

For the calculation of *capacitance* C , the potential difference V is determined as a function of the equivalent charge Q from Eq. (2.3-71):

$$\begin{aligned} V &= \phi(x = -d/2 + r_0) - \phi(x = d/2 - r_0) \\ &= \frac{Q/L}{2\pi\epsilon} \ln \left[\frac{a/2 + d/2 - r_0}{a/2 - d/2 + r_0} \cdot \frac{a/2 + d/2 - r_0}{a/2 - d/2 + r_0} \right] \end{aligned}$$

$$= \frac{Q/L}{\pi\epsilon} \ln \frac{a/2 + d/2 - r_0}{a/2 - d/2 + r_0}$$

The capacitance is equal to the ratio $C = Q/U$:

$$C = \frac{\pi\epsilon L}{\ln \frac{a/2 + (d/2 - r_0)}{a/2 - (d/2 - r_0)}} \quad (2.3-73)$$

With the distance a between the charges according to Eq. (2.3-70), the capacitance can be written as function of the geometric quantities d and r_0 :

$$C = \frac{\pi\epsilon L}{\ln \left[\frac{d}{2r_0} + \sqrt{\left(\frac{d}{2r_0} \right)^2 - 1} \right]} \quad (2.3-74)$$

Note: The deviation of Eq. (2.3-74) from Eq. (2.3-73) requires some intermediate steps. Thereby it is reasonable to cancel the expression $(d/2 - r_0)^{-1/2}$ out in the argument of the logarithm, and to make the denominator rational by expanding the fraction.

For large distances d and accordingly for small radii r_0 , i.e. for $d \gg r_0$, Eq. (2.3-74) is simplified:

$$C \approx \frac{\pi\epsilon L}{\ln \frac{d}{r_0}} \quad (2.3-75)$$

Note: This approximation can also be derived directly from Eq. (2.3-73), if the distance a between the charges is assumed to be equal to the distance d between the conductors for large distances d , Eq. (2.3-70). Therefore, we find for the numerator in the argument of the logarithm

$$a/2 + d/2 - r_0 \approx d - r_0 \approx d.$$

The denominator is

$$a/2 - d/2 + r_0 = -b + r_0 \approx r_0,$$

as the distance b between line charge and conductor axis is small in comparison with the conductor's radius r_0 .

The validity limits of the approximation Eq. (2.3-75) result from an *error estimation* for different ratios d/r_0 :

d/r_0	2.5	5	10	20
C_{approx}/C	0.757	0.973	0.996	0.9992
Error in %	24.3	2.7	0.4	0.08

I.e. for many electrode arrangements in high voltage engineering, the simplified Eq. (2.3-75) can be used, if the distance d of the conductors is much greater than the radius r_0 .

Maximum field strength results from Eq. (2.3-72) at the conductor surface at $x = d/2 - r_0$.

In order to get an exact solution, Q is replaced by $Q = C \cdot V$ with C according to Eq. (2.3-74):

$$E_{\text{max}} = \frac{V \cdot \sqrt{\left(\frac{d}{2r_0} \right)^2 - 1}}{(d - 2r_0) \cdot \ln \left[\frac{d}{2r_0} + \sqrt{\left(\frac{d}{2r_0} \right)^2 - 1} \right]} \quad (2.3-76)$$

For $d \gg r_0$, i.e. for large distances d or small conductor radii r_0 , Eq. (2.3-76) is simplified:

$$E_{\text{max}} \approx \frac{V}{2r_0 \cdot \ln \frac{d}{r_0}} \quad (2.3-77)$$

For thin wires, the *inception voltage for corona discharges* can be derived, if the inception field strength E_i for discharges is known:

$$V_i \approx E_i \cdot 2r_0 \cdot \ln(d/r_0) \quad (2.3-78)$$

The validity limits of Eq. (2.3-77) and (2.3-78) result from an *error estimation* for different ratios d/r_0 :

d/r_0	5	10	20	40
E_{approx}/E	0.637	0.813	0.904	0.951
Error in %	36.3	18.7	9.6	4.9

Thus, the approximate Eqs. (2.3-77) and (2.3-78) for the maximum field strength and for the corona inception voltage are only accurate enough for large ratios d/r_0 . Therefore, in general the exact solution from Eq. (2.3-76) must be used.

Example 2: Cylinder-to-Plane

A common high voltage electrode arrangement is a cylindrical conductor, which is led at a

height h above or beside a conducting plane. This case can be reduced to the former example of parallel cylinders, if the conducting plane is regarded as a plane of symmetry or an equipotential surface with the potential $\varphi = 0$ and if the arrangement is complemented symmetrically with a second cylinder (image conductor), Figure 2.3-26. The capacitance C of the arrangement is twice the capacitance C' of the associated parallel cylinders. With Eq.

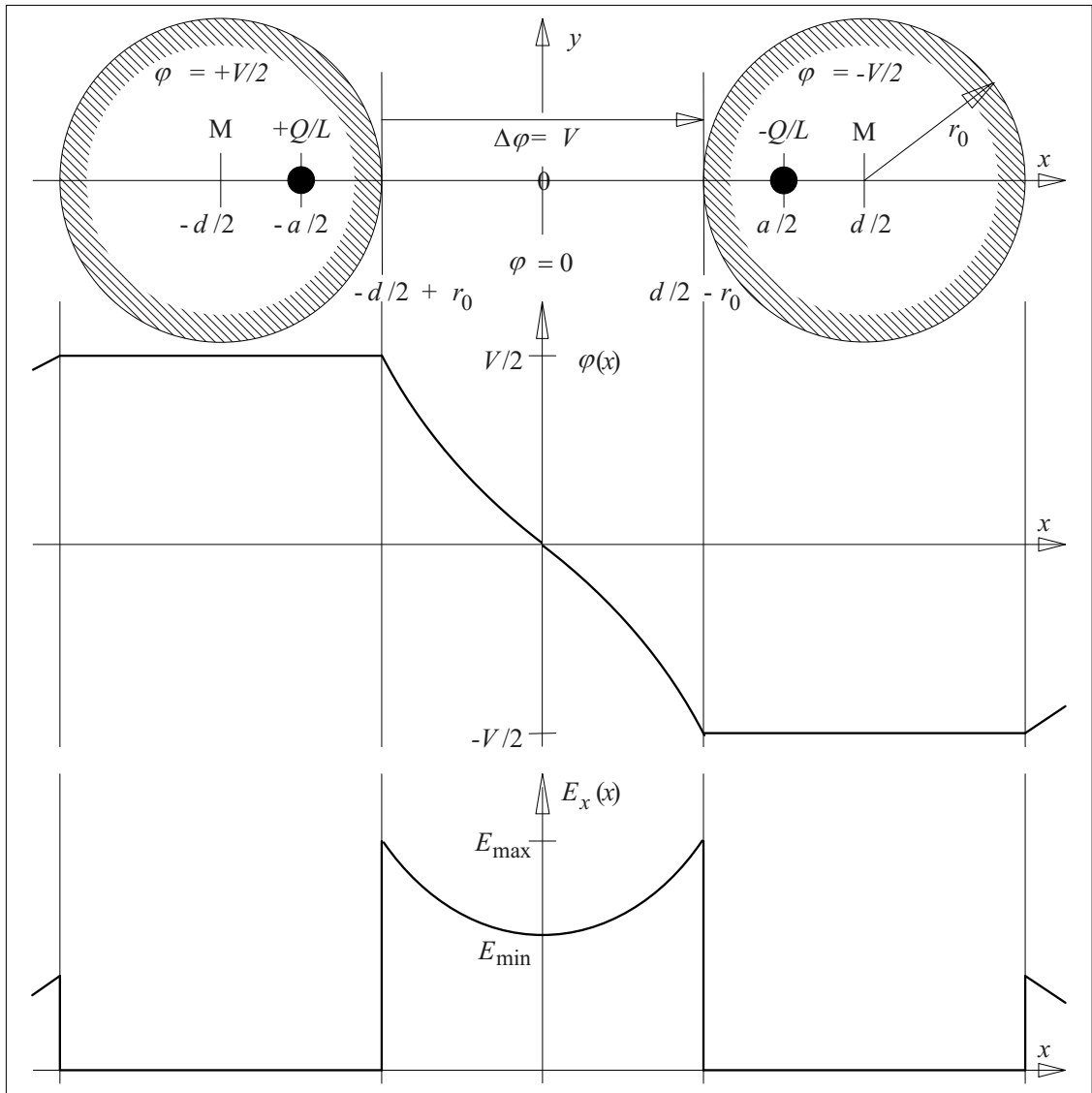


Figure 2.3-25: Parallel cylindric conductors: Potential and field strength profiles along the connecting line of the conductor centre points (x -axis) in the x,y -plane. The profiles within the conductors can not be determined from the equivalent charges.

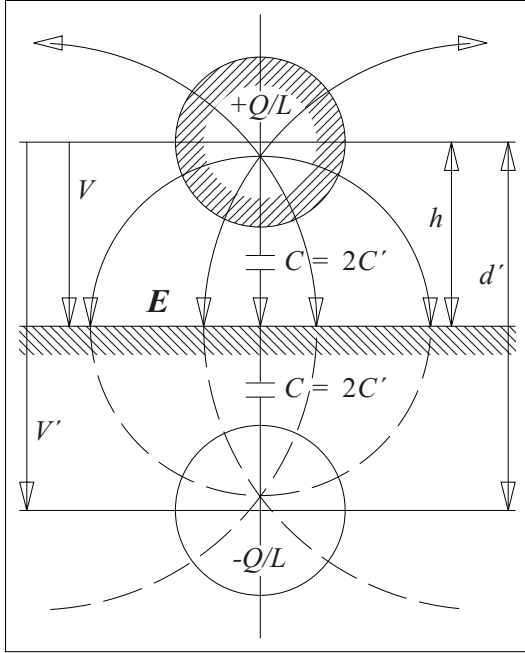


Figure 2.3-26: Cylindric conductor above a conducting plane. Field calculation by means of a symmetric image charge.

(2.3-75) and $d' = 2h \gg r_0$ the capacitance is

$$C \approx \frac{2\pi\epsilon L}{\ln \frac{2h}{r_0}} \quad (2.3-79)$$

The *maximum field strength* is obtained from Eq. (2.3-76) or (-77), if the voltage V and distance d are replaced by $V' = 2V$ and $d' = 2h \gg r_0$:

$$E_{\max} \approx \frac{V}{r_0 \cdot \ln \frac{2h}{r_0}} \quad (2.3-80)$$

For the *corona inception voltage* of a thin wire above a conducting plane we find

$$V_i \approx E_i \cdot r_0 \cdot \ln(2h/r_0) \quad (2.3-81)$$

Example 2a: Cylinder-to-Plane (numerical example)

The diameters and distances of cylindrical conductors above conducting planes shall be dimensioned for application in air ($\hat{E} = 30 \text{ kV/cm}$, $\epsilon_r = 1$) and in insulating oil ($\hat{E} = 150 \text{ kV/cm}$, $\epsilon_r = 2.2$) for the peak voltages $\hat{V} = 10 \text{ kV}$, 100 kV and 1 MV without the field intensities

exceeding $2/3$ of the breakdown voltages. Furthermore, the capacitance per unit length of the configuration shall be calculated. In all cases the ratio $h/r_0 = 10$ shall be assumed to be equal.

Solution: Because of the ratio $d/r_0 = 20$, approximate Eq. (2.3-80) for the *maximum field strength* will provide an error of approx. 10 % (see above estimate). Therefore, the exact Eq. (2.3-76) is evaluated: If $2r_0$ is factored out in the denominator, the equation can be solved for r_0 . For d and V the terms $2h$ and $V' = 2V$ have to be inserted:

$$\begin{aligned} r_0 &= \frac{\frac{2\hat{V}}{0.67\hat{E}_D} \sqrt{10^2 - 1}}{2(10 - 1) \ln \left[10 + \sqrt{10^2 - 1} \right]} \\ &= 0.5540 \cdot \hat{V} / \hat{E}_D \end{aligned}$$

The capacitance can be estimated from Eq. (2.3-79) with a small error.

Voltage $\hat{V}$:	10 kV	100 kV	1 MV
Air:			
r_0	2 mm	1.9 cm	18.5 cm
h	2 cm	19 cm	1.85 m
C/L	18.5 pF/m	18.5 pF/m	18.5 pF/m
Oil:			
r_0	0.4 mm	3.7 mm	3.7 cm
h	3.7 mm	3.7 cm	37 cm
C/L	40.8 pF/m	40.8 pF/m	40.8 pF/m

Note: As shown in all the examples with the spherical electrode (Section 2.3.1.2), with the cylindrically symmetrical tubular conductor (Section 2.3.1.3) and with the cylinder-to-plane arrangement (in the current example) it is also shown here that air-insulated equipment for the MV-range needs insulation distances and radii of curvature of the order of meters.

Much more compact *dimensions* are possible with electrically strong materials (e.g. insulating oil or sulfur hexafluoride gas SF_6).

Attention: The *electric strengths* assumed to be constant in these examples for simplicity, are not constant in reality. They depend, for example, on the type and duration of the electric field stress, on the insulating material thickness, on the insulating volume, on the electrode surface, on the inhomogeneity of the field or on environmental influences (pressure, temperature, water content,) for instance.

The *capacitance per unit length* does not change with the dimensions h and r_0 because of the assumption of a constant ratio h/r_0 , which determines the capacitance, in this example.

Example 3: Overhead ground wire

(Shielding effect and field stress enhancement)

Grounded wires above overhead lines are used to protect the phases against direct lightning strikes. Here, it shall be investigated, to what extent the vertical electrostatic field in the atmosphere (i.e. in the air) is influenced by a grounded wire (radius r_0 , height h above ground), Figure 2.3-27.

The original *field in the air* E_0 is assumed to be uniform; it is directed in the negative x -direction. The potential is

$$\varphi_1 = E_0 \cdot x.$$

In the ground wire a charge Q is influenced whose field E_S is superimposed on the original field E_0 . The *additional field of the charges in the wire* against the grounded plane can be calculated from the superposition of the fields associated with Q and with an image charge $-Q$ on the x -axis at $x = -h$. According to Eq. (2.3-67), the potential is

$$\varphi_2 = \frac{Q}{2\pi\epsilon L} \ln \frac{r_2}{r_1}.$$

At the *surface of the grounded wire* (and in the plane of symmetry, i.e. on the ground surface), the sum of the potentials must be zero. This condition can be used to calculate the magnitude of the influenced charge Q :

$$\varphi = \varphi_1 + \varphi_2 = 0$$

$$\varphi = E_0 \cdot x + \frac{Q}{2\pi\epsilon L} \ln \frac{r_2}{r_1} = 0$$

For all points on the wire surface, the distances to the equivalent charges $+Q$ and $-Q$ are $r_1 \approx r_0$ and $r_2 \approx 2h$ approximately. Because of the large height $h \gg r_0$, the equivalent charges are close to the axes of the wire and its image. With $x \approx h$ the charge is

$$Q = -2\pi\epsilon L \frac{E_0 \cdot h}{\ln \frac{2h}{r_0}}. \quad (2.3-82)$$

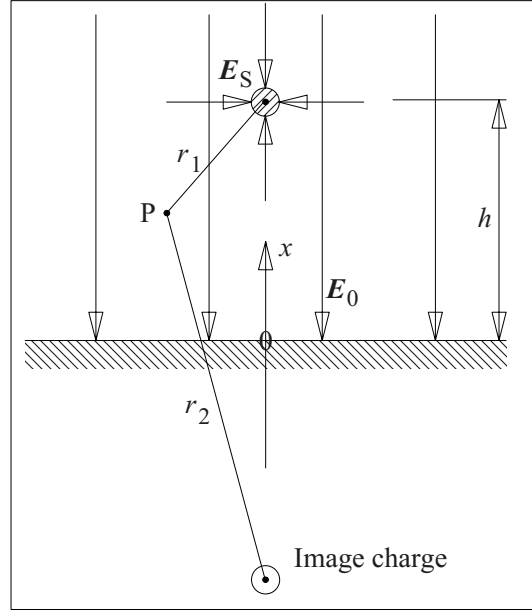


Figure 2.3-27: Distortion of the electric field in air by a conducting overhead ground wire.

The *field strength* on the x -axis is the derivate of the potential with respect to x according to Eq. (2.3-72) or it is the superposition of the field strengths according to Eq. (2.3-17). Q is inserted from (2.3-82):

$$\begin{aligned} E_x(x) &= E_0 + E_{+Q} + E_{-Q} \\ &\stackrel{(17)}{=} E_0 + \frac{Q}{2\pi\epsilon L} \left(\frac{1}{h-x} + \frac{1}{h+x} \right) \\ &\stackrel{(82)}{=} E_0 - \frac{E_0 \cdot h}{\ln \frac{2h}{r_0}} \left(\frac{1}{h-x} + \frac{1}{h+x} \right) \end{aligned} \quad (2.3-83)$$

Note: The discussion of the signs shows that the original field E_0 and the additional field of the charges have opposite directions underneath the ground wire ($0 < x < h$). Above the ground wire ($x > h$), the field E_0 in the air and the field contribution of the upper equivalent charge $+Q$ are superimposed with the same sign; the field contribution of the image charge $-Q$ is in the opposite direction, Figure 2.3-27.

The field strength at the *ground surface* ($x = 0$) is

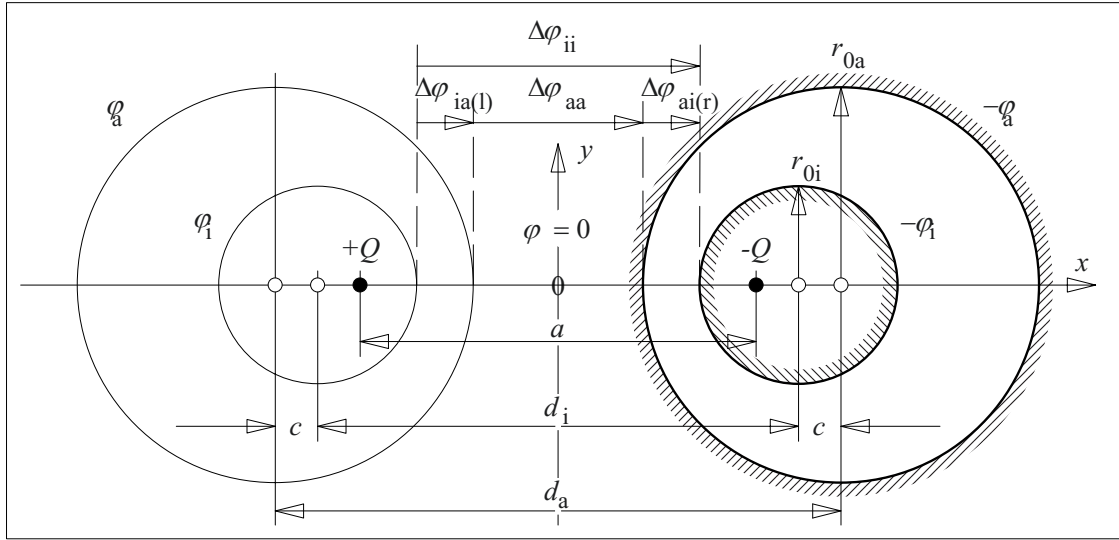


Figure 2.3-28: Calculation of eccentric conductors with equivalent line charges in parallel.

$$E_x(0) = E_0 \left(1 - \frac{2}{\ln \frac{2h}{r_0}} \right). \quad (2.3-84)$$

Note: For a ratio $h/r_0 = 1000$ the field strength is $E_x(0) = 0.74 E_0$, i.e. there is only a weak shielding of the original field at the ground surface. Improved shielding efficiency is achieved by a *screen grid*, e.g. by an arrangement of parallel grounded wires at small intervals.

At the *upper side of the wire*, the contribution associated with upper equivalent charge Q predominates according to Eq. (2.3-82). The contribution caused by the distant image charge $-Q$ and the original field E_0 can be neglected. With the conditions $x = h + r_0$ and $2h/r_0 \gg 1$ Eq. (2.3-83) provides

$$E_x(h + r_0) \approx E_0 \frac{h/r_0}{\ln \frac{2h}{r_0}}. \quad (2.3-85)$$

Note: For a ratio $h/r_0 = 1000$ there is a field stress enhancement of $E/E_0 = 132$. In cases of very high field strengths E_0 , discharges are possible at sharp edges of grounded conductors. Especially during a lightning discharge, a discharge channel propagates from the cloud

towards the ground and causes a very high increase of the local electric field strength in a limited field region. This can be regarded as an increase of the primary field E_0 which initiate *upward discharges* starting from overhead line wires, lightning conductors or other grounded structures. The discharges propagate upwards, meet the downward discharge within a limited range and cause a conducting path to the ground.

Example 4: Eccentric tubular conductor

The electric field between eccentric tubular conductors (cylinders) can be calculated with parallel line charges, if the outer and the inner conductor are interpreted as equipotential surfaces in the field of two mirror-symmetric line charges, Figure 2.3-23 and 2.3-28.

The cylinder radii r_{0i} and r_{0a} and the lateral offset of the cylinder axes c (eccentricity) are given. The distance a of the equivalent charges and distances d_i and d_a of the center points are unknown. Therefore, the Equations (2.3-70) ff cannot be applied directly.

The solution can be based on the fact that the charge distance a is equal for both the arrangement with the large cylinders (r_{0a} , d_a) and for the arrangement with the small cylin-

ders (r_{0i} , d_i). With Eq. (2.3-70) and Figure 2.3-28 we get the following solution

$$\begin{aligned} a^2 &= d_i^2 - 4r_{0i}^2 = d_a^2 - 4r_{0a}^2 \\ \text{I.e.:} \\ d_a^2 - d_i^2 &= 4r_{0a}^2 - 4r_{0i}^2 \\ (d_i + 2c)^2 - d_i^2 &= 4r_{0a}^2 - 4r_{0i}^2 \\ d_i &= (r_{0a}^2 - r_{0i}^2 - c^2)/c \quad (2.3-86) \end{aligned}$$

From this all unknown geometric quantities in Figure 2.3-28 are determined. The charge distance a is determined from Eq. (2.3-70), the distance d_a is $d_a = d_i + 2c$.

Instead of a difficult general calculation, a *numerical evaluation* with specific numerical values is recommended here.

Numerical example:

It shall be investigated how much capacitance and maximum field strength are changed for an arrangement with eccentric tubular conductors ($r_{0i} = 5$ cm, $r_{0a} = e \cdot r_{0i} = 13.59$ cm, $c = 1$ cm) in comparison with coaxial configurations.

From Eq. (2.3-86) we get $d_i = 158.73$ cm. From this $d_a = 160.73$ cm and $a = 158.41$ cm are derived. The *maximum field strength* at the surface of the inner cylinder can be determined from Eq. (2.3-76) with $d = d_i$ and $r_0 = r_{0i}$, if the voltage V is interpreted as potential difference $\Delta\varphi_{ii}$ between the two inner cylinders.

$$E_{\max} = \Delta\varphi_{ii} / 32.45 \text{ cm} \quad (*)$$

The potential difference $\Delta\varphi_{ii}$ has to be related to the potential difference $\Delta\varphi_{ai(r)}$ between the outer and inner cylinders on the right side: The x -axis intersects the inner cylinders at $x_i = \pm(d_i/2 - r_{0i}) = \pm 74.37$ cm and the outer cylinders at $x_a = \pm(d_a/2 - r_{0a}) = \pm 66.78$ cm. For points x_i and x_a on the negative x -axis the potentials can be calculated with Eq. (2.3-71):

$$\varphi_i = 3.458 \cdot Q / (2\pi\epsilon L)$$

$$\varphi_a = 2.464 \cdot Q / (2\pi\epsilon L)$$

From this, the potential differences are given:

$$\begin{aligned} \Delta\varphi_{ii} &= (3.458 + 3.458) \cdot Q / (2\pi\epsilon L) \\ &= 6.916 \cdot Q / (2\pi\epsilon L) \\ \Delta\varphi_{ai(r)} &= (-2.464 + 3.458) \cdot Q / (2\pi\epsilon L) \\ &= 0.994 \cdot Q / (2\pi\epsilon L) \end{aligned}$$

The ratio of the potential differences is

$$\Delta\varphi_{ii} / \Delta\varphi_{ai} = 6.958.$$

The *maximum field strength* is calculated from Eq. (*):

$$\begin{aligned} E_{\max} &= \Delta\varphi_{ii} / 32.45 \text{ cm} \\ &= \Delta\varphi_{ai} / 4.664 \text{ cm}. \end{aligned}$$

In the cylindrically symmetric case, maximum field strength according to Eq. (2.3-22) is

$$E_{(\text{cyl})\max} = \Delta\varphi_{ai} / 5 \text{ cm}.$$

The *field stress enhancement* caused by the eccentricity $c = 1$ cm is 7.2 %:

$$E_{\max} / E_{(\text{cyl})\max} = 1.072$$

Note: The *capacitance* C_{ai} between the inner and the outer cylinder can be calculated, if the capacitances C_{ii} and C_{aa} between the cylinders of the same size are calculated with Eq. (2.3-74). C_{ii} can then be regarded as a series circuit consisting of C_{ia} , C_{aa} and C_{ai} , Figure 2.3-28.

From this the *magnitude of the equivalent charge* $Q = C_{ai} \cdot \Delta\varphi_{ai}$ is also determined. Eq. (2.3-71) and (-72) then allow one to calculate *potential and field strength profiles* along the x -axis.

Example 5: Three-phase overhead line ("Working capacitance")

A three-phase overhead line is a so called *multi-phase system*, consisting of a number of parallel cylindrical conductors with different potentials and insulated against each other. The calculation of multi-phase systems is possible by means of equivalent line charges and their image charges. For a detailed analysis the basic literature can be consulted [2], [4].

As an example, a *three-phase overhead line* connected to a three-phase AC voltage system shall be considered (complex r.m.s. values of the phase voltages: $\underline{V}_{10}$, $\underline{V}_{20}$, $\underline{V}_{30}$). Perfect *symmetry* of the voltages, the conductor properties (line parameters per unit length) and the currents ($\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$) is assumed.

During the calculation of three-phase systems, lines and cables are described by line impedances determined by series resistances, series inductances, parallel capacitances and parallel conductances.

The charge simulation method allows the **calculation of a so-called “working capacitance”** of a three-phase system. This is *not* the capacitance between oppositely charged conductors, such an arrangement does *not* exist in a three-phase system.

The working capacitance C_b is defined by the single-phase capacitive charging current $\underline{I}_{C1}$ in a symmetric three-phase overhead line without load. In the positive-sequence network (i.e. in a transformed single-phase equivalent circuit) the following relationship is established:

$$\underline{I}_{C1} = j\omega C_b \cdot \underline{V}_{10} \quad (2.3-87)$$

Physically, the charging current $\underline{I}_{C1}$ is not only fed from the displacement field that is associated with the phase-to-ground voltage $\underline{V}_{10}$. The fields between the considered phase L1 and the other phases L2 and L3 cause additional displacement currents to be coupled in, i.e. there are additional influences of the phase-to-phase voltages $\underline{V}_{12}$ and $\underline{V}_{31}$, Figure 2.3-29. In order to calculate with the simple Eq. (2.3-87) despite this, it is necessary to define a *working capacitance* C_b whose magnitude takes into account the influence of all interferences.

Note: The simple assumption of a single-phase equivalent circuit, consisting solely of the so-called positive-sequence network, and considering capacitive couplings by the magnitude of the working capacitance, is only valid in the case of *perfect symmetry*. This means that the three-phase system has to be built symmetrically and that it has to be operated symmetrically too.

From the physical viewpoint, the single-phase equivalent circuit (the so-called positive-sequence network) is *not* identical with the phase L1 alone. Capacitive and magnetic couplings to the neighboring phases are taken into account by the magnitudes of the working capacitances and working inductances.

In the case of *asymmetric* three-phase systems, the three coupled circuits L1, L2 and L3 are transformed into three *decoupled* circuits (positive-sequence network, negative-sequence network and zero-sequence network) in order to allow a simpler and clearer calculation (*method of symmetrical components* [20]). A working capacitance can no longer be specified because the condition of symmetrical voltages and fields is no longer fulfilled.

In the special case of *perfect symmetry*, the single-phase equivalent circuit is identical with the positive-sequence network. According to Eq. (2.3-87) $1/(j\omega C_b) = \underline{V}_{10}/\underline{I}_{C1}$ is the “positive-sequence impedance” of the unloaded line (resistive and inductive components are neglected).

The working capacitance C_b shall be calculated from the ratio of the charge q_1 on line L1 to the phase voltage v_{10} . The quantities q_1 and v_{10} are the instantaneous values of the time variant quantities. The charging current $i_{C1}(t)$ or $\underline{I}_{C1}$ has to carry the charge q_1 to and from the conductor. The influence of the ground is taken into account by image charges, Figure 2.3-29.

The voltage v_{10} is equal to the potential φ_1 , which is established by the superposition of the contributions from all equivalent charge pairs:

$$\varphi_1 = \varphi_1(q_1, -q_1) + \varphi_1(q_2, -q_2) + \varphi_1(q_3, -q_3)$$

For overhead lines, the charge distances a_{12} , a_{13} , D_{12} , D_{13} and $D_{11} \approx 2h$ are very large in comparison to the conductor radius r_{01} . The potential at the surface of conductor L1 is determined with Eq. (2.3-67):

$$\varphi_1 = \frac{q_1}{2\pi\epsilon L} \ln \frac{2h_1}{r_{01}} + \frac{q_2}{2\pi\epsilon L} \ln \frac{D_{12}}{a_{12}} + \frac{q_3}{2\pi\epsilon L} \ln \frac{D_{13}}{a_{13}}$$

The distance from any charge (with the exception of q_1) to the surface of conductor L1 is approximately equal to the charge distance to the charge q_1 . The distance from q_1 to the conductor's surface is approximately equal to r_{01} .

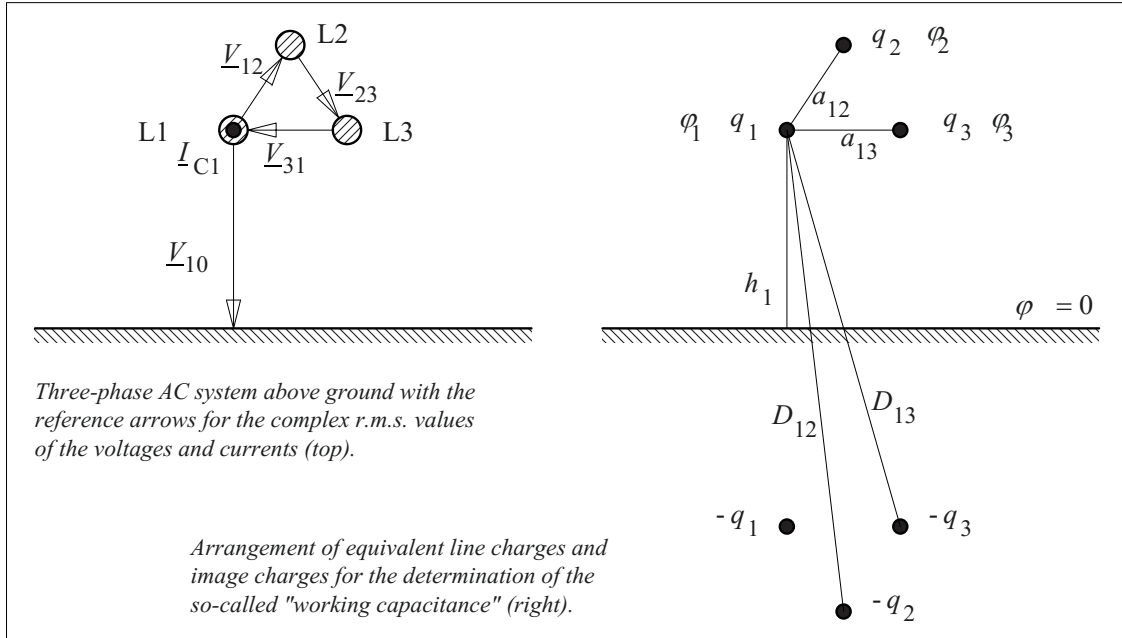


Figure 2.3-29: Calculation of the "working capacitance" for a symmetric three-phase system with the charge simulation method. The influence of the ground is taken into account by image charges.

In a geometrically perfectly symmetric system, the equivalent distances are equal to each other. In practice, cyclic exchanging of the phase positions compensates for the asymmetries:

$$\begin{aligned} h_1 &= h_2 = h_3 = h \\ r_{01} &= r_{02} = r_{03} = r_0 \\ D_{12} &= D_{23} = D_{31} = D = 2h \\ a_{12} &= a_{23} = a_{31} = a \end{aligned}$$

With this the expression for the potential of conductor L1 is simplified:

$$\varphi_1 = \frac{1}{2\pi\epsilon L} \left[q_1 \ln \frac{D}{r_0} + (q_2 + q_3) \ln \frac{D}{a} \right]$$

In a symmetric three-phase system the sum of the charges is zero:

$$q_1 + q_2 + q_3 = 0$$

This gives the condition

$$q_2 + q_3 = -q_1.$$

For the potential φ_1 it follows that

$$\varphi_1 = \frac{q_1}{2\pi\epsilon L} \left[\ln \frac{D}{r_0} - \ln \frac{D}{a} \right]$$

$$\varphi_1 = \frac{q_1}{2\pi\epsilon L} \ln \frac{a}{r_0}.$$

From this the working capacitance is derived:

$$C_b = \frac{q_1}{\varphi_1} = \frac{2\pi\epsilon L}{\ln \frac{a}{r_0}} \quad (2.3-88)$$

It is worth noting that the working capacitance, which could possibly (but misleadingly) be understood as capacitance between conductor L1 and ground, does *not* depend on the distance h between conductor and ground. The working capacitance is exclusively dependent on the distance a between conductors and on the conductor radius r_0 .

For overhead lines with *conductor bundles*, the radius r_0 is to be replaced by the much larger equivalent radius R' according to Eq. (2.3-40), i.e. it gives a greater working capacitance than

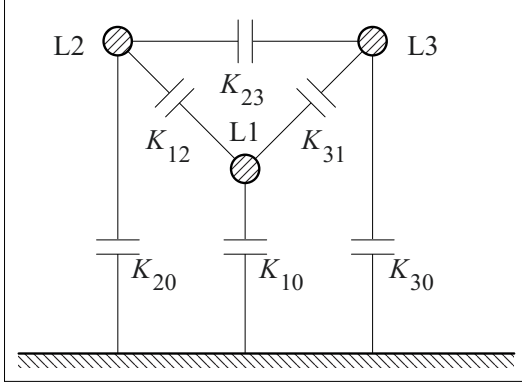


Figure 2.3-30: Coupling and ground capacitances (capacitance coefficients) of a three-phase system.

for single conductors. It can be calculated from Eq. (2.3-88).

If several three-phase AC systems are operated in close proximity to each other, e.g. on the same tower, the working capacitance is influenced. The former calculation for φ_1 has to be complemented with further terms associated with the additional AC conductors. Owing to the relatively large distances, they are generally of minor importance.

Generally, the charge simulation method also allows one to calculate the working capacitance of a *three-phase cable* or a *three-phase gas-insulated line*, for which the distances between the conductors are comparable with the conductor's radii [2]. In practice, values measured and specified by manufacturers are used, but they are only valid for a specific product. *High and very high voltage cables* are designed as single-phase cables with cylindrically symmetric fields, so that the working capacitance corresponds to the line-to-ground capacitance according to Eq. (2.3-20).

The *typical magnitude* of the working capacitance per unit length is approximately $C_b/L \approx 10$ nF/km for overhead lines and $C_b/L \approx 120$ nF/km for single-phase polymer cables (with $\varepsilon_r = 2.2$ and $R_a/R_i = e$, Eq. (2.3-20)). For oil-impregnated paper cables and for cables with a smaller radius ratio R_a/R_i (e.g. medium voltage cables with a large conductor cross-section), significantly higher values can occur.

Note: Because of the high capacitive reactive power, *economic AC power transmission with cables* is normally limited to lengths of a few 10s of km.

The **measurement of the working capacitance** C_b is performed via partial capacitances, Figure 2.3-30. The charging current I_{C1} is constituted from the superposition of all displacement currents that are coupled into L1 and which are calculated from the capacitance coefficients K_{lj} and the associated potential differences V_{lj} :

$$I_{C1} = j\omega[K_{10}V_{10} + K_{12}V_{12} + K_{31}(-V_{31})]$$

Because of the symmetry, $K_{12} = K_{31}$:

$$I_{C1} = j\omega[K_{10}V_{10} + K_{12}(V_{12} - V_{31})]$$

By means of a vector diagram it can be shown that $V_{12} - V_{31} = 3 V_{10}$ in a symmetric three-phase system. Thereby we get

$$I_{C1} = j\omega [K_{10} + 3 \cdot K_{12}] V_{10}.$$

The comparison with Eq. (2.3-87) gives the working capacitance:

$$C_b = K_{10} + 3 \cdot K_{12} \quad (2.3-89)$$

The capacitance to ground K_{10} and the coupling capacitance K_{12} are determined from two measurements:

During the *first measurement*, L2 and L3 are grounded, i.e. K_{20} and K_{30} are short-circuited. The measured capacitance C^* between L1 and ground is

$$C^* = K_{10} + K_{12} + K_{31} = K_{10} + 2 \cdot K_{12}.$$

During the *second measurement* the conductors L1, L2 and L3 are connected to each other. The measured capacitance C^{**} between L1-L2-L3 and the ground is now

$$C^{**} = K_{10} + K_{20} + K_{30} = 3 \cdot K_{10}.$$

For the partial capacitances we find

$$K_{10} = C^{**}/3 \quad \text{and} \quad K_{12} = C^*/2 - C^{**}/6.$$

From this the working capacitance can be expressed as a function of the measured values C^* and C^{**} :

$$C_b = 3 \cdot C^* / 2 - C^{**} / 6 \quad (2.3-90)$$

For the *calculation of field strengths*, the magnitude of the equivalent charges can be determined with Eq. (2.3-88). The calculation is performed *at a specific point in time* that is characterized by specific instantaneous values of the potentials (or phase-to-ground voltages) $\varphi_1, \varphi_2, \varphi_3$ and by the instantaneous values of the associated equivalent charges q_1, q_2, q_3 , Figure 2.3-31. Nevertheless, the analytic calculation of field strengths from the gradient of the resulting potential (or from the vectorial superposition of the different field components) is very complex. Furthermore, the result is only valid for the considered instant. For other points in time, other field distributions, i.e. other locations, directions and magnitudes of the maximum field strength, are produced.

Note: The maximum voltage difference between the phases L1 and L2 is given for a sinusoidal voltage $v_{10}(t) = \sin \omega t$ at the time point $\omega t = 60^\circ$ with magnitude $\sqrt{3} \cdot \sqrt{2} \cdot V_{ph}$. The potential of phase L3 is zero at this time. If the conductors are arranged in a triangular configuration (i.e. in an equilateral triangle) and if ground-

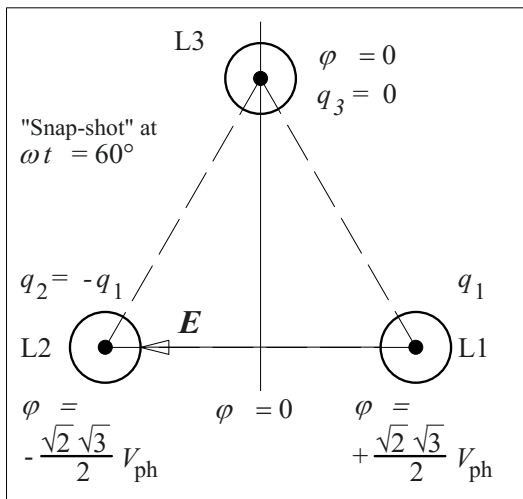


Figure 2.3-31: "Snap-shot" for the instant of maximum field strength at the surfaces of L1 and L2 (influence of the ground is neglected).

influences are neglected, the maximum field strength is located at the conductor surfaces of L1 and L2 close to the connecting line between these two phases, Figure 2.3-31. Because of

$$q_3(60^\circ) = 0$$

and

$$q_1(60^\circ) = -q_2(60^\circ) = C_b \cdot \Delta\varphi/2$$

L1 and L2 can be approximated as parallel cylindrical conductors, Figure 2.3-25.

2.3.6 Similarity Relations, Field Efficiency Factor (Schwaiger's Utilization Factor)

In the above sections, the common analytical methods for the calculation of electrostatic (and quasi-static) fields were described. Of course, the high voltage problems and examples discussed are not complete, they are more exemplary in character, in order to introduce the methods and the ways of thinking.

It is clear that there is no standard procedure that always gives the desired result. It is often necessary to have a good deal of intuition, training and experience in order to find the best calculation methods and appropriate simplifications.

It is a substantial improvement for a quick and practical solution if one can avoid doing one's own complex calculation, i.e. if *already available results can be used*.

Such calculation results are given in the basic literature on electrical engineering theory, e.g. for capacitances of different electrode configurations [2].

In high voltage engineering, the *central* question has also to be answered:

"What is the *maximum field strength* in the given insulation arrangement?"

The result can be specified independently of the applied voltage, if the maximum field strength E_{max} is given as a multiple of the mean field strength E_0 between the electrodes.

E_0 is equal to the uniform field strength in a parallel-plate capacitor with the same electrode distance s :

$$E_{\max} = \frac{1}{\eta} E_0 \quad (2.3-91)$$

E_0 can also be regarded as the mean field strength between the electrodes:

$$E_{\text{mean}} = \frac{1}{s} \int_{p1}^{p2} E \, dx = \frac{V}{s} = E_0$$

The maximum field strength for a given voltage V is determined from Eq. (2.3-91) by inserting $E_0 = V/s$.

The factor $\eta = E_0/E_{\max}$ is the *field efficiency factor* or *utilization factor* according to Schwaiger [21], it describes the “degree of uniformity” of the field. The inverse quantity $1/\eta$ is referred to as the *degree of non-uniformity*. In a uniform field $E_{\max} = E_0$ and the field efficiency factor is $\eta = 1$. In a very strongly non-uniform field $E_{\max} \gg E_0$ and the field efficiency factor or utilization factor is $\eta \ll 1$.

Example: Hemispherical electrode

The maximum field strength at the surface of a hemispherical electrode on a conducting plane was calculated in Section 2.3.5.1, Figure 2.3-18. The result is $E_{\max} = 3 E_0$, i.e. Schwaiger’s field efficiency factor or the utilization factor is $\eta = 0.333 = 33.3 \%$.

Field efficiency factors are available in catalogue-like tables from literature for a very large number of electrode arrangements [4], [22], [23]. The determination of η is performed in three steps:

1. At first, the *type of electrode arrangement* is selected from the tables, e.g. cylinder-to-plane, sphere-to-sphere, toroid-to-plane, disk-to-disk, ...
2. For each of the electrode types a specific *geometry factor* p is given as a function of geometric parameters. It is mainly to be determined from the electrode distance s and the relevant radius of curvature r :

$$p = f(\text{geometry}) = f(s, r) \quad (2.3-92)$$

If additional radii R , heights h , or distances d are necessary for the description of the electrode arrangement, additional *parameters* have to be determined. Mostly they are related to r , e.g. R/r , h/r or d/r .

3. For every electrode type a *curve* is given, which shows the field efficiency factor η as a function of the geometry factor p :

$$\eta = f(p) \quad (2.3-93)$$

If the definition of a further parameter is required, a single curve is not sufficient. A *set of curves* for different values of the additional parameter has to be used.

Example:

Cylindrical and spherical capacitors

In the geometry catalogue, cylindrical and spherical capacitors are described with the same geometry factor $p = (r + s)/r$, because of their identical sectional view, Figure 2.3-32. Nevertheless, there are different curves for the fields with two-dimensional geometry (coaxial cylinders) and for the fields with rotational symmetry (concentric spheres).

Analytical expressions for the field efficiency factors, which are graphically represented in Figure 2.3-33, can be derived from the equations (2.3-22) and (2.3-14) for the maximum field strengths, if the radii R_1 and R_2 are replaced by r and $(r + s)$, the voltages V are replaced by $E_0 \cdot s$ and the geometry factor is introduced with $p = (r + s)/r$:

$$\eta_{\text{cyl.}} = \frac{1}{p-1} \ln p \quad \text{and} \quad \eta_{\text{spher.}} = \frac{1}{p} \quad (2.3-94)$$

For practical applications, these equations are given as *tables of values* or as *graphs*. The geometry factor p is calculated from the given geometry and η is taken from the associated diagram, Figure 2.3-33.

Numerical example: For a coaxial, air-insulated tubular conductor with $D_a = 30 \text{ cm}$ and $D_i = 10 \text{ cm}$ the maxi-

mum possible voltage $\hat{V}$ shall be determined, if $\hat{E}_{\max} = 15 \text{ kV/cm}$ is accepted.

From the diameters $r = 5 \text{ cm}$, $s = 10 \text{ cm}$ and $p = 3$ is calculated. The field efficiency factor $\eta \approx 0.55$ can be taken from Figure 2.3-33. With Eq. (2.3-91) we find

$$\hat{E}_0 = \eta \cdot \hat{E}_{\max} = 8.25 \text{ kV/cm} \quad \text{and}$$

$$\hat{U} = \hat{E}_0 \cdot s = 82.5 \text{ kV}.$$

Note: If a considered electrode arrangement can be related to an already filed and evaluated arrangement, the determination of field efficiency factors is very advantageous for a *quick approximation* of numerical values, especially for electrode arrangements, which cannot be treated analytically or are difficult to treat analytically.

The *limited accuracy* of the method is particularly a disadvantage. In very non-uniform fields the numerical values of the field efficiency factor η are very small and can be read from a diagram (as Figure 2.3-33 for instance) only with very low accuracy. An analytical solution is therefore preferable, and furthermore it is suitable for *optimization* processes (see Section 2.3.1.2 and 2.3.1.3).

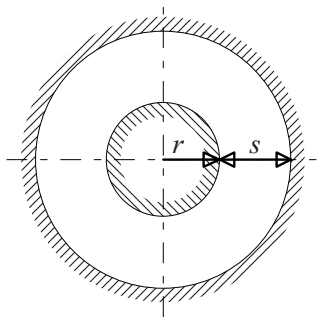
Type of electrode (sectional view)	
Geometry factor	$p = \frac{r+s}{r}$
Plane field arrangement	Coaxial cylinders Fig. 2.3-33, curve (1)
rotationally symmetric field arrangement	concentric spheres Fig. 2.3-33, curve (2)

Figure 2.3-32: Electrode configuration and geometry factors with reference to the curves for the determination of the field efficiency factors (example).

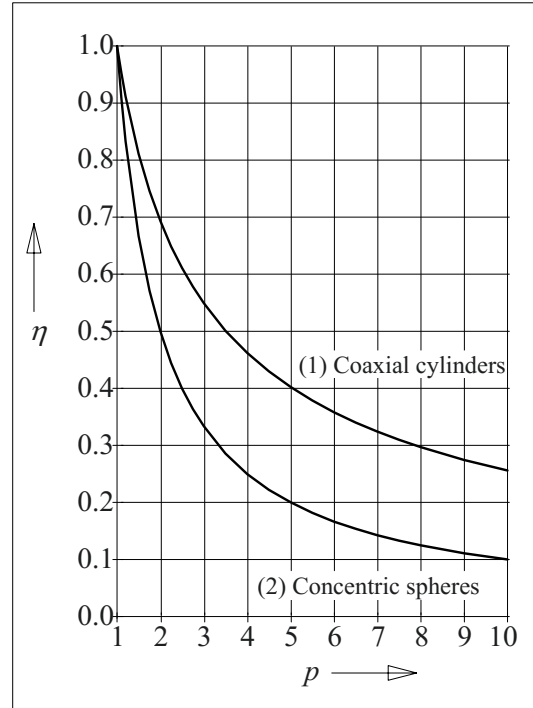


Figure 2.3-33: Field efficiency factor as a function of the geometry factor for arrangements with coaxial cylinders (1) and concentric spheres (2).

Figure 2.3-33 illustrates an interesting *relation between plane and rotationally symmetric arrangements* with identical sectional views: because of the additional curvature, the field efficient factor of the spherical arrangement decreases drastically in comparison with the cylindrical configuration. In the analytical expressions for η , the factor p can be eliminated and the relation

$$\eta_{\text{cyl.}} = -\frac{\eta_{\text{spher.}}}{1 - \eta_{\text{spher.}}} \ln \eta_{\text{spher.}} \quad (2.3-95)$$

between spherical and cylindrical arrangements is derived, Figure 2.3-34.

According to [22] this is also a useful approximation for the *general relation* between the field efficiency factors of plane field configurations and the equivalent rotationally symmetric configurations for $\eta_{\text{plane}} > 0.6$ (or $\eta_{\text{cyl.}} > 0.6$). For stronger non-uniform fields

with smaller field efficiency factors, Figure 2.3-34 can only be used for rough approximations.

Example: Sphere gap

The maximum field strength in a *sphere gap* with $r_0 = 0.2 d$ shall be estimated (see Section 2.3.5.2, example “sphere gap”).

At first, the field efficiency factor of an *equivalent plane configuration* with the same sectional view shall be determined. It is an arrangement of two parallel cylindrical conductors with $r = r_0$ and with the electrode distance (flashover distance) $s = 0.6 d = 3 r$.

From the charge simulation method we obtain with Eq. (2.3-76)

$$\begin{aligned} E_{\max(\text{plane})} &= 1.462 V/(d - 2r_0) \\ &= 1.462 E_0 s/(3 r_0) \\ &= 1.462 E_0 . \end{aligned}$$

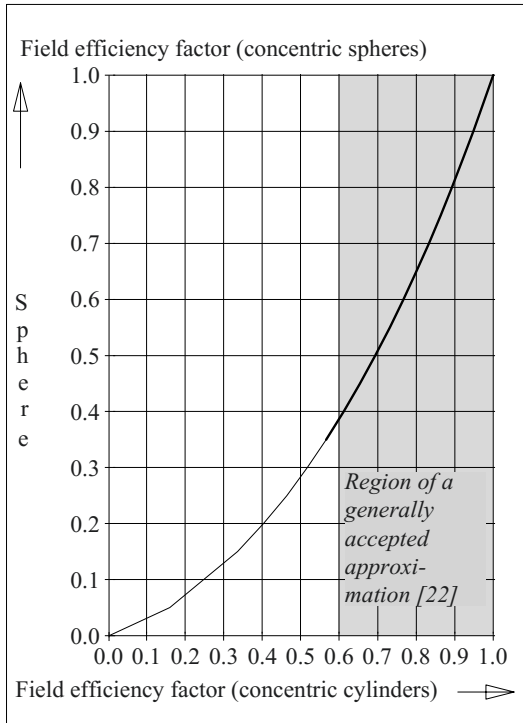


Figure 2.3-34: Comparison of the field efficiency factors of plane and rotationally symmetric electrode arrangements for the example of cylindrical and spherical capacitors.

Thus

$$\eta_{\text{plane}} = 1/1.462 = 0.684 .$$

From Figure 2.3-34 the field efficiency factor for the equivalent arrangement with rotational symmetry

$$\eta_{\text{rot}} \approx 0.48$$

is taken. As expected, the arrangement is significantly less uniform. The maximum field strength

$$E_{\max(\text{rot})} \approx V/(s \cdot 0.48) = 2.1 \cdot V/s ,$$

is in good agreement with $E_{\max} = 2.21 \cdot V/s$, calculated with the charge simulation method in Section 2.3.5.2 (sphere gap example).

It can be seen from the above numerical example that Figure 2.3-34 and Eq. (2.3-95) can be useful tools for the calculation of rotationally symmetric arrangements, if the field efficiency factor of the equivalent plane arrangement can be determined easily. Nevertheless, the method is only an approximation.

2.3.7 Measurement of Stationary Conduction Fields

Normally, the electric fields to be determined cannot be measured directly, or there are no suitable measuring methods (see Chapter 6). Therefore, we are dependent on the indirect determination of stresses by calculation.

In addition to the described analytic methods, procedures were established for the point-by-point *measurement of static conduction fields*, in order to determine complex potential distributions. In practical applications, these methods are nowadays largely replaced by more flexible and more accurate numerical field calculation methods.

Nevertheless, the analogy between steady-state conduction fields (at DC voltage) and quasi-static (slowly varying) displacement fields (at AC voltage) is of basic importance and of great educational value.

2.3.7.1 Analogy between Dielectric Displacement Field and Static Conduction Field

The determination of *potential fields* by the measurement of static conduction fields is based on the analogy with the slowly changing dielectric displacement fields, see also Section 2.1.4. This means that permittivity ε and displacement density $\mathbf{D}$ have to be replaced by the conductivity κ and the conduction current density $\mathbf{J}$, Eq. (2.1-19) and (-20):

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \text{is equivalent to} \quad \mathbf{J} = \kappa \mathbf{E} \quad (2.3-96)$$

For both kinds of fields, the electric field strength $\mathbf{E}$ is determined from formally identical equations. Instead of the charge Q as the source of the field, the current I is injected into the arrangement:

$$Q = \iint \mathbf{D} \cdot d\mathbf{A} \quad \text{is equivalent to} \quad I = \iint \mathbf{J} \cdot d\mathbf{A} \quad (2.3-97)$$

The electric field strength $\mathbf{E}$ and the derived quantities, potential φ and voltage V , are equivalent for the two different kinds of fields. In particular, Laplace's Equation (2.3-31) ff without space charges and current sources in the insulating volume

$$\Delta \varphi = 0 \quad (2.3-98)$$

is equally valid in both cases.

This means that the described field calculations for the electrostatic fields caused by charges are also valid for static conduction fields. Conversely, potential distributions that were measured in static conduction fields are valid for quasi-static displacement fields generated by charges.

Note: The fields, which are calculated with fixed (static) charges, are often referred to as “static electric fields”. However, this is an auxiliary picture only, since the static case cannot exist in a real insulating material

because of its (residual) conductivity. A *stationary conduction field* will inevitably develop.

Nevertheless, the electrostatic field caused by charges is a good approximation for slowly changing displacement fields in insulating materials with very low (residual) conductivity, if the conduction current density $\mathbf{J}$ can be neglected in comparison with the displacement current density $\partial \mathbf{D} / \partial t$, see also Section 2.1.4.4.

Basically, there are two methods of interest for the measurement of conduction fields, two-dimensional measurement on semi-conductive paper and three-dimensional measurement in semi-conductive liquids.

2.3.7.2 Measurements on Semi-conductive Paper (“Resistive Paper”)

Two-dimensional conduction fields can be generated with the aid of semi-conductive paper by means of conductive electrodes, which are pressed onto the paper or which are painted with conductive varnish. The edge of the paper must be far away from the field region of interest in order to avoid field distortions by the artificial boundaries.

After the application of a DC voltage to the electrodes, the measurement of potential magnitudes is performed for all points of interest by means of a metal probe, which is put on the paper, point after point.

Normally, the measurement is performed in a *bridge circuit* with a null indicator in order to achieve a measurement without any reaction.

During the measurement it is useful to adjust the bridge to a distinct potential value so as to enable following the associated equipotential line on the surface of the paper by means of the probe. By suitably marking the points, an equipotential plot is generated.

Measurements on semi-conductive paper allow us to consider different conductivities κ (and different permittivities ε) by stacking papers in different numbers. However, good contact between the sheets is required.

2.3.7.3 Measurements in Semi-conductive Liquids (“Electrolytic Tank”)

Any *three-dimensional* field arrangement can be measured (*field plotter*) point-by-point by lowering the electrode arrangement into a semi-conductive liquid (e.g. in a water-based electrolyte). In principle the original electrode itself can be investigated, if an electrolytic tank of sufficient size is available.

The field limitations at the basin walls must not have any influence on the field in the region of interest. Therefore, basin dimensions have to be large in many cases.

The simulation of different permittivities with liquids of different conductivities, which must be in contact at their interfaces without any mixing, is not readily achievable.

A three-dimensional field pattern requires a large quantity of data to be measured. Therefore an automatic measurement process with positioning of the measurement probe (“field plotter”) is recommended.

Of course, the probe that is immersed inserted in the liquid must be insulated against the liquid, with the exception of actual measurement tip.

2.4 Conduction and Displacement Fields in Inhomogeneous Dielectrics

Matter in the electric field has a significant influence on the formation of the field and potential distribution:

- Additional fields are caused by polarization, i.e. by displacement of charge carriers (ions, charged atoms, molecules and molecule groups) or by orientation of existing dipoles in the electric field. This is described by the *permittivity* ε of the insulating material.

- The movement of charge carriers in the electric field causes a so-called conduction field. This is described by the so-called (*residual*) *conductivity* κ of the insulating material.

In Section 2.3, the fields in *homogeneous dielectrics* were calculated with constant permittivities ε and constant conductivities κ , i.e. it was assumed either that there was a perfectly homogeneous dielectric in the field volume, or that there was absolutely no matter (perfect vacuum). Dependences on environmental parameters (e.g. temperature), field dependences (non-linearities) and dependences on direction (isotropy) are not considered.

Under these conditions, there is absolutely no influence of the material parameters ε and κ on the potential distribution and on the magnitude and direction of the electric field E .

Nevertheless, in reality the field quantities D and J depend on material properties. Thereby the capacitance C of the electrode arrangement

$$C = \frac{Q}{U} = \frac{\oiint D \, dA}{U} = \varepsilon \frac{\oiint E \, dA}{U} \quad (2.4-1)$$

is also dependent on the permittivity ε . In addition, the volume resistance R and the conductance G of the electrode arrangement is a function of the conductivity κ :

$$G = \frac{1}{R} = \frac{I}{U} = \frac{\oiint J \, dA}{U} = \kappa \frac{\oiint E \, dA}{U} \quad (2.4-2)$$

Note: From these equations the “self-discharging time constant” of the insulating material is derived as

$$\tau_d = RC = \varepsilon/\kappa \quad (2.4-3)$$

(see also Section 2.1.4.3, example of the self-discharging of a dielectric). I.e. for a given capacitance C the resistance R can be calculated directly, if τ_d is known.

Homogeneous insulating materials can only be found in some areas in a high voltage engi-

neering insulation system, e.g. as air insulation in overhead lines, as pressurized gas insulation in enclosed switchgear (GIS) or as cable insulation in coaxial cables. **Insulation systems** in service always need *further insulation components*, e.g. string insulators (overhead lines), post insulators (enclosed switchgear) or cable terminations (cables).

Generally, it is not sufficient to regard the homogeneous insulation parts only, also the parts with *different insulating materials* must be considered. Complex insulation systems (e.g. in transformers, bushings, cable fittings) always consist of a number of components with different insulating materials (e.g. oil, impregnated paper, impregnated pressboard, polymeric films, porcelain, epoxy resin, silicone or air).

Field and potential distributions in arrangements with a number of insulating materials can differ significantly from field and potential distributions in homogeneous arrangements. Especially at the interfaces, there are refractions of field vectors, refractions of equipotential lines and discontinuities of field quantities.

In the following the physical reason and the mathematical description of *polarization* and *conductivity* in insulating materials is discussed (Section 2.4.1). This allows one to calculate the *basic insulation structures* with interfaces orthogonal, parallel and inclined to the field direction (Section 2.4.2).

The use of *analytical field calculation methods* for insulating systems (Section 2.4.3) allows the calculation of some important *special cases*, e.g. for layered capacitor insulations, coated electrode surfaces, barrier systems, ruptures and slots, bubbles and voids, and for triple-points and interstices at the electrode surfaces.

At first, the discussion refers to the quasi-static *dielectric displacement field* (for an AC voltage) and to materials with different permittivities ϵ . Because of the analogies described in Section 2.3.7.1 the results can be transferred to

the stationary *conduction field* (for a DC voltage) and to materials with different conductivities κ (Section 2.4.4).

2.4.1 Conductivity and Polarization

In most cases, the atomic structure of matter, i.e. the presence of charged protons and electrons, cannot be detected directly because of a statistically uniform distribution of the charges. They are either mobile (free) or immobile (fixed).

2.4.1.1 Conductivity

The forces of the electric field accelerate *mobile charges* and impacts slow them down. Statistically averaged, there is constant *drift velocity* $\mathbf{v}$ and a constant *current density* $\mathbf{J}$, which are proportional to the electric field strength $\mathbf{E}$ [24], [25]. The Material Equation (2.1-20) describes this relation with the proportionality coefficient κ (*conductivity*):

$$\mathbf{J} = \kappa \mathbf{E} \quad (2.4-4)$$

Note: In gases, the linear relation is no longer valid for high field strengths. At first, there are saturation effects and then, the current increases again because of the production of further charge carriers by impacts. (see Chapter 3).

In liquid and solid insulating materials, Eq. (2.4-4) can often be used as a good approximation. Depending on the type of the mobile charge carriers, *ion conductivity* and *electron conductivity* are distinguished.

Conductivities of insulating materials strongly depend on the materials used, impurities, manufacturing processes and service conditions (e.g. on temperature, sometimes also on stress duration and field strength). For example, conductivities often increase exponentially with the temperature. The differences between different insulating materials can be many orders of magnitude. A more accurate assessment follows in Chapter 4.

The reliability of a field calculation for a stationary conduction field (i.e. for a steady-state DC voltage) depends very strongly on the reliability of the conductivity values used. For practical applications, special attention must be given to the determination of *relevant* conductivity values.

2.4.1.2 Polarization

The forces of the electric field can displace *immobile* positive and negative charge carriers against each other, and polarization of the insulating material is caused, Figure 2.4-1. There are a number of different polarization mechanisms [24], [25]:

- The displacement of the negative electron shell relative to the positive nucleus deforms the atom. It is called *electron polarization* or *polarization by deformation*.
- The displacement of atoms carrying different charges deforms molecules. It is called *atom polarization* or *polarization by deformation* as well.
- The displacement of differently charged lattice elements in a crystal lattice causes the *lattice polarization*.
- The orientation of polar molecule groups, molecules or particles (so-called electric dipoles) is called *molecular polarization* or *orientation polarization*.
- Furthermore, the accumulation of charge carriers at macroscopic or microscopic interfaces between materials with different conductivities causes polarization of the dielectric, i.e. the so-called *interfacial polarization*.

The influence of different polarization processes is always the same: From the superposition of many dipole fields, an additional electric field E_{Dip} is generated, which is superimposed on the original field E_0 of the same arrangement without insulating material (“vacuum field”), Figure 2.4-2b:

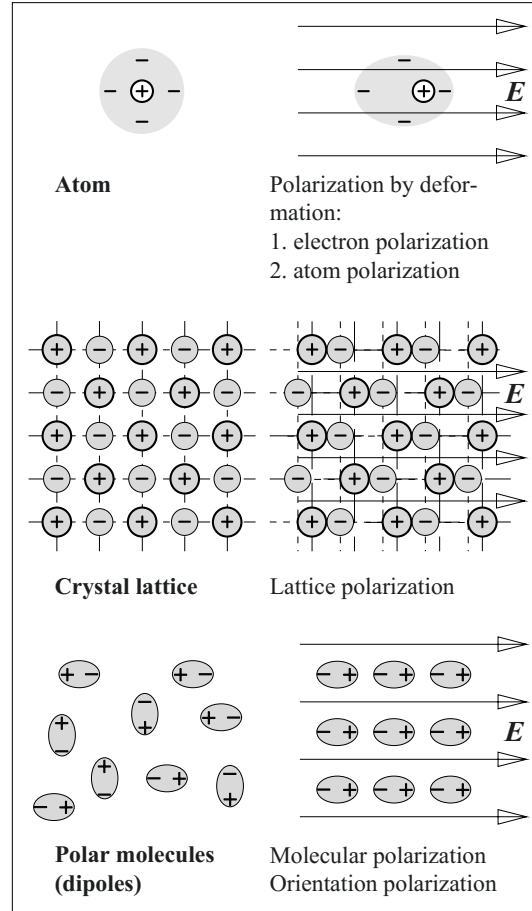


Figure 2.4-1: Polarization of insulating materials by the forces of an electric field (right).

$$E = E_0 + E_{\text{Dip}} \quad (2.4-5)$$

The dipole field, generated by the displaced charges, is oppositely directed to the original vacuum field. Therefore the magnitude of the resulting field is

$$E = E_0 - E_{\text{Dip}}. \quad (2.4-6)$$

The physical relations shall be explained by means of a thought experiment:

An insulating material is inserted into a capacitor with the charge Q , Figure 2.4-2a and -2b. Thereby the charge Q on the electrodes is not changed, if the capacitor is not connected to an external voltage source, i.e. *constant*

Figure 2.4-2a:
Electrostatic
field in vac-
uum.

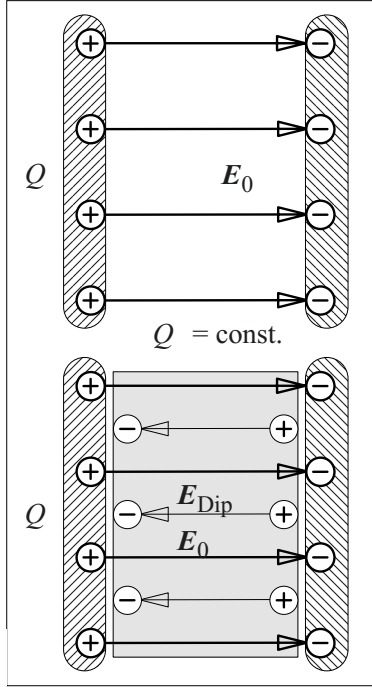


Figure 2.4-2b:
Additional
electric field
caused by the
polarization
of a dielectric
for a given
constant
charge Q .

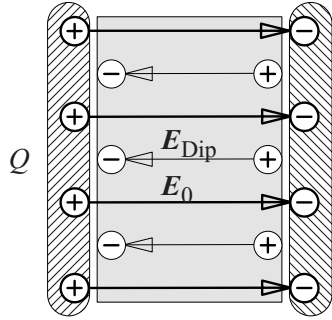


Figure 2.4-3a:
Electrostatic
field in vac-
uum.

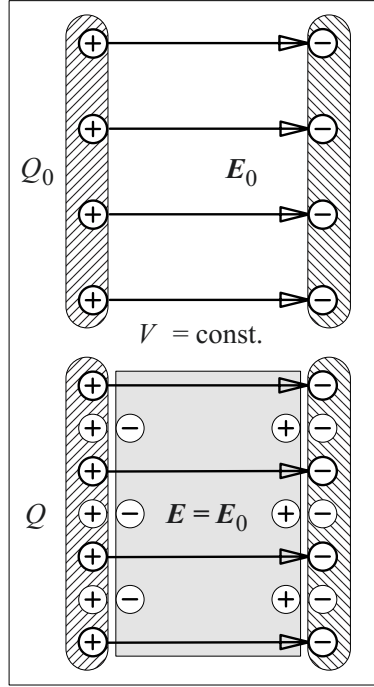
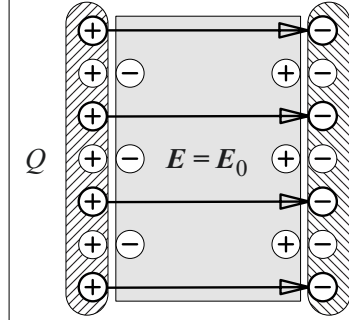


Figure 2.4-3b:
Binding of
additional
charges on the
electrodes by
polarization
of a dielectric
at constant
voltage.



charge $Q = D \cdot A$ and constant dielectric displacement density

$$D = \epsilon_0 \cdot E_0 = \text{const.}$$

are assumed. With Eq. (2.4-6) D is

$$\begin{aligned} D &= \epsilon_0 \cdot (E + E_{\text{Dip}}) \\ &= \epsilon_0 \cdot E + P. \end{aligned}$$

The term $P = \epsilon_0 \cdot E_{\text{Dip}}$ is referred to as *(di)electric polarization*. It has the same dimension as the electric displacement density D . The vector $\mathbf{P} = -\epsilon_0 \cdot \mathbf{E}_{\text{Dip}}$ can be seen as the fraction of the displacement density $\mathbf{D}$, for which the electric field is compensated by the polarized charges. Generally the displacement density is

$$\mathbf{D} = \epsilon_0 \cdot \mathbf{E} + \mathbf{P}. \quad (2.4-7)$$

The fraction $\epsilon_0 \cdot \mathbf{E}$ is associated with those charges on the electrode, which are *not* compensated by the polarized charges in the insulating material. Accordingly, they generate an electric field $\mathbf{E}$ which is reduced in comparison with $\mathbf{E}_0$, see also Eq. (2.4-5) and (-6).

Usually, the influence of the polarization, i.e. the influence of the insulating material, is described by a factor ϵ_r , the so-called relative permittivity (relative dielectric constant). Thus, the general *Material Equation* or the *Constitutive Relation* (2.1-2) and (-19) is defined:

$$\mathbf{D} = \epsilon_0 \cdot \epsilon_r \cdot \mathbf{E} \quad (2.4-8)$$

The absolute permittivity of vacuum ϵ_0 (electric constant) and the relative permittivity ϵ_r are often combined as permittivity ϵ (dielectric coefficient/ constant):

$$\epsilon = \epsilon_0 \cdot \epsilon_r \quad (2.4-9)$$

From the equality of the Equations (2.4-7) and (-8) the polarization $\mathbf{P}$ is derived:

$$\mathbf{P} = \epsilon_0 \cdot (\epsilon_r - 1) \cdot \mathbf{E} \quad (2.4-10)$$

In vacuum there is no polarization, i.e. $\mathbf{P} = 0$ and $\epsilon_r = 1$. In the presence of matter we always find $\epsilon_r > 1$.

According to Eq. (2.4-1), the insertion of a dielectric material causes an *increase of capacitance*:

$$C = \epsilon_r \cdot C_0 \quad (2.4-11)$$

Note: Up to now it was assumed that a capacitor both without and with a dielectric material carries a defined *constant charge* Q . In this case, the insertion of the dielectric is associated with a polarization that reduces the field strength $E = E_0/\epsilon_r$ and the capacitor voltage V , Figure 2.4-2.

Similar reasoning can also be performed for a capacitor with a *constant voltage* V and a constant field strength E , both sustained by an external voltage source. In this case, the insertion of a dielectric is associated with a polarization that binds additional charges on the electrodes in addition to the existing electrode charge Q_0 , Figure 2.4-3. The additional charges must be supplied by a current from the connected source. The increase of the charge Q on the electrodes corresponds to the increase of the displacement density

$$D = \epsilon_0 \cdot \epsilon_r \cdot E = \epsilon_r \cdot D_0. \quad (2.4-12)$$

Then the polarization P in Eq. (2.4-7) can be interpreted as the displacement density, which is associated with the additional charges that are bound on the electrodes.

The values of the relative permittivities depend strongly on the relevant *polarization mechanisms*, Fig 2.4-4. In the following some typical values (for room temperature and power frequency $f = 50/60$ Hz) are discussed:

- In a perfect *vacuum*, there is no polarizable matter. Therefore the permittivity is $\epsilon_r = 1$.
- In *gases*, there is little matter in comparison with liquids and solids, and the atoms or molecules do not have a polar character. Because of electron polarization there is a small, often negligible increase of the relative permittivity. For *ambient air* we find $\epsilon_r = 1.0006$.
- Materials with *symmetric, non-polar molecules* have comparatively small permittivities, caused by electron, atom or lattice polarization. For *mineral oil* and for *polyethylene (PE)* the relative permittivity is approximately $\epsilon_r = 2.2$ to 2.3.
- *Asymmetric and more complex molecules* often have high dipole moments. Because of molecular polarization (orientation polarization) there are higher relative permittivities.

Some values are $\epsilon_r = 3.5$ for *polyvinyl chloride (PVC)*, $\epsilon_r = 3.5 \dots 4$ for *epoxy resin (EP)*, $\epsilon_r = 5$ for *castor oil* and up to $\epsilon_r = 7$ for *cellulose fibers*.

- Liquids with *polar molecules of high mobility* have very high permittivities because of molecular polarization (orientation polarization). We find $\epsilon_r = 40$ approximately for *glycerin* and $\epsilon_r = 81$ for *water*.

Note: Both water and glycerin, have a comparatively high ionic conductivity. Therefore, they can be used as dielectrics for very short impulse stresses only.

- Extreme relative permittivities $\epsilon_r > 1000$ can be observed in so-called *ferroelectrics*. Close to the transformation temperature of a crystal structure the binding conditions can change so as to cause a so-called “polarization catastrophe”, i.e. an extreme increase of the permittivity, under the influence of the electric field [25]. This effect is strongly dependent on temperature and field strength; it occurs only in the direction of certain crystal axes. For *barium titanate* $\epsilon_r = 3000 \dots 7000$ approximately.

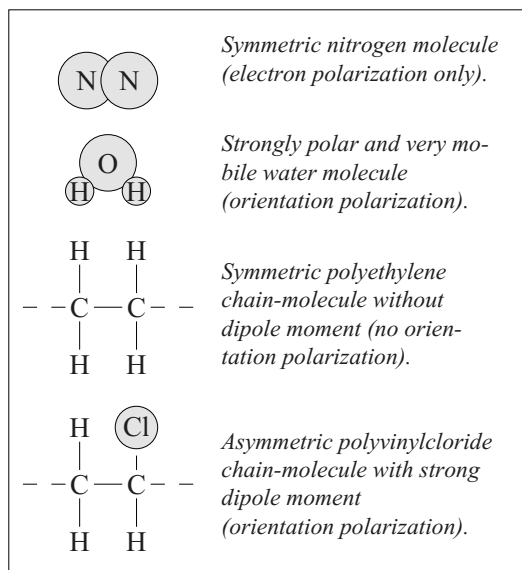


Figure 2.4-4: Examples for polarization mechanisms in insulating materials.

Permittivities are not constant quantities; they vary mainly with *temperature* T and *frequency* f of the electric field, see Figure 2.4-5 and Section 4.2.

With increasing *temperature* the mobility of given dipoles increases on the one hand, and on the other hand the Brownian motion and thermal agitation cause an increasing destruction of dipole orientation by collisions. Therefore, an increasing temperature can be associated with an increasing relative permittivity ϵ_r at first, because of the increasing mobility of the dipoles formerly “frozen” at lower temperatures. This is often accompanied by a structural change of the insulating material. Further increasing temperatures then result in decreasing relative permittivities, Figure 2.4-5 (top) and Figure 4.2-13.

With increasing *frequency*, the polarization is influenced by the mass inertia and interaction of the dipoles, which is maximal for the orientation of larger dipoles and minimal for the electron polarization. With increasing frequency, the dipoles cannot follow the field changes without delay, because of their mass inertia. Therefore, the relative permittivity is strongly dependent on frequency (dispersion): Generally ϵ_r decreases with increasing frequency in steps, which are associated with the stepwise drop-out of different polarization processes, Figure 2.4-5 (bottom), 4.2-3 and 4.2-13.

Note: Similar to the resistive losses in a conduction field (current losses), *polarization losses (dissipation, dielectric losses)* occur during the polarization process. These losses are caused by collisions and energy dissipation during the repetitive re-orientation of the dipoles with the frequency of the applied field. At low frequencies, the polarization losses are small, because of the low repetition rate. At high frequencies, there is no polarization any more and no dissipation consequently. Maximum losses are produced in the range of the transition frequency, Figure 2.4-5 bottom, see Chapter 4.

For sinusoidal AC fields the dielectric displacement current and a fictitious loss current (describing the losses both by conduction current and by polarization) can be described in frequency domain by a *complex relative permittivity*. The real part equals ϵ_r and the imaginary part describes the losses, see Section 4.2.4.

2.4.2 Multi-dielectric Arrangements

For multi-dielectric arrangements, special boundary conditions for field quantities of slowly changing fields at dielectric interfaces can be derived from Maxwell's Equations (Section 2.4.2.1). Multi-dielectric arrangements with interfaces orthogonal, parallel and inclined to the field direction are discussed for the dielectric displacement field, which is normally assumed for alternating fields in insulating materials (Section 2.4.2.2 to 2.4.2.4). The stationary conduction field at DC voltages is discussed analogously in Section 2.4.4.

2.4.2.1 Boundary Conditions at Interfaces

The interface between two different insulating materials is considered, Figure 2.4-6.

From the integration of the electric field strength E along a very small closed path P_1 - P_2 - P_3 - P_4 - P_1 on both sides of the interface, Faraday's law gives according to Eq. (2.1-32)

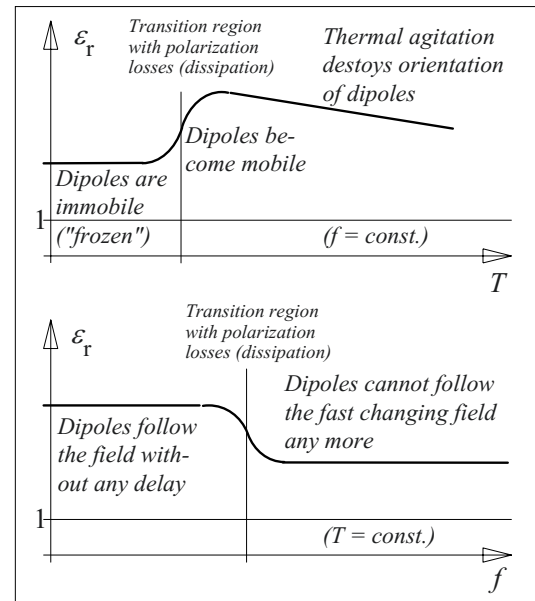


Figure 2.4-5: General dependence of the relative permittivity on the parameters temperature T and frequency f for a dielectric with orientation/ molecular polarization (schematic), cf. Figure 4.2-13.

$$\oint \mathbf{E} \, ds = E_{1t} \cdot s + (-E_{2t}) \cdot s = 0.$$

Therefore, the *tangential components* of the *electric field strength* are equal on both sides of the interface:

$$E_{1t} = E_{2t} \quad (2.4-13)$$

If the line P_1 - P_2 - P_3 - P_4 - P_1 is regarded as contour of a closed surface, it can be concluded from Gauss's law/ continuity Eq. (2.1-35) that all the current entering the enclosed volume on one side of the interface has to leave it on the other side. This condition is expressed by the continuity of the *normal components* of *current densities* (both conduction and displacement current density):

$$J_{1n} + \partial D_{1n} / \partial t = J_{2n} + \partial D_{2n} / \partial t \quad (2.4-14)$$

In many cases, it is possible to confine oneself to the *special cases* of the stationary conduction field (without displacement current) and the dielectric displacement field (without conduction current).

Therefore, in the case of the **stationary conduction field** (at DC voltage), the *normal component of the conduction current density $\mathbf{J}$* continuously passes through the interface:

$$J_{1n} = J_{2n} \quad (2.4-15)$$

For the **dielectric displacement field** (at AC voltage, if the conduction current can be neglected) the *normal component of the displacement density $\mathbf{D}$* continuously passes through the interface:

$$D_{1n} = D_{2n} \quad (2.4-16)$$

In the following, the *dielectric displacement field* alone is discussed. It can normally be assumed in insulating materials for alternating fields at power frequency and above.

Because of the analogy of the Equations (2.4-15) and (-16), the results can be transferred to

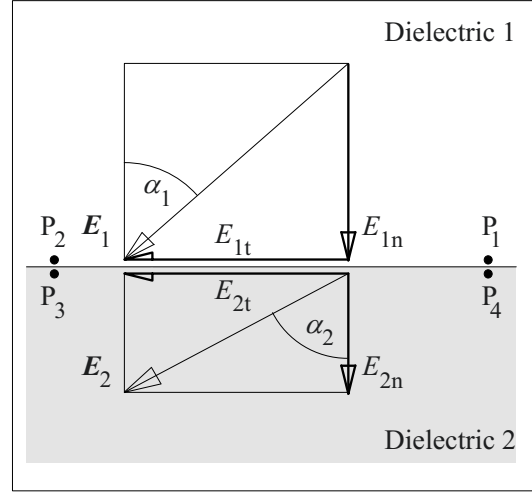


Figure 2.4-6: Vectors of the electric field strength at an interface between two dielectrics.

the stationary conduction field for DC fields (Section 2.4.4). For this purpose, in particular the ratio of the permittivities ϵ_1/ϵ_2 must be replaced by the ratio of the conductivities κ_1/κ_2 .

2.4.2.2 Interface Orthogonal (Normal) to the Field („Field Displacement“)

For “sandwiched” dielectrics, if the interface between two dielectric layers (with permittivities $\epsilon_1 = \epsilon_0 \cdot \epsilon_{r1}$ and $\epsilon_2 = \epsilon_0 \cdot \epsilon_{r2}$) is orthogonal to the electric field, the displacement density continuously passes through the interface, Figure 2.4-7. The magnitudes of the field quantities D and E are identical with the magnitudes of the normal components. According to Eq. (2.4-16) the displacement densities $D_1 = D_2$ are equal, i.e.

$$\frac{E_1}{E_2} = \frac{\epsilon_2}{\epsilon_1} \quad (2.4-17)$$

The magnitudes of the field strengths and the permittivities are inversely related to each other.

The dielectric with the *lower permittivity* is stressed with a *higher field strength* than the material with the higher permittivity. This effect is called “**field displacement**” into the dielectric with lower permittivity.

Note: The *field displacement* is of fundamental importance in high voltage engineering. For instance air or gas-filled insulating layers, which have a comparatively low electric strength, are stressed with strongly enhanced field strengths because of the field displacement effect. Gas-filled *gaps, cracks, cavities, shrink holes* and *voids* are some of the most frequent/common reasons for defective insulations and partial discharges, Figure 2.4-8. *Partial discharges* (PD) mostly cause a slowly progressing *erosion* of the insulating material, finally leading to breakdown.

In non-uniform fields, the field displacement effect can be used to reduce the stress on regions with high field stresses and to displace some of the stress into regions with lower field strengths.

For the partially uniform field of the parallel-plate capacitor according to Figure 2.4-7, the voltage is

$$V = V_1 + V_2 = E_1 \cdot d_1 + E_2 \cdot d_2.$$

With Eq. (2.4-17) the field strengths are

$$E_1 = \frac{V}{d_1 + d_2 \frac{\epsilon_{r1}}{\epsilon_{r2}}} \quad (2.4-18)$$

and

$$E_2 = \frac{V}{d_1 \frac{\epsilon_{r2}}{\epsilon_{r1}} + d_2}. \quad (2.4-19)$$

Example: Epoxy resin plate in an oil-insulated parallel-plate capacitor

An epoxy resin plate ($d_2 = 12$ mm, $\epsilon_{r2} = 4.4$) is inserted into an oil-insulated parallel-plate capacitor ($d = 20$ mm, $\epsilon_{r1} = 2.2$). The field changes shall be calculated for $\hat{V} = 80$ kV AC.

Prior to the insertion of the plate, the field strength is $\hat{E}_0 = \hat{V}/d = 4$ kV/mm. After the insertion of the plate, the field strengths for the oil gap and for the epoxy resin plate are $\hat{E}_1 =$

5.71 kV/mm = $1.43 \cdot \hat{E}_0$ and $\hat{E}_2 = 2.86$ kV/mm = $0.71 \hat{E}_0$, calculated with Eq. (2.4-18) and (-19) and with $d_1 = 8$ mm. The field strength in the oil is increased by 43 % through the insertion of the plate.

Note: The *highest* field displacement occurs for a very thin oil gap. With $d_1 \ll d_2 \cong d$ the field strength is $\hat{E}_1 \cong \hat{V}/(0 + d \epsilon_{r1}/\epsilon_{r2}) = \hat{E}_0 \cdot \epsilon_{r2}/\epsilon_{r1} = 2 \cdot \hat{E}_0$.

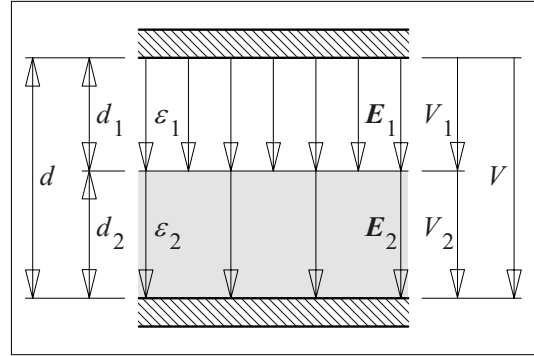


Figure 2.4-7: Two dielectrics in a parallel-plate capacitor with an interface orthogonal to the electrical field ("field displacement").

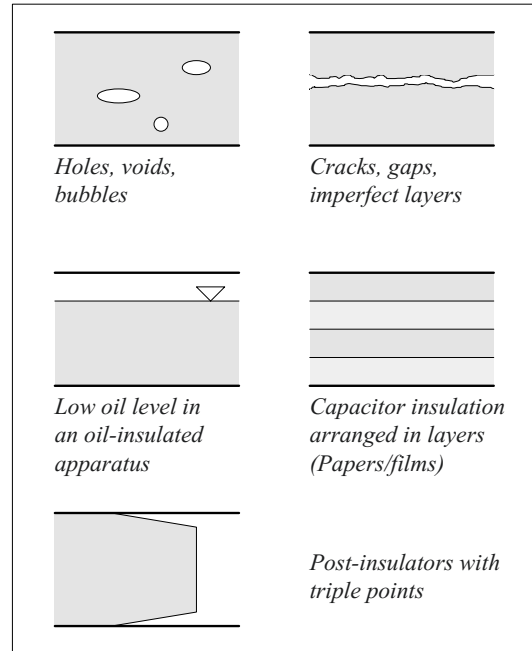


Figure 2.4-8: Examples of field displacement effects in high voltage engineering.

Example: Insulating barrier in an air-insulated parallel-plate capacitor

An insulating barrier ($d_2 = 4$ cm, $\epsilon_{r2} = 5$) shall be introduced into an air-insulated parallel-plate capacitor ($d = 5$ cm). The (macroscopic) partial discharge inception field strength E_{pd} is 15 kV/mm. The partial discharge inception voltages V_{pd} shall be calculated, in order to investigate whether the electric strength can be increased by the insertion of the barrier.

Without a barrier, V_{pd} is 75 kV. In a uniform field, the inception of discharges is directly accompanied by a breakdown. With a barrier, V_{pd} is 27 kV according to Eq. (2.4-18). At this voltage the air gap breaks down (partial breakdown), but the barrier prevents a total breakdown at first. After some time, the permanent discharge activity in the air gap will erode most of the insulating barriers (made of organic materials). Thus, the insulating barrier *deteriorates* the quality of the insulation!

Note: In high voltage insulations, the effect of electric strength deterioration because of field displacement effects is countered by a complete *impregnation* of all voids and gaps with an electrically strong insulating oil.

2.4.2.3 Interface Parallel to the Field (Tangential Interface)

If the interface between two dielectrics and the electric field are in parallel (tangential), the field strength is equal on both sides. E is tangential to the interface and it is continuous according to Eq. (2.3-13), Figure 2.4-9:

$$E_1 = E_2 = E_0 = V/d \quad (2.4-20a)$$

According to the material Eq. (2.4-8), the displacement densities on both sides of the interface are determined by

$$D_1/\epsilon_1 = D_2/\epsilon_2. \quad (2.4-20b)$$

Therefore, the displacement densities in the two dielectrics are different because of differ-

ent polarizations. Consequently, there is more charge on the electrode surface areas adjacent to the material with the higher permittivity than on the electrode surface areas adjacent to the material with the lower permittivity.

Because of the equal field strengths in both materials, it could be supposed that an interface in parallel to the electric field should be a good insulation arrangement. Nevertheless, it can often be observed that the **electric strength** of a tangential interface is *lower* than the strength of the two adjacent materials.

This seemingly paradoxical interfacial problem can have different causes depending on the type of interface:

- Material properties close to the surface often are different from the properties of the bulk material. Often, electrons are only *weakly bound* in impurity traps. Therefore, additional charge carriers are available for the generation of *surface avalanches and discharges*.
- Additionally, deposits of *foreign conductive and semi-conductive layers* can cause potential shifts and field stress enhancements at the surface. A flashover triggered as a result of this is referred to as "*pollution flashover*".

Note: DC stresses are especially critical because the semi-conductive pollution layers can have a very high conductivity in comparison with the insulating material.

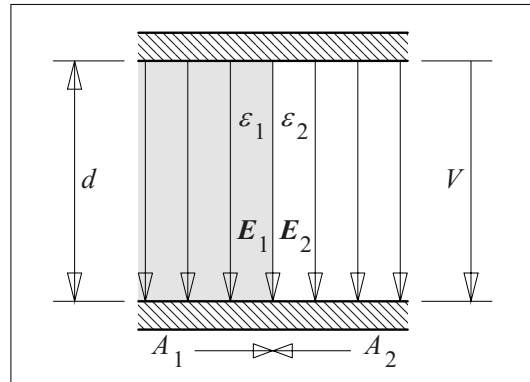


Fig. 2.4-9: Two dielectrics in a parallel-plate capacitor with an interface parallel to the field ("tangential interface").

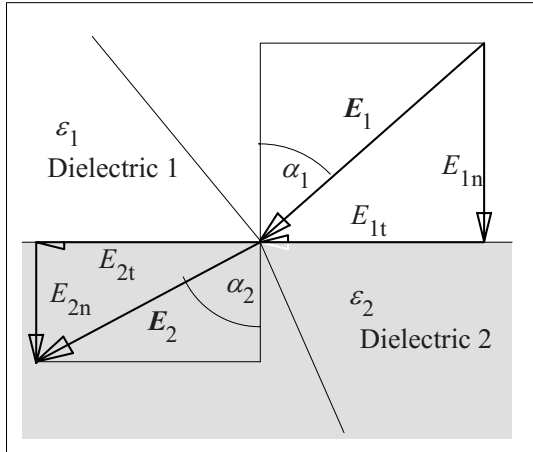


Figure 2.4-10: Vectors of the electric field strength, inclined to the interface between two dielectrics.

Also in the case of *AC stresses*, heavy pollution of insulator surfaces together with humidity can cause field distortions and flashovers. Insulators are therefore equipped with *shed profiles*, which guarantee relatively clean and dry zones, see also Figure 1-1 for example. In extreme cases *hydrophobic (water-repellent) materials* are used, e.g. silicone elastomers.

- Insulator surfaces are never totally smooth and parallel to the field. Because of the microscopic *surface roughness*, there are microscopically tiny areas with interfaces orthogonal to the field, so that *local field stress enhancements* can occur through field displacements.

2.4.2.4 Interface Inclined (at an Angle) to the Field (“Refraction Law”)

If the vector of the electric field strength $\mathbf{E}$ and the area vector $\mathbf{A}$ (orthogonal to the interface) enclose an angle α between 0° and 90° , the electric field is “*inclined to the interface*”, Figure 2.4-10. There are different angles α_1 and α_2 on both sides of the interface, i.e. field lines and equipotential lines are subject to *refraction* at the interface.

The so-called “refraction law” for the dielectric displacement field is derived from Eq. (2.4-13) and (-16). As the normal components

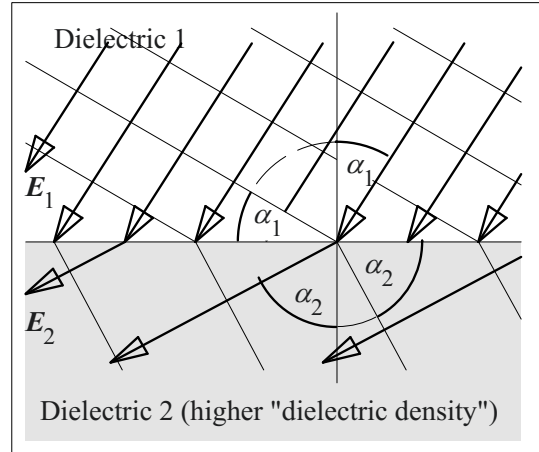


Figure 2.4-11: “Refraction” of field and equipotential lines at the interface between two dielectrics.

of the displacement density $\mathbf{D}$ are continuous at the interface, this gives

$$D_{1n} = D_{2n}$$

and

$$\varepsilon_{r1} \cdot E_{1n} = \varepsilon_{r2} \cdot E_{2n}.$$

Furthermore, the tangential component of the electric field strength is continuous, i.e.

$$E_{1t} = E_{2t}.$$

The division of the two continuity conditions gives

$$E_{1t}/(E_{1n} \cdot \varepsilon_{r1}) = E_{2t}/(E_{2n} \cdot \varepsilon_{r2}).$$

The quotient of tangential and normal components is equal to the tangent of the angle α_1 or α_2 , Figure 2.4-10:

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \quad (2.4-21)$$

Eq. (2.4-21) is called “refraction law” for the electric field lines and for the orthogonal equipotential lines.

It states that the angle α between field vector $\mathbf{E}$ and the surface normal vector $d\mathbf{A}$ decreases or increases with the permittivity ε . In the material with the higher permittivity (higher “dielectric density”), the field lines are refracted

away from the surface normal. In the material with the lower permittivity (lower “dielectric density”) the field lines are refracted towards the surface normal. If the *permittivities* are *very different*, the field lines emerge from the material with the much higher permittivity nearly *normal to the surface*.

Equipotential lines are perpendicular to field lines. In the material with the higher permittivity they are refracted towards the surface normal, and in the material with the lower permittivity they are refracted away from the surface normal resp., Figure 2.4-11. Therefore, the angles α_1 and α_2 can also be regarded as angles between the equipotential lines and the surface contour itself. The angles in Figure 2.4-11 represent a ratio of approximately $\epsilon_{r1}/\epsilon_{r2} = 1/3$.

In the case of two-dimensional plane fields, the methods of **graphical field mapping** can be applied. According to Eq. (2.3-35), the concept of *equal partial capacitances*, for the rectangular cross sections defined by field and equipotential lines, would result in the condition

$$\epsilon_r \cdot b/a = \text{const.} \quad (2.4-22a)$$

Therefore, the aspect ratio of the rectangles would be changed in proportion to the permittivities.

For the practical mapping, however, it is more convenient to assume *equal rectangles* (e.g. quadratic boxes) in the whole field volume:

$$b/a = \text{const.} \quad (2.4-22b)$$

Thereby, the distance of field lines is proportional to the distance of the equipotential lines and gives a graphic impression of the *electric field strength magnitude*, Figure 2.4-11.

For *capacitance estimation*, the different partial capacitances must be calculated in the different regions according to Eq. (2.3-35), and then they are combined in an equivalent network. Alternatively, elements with the same

partial capacitance can be marked off in a field/equipotential-line plot, Figure 2.4-12 (bold field and potential lines).

2.4.3 Analytical Calculation of Multilayer Dielectric Arrangements

Generally, multilayer dielectric arrangements are calculated with the methods described in Section 2.3. Additionally, the boundary conditions according to Eq. (2.4-13) and (-16) must also be fulfilled.

Some important multilayer dielectric arrangements are discussed below:

Section 2.4.3.1 describes plane, cylindrically symmetric and spherically symmetric multilayer insulations. The field vectors shall be orthogonal to the interfaces (transverse multilayer dielectrics).

Weak links or defects in insulations often occur as multi-layer arrangements, e.g. as gaps and cracks (Section 2.4.3.2), as interstices and triple-points (Section 2.4.3.3) or as holes, voids and inclusions (Section 2.4.3.4). The interfaces are subject to electric field forces (Section 2.4.3.5).

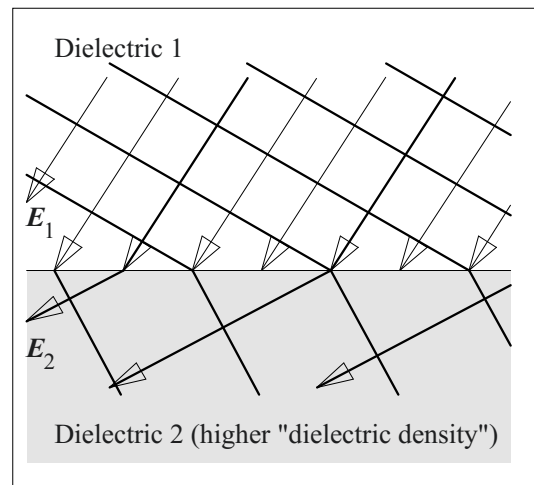


Figure 2.4-12: Outline of elements with equal partial capacitances (bold lines), for capacitance estimation in a two-dimensional arrangement.

2.4.3.1 Plane, Cylindrically Symmetric and Spherically Symmetric Multi-layer Arrangements

Very often, insulations are designed as multi-layer arrangements, which are stressed as nearly normally to the interfaces as possible in order to avoid tangential stresses along weak interfaces.

Examples:

1. **Capacitor dielectrics** consist of a number of layers of thin papers or polymer films. Voids and gaps are completely filled with an impregnating fluid. The dielectric can mostly be regarded as a plane multi-layer arrangement.

2. **High voltage conductors** between the windings of a transformer and the bushings are led through the insulating oil and they are wrapped with a paper layer to improve the insulation. At very high voltages a subdivision of the oil gap by cylindrical pressboard barriers may be necessary in order to guarantee sufficient electric strength.

3. **High voltage bushings** are cylindrically symmetric multi-layer arrangements, consisting of a main insulation body (core), subsidiary insulation gaps, housing insulators and ambient media, see Figure 1-1. Nevertheless, the direction of the electric field can approximately be assumed to be radial in certain regions (between the grading layers).

In the following, the analytical calculation of *basic multi-layer insulations* in plane, cylindrically symmetric and spherically symmetric arrangements is discussed, Figure 2.4-13. The *dielectrics* are always layered *transversely* to the electric field, i.e. E and the displacement density D are always normal (orthogonal) to the interfaces (parallel to the area vector A). Consequently, the interfaces are regarded as equipotential surfaces.

Because of the continuity of the displacement density,

$$D_k(x_k) = D_{k+1}(x_k) \quad (2.4-23)$$

applies for any interface at $x = x_k$ (or $r = r_k$), and the electric field strength changes abruptly at the *interface*, with the reverse ratio of the permittivities:

$$E_{k+1}/E_k = \varepsilon_k/\varepsilon_{k+1} \quad (2.4-24)$$

In Figure 2.4-13 a gradation of the permittivity ratios was chosen according to 6 : 4 : 2 : 1, both in x - and in r -direction respectively. For all three arrangements, the electric field strength increases by 50 % at the first interface (at x_1 and at r_1) and by 100 % at the two following interfaces, Figure 2.4-13 (middle).

In a *plane arrangement* there is a uniform field within a single dielectric layer, i.e. E is constant section by section. Because of the field displacement, the field strength E_N in the dielectric N is 600 % of the field strength E_1 in the dielectric 1.

In a *cylindrically symmetric arrangement* of length z

$$E_k(r) = \frac{Q/z}{2\pi\varepsilon_0\varepsilon_{rk}} \cdot \frac{1}{r} \quad (2.4-25)$$

generally applies for any dielectric layer k , according to Eq. (2.3-17).

The field strength profile in Figure 2.4-13 is a result of decreasing field strengths $E_k(r) \sim 1/r$ in the considered dielectrics and abrupt field strength changes at the interfaces. In comparison with the plane arrangement there is a homogenization of the stress, i.e. on the outer side there is a decrease and on the inner side there is an increase of the electric field stress.

According to Eq. (2.3-2),

$$E_k(r) = \frac{Q}{4\pi\varepsilon_0\varepsilon_{rk}} \cdot \frac{1}{r^2} \quad (2.4-26)$$

generally applies for *spherically symmetric arrangements*. The field strength profile in Figure 2.1-13 is a result of decreasing field strengths $E_k(r) \sim 1/r^2$ in the considered dielectrics and abrupt field strength changes at the interfaces. In comparison with the plane and cylindrical arrangements, there is a strong decrease at the outer side. Dielectric no. 1, which is weakly stressed in the other arrangements, is exposed to the highest stress here.

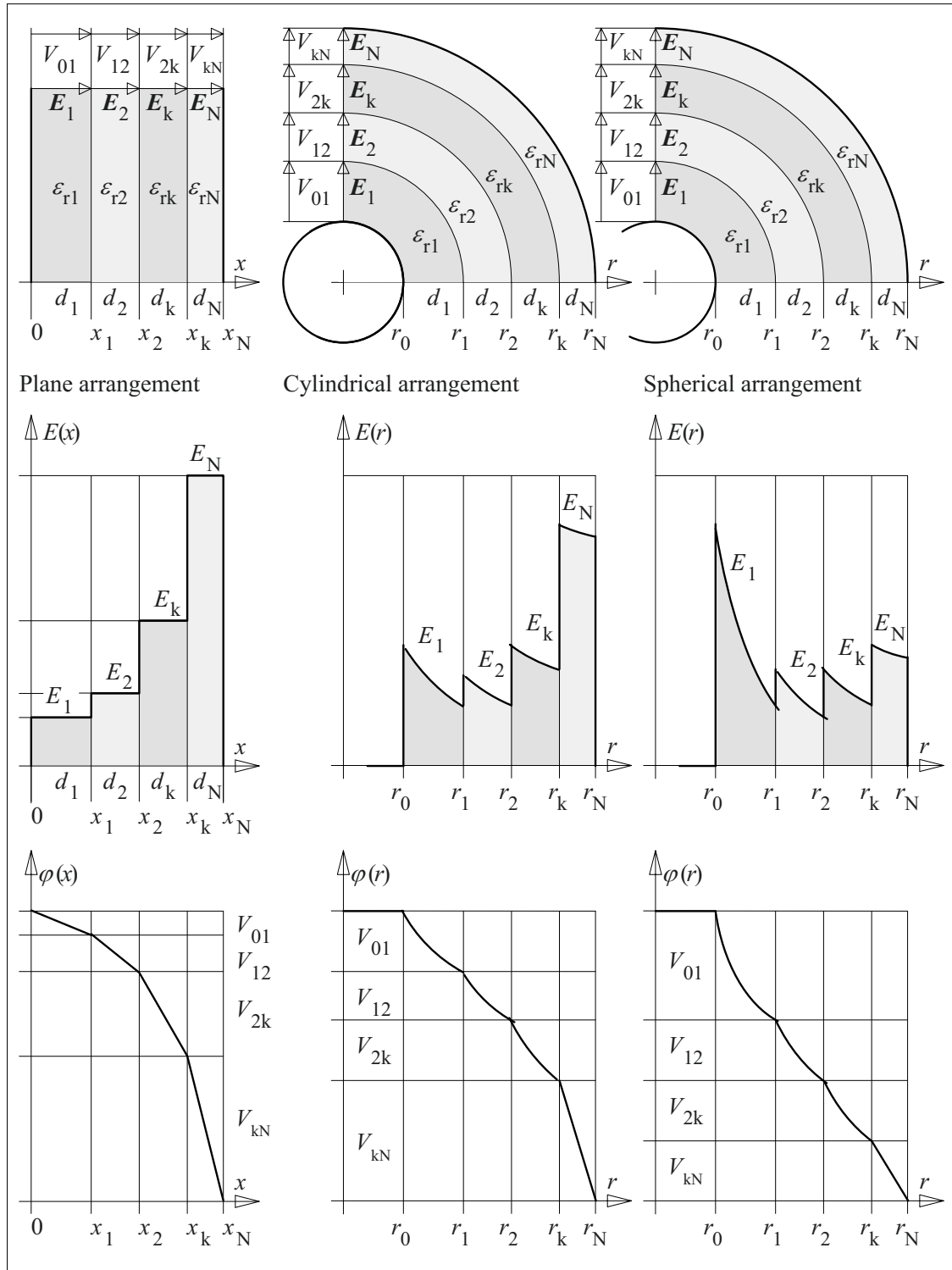


Figure 2.4-13: Plane, cylindrical and spherical multi-layer insulation arrangement (top) with the related field strength profiles (middle) and the potential profiles (bottom). The profiles in the figure are in accordance with a permittivity ratio of approximately 6 : 4 : 2 : 1 in x - or in r -direction resp..

Note: Theoretically, *constant field strengths* can also be achieved in cylindrically and spherically symmetric arrangements, if the relative permittivities according to Eq. (2.4-25) and (-26) are assumed to decrease continuously with

$$\varepsilon_r(r) \sim 1/r \quad \text{and} \quad \varepsilon_r(r) \sim 1/r^2$$

(refractive potential grading). Unfortunately, there is no technically practical solution for this. The approximation of the perfect permittivity profiles by discrete layers with appropriate permittivities will normally not be possible: In practice, the choice of insulating materials is very much limited because of many constraints (e.g. mechanical, thermal or chemical stresses, costs etc.). Optimizations problems often have to be solved by application of certain materials with given properties.

Example: Coated spherical electrode

For the spherically symmetric multi-layer arrangement according to Figure 2.4-13 (right) there is a high stress on the inner dielectric 1 (e.g. epoxy resin coating with $\varepsilon_{r1} = 6$) and a significantly lower stress on the outer dielectric N (e.g. air gap with $\varepsilon_{rN} = 1$). Because of the much higher electric strength of the epoxy resin, it would not make sense to seek to homogenize the field strength profile. Multi-layer insulations should be designed in such a way that the materials with the *higher electric strength* are *carrying a higher dielectric stress* than the electrically weaker materials.

For the **calculation of the plane arrangement** a constant displacement density can be assumed, Figure 2.4-13 (left):

$$D = Q/A = \text{const.}$$

The field strengths in the different layers are

$$E_k = Q/(A \varepsilon_k).$$

With the total voltage

$$V = \sum_{i=1}^N E_i d_i = \frac{Q}{A} \cdot \sum_{i=1}^N \frac{d_i}{\varepsilon_i},$$

the charge density

$$\frac{Q}{A} = V / \sum_{i=1}^N \frac{d_i}{\varepsilon_i}$$

and the *field strength* E_k in any layer k

$$E_k = \frac{V}{\varepsilon_k \sum_{i=1}^N \frac{d_i}{\varepsilon_i}} = \frac{V}{\varepsilon_k \left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \dots + \frac{d_N}{\varepsilon_N} \right)} \quad (2.4-27)$$

is calculated. The *potential distribution* results from the sectional integration of the electric field strengths, Figure 2.4-13 (bottom left).

The *capacitance* of the plane arrangement is determined from the series connection of the partial capacitances

$$C_k = \varepsilon_k A / d_k$$

or from the above-mentioned relation between Q and V with $\varepsilon_k = \varepsilon_0 \varepsilon_{rk}$:

$$C = \frac{\varepsilon_0 \cdot A}{\sum_{i=1}^N \frac{d_i}{\varepsilon_{ri}}} = \frac{\varepsilon_0 \cdot A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \dots + \frac{d_N}{\varepsilon_{rN}}} \quad (2.4-28)$$

For the **calculation of the cylindrically symmetric arrangement**, the dependence of the displacement density on the radius r must be considered, Figure 2.4-13 (middle). Eq. (2.4-25) for the field strength $E_k(r)$ in any layer k results from

$$D = Q / A(r) = Q / (2\pi r \cdot z).$$

The length of the arrangement is z . The integration of $E_k(r)$ in the radial direction provides the partial voltage

$$V_{(k-1)k} = [Q / (2\pi \varepsilon_k \cdot z)] \cdot \ln(r_k / r_{k-1}).$$

The summation of the partial voltages gives the total voltage V . After that, Q can be expressed as a function of V and can be eliminated in Eq. (2.4-25):

$$E_k(r) = \frac{V}{r \cdot \varepsilon_k \sum_{i=1}^N \left\{ \frac{1}{\varepsilon_i} \cdot \ln \frac{r_i}{r_{i-1}} \right\}} \quad (2.4-29)$$

This expression is valid in any layer k , i.e. for $r_{k-1} < r < r_k$.

The *potential distribution* results from the sectional integration of the electric field strengths, Figure 2.4-13 (bottom middle).

The *capacitance* of the cylindrical arrangement is determined from the series connection of the partial capacitances according to Eq. (2.3-20)

$$C_k = 2\pi\epsilon_k z / \ln(r_k/r_{k-1})$$

or from the relation $C = Q/V$ with $\epsilon_k = \epsilon_0\epsilon_{rk}$:

$$C = \frac{2\pi\epsilon_0 z}{\sum_{i=1}^N \left\{ \frac{1}{\epsilon_{ri}} \cdot \ln \frac{r_i}{r_{i-1}} \right\}} \quad (2.4-30)$$

Also for the **calculation of the spherical arrangement**, it has to be regarded that the displacement density depends on the radius r , Figure 2.4-13 (right). Eq. (2.4-26) for the field strength $E_k(r)$ in any layer k results from

$$D = Q / A(r) = Q / (4\pi r^2)$$

The integration of $E_k(r)$ in radial direction provides the partial voltage

$$V_{(k-1)k} = (1/r_{k-1} - 1/r_k) \cdot Q / (4\pi\epsilon_k).$$

The summation of the partial voltages gives the total voltage V . After that, Q can be expressed as a function of V and can be eliminated in Eq. (2.4-26):

$$E_k(r) = \frac{V}{r^2 \cdot \epsilon_k \sum_{i=1}^N \left\{ \frac{1}{\epsilon_i} \left(\frac{1}{r_{i-1}} - \frac{1}{r_i} \right) \right\}} \quad (2.4-31)$$

This expression is valid in any layer k , i.e. between $r_{k-1} < r < r_k$.

The *potential distribution* results from the sectional integration of the electric field strengths, Figure 2.4-13 (bottom right).

The *capacitance* of the spherical arrangement can be determined from the series connection of the spherically symmetric partial capacitances according to Eq. (2.3-12)

$$C_k = 4\pi\epsilon_k / (1/r_{k-1} - 1/r_k)$$

or from the relation $C = Q/V$ with $\epsilon_k = \epsilon_0\epsilon_{rk}$:

$$C = \frac{4\pi\epsilon_0}{\sum_{i=1}^N \left\{ \frac{1}{\epsilon_{ri}} \left(\frac{1}{r_{i-1}} - \frac{1}{r_i} \right) \right\}} \quad (2.4-32)$$

Example: Coated cylindrical conductor

For a cylindrical high voltage conductor ($r_0 = 2$ cm) the thickness or the **radius** r_1 of a cast-resin coating ($\epsilon_{r1} = 5$) shall be determined in such a way that the maximum field strength in the surrounding gas becomes minimal. The conductor is led in a coaxial air-filled ground conductor ($r_2 = 10$ cm), Figure 2.4-14 (top). The peak value of the applied AC voltage shall be $\hat{V} = 100$ kV.

The maximum field strength in the gas occurs

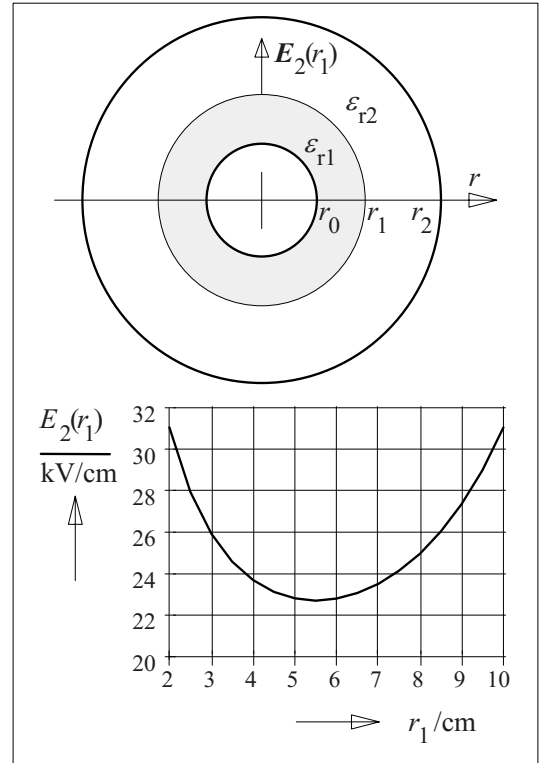


Figure 2.4-14: Cast-resin coated conductor in a gas-filled tubular ground conductor (top) and maximum field strength in the gas as a function of the coating-thickness/ radius (bottom).

at the surface of the cast-resin coating at $r = r_1$. The calculation of $E_{\text{Gas(max)}} = E_2(r_1)$ is performed according to Eq. (2.4-29). The dimensioning of the cast-resin coating (radius r_1) for minimum field strength at $r = r_1$ can generally be performed as extreme value determination by differentiation, see Chapter 2.3.1.2. For clarity, however, a numerical solution is preferred in this case, Figure 2.4-14 (bottom).

As a solution we have a coating thickness of 3.5 cm, i.e. at $r_1 = 5.5$ cm there is a field-strength minimum $\hat{E}_2(r_1) = 23$ kV/cm, which is below the breakdown strength of air at standard atmospheric conditions ($\hat{E}_D = 30$ kV/cm). For $r_1 \rightarrow r_0 = 2$ cm and for $r_1 \rightarrow r_2 = 10$ cm there are field strengths above $\hat{E}_D$. The diagram shows that a coating with a thickness of 1 cm already gives 60 % of the maximum possible field-strength reduction.

The application of SF_6 would increase the electric strength by a factor of approximately three and it would be possible to apply voltages approximately three-times higher.

Note: Similar conductor configurations can be found with conductors that are located under oil and wrapped with oil-impregnated paper.

Example: Coated high-voltage electrode

A spherical high voltage electrode with the radius $r_1 = 3$ cm shall be compared with a smaller electrode with the radius r_0 . By means of a cast-resin coating with $\epsilon_{r1} = 5$, the radius is increased to $r_1 = 3$ cm. The counter-electrode is assumed to be far away, $r_2 \gg r_1$, Figure 2.4-15.

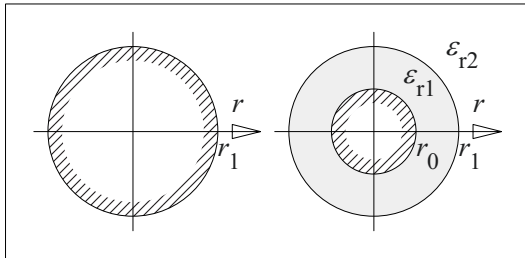


Figure 2.4-15: Spherical metallic electrode (left) and coated electrode (right) with the same external radii.

The **admissible voltages**, which can be applied to the two electrodes, shall be determined for a maximum admissible field strength $\hat{E}_{\text{Resin}} = 200$ kV/cm in the cast-resin coating and $\hat{E}_{\text{Air}} = 20$ kV/cm in the ambient air.

1.) According to Eq. (2.3-8), the voltage at the **metallic spherical electrode** must not exceed

$$\hat{V} = \hat{E}_L \cdot r_1 = 60 \text{ kV}$$

2.) The **coated electrode** has to be dimensioned in such a way that the field strength at the conductor's surface at $r = r_0$ is precisely $E_1(r_0) = \hat{E}_{\text{Resin}} = 200$ kV/cm, and that the field strength at the coating's surface at $r = r_1$ is precisely $E_2(r_1) = \hat{E}_{\text{Air}} = 20$ kV/cm. With Eq. (2.4-31) and $r_2 \rightarrow \infty$ two conditions are derived:

$$\hat{E}_1(r_0) = \hat{E}_{\text{Resin}} = \frac{\hat{V}}{r_0^2 \left\{ \frac{1}{r_0} + \left(\frac{\epsilon_1}{\epsilon_2} - 1 \right) \frac{1}{r_1} \right\}}$$

and

$$\hat{E}_2(r_1) = \hat{E}_{\text{Air}} = \frac{\hat{V}}{r_1^2 \frac{\epsilon_2}{\epsilon_1} \left\{ \frac{1}{r_0} + \left(\frac{\epsilon_1}{\epsilon_2} - 1 \right) \frac{1}{r_1} \right\}}$$

If the ratio of the two field strengths is calculated, the voltage and the bracket in the denominator are eliminated:

$$\hat{E}_{\text{Resin}} / \hat{E}_{\text{Air}} = (r_1^2 \cdot \epsilon_2) / (r_0^2 \cdot \epsilon_1)$$

The solution for r_0 is $r_0 = 0.42$ cm. With this value, the maximum voltage $\hat{V}$ can then be calculated from one of the two conditions mentioned above:

$$\hat{V} = 132 \text{ kV}$$

Thus, the admissible voltage at the coated electrode is twice as high as the admissible voltage at the metallic electrode. Nevertheless, on larger electrodes it is difficult to produce thick coatings, which are free from defects and which are able to carry the major fraction of the voltage. For practical applications, large metallic electrodes and toroids are used, if there is enough space in the ambient air.

2.4.3.2 Gaps and Cracks

Gaps and cracks in highly stressed insulations are defects, which must always be avoided. Gas-filled gaps remain between insulation layers for example, if the impregnation of the residual interstices is incomplete. Cracks often are caused by material ageing after long periods, and they mostly originate from mechanical and thermal stresses. Cracks can also originate from shrinkage stress during the curing of cast-resin insulation bodies.

Gaps and cracks **parallel to the electric field** are particularly critical because they bridge a major part of the insulation distance (up to the whole insulation distance) with an *interface* of very *low electric strength* and with *tangential stress*. Normally, the macroscopic field distribution is not influenced very much, but within the gap and at the interfaces there are microscopic field stress enhancements and significantly reduced electric strengths, see Section 2.4.2.3 (Parallel/ tangential multilayer dielectric).

Example: Glass-fiber reinforced plastics (GRP) have an extraordinarily enhanced mechanical strength because of glass fibers, which are embedded in the polymer matrix. Rods and tubes made of reinforced epoxy resin are used as mechanically and electrically stressed parts of suspension insulators, post insulators and housing insulators. These applications require a durable chemical bonding of the resin and the glass surfaces, free of any voids and defects. The bonding can be achieved by the application of a facing (primer) on the glass surface (*silane glass primer*). Incomplete or defective priming results in detachment of the fibers from the resin. In the very long cracks and cavities that arise humidity can be accumulated, which results in a significant decrease of the dielectric strength.

Gaps and cracks **orthogonal to the electric field** can approximately be regarded as multi-layer arrangements with interfaces normal to the electric field, Section 2.4.2.2. The field strength E_i in a gas-filled crack or gap ($\epsilon_{ti} = 1$) is enhanced by the factor $\epsilon_r/\epsilon_{ti} = \epsilon_r$ in comparison with the original field strength, because of the field displacement effect according to Eq. (2.4-17):

$$E_i = \epsilon_r \cdot E \quad (2.4-33)$$

Because of the low electric strength of air-filled gaps, the inception voltage of *partial discharges* is very low. The discharges can erode insulating materials and they can eventually cause breakdown (*breakdown by erosion*).

Example: Detachment of a dielectric

The epoxy casting resin in a cylindrical capacitor ($R_2 = 5$ cm, $R_1 = R_2/e$, $\epsilon_r = 4$) shrinks during curing onto the inner conductor, leaving a circumferential gap with the width d_i between 0 and 1 mm, Figure 2.4-16. The r.m.s. value of the applied voltage V_{pdi} , for which inception of partial discharges is expected, shall be calculated.

The electric strength of air under standard atmospheric conditions is approximately $\hat{E} = 30$ kV/cm = 3 kV/mm; it decreases with increasing distances, see Figure 3.2-15. Thus, the strength of the air gap is lowest for the highest gap width $d_i = 1$ mm. For this distance, the strength $\hat{E}(1 \text{ mm}) \geq 4$ kV/mm. If a constant field strength is assumed in the circumferential gap, discharge inception is expected to be at $d_i = 1$ mm.

The inception voltage V_{pdi} is calculated from Eq. (2.3-21) for the field strength at the outer radius $r = R_2$ and from Eq. (2.4-33) for the field stress enhancement in the gap:

$$V_{pdi} = E \cdot R_2 \cdot \ln(R_2/R_1) = (E_i/\epsilon_r) \cdot R_2 \cdot \ln(R_2/R_1)$$

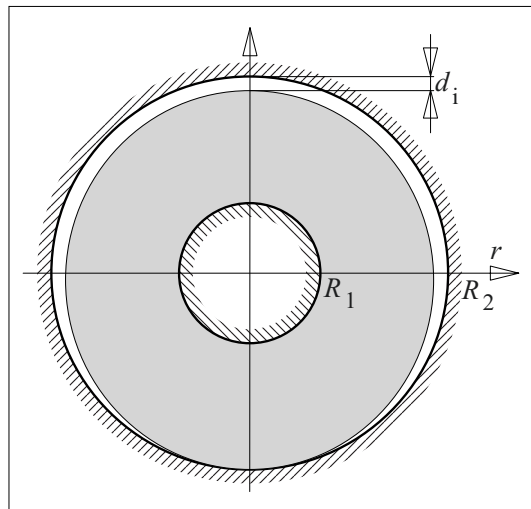


Figure 2.4-16: Detachment of a dielectric from the outer cylindrical conductor during the shrinking onto the inner cylindrical conductor.

With the partial-discharge inception field strength $\hat{E}_i \geq 4 \text{ kV/mm}$, the *partial-discharge inception voltage* is $\hat{V}_{\text{pdi}} \geq 5 \text{ kV}$ (peak value) or $V_{\text{pdi}} \geq 3.5 \text{ kV}$ (r.m.s. value).

Note: The discharge inception at $V = 3.5 \text{ kV}$ represents practically an extreme *loss of electric strength*. Without the formation of a gap, the highest relevant field strength would occur at $r = R_1$. If the admissible field strength in the epoxy resin is $\hat{E}_{\text{max}} = 40 \text{ kV/mm}$, the admissible maximum voltage would be $\hat{V}_{\text{max}} = 74 \text{ kV}$ (peak value) or $V_{\text{max}} = 52 \text{ kV}$ (r.m.s. value) according to Eq. (2.3-22).

Note: Resin-bonded paper bushing

Formerly used RBP bushing cores are “solid” insulation bodies (“hard paper”) wrapped with Kraft paper and bonded or laminated with phenolic resin without being fully impregnated (RBP, *resin-bonded paper*). The bushing cores could not be fully impregnated without residual air volumes in order to avoid high mechanical stresses and cracks during the curing process. Therefore, partial discharges could occur already at the service voltage both orthogonal and parallel to the paper layers, but the phenolic resin has a durability that is sufficient in many cases. Nevertheless, according to modern criteria, permanent partial discharges and erosion are a significant quality defect because surface discharges parallel to the paper layers, partial breakdowns and full breakdowns cannot be excluded.

Nowadays, cavity-free and discharge-free **RIP insulating bodies** are used (RIP, *resin-impregnated paper*) as bushing cores. They are wrapped with crepe-paper, dried, impregnated completely with liquid epoxy resin under vacuum and cured.

Example: Capacitor dielectric of polymeric films

A capacitor dielectric is wound on from polypropylene film (thickness $12 \mu\text{m}$, $\epsilon_r = 2.2$). Between adjacent layers, there are non-impregnable air-filled gaps with a maximum thickness of $7 \mu\text{m}$. The admissible voltage for four-layer insulation shall be estimated.

As the electric strength decreases with increasing gap width, discharge inception is expected for the maximum width $d_i = 4 \mu\text{m}$. According to Paschen’s law for air, the approximate electric strength of the gap is $\hat{V}_i \geq 360 \text{ V}$ or $\hat{E}_i \geq 90 \text{ V}/\mu\text{m}$, see Section 3.2.2.4.

According to Eq. (2.4-33), the field strength in the polymeric dielectric is approximately $\hat{E} = \hat{E}_i/\epsilon_r \geq 41 \text{ V}/\mu\text{m}$. Therefore, the whole dielectric with a thickness $d = 4 \cdot 12 \mu\text{m} = 48 \mu\text{m}$ can be stressed with a voltage in the range of $\hat{V} \geq 48 \mu\text{m} \cdot 41 \text{ V}/\mu\text{m} = 2 \text{ kV}$. This is a rough estimation of the partial discharge inception voltage

only, a more accurate calculation of the multi-layer insulation according to Eq. (2.4-27) would not be very useful.

Note 1: Higher voltages are possible, if the maximum *air-gap width* can be reduced. Then it has to be ensured that the field strength in the polymeric films does not exceed the respective electric strength.

Note 2: The partial discharge behavior in capacitor dielectrics made of films or papers is essentially determined from the *edges of the metal foils*, which are wrapped as electrodes together with the dielectric layers. At the edges, there are strong field distortions, field stress enhancements and interstices without films or papers. Therefore, the *impregnation* of high-voltage capacitors is always necessary.

2.4.3.3 Interstices (Triple-Points)

Tangentially stressed interfaces are particular weak points of an insulation arrangement, Figure 2.4-17 (left). Therefore, this “*support-type arrangement*” is avoided where possible and the interface is arranged orthogonal to the electric field forming a “*creepage surface*”, Figure 2.4-17 (right). Thus, tangential stresses are significantly reduced, and they decrease outwards to negligible small values. Because of the proximity of three materials, the considered microscopic region is called “*triple point*” or interstice between electrode and dielectric plate.

Unfortunately, there is an increased *normal electric field stress* in the interstice between insulating plate and bent electrode because of the field displacement effect. If the material in the interstice (e.g. air) has only a weak electric strength, the *partial discharge* inception volt-

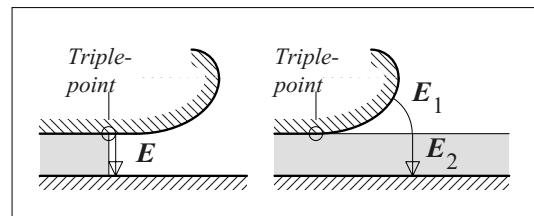


Figure 2.4-17: Insulating plate between electrodes: “Support-type arrangement” with tangential stress of the interface (left) and “creepage surface” with normal stress of the air-filled interstice (right).

age can be very low. At (significantly) higher voltages, the discharges can grow into *creeping/ surface discharges* and result in *surface flashover*. Therefore, the insulation arrangement is called a “**creepage surface**”.

Note: The creepage surface is a *basic problem* of high-voltage engineering, which cannot be avoided in many technical arrangements. Many technological measures are taken therefore, in order to prevent discharge inception in the interstices close to triple-points and to prevent the inception of surface discharges [26].

For a rough estimation of the *partial discharge*

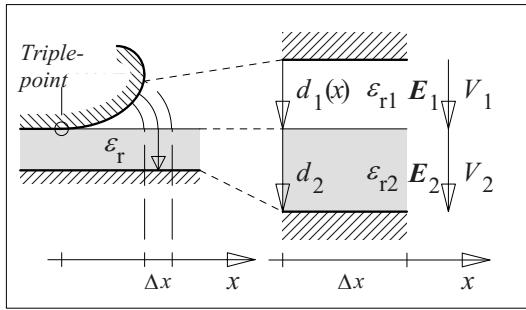


Figure 2.4-18: “Creepage surface” with highly stressed interstice (left) and an equivalent model of a small section for an approximate calculation (right).

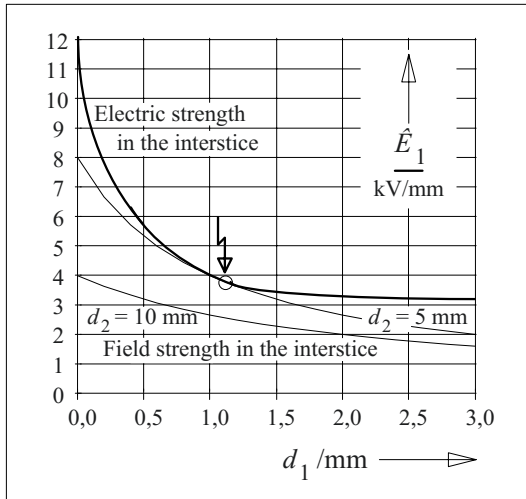


Figure 2.4-19: Field strength in an interstice as function of air-gap width at 8 kV (peak value) for insulating plates with a thickness of 5 mm and 10 mm (bottom) compared with the electric strength (top). The permittivity ratio is assumed as 1:5.

inception voltage V_{pdi} , it is simply assumed that there are small sections with regionally uniform field, Figure 2.4-18. The interface is orthogonal to the field, being uniform but different on both sides of the interface. The gap width d_1 in the interstice increases with the distance x from the triple-point. A section Δx is considered with approximately uniform field regions 1 (interstice) and 2 (insulating plate). According to Eq. (2.4-18), the field strength in the interstice is

$$E_1(d_1) = \frac{V}{d_1 + d_2 \frac{\epsilon_{r1}}{\epsilon_{r2}}} \quad (2.4-34)$$

Example:

Electrode edge on an insulating plate

An electrode edge on an insulating plate according to Figure 2.4-18, is discussed. Figure 2.4-19 shows the numerical analysis of Eq. (2.4-34) for a total voltage $\hat{V} = 8$ kV ($V = 5.7$ kV r.m.s.), for the insulation thicknesses $d_2 = 5$ and 10 mm and for the permittivity ratio $\epsilon_{r1}/\epsilon_{r2} = 1/5$.

There is a *decreasing field strength in the interstice* with increasing gap width d_1 . If the insulating thickness d_2 is doubled from 5 mm to 10 mm, the field strength at $d_1 = 0$ is reduced to half the magnitude, but a further decrease across d_1 is slower.

Figure 2.4-19 also shows a curve of the *electric strength in the interstice*. The increase of strength with decreasing air-gap width d_1 is typical for many insulating materials, e.g. for air, SF_6 and insulating oil. The curve in the picture is approximately valid for *air* at atmospheric pressure and room temperature.

For an insulating plate with thickness $d_2 = 5$ mm, the field strength in the gap reaches the electric strength of the gap at approximately $d_1 = 1.2$ mm, and partial discharges occur. Obviously, the voltage $\hat{V} = 8$ kV ($V = 5.7$ kV r.m.s. value) is the *partial discharge inception volt-*

age $\hat{V}_{\text{pdi}}$ (V_{pdi} r.m.s.). Normally, r.m.s. values are given.

If the insulation thickness is doubled (i.e. $d_2 = 10$ mm), there are no discharges at $\hat{V} = 8$ kV ($V = 5.7$ kV). Nevertheless, Figure 2.4-19 shows that the curve of the electric strength is reached if the voltage (i.e. the field strength) is only increased by about 40 %.

Note: Obviously, there is *no linear relation* between the *insulation thickness* d_2 and the peak value of the *partial discharge inception voltage* $\hat{V}_{\text{pdi}}$:

$$\hat{V}_{\text{pdi}} \sim d_2^{0.5}$$

According to Eq. (2.4-34), the field strength in the interstice depends on the product $d_2 \cdot \varepsilon_{r1}/\varepsilon_{r2}$. In accordance with the described **model**, the peak value of the partial discharge inception voltage for **surface discharges** is

$$\frac{\hat{V}_{\text{pdi}}}{\text{kV}} = \sqrt{2} K \left(\frac{d_2}{\text{cm}} \cdot \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \right)^a. \quad (2.4-35)$$

A theoretical proportionality factor $K = 18$ (for air) could be determined from Figure 2.4-19. Unfortunately, **experiments** show that the factor can be significantly smaller. Probably, the theoretical model (with regionally uniform fields according to Figure 2.4-18) is too simple. Furthermore, surface effects and different electrode shapes are not taken into account. Nevertheless, the general dependences of Eq. (2.4-35) are in good agreement with experiments for the exponent $a = 0.44 \dots 0.5$ [22], [23].

At *sharp electrode edges* the factors are approximately $K = 8$ for *air* and $K = 21$ for *SF₆* [23]. For *insulating oil* a factor $K = 20$ can be derived from [23].

Note: For different electrode edges under *oil*, factors between 21.6 (paper wrapped conductor on paper insulation) and 15.6 (for sharp electrode edges on paper insulation) are reported without particularly taking into

account the ratio $\varepsilon_{r1}/\varepsilon_{r2} \simeq 1/2$, which is already included in these factors [22].

Example: Edges of metal foil electrodes in capacitor insulations

For wound capacitors the metal foil electrodes and the insulating films or papers are wound together; remaining gaps and voids are filled completely with an impregnating medium, Figure 2.4-20. The electrical connection of the foils, which are displaced to left and right relative to each other, is either made at the ends with metallic tabs or made by large-area end contacts via all protruding foil edges, Figure 2.4-20 (top).

Very high field strengths occur in the interstices between the dielectric layers at the edges of the metallic foils. The critical point is not the normal (radial) field stress in the impregnating medium at the bent electrodes (as in the former example). In this case, the *tangential* (axial) *stress on the dielectric interfaces* is mainly problematic, which arises because of extreme field stress enhancements at the strongly curved electrode edges, Figure 2.4-20 (bottom).

An approximate calculation is performed for an equivalent cylindrically symmetric arrangement with $R_1 = d_M/2$ and $R_2 = d_M/2 + d_1$. The curved electrode edge with a very small *radius of curvature* $R_1 = d_M/2$ is regarded as the “inner conductor”; the adjacent metallic foils are regarded as the “outer conductor” and they are replaced by an *auxiliary cylinder* with the radius $R_2 = d_M/2 + d_1$. To a first approximation, the multi-layer arrangement of the dielectrics has no influence on the maximum magnitude of the electric field strength in the impregnating medium because of the tangential direction of the field E_{edge} at the electrode edge, Figure 2.4-20 (bottom). This means that the field is parallel (tangential) to the surface of the films or papers, see Figure 2.4-9.

With Eq. (2.3-22), the field strength at the edge of the foil (edge field strength) is given by

$$E_{\text{Rand}} = \frac{U}{R_1 \ln \frac{R_2}{R_1}} = \frac{E_0 d_1}{\frac{d_M}{2} \ln(1 + \frac{2d_1}{d_M})},$$

and the field strength enhancement is the reciprocal of the field efficiency factor η

$$\frac{E_{\text{Rand}}}{E_0} = \frac{1}{\eta} = \frac{\frac{2d_1}{d_M}}{\ln(1 + \frac{2d_1}{d_M})}. \quad (2.4-36)$$

The numerical analysis of Eq. (2.4-36) shows that significant field stress enhancements can occur, even for round edges, Figure 2.4-21 (lower curve). If further enhancements by im-

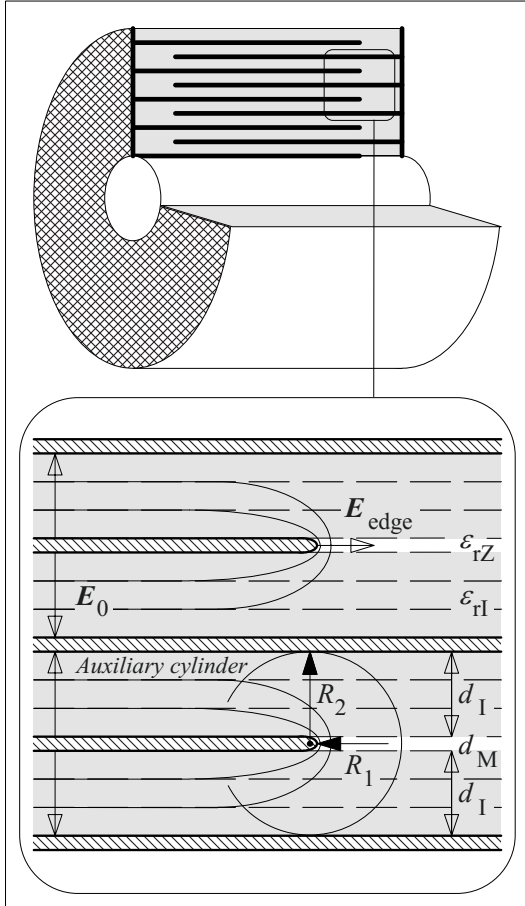


Figure 2.4-20: Roll-type capacitor with large-area end contacts of the axially shifted metal foils (top) and sectional view for the right-hand edge of the foils with equipotential lines (bottom).

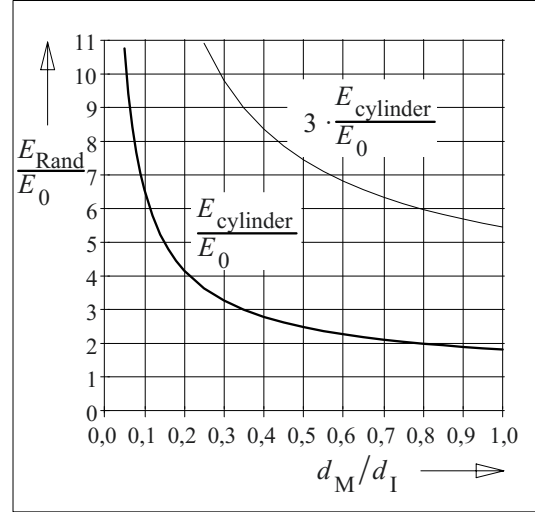


Figure 2.4-21: Field enhancement in a roll-type capacitor element at the edges of a metallic foil electrode. Lower curve: Calculation with the assumption of a cylindrically symmetric field. Upper curve: Taking into consideration additional field enhancement by imperfections of the surface.

perfections of the surface are considered, a factor of 3 is realistic according to Eq. (2.3-62), and field stress enhancements are even more extreme, Figure 2.4-21 (upper curve).

Numerical example:

A capacitor consists of paper insulated windings with an insulation thickness $d_I = 50 \mu\text{m}$, which are impregnated with mineral oil. The edges of the $6 \mu\text{m}$ thick aluminum foils are folded, in order to guarantee a smooth curvature at the edges. Partial discharge inception was measured at a r.m.s. AC-voltage of 3 kV. The field stresses between the foils and at the edges of the foils shall be calculated.

The field strength between the foils *within the papers* is $E_0 = 3 \text{ kV} / 50 \mu\text{m} = 60 \text{ kV/mm}$ for the uniform field region. Because of field displacement, there is a higher stress *in the impregnating oil-filled gaps*, $E_{0\text{-oil}} = \epsilon_{r\text{-paper}}/\epsilon_{r\text{-oil}} \cdot E_0 \simeq 120 \text{ kV/mm}$. At the edges a field stress enhancement $E_{\text{edge}}/E_0 = 3.7$ is calculated from Eq. (2.4-36) or Figure 2.4-21 with $d_M = 2 \cdot 6 \mu\text{m} = 12 \mu\text{m}$ (thickness is doubled at the folded edges) and $d_M/d_I = 0.24$. This gives the edge field strength as 220 kV/mm . Such an electric strength can be expected from oil gap widths in a range of a few μm [27]. However, the estimated very high field strength only occurs very close to the strongly curved edge. The field strength decreases very strongly with increasing distance $\sim 1/r$, i.e. at a distance of $6 \mu\text{m}$ ($r = 12 \mu\text{m}$) it is 110 kV/mm and at a distance of $18 \mu\text{m}$ ($r = 24 \mu\text{m}$) it is just 55 kV/mm .

Note: Normally, the calculation of edge field strengths and partial discharge (PD) inception voltages is not possible for practical applications, because of many unknown parameters. Therefore, *experiments with different insulation designs* are necessary, in order to determine the acceptable stress. For sharp-edged cut aluminum foils, the PD inception voltage would decrease from 3 kV to 2.5 kV ($E_0 = 50$ kV/mm) in the above-mentioned example. On the other hand, a significant increase of PD inception field strength could be achieved by means of special synthetic insulating liquids.

Theoretically, the **volume** of a capacitor can be minimized by choosing an optimal thickness d_M for the metal foils:

For $d_M \rightarrow 0$ the field stress enhancement factor becomes infinite, i.e. the admissible field strength and the energy density approach zero. For $d_M \gg d_I$ the dead volume of the foil v_M is very much greater than the energy storage volume of the dielectric v_I and the energy density approaches zero. In-between there must be a *maximum of overall energy density*.

$$w = 0.5 \varepsilon E_0^2 v_I / (v_I + v_M) \quad (2.4-37)$$

This equation can be used for the determination of the maximum energy density w , if the admissible edge field strength is given and if the equations for the volumes and Eq. (2.4-36) for E_0 are used:

The derivate of w with respect to the ratio d_I/d_M is set equal to zero. The resulting transcendental equation is solved iteratively with $d_I/d_M = 0.24$. This means that the metallic foil should theoretically be about four times as thick as the insulation.

In practice, the optimum can be assumed for *much thinner foils*: The admissible edge field strength is *not constant*. It increases significantly with decreasing radius of curvature. The best insulation design has to be determined experimentally, as mentioned above.

2.4.3.4 Dielectric Cavities and Spheres

Completely closed *cavities* in a medium with higher permittivity are defects, they can be observed e.g. as *bubbles* in insulating liquids, as *shrink holes* in epoxy resin bodies or as *voids* in porcelain insulators, Figure 2.4-22.

Dielectric spheres in a material with lower permittivity can also be defects, e.g. *non-conductive particles* in oil or in gas.

The basic effect of field displacement was already discussed for gaps and cracks in Section 2.4.3.2. For spherical defects bounded on all sides, the field distortion is less pronounced.

In solving Poisson's/ Laplace's Equation (2.3-34) for the spherically symmetric arrangement, Figure 2.4-22, it has to be considered as a boundary condition that there is a uniform field E_0 at infinite distance. Furthermore, the boundary conditions of Eq. (2.4-13) and (-16) have to be fulfilled at the sphere's surface. The solution is a **uniform field within the sphere** [2]:

$$E_1 = E_0 \cdot 3 \varepsilon_2 / (\varepsilon_1 + 2 \varepsilon_2) \quad (2.4-38)$$

Outside of the sphere, at the sphere's surface on the x -axis (which is determined by the field vector E_0), the solution is

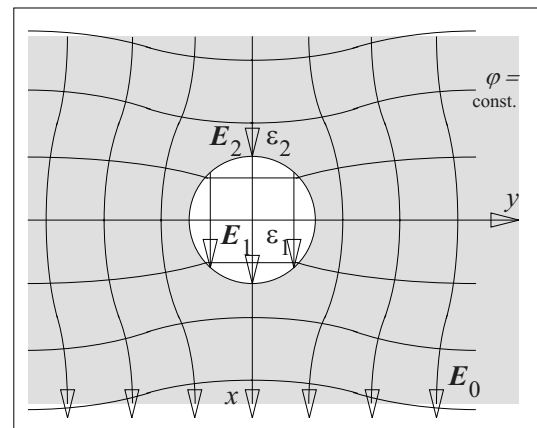


Figure 2.4-22: "Dielectric sphere" as a model of a cavity in an insulating material or as a model of a dielectric particle.

$$E_2 = E_0 \cdot 3 \varepsilon_1 / (\varepsilon_1 + 2 \varepsilon_2). \quad (2.4-39)$$

The comparison of Eq. (2.4-38) and (-39) shows that the magnitude ratio of the field vectors *normal* to the interface is the reciprocal of the permittivity ratio, see Eq. (2.4-17), transversely layered dielectric. The continuity of the *tangential* components $E_1 = E_2$ applies on the y -axis at the sphere's surface.

In the case of a **dielectric cavity** with a lower permittivity $\varepsilon_1 < \varepsilon_2$ the field strength E_1 in the cavity is enhanced in comparison with E_0 . The maximum value is $E_1 = 1.5 \cdot E_0$ for $\varepsilon_1 \ll \varepsilon_2$, according to Eq. (2.4-38), i.e. in a spherical cavity there is a *limited field stress enhancement only*. It is more serious that the gas-filled cavity normally has *weak electric strength* in comparison with a highly stressed surrounding dielectric. Therefore partial discharge inception voltages are significantly lower than for a solid or liquid insulating material without any defects.

In the case of a **dielectric particle**, with a permittivity ε_1 bigger than the permittivity ε_2 of the surrounding material, the field strength E_2 outside of the sphere is higher than within the sphere. The maximum value is $E_2 = 3 \cdot E_0$ on the x -axis at the sphere's surface for $\varepsilon_1 \gg \varepsilon_2$, according to Eq. (2.4-39). Therefore, dielectric particles can cause significant field strength enhancements in liquid and in gaseous media, and they can reduce the electric strength, especially in liquids.

2.4.3.5 Electric Forces at Interfaces

Often it is particularly troublesome that particles can follow the **electric field forces** and accumulate in the region of highest field strength.

The mechanical tensile stress exerted by an electric field **orthogonal to an interface** is [2]

$$\sigma = \partial F / \partial A = \frac{1}{2} \cdot E_1^2 (\varepsilon_2 - \varepsilon_1) \varepsilon_1 / \varepsilon_2. \quad (2.4-40)$$

The force acts from the higher towards the lower permittivity parallel to the field (*longitudinal* tensile stress).

In a *non-uniform field*, the forces on opposite sides of a dielectric body are no longer equal. The resulting force pulls the body towards increasing field strength.

Example:

In **insulating oil**, fibrous impurities are aligned parallel with the field lines, especially in the non-uniform regions of the field. This reduces the electric strength of long oil gaps significantly (*fiber-bridge breakdown, suspended solid particle mechanism*).

Also in **gas-insulated switchgear**, the electric strength is reduced by the presence of dielectric (and conductive) particles [28].

Also, the field component E_t **tangential to an interface** exerts a force orthogonal to the interface and towards the lower permittivity. The so-called *lateral pressure* is

$$\begin{aligned} \sigma &= \partial F / \partial A \\ &= \frac{1}{2} \cdot E_t^2 (\varepsilon_2 - \varepsilon_1). \end{aligned} \quad (2.4-41)$$

The tensile stress on **metallic electrode surfaces**

$$\begin{aligned} \sigma &= \partial F / \partial A \\ &= \frac{1}{2} \cdot E_n^2 \varepsilon \end{aligned} \quad (2.4-42)$$

results from the field that is always acting orthogonally to the surface and parallel to the field.

Note: Eqs. (2.4-40) to (-42) can each be deduced from an energy balance for an imaginary displacement of the interface by an infinitesimal shift Δx by the desired force F . This results in a change of electric field energy, which is equal to the exerted mechanical work $F \cdot \Delta x$. The mechanical pressure σ or the tensile stress σ is determined if the force F is divided by the area A [2].

2.4.4 Direct Voltage and Transients

There is a perfect *analogy* between the stationary conduction field at a pure DC voltage and the formerly discussed dielectric displacement field.

From this analogy, the *principles of the conduction field* are deduced (Section 2.4.4.1). They can be used for calculation of some typical *examples* of insulation systems stressed with DC voltage (Section 2.4.4.2). In many cases, there are no stationary conditions: If a DC voltage is switched on, reversed in polarity or changed in magnitude, a displacement field is generated. Then, a transient process takes place, approaching a new stationary condition (Section 2.4.4.3).

2.4.4.1 Analogies to the Dielectric Displacement Field

The basic Material Equations (2.1-19) and (2.1-20) contain a perfect analogy between the fields of the dielectric displacement density $\mathbf{D}$ and the conduction current density $\mathbf{J}$.

The corresponding equations and boundary conditions are compared against each other below for the displacement field (left) and the conduction field (right):

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \mathbf{J} = \kappa \mathbf{E} \quad (2.4-43)$$

The continuity of the normal components for the field quantities $\mathbf{D}$ and $\mathbf{J}$ is given with Eq. (2.4-15) and (2.4-16):

$$D_{1n} = D_{2n} \quad J_{1n} = J_{2n} \quad (2.4-44)$$

According to Eq. (2.4-13) the tangential component of the electric field strength $\mathbf{E}$ is also continuous at interfaces, both for the displacement field and for the conduction field:

$$E_{1t} = E_{2t} \quad E_{1t} = E_{2t} \quad (2.4-45)$$

Instead of a capacitance C in the displacement field, there is a conductance $G = 1/R$ (recipro-

cal of the resistance) in the conduction field. The following equations describe a parallel-plate capacitor e.g.:

$$C = \varepsilon A/d \quad G = 1/R = \kappa A/d. \quad (2.4-46)$$

The comparison shows that all relationships for the dielectric displacement field are also valid for the stationary conduction field, if the permittivity ε is replaced by the conductivity κ , the displacement density $\mathbf{D}$ by the conduction current density $\mathbf{J}$ and the capacitance C by the conductance G . This is also valid for the deduced Eqs. (2.4-17) to (2.4-32), which are related to interfaces orthogonal, parallel and inclined to the electric field direction.

For the **interface orthogonal to the electric field**, the continuity of the conduction current density $\mathbf{J}_1 = \mathbf{J}_2 = \mathbf{J}$ orthogonal to the interface is valid. By analogy with Eq. (2.4-17), it is concluded that

$$\frac{E_1}{E_2} = \frac{\kappa_2}{\kappa_1}. \quad (2.4-47)$$

The *field strength* magnitudes and the *conductivities* are in *inverse ratio* to each other. Analogously with the dielectric field displacement, the material with the lower conductivity is stressed with a higher field strength than the material with the higher conductivity.

Note: Conductivities often differ by several orders of magnitude. Thus, the material with the higher conductivity is almost completely without stress, but the material with the lower conductivity is stressed with nearly the whole voltage. This is an almost *complete field displacement*. Figure 2.4-23 shows field and potential distribution for the conductivity ratio $\kappa_1 : \kappa_2 = 1 : 10$.

The normal components of the conduction current density J_n are certainly continuous at the interface, but the normal components of the displacement density D_n are not continu-

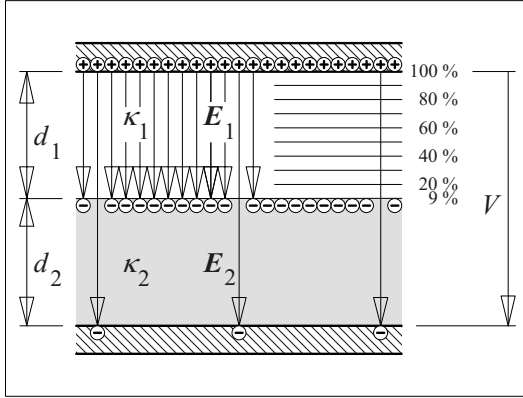


Figure 2.4-23: Field and potential distribution in two dielectrics with an interface orthogonal to the electric DC field (conductivity ratio 1 : 10).

ous. The difference between the displacement densities D_{1n} and D_{2n} is equal to a *surface charge density* σ at the interface. This effect is called *interfacial polarization*, Figure 2.4-23:

$$\begin{aligned}\sigma &= D_{2n} - D_{1n} \\ &= \varepsilon_2 E_2 - \varepsilon_1 E_1 \\ &= E_1 (\varepsilon_2 \kappa_1 / \kappa_2 - \varepsilon_1)\end{aligned}\quad (2.4-48)$$

In the case of a short circuit at the electrodes, the surface charge (interfacial polarization) does not disappear immediately, it decreases with the time constant $R_2 C_1$, which is determined by the geometries and by the material properties κ_2 and ε_1 , see also Figure 2.1-16. If the short circuit is opened too early, an *unexpected and therefore dangerous re-charging of the electrodes* (a so-called “*recovery voltage*”), can occur (see Section 2.4.4.3).

Example: Capacitor with mixed dielectric

In capacitor dielectrics, made of oil-impregnated paper layers and high-resistive polymeric films, there is an almost complete field displacement into the polymeric films. A numerical example was already discussed in Section 2.1.4.2. The example shows that the polymeric films almost entirely produce the insulation. The paper layers are mainly used as impregnation wicks.

For an **interface parallel to the electric field**, the tangential field E is theoretically not influenced by the adjacent materials, i.e. $E_1 = E_2 =$

E according to Eq. (2.4-45). The current densities are different on both sides of the interface because of the different conductivities: $J_1 = \kappa_1 E$ and $J_2 = \kappa_2 E$. According to Eq. (2.4-46) there are different area-related conductances and resistances on both sides of the interface.

It should be noted that for DC voltage stress, the interface parallel to a DC field is especially critical: *Conductive deposits and pollution layers* (e.g. caused by contaminations, impurities or wetting) can cause field distortions and extreme field stress enhancements if there are only slight non-uniformities in the layer, Figure 2.4-24.

For an **interface inclined to the electric field**, the different conductivities cause a refraction of the DC field lines and DC equipotential lines in the stationary conduction field (refraction law) by analogy with Eq. (2.4-21):

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\kappa_1}{\kappa_2} \quad (2.4-49)$$

α_1 and α_2 are the angles between the area vector A (orthogonal to the interface) and the field vectors E_1 and E_2 , Figure 2.4-25.

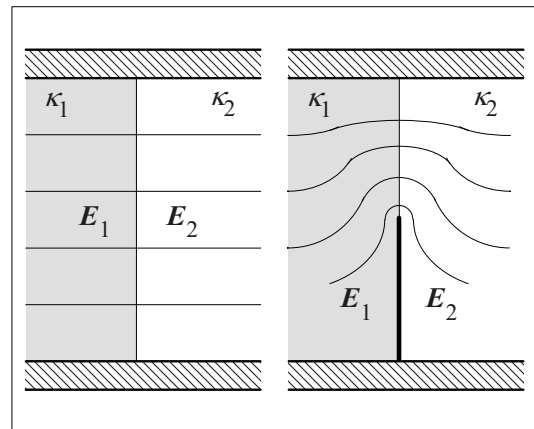


Figure 2.4-24: Interface parallel to a DC field. Left: Ideal potential distribution. Right: Potential distribution with a non-uniform conductive pollution layer causing extreme local field enhancements.

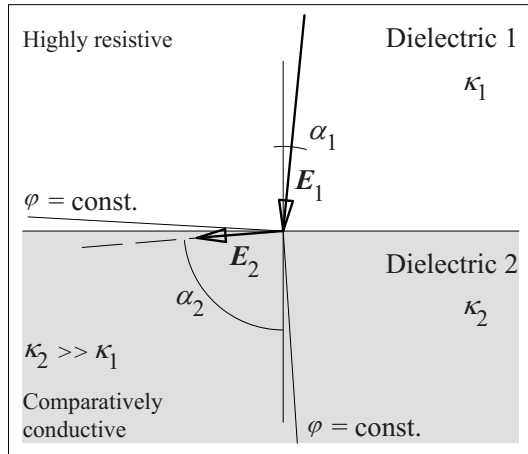


Figure 2.4-26: Refraction of field lines and potential lines in a stationary conduction field at the interface between insulating materials with very different conductivity.

In many practical applications, *conductivities* on both sides of the interface are *very different*. For $\kappa_2 \gg \kappa_1$, the angle α_2 approaches 90° , even for very small angles α_1 . In the *more conductive material 2*, the field lines are almost parallel and the potential lines are nearly orthogonal to the interface, Figure 2.4-26 (bottom). In the *more resistive material 1*, the field lines are almost orthogonal and the equipotential lines are nearly parallel to the interface, Figure 2.4-26 (top).

Note: This circumstance can clearly be explained by the fact that a *current* can only flow nearly parallel to the interface in the comparatively conductive material. Therefore, field lines must orient themselves almost parallel and equipotential lines almost orthogonal to the interface. In the highly resistive material, the field lines are almost orthogonal to the interface, which is similar to the situation close to a *conductive electrode*.

Example: In *oil-insulated equipment for high DC voltages*, the potential distribution in oil can be controlled by forming a uniform *oil duct* of higher conductivity between highly resistive *pressboard barriers* and other highly resistive insulating components (e.g. for bushings) [7].

For inclined layered dielectrics there is also a *surface charge* at the interface. It can also be calculated from the difference of the *normal components* of the displacement density $\mathbf{D}$.

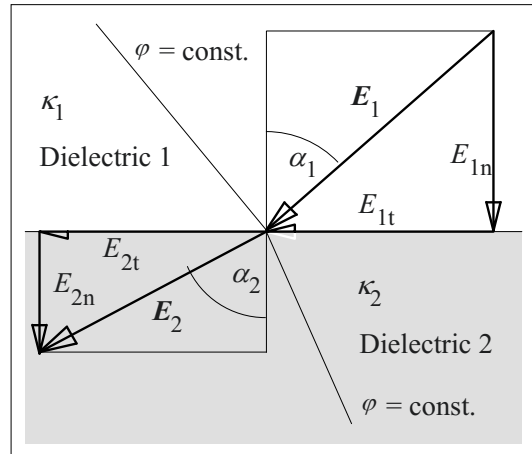


Figure 2.4-25: Vectors of the electric field strength and potential lines at an interface inclined to the electric field for two dielectrics with different conductivities ("Refraction" of field lines and potential lines at an inclined interface for a stationary conduction field).

The *calculation* of DC voltage fields is not only complicated by the possibility of large differences of the conductivities. Furthermore, it is often difficult to get *reliable numerical values*, since conductivity is dependent on the exact material composition, on manufacturing process technology and very strongly on the temperature. Some examples are described below:

- Different porcelain mixtures have different conductivities.
- The conductivity of oil-impregnated paper increases with the water content.
- The conductivity ratio in an oil-pressboard insulation may be 100 : 1 at 20 °C (test temperature). At 90° C (service temperature) the ratio is just 10 : 1.

It was already mentioned in Section 2.4.1.1 that in practice it is very important to determine reliable and applicable conductivity values. Because of the high degree of possible variations, a field calculation with wrong conductivity values can lead to completely wrong results.

2.4.4.2 Typical DC fields

Some examples for typical DC fields shall be discussed below. Because of high conductivity differences, strong temperature dependences and the sensitivity to pollution layers, there are field distributions which are completely different from a comparable AC field.

Example 1: **Capacitor with mixed dielectric**

The example of a *DC capacitor* with a mixed dielectric, made of *polymeric films* and *oil-impregnated paper with a hundredfold greater conductivity*, has already been discussed several times (Sections 2.1.4.2 and 2.4.4.1). It was shown that nearly all the voltage has to be insulated by the electrically strong polymeric films. The paper layers are relieved of the electrical stress to a large extent because of their significantly higher conductivity.

It is disadvantageous that the paper volume does not contribute to the capacitive storage volume. Therefore, it is desirable for weight reasons to design the insulation without any paper, which is only used as “impregnating wick”. Then, good impregnation has to be guaranteed by adequate surface texture of the films.

Note: For *AC voltage*, the papers are stressed with a field strength, which is half as high as in the polymeric films because of the field displacement, see Eq. (2.4-17) with $\epsilon_2/\epsilon_1 = 2$. Nevertheless, the field strength in the paper can be the critical quantity that limits the voltage, because of the very high electric strength of polymeric films. Thus, the design does not make full use of the excellent electric strength of the polymeric films, and it is desirable therefore to replace the paper by polymeric films (*all-film dielectric*).

Example 2: HVDC cable

In a high voltage DC cable with a homogeneous dielectric there is a cylindrically symmetric field. According to Eq. (2.3-21), the field strength decreases proportional to $1/r$ between the inner and outer conductors, Figure 2.4-27 (curve 1). During service operation, the inner conductor is heated by the ohmic losses due to

the current. As a result, there is a *temperature gradient* $T(r)$ from the inside to the outside. Because of the strong temperature dependence of the conductivity, a *conductivity gradient* is also caused. This results in a continuous field displacement from inside to outside. Depending on the conductor's temperature and the insulating material, the *field profile* is more or less *equalized*, Figure 2.4-27 (curves 2 and 3).

Nevertheless, the designer of the cable has to take into account that the cable has to withstand the voltage not only in the warmed-up service operation, but also in the *cold starting condition*.

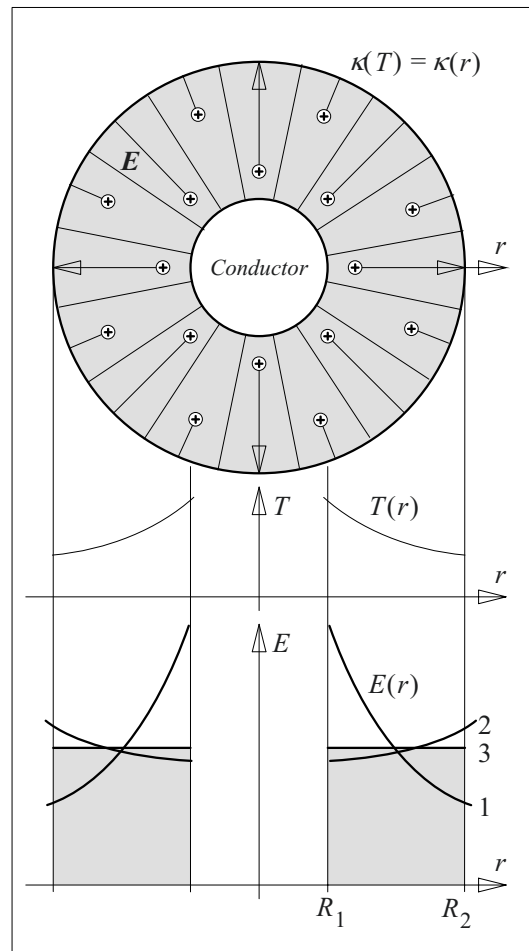


Figure 2.4-27: DC cable with a conductivity gradient caused by a temperature gradient and modification of the initial field distribution (curve 1) due to the space charge accumulation (curves 2 and 3).

The continuous variation of conductivity is accompanied by the accumulation of charge in the insulating material. In contrast to multi-layer dielectrics, the charge is not accumulated at the interfaces, here it is distributed as *space charge* over the entirety of the *inhomogeneous insulating material*. This leads finally to the deviation of the field stress profile from the initial profile $\sim 1/r$.

The space charge is of high importance for the operation of the cable because the remaining charge can cause very high *field stress enhancements* after a *polarity reversal*. Furthermore, space charge can cause a dangerous *re-charging* of the cable, if a short circuit between inner and outer conductor is opened again. Because of the high capacitance of long cables, even relatively low “*recovery voltages*” can accumulate significant and dangerous amounts of charge.

Example 3: HVDC Bushing

A high voltage electrode in oil shall be connected via a capacitively graded bushing, both for AC and DC, Figure 2.4-28. At *AC voltage*, the capacitive grading layers have approximately the intended potentials, because of their mutual capacitances. In this way the tangential stress at the bushing surface is significantly reduced, Figure 2.4-28 (top).

Also at *DC voltage*, the intended potential distribution is approximately achieved *within* the bushing core, because of the mutual resistances of the grading layers, i.e. the grading is now resistive and no longer capacitive. *Outside* the bushing, there is a

completely different potential distribution within the oil, Figure 2.4-28 (middle). The distribution is mainly determined by the electrode geometry, and the highly resistive bushing acts as a highly resistive boundary of the comparatively conductive oil volume. In this way, a very high *tangential stress* of the bushing surface can occur.

This field concentration can be avoided by a very large electrode diameter in very large oil tanks (cylinder). However, this is not generally an economic solution.

In the case of the given narrow installation conditions, the tangential field strength can also be reduced by a system of highly resistive

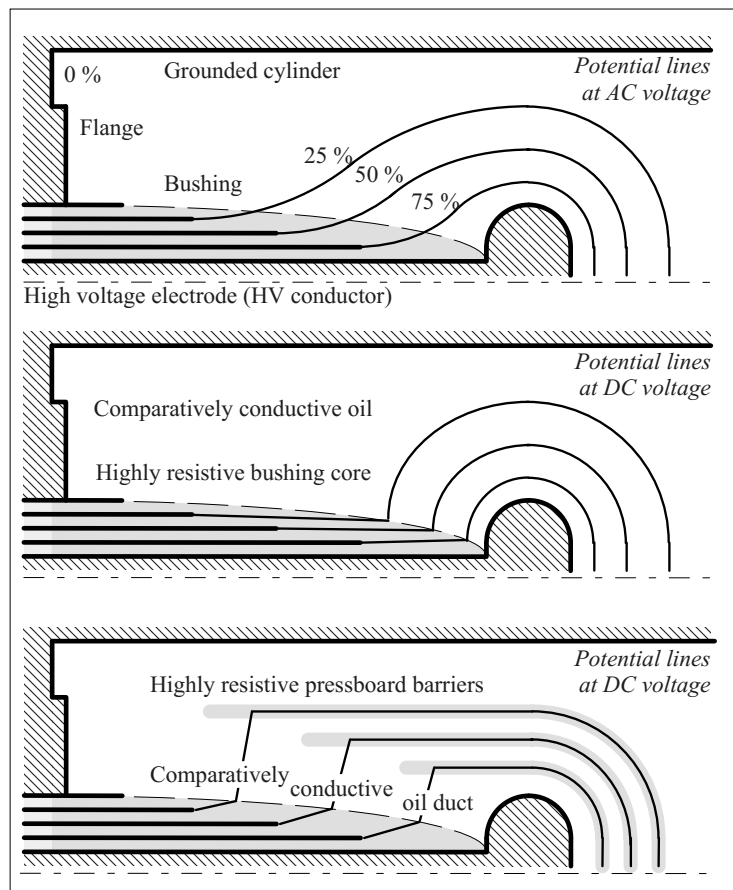


Figure 2.4-28: Connection of a HV electrode in oil via a capacitively graded bushing at AC voltage (top) and DC voltage (middle and bottom). Improved potential distribution at DC voltage by highly resistive pressboard barriers (bottom) [7].

cylindrical *pressboard barriers* with different lengths (pressboard barrier system), Figure 2.4-28 (bottom). In this way a *uniform oil duct* shall be formed, with a current flow between high voltage and ground and with an almost uniform potential distribution.

The grading capability of the barriers at DC voltage is based on the *external potential grading* in the oil duct, which is adjusted to the *internal grading* of the bushing's grading layers. The internal bushing itself cannot influence the stationary conduction field outside the bushing [7], [10].

At *elevated temperature*, the conductivity differences between the materials and the grading capability of the barriers are reduced. A calculation with sufficient accuracy can only be achieved by *numerical field calculations* (see Section 2.5) with correct conductivity values.

According to the refraction law Eq. (2.4-49), it is concluded that the potential lines *in the oil duct* emanate from the highly resistive materials (bushing and barriers) almost orthogonally, see Figure 2.4-26. *Around the electrode*, the interfaces are orthogonal to the field (and parallel to the equipotential lines). Here, the field is displaced from the comparatively conductive oil gaps into the highly resistive barriers. Therefore, the thickness and number of the barriers must be such that the barriers can withstand the whole DC voltage.

Note: The barriers have an important function also at *AC voltage*: Although the influence of thin barriers on the AC field strength in the oil gaps is small, the electric strength of these gaps is significantly improved by a *subdivision into smaller gaps*.

Example 4: HVDC wall bushing

On the outdoor insulators of wall bushings, *pollution layers* develop by deposition of dust and dirt. The exposure to *water* by rain or by moisture condensation causes a comparatively high *surface conductivity*, Figure 2.4-29.

At *AC voltage*, the field distortion by conduction currents on the surface (creepage currents) is normally negligible, because of the comparatively high capacitive displacement currents.

At *DC voltage*, wet pollution layers, which have a significantly higher conductivity than the bushing insulator, cause very strong field distortions, especially if the pollution layer does not cover the surface completely and uniformly.

In *HVDC installations* for outdoor sites, the *non-uniform rain* on bushing insulators (e.g. in the lee side of a building) is especially critical at higher voltages, Figure 2.4-29. The high voltage potential can be shifted along the surface for long distances down to the transition zone between the dry and the wet surface. This is comparable with a sharp electrode on a

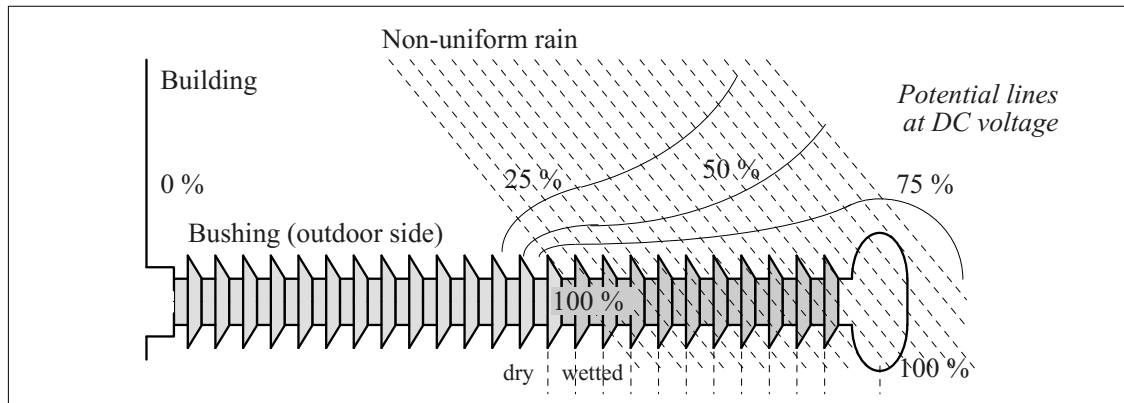


Figure 2.4-29: Air-side of an HVDC wall bushing with the formation of a wet and conductive surface layer. Because of the non-uniform rain only a part of the surface is bridged at DC voltage, see figs. 2.4-1 and -2.

creepage surface (Figures 2.4-17, -18 and -24) with extreme tangential and radial field stress enhancements, which can cause a flashover (comparatively best-case) or a radial breakdown of the bushing (worst-case).

Therefore, it is very often necessary, to apply *hydrophobic* (water-repellent) silicone paste on the insulator surface, in order to avoid the formation of conductive liquid films on the easily wettable porcelain surface. The application and regular renewal of the silicone paste can be avoided, if the porcelain insulator is replaced by a **composite insulator** made of a glass-fiber-reinforced plastic (GRP) tube (i.e. reinforced epoxy resin tube) with *elastomeric silicone rubber* (SIR) sheds [7], [8], [9], [10], see Section 5.3.4 with Figure 5.3-18.

Example 5: Energy storage capacitor

Energy storage capacitors are charged with *DC voltage*, and they are in general discharged by *electric pulses* or damped *high-frequency oscillations*.

During the charged condition, i.e. during a *steady-state DC stress*, the potential distribution close to the edge of a foil is significantly different from the AC distribution shown in Figure 2.4-20, see Figure 2.4-30. The impregnating gap that ends at the interstice at the edge of the foil, is filled with oil and normally has a higher conductivity κ_Z than the adjacent insulating films with κ_I . Thereby, a comparatively uniform gap is formed, where a conduction current can flow through the oil and grade the potential, Figure 2.4-30 (bottom). Thereby, the *stress at the edges* of the foil is significantly *reduced*.

Therefore, the *DC strength* of a capacitor dielectric is in practice significantly higher than the AC strength. A factor of about three can often be assumed.

The critical stress in an energy storage capacitor does not arise during the steady state DC stress, but during the fast *discharge* impulse or the *discharge* oscillation. The associated time-

variant displacement field is more like Figure 2.4-30 (top). Furthermore, *space charges* accumulate at the interfaces of the polymeric films during the preceding steady-state DC phase. During an oscillating discharge, there are multiple *polarity reversals*, and a time-varying displacement field is superimposed to the steady-state space charge field. Thus, the electric field stress at the edges of the foils is much higher than for a pure DC or AC stress alone, see Section 7.3.3.

The *lifetime* of an energy storage capacitor or an impulse capacitor is therefore given by the number of possible discharges depending on the charging voltage, the frequency of the discharge oscillations and the relative magnitude of the first amplitude with reversed polarity (back-swing ratio) [29].

2.4.4.3 Transient Processes

The above-mentioned DC voltage stress assumes a steady-state condition, which requires very long times between hours and days for highly resistive insulating materials. According to Eq. (2.1-41), times should be much

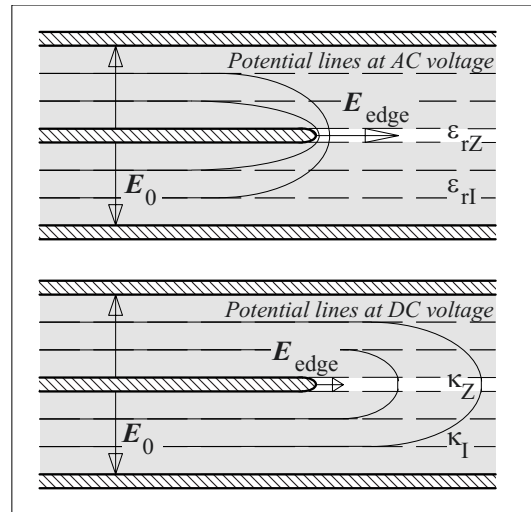


Figure 2.4-30: Electric field stress at the edges of the metallic foils in a capacitor dielectric at AC voltage (top) and reduced field stress at DC voltage (bottom) due to a comparatively high conductivity in the oil-filled interstice between the plastic films.

longer than the self-discharging time constants of the relevant materials:

$$t \gg \tau_d = \varepsilon/\kappa \quad (2.4-50)$$

For the **application of a DC voltage**, the following phases must be distinguished (see Figure 2.1-16):

a) If the DC voltage is applied as a *step* within a very short time (in comparison with the relevant time constants of the dielectric system) a *dielectric displacement field* can be assumed at first. It is determined by the *permittivities* ε . Geometrically simple configurations can be described by equivalent networks consisting of *capacitances* only.

b) Then, a *transient process* takes place, which consists of charging and discharging processes among the different dielectrics. A mathematical description requires the Material (Constitutive) Equations $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{J} = \kappa \mathbf{E}$ together with the Continuity Equation (2.1-35) in their general form. Both conduction current density $\mathbf{J}$ and displacement current density $\partial \mathbf{D} / \partial t$ have to be considered.

Geometrically simple arrangements can often be described by equivalent networks consisting of *capacitances* C (for the description of the displacement current) and *resistances* R (for the description of the conduction current). Voltages and currents are then calculated with the methods of *network analysis*. The *Laplace transform* is very useful for this purpose [2], [30], [31].

Note: The description of a material by a single permittivity (capacitance) and a single conductivity (resistance) neglects that the **polarization process** of the material takes time and can continue for comparatively long times until the steady state is reached. Polarization processes are therefore described by a more complex equivalent circuit, which contains RC elements with different time constants for describing the different polarization mechanisms, see Section 4.3.

c) After the decay of the transient process, a *steady state* is reached, which depends on the *conductivities* (or resistivities) of the insulating materials only (Section 2.4.4.1 and 2.4.4.2). Geometrically simple arrangements can be described by an equivalent network consisting of *resistances* only.

In DC applications it often happens that a **given state** is changed **into another state** by a transient process. Examples are the transients after a *polarity reversal* (e.g. during an HVDC voltage test), after an *increase* or *decrease* of the DC voltage magnitude, after a *short circuit*, after a *discharging process* or during the development of a *recovery voltage*.

A **calculation** of the mentioned transitions can be performed in the following steps:

a) At first, the *initial state* has to be calculated. In the easiest case, this is the steady state. In an *equivalent network*, the initial state is given by the initial charge state of the equivalent capacitances. The initial state of a *complex arrangement*, which cannot be described by an equivalent network any more, has normally to be described by a numerically calculated *field pattern* or an *equipotential line plot*.

b) The *subsequent voltage step* can be described by a voltage source in an equivalent network. In more complex arrangements, which are described by field or equipotential plots, the *dielectric displacement field* associated with the voltage step can be superimposed on the initial field distribution in the form of a field plot. This gives the dielectric stress directly after the voltage step [7], [10].

c) The *transient process* can be determined by a transient network analysis in an equivalent circuit. For geometrically complex arrangements numerical field calculations have to be based on transient field theory. For practical applications it is often sufficient to calculate the *steady end state*.

Hereafter, some practical examples are discussed:

lent capacitance C_1 (polymeric films) is nearly the whole voltage (approx. $0.99 \cdot V$), whereas C_2 (papers) is only charged to $0.01 \cdot V$, Figure 2.4-31 (left).

During a *short circuit* of the capacitor at the external terminations, the charge $Q_1 \approx C_1 \cdot V$ is distributed among the two parallel partial capacitances C_1 and C_2 , which have the same voltage, but with opposite polarity. The voltage between the outer terminations is thus zero. Theoretically, the voltages are $v_1' = -v_2' = (1/3) \cdot (C_1 \cdot V) / C_1 = V/3$ for $C_2 = 2 C_1$, if $Q_2 = C_2 \cdot 0.01 \cdot V$ is neglected, Figure 2.4-31 (middle). The difference of the capacitively stored energies, before and after the short circuit, is dissipated as ohmic loss in the resistance of the short circuit. If the short circuit is not opened again, the parallel capacitances C_1 and C_2 are exponentially discharged via $R_2 \ll R_1$ with the time constant $\tau = (C_1 + C_2) \cdot R_2$.

If the *short circuit is opened again*, immediately after the connection of the terminals, however, the partial capacitances, which are charged to $v_1' = V/3$ and $v_2' = -V/3$, can only be discharged by self-discharging via the associated equivalent resistances with the self-discharging constants $\tau_1 = R_1 C_1 = \varepsilon_1 / \kappa_1$ and $\tau_2 = R_2 C_2 = \varepsilon_2 / \kappa_2$. In the given example, the exponential discharging of C_2 would be fifty times faster than the exponential discharging of C_1 , so that the resulting voltage at the open terminals is $v'(t) = v_1'(t) + v_2'(t)$, which is called “*recovery voltage*”, Figure 2.4-31 (right).

Note: This explanation of the recovery voltage is based on charges that are stored at the interfaces in the dielectric and on charge reversals between *different materials*. A similar recharging takes place, if there are *space charges stored* in the dielectric (see the example about HVDC cables in Section 2.4.4.2) or if charge is stored by means of *polarization effects*, Section 4.3.2.1.

Note: Charged capacitors and recovery voltages are some of the **main dangers for ex-**

perimental work with high voltages. Equipment with high capacitances (e.g. capacitors, cables) must therefore have a *permanent short circuit*. Also in a *series-connection* of capacitors all the single units must have an *individual short circuit*: A short circuit at the end terminals only would not prevent the individual capacitors to be (oppositely) charged. For safety reasons, it should not be assumed that the individual capacitors are identical (if only because of temperature gradients could cause differences) and that they are charged and discharged simultaneously.

Note: There are attempts to use parameters of recovery voltages for *dielectric diagnosis*, e.g. for transformers and cables [32], [33], see Section 6.4.7.5.

Example 3:

Transient enhancements of field strength in multi-layer dielectrics

After the application of a DC voltage to a multi-layer dielectric, the transition from the initial capacitive to the final ohmic potential distribution can be accompanied by *temporarily enhanced and unexpected stresses* which cannot be recognized from the initial and final states.

We discuss a three-layer dielectric consisting of a plastic barrier separating two oil-gaps with different oil qualities (aged and new oil), Fig 2.4-32. It is assumed that the *self-discharging time constants* of the aged oil, the plastic barrier and the new oil have the relation 1 : 100 : 10. The relation of the *equivalent capacitances* shall be assumed to be 1 : 2 : 2. With $\tau = RC$, the relation of the *equivalent resistances* is 1 : 50 : 5.

After the application of the DC voltage, a *capacitive voltage distribution* occurs at first, because of the dielectric displacement field, Figure 2.4-33. Dielectric 1 is stressed with half the voltage, dielectrics 2 and 3 each carry a quarter of the voltage V .

Because of the short self-discharging time constant τ_1 of dielectric 1, C_1 is *discharged quickly* and $v_1(t)$ decreases very rapidly. Thus,

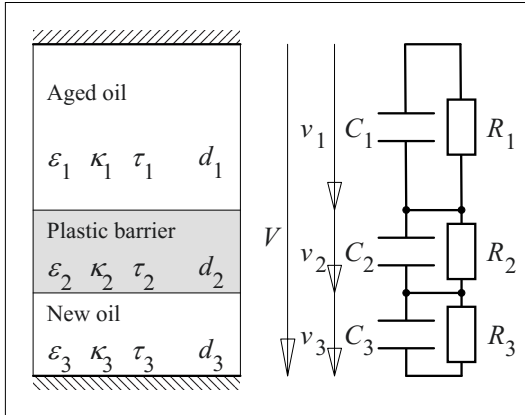


Figure 2.4-32: Three-layer dielectric with different self-discharging time constants (e.g. with the ratios 1 : 100 : 10), see fig. 2.4-33 with the corresponding voltage curves after the application of a DC voltage V .

the other dielectrics 2 and 3 have to carry an additional fraction of the total voltage V . The equivalent capacitances C_2 and C_3 are recharged via R_1 at first. Therefore the voltages $v_2(t)$ and $v_3(t)$ increase simultaneously at first, Figure 2.4-33.

Then, the lower self-discharging time constant of dielectric 3 becomes apparent by a further discharging of C_3 together with a further recharging of C_2 via R_3 to even higher voltages.

The voltage $v_3(t)$ is increasing at first and decreasing in the long term. In between there is a distinct *maximum*.

This maximum is significantly higher than the initial voltage at this material, and it is many times higher than the final steady-state voltage. Therefore, we have a possible *overstress* of dielectric 3 in the course of transient process, which is often ignored.

In the *steady-state condition* the voltage is distributed according to the equivalent resistances, i.e. dielectric 1 is stressed with 2 %, dielectric 2 with 89 % and dielectric 3 with 9 % of the total voltage V .

Note: Compared to oil, the high stress on the plastic barrier may still be tolerable because solid materials can normally be stressed with much higher field strength than liquids.

The exact level of the overstress in dielectric 3, which is qualitatively described here, can be calculated by means of a *network analysis*. The result depends very strongly on the nature of the observed multilayer dielectric. Especially *critical* arrangements can often be found, if the initial and the final distribution are very different.

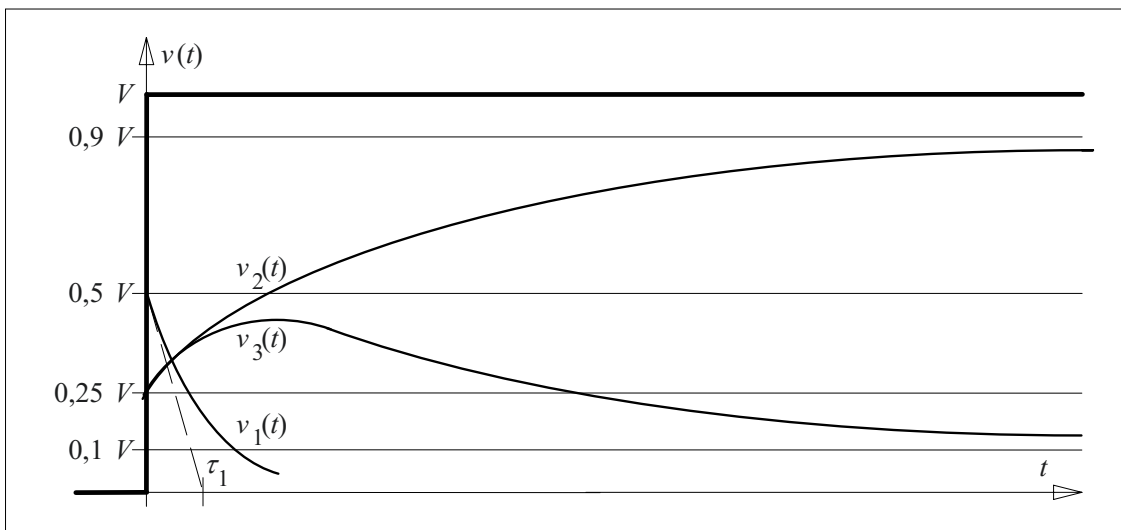


Figure 2.4-33: Voltage curves for a three-layer dielectric according to fig. 2.4-32, with a temporary voltage enhancement at dielectric 3 after the application of a DC voltage.

Example 4:**Polarity reversal of a DC voltage**

Very complex field distributions, field migrations and significant field stress enhancements can occur in connection with the *polarity reversal* (PR) of a DC voltage in an oil-board barrier system, as used in HVDC insulation, e.g. in *converter transformers* or *smoothing reactors*. Normally, these fields can no longer be described by equivalent circuits, and it is necessary to perform numerical field calculations, Section 2.5.

A plane arrangement with two high-resistive barriers between two plane electrodes in comparatively conductive oil is discussed, in order to give a qualitative explanation of the conditions when reversing the voltage, Figure 2.4-34. The insulation arrangement can be regarded as a very simplified *model of a barrier system* according to Figure 2.4-28.

If a *negative DC voltage* is applied to the upper electrode, a *steady-state conduction field* in the oil develops parallel to the interfaces between oil and barriers, Figure 2.4-34a. It can be related to a conduction current from the lower to the upper electrode, conducted between the highly resistive barriers. Within the oil duct, the distance between the equipotential lines is great and the field strength is low, if the oil duct is long enough. Thus, the barrier system can grade the field in the oil duct. The barriers are stressed with nearly the whole DC voltage, in the regions where they do not overlap. The barriers have to be designed accordingly, e.g. by multi-layer design with sufficient thickness.

The *polarity reversal* can be described by the superposition of a *positive voltage step* with *twice the amplitude* of the DC voltage. It causes a strong dielectric *displacement field*, which is superimposed on the initial conduction field [7], Figure 2.3-34b. To a first order approximation, the field displacement from the barriers into the oil is neglected.

The *superposition* results in a *very high stress* in the oil duct, Figure 2.4-34c. Only a minor

part of the stress is taken off from the barriers. Interestingly, there are “islands” with a potential higher than 100 % and lower than 0 % of the applied DC voltage. These islands are caused by positive and negative *surface charges* that were accumulated on the surfaces of the barriers during the preceding steady state, see also Figure 2.4-23.

After the polarity reversal, a *transition process* takes place, during which the capacitances of the barriers are essentially recharged via the series resistance of the oil ducts. Finally, a new stationary field distribution is formed for a positive DC voltage that is equal and opposite to the original, Figure 2.4-34d.

Note: Experience with DC voltage tests of oil-insulated equipment with barrier systems shows that the *minutes after the polarity reversal* are often critical. During this time partial discharges can occur, which disappear later on. This is an indication of high and decreasing field strengths directly after the polarity reversal, see Figure 2.4-34c.

2.4.5 Field Grading at Interfaces

Insulation arrangements with interfaces tangential to strong electric fields are especially critical in high voltage engineering for any kind of voltages, AC, DC and impulse stresses. Most of the problems arise at the interfaces to the electrically weak air, i.e. at the so-called surfaces.

Electrical discharges can be triggered, both by the normal field in the air-filled interstice at a triple point (see Section 2.4.3.3) and by the tangential field at the edge of an electrode (see Section 3.2.6), Figure 2.4-35 top (right and left). Inception voltages are comparable for both cases, Eq. (2.4-35) and (3.2-72). If a discharge is triggered by an AC voltage, strong tangential field components cause powerful surface discharges, because of high stray lateral capacitances and high capacitive AC currents in the discharge channel. Thus, insulating materials can severely be damaged by erosion at the surface, Section 3.2.6.

Arrangements with high tangential field strengths and with high lateral capacitance (i.e. with small insulation thicknesses) are susceptible to surface discharges. They can be found in many insulation arrangements, and they are called “creepage surfaces”, Figure 2.4-35 (bottom). Some common cases are given e.g.

- in arrangements with rotational symmetry (e.g. for cables, Section 7.1.1 or for bush-

ings, Section 7.1.2),

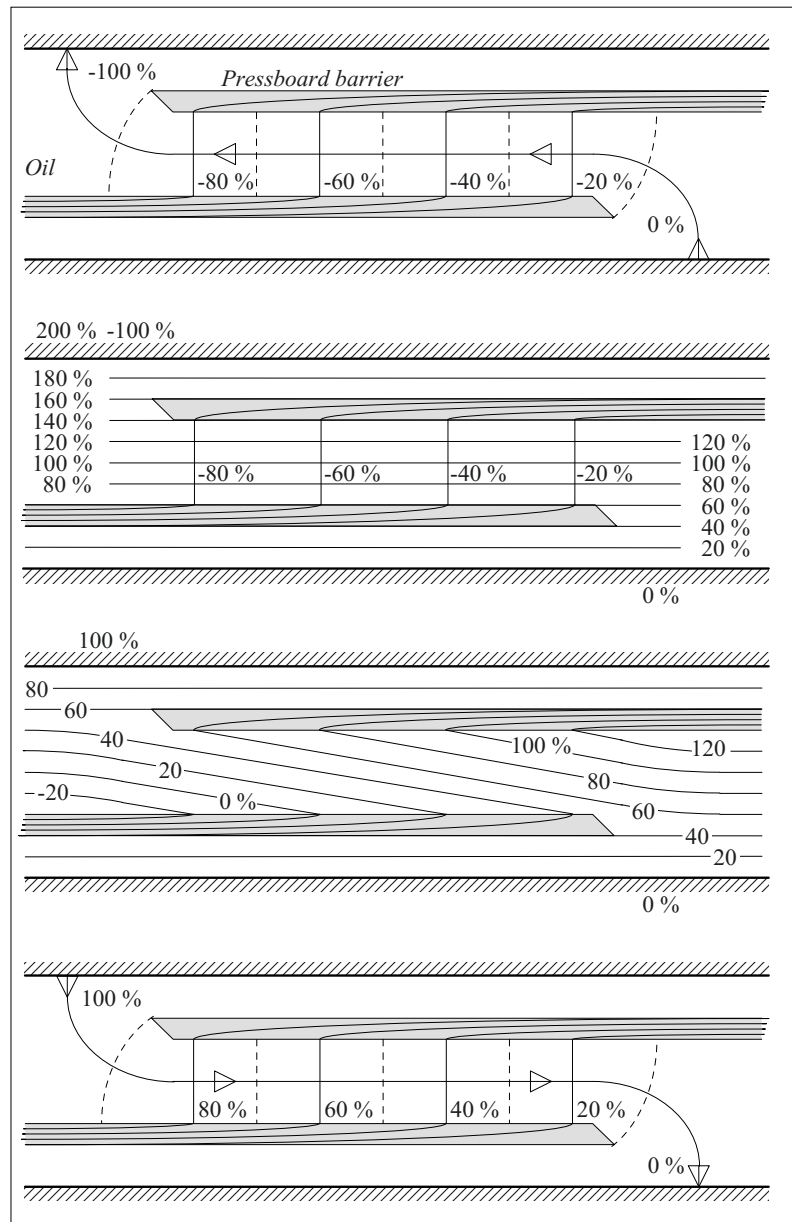
- in plane arrangements (e.g. at the edges of a parallel-plate capacitor, or for thin insulating foils, Figure 2.4-20)
- and for the insulation surface of insulated rectangular conductors (e.g. for insulated busbars or stator winding insulations, Figure 7.1.6-4).

Figure 2.4-34a: Steady-state conduction field with a negative DC voltage at the upper electrode.

Figure 2.4-34b: Superposition of a dielectric displacement field with opposite polarity and twice the amplitude at polarity reversal. The field displacement at the barriers is neglected.

Figure 2.4-34c: Resulting field directly after the polarity reversal. Because of the surface charges at the interfaces, there are “islands” with potentials higher than 100 % and lower than 0 %.

Figure 2.4-34d: Steady-state conduction field after polarity reversal and after the decay of the transient process, i.e. after the re-charging of the barrier capacitances via the resistances of the oil ducts.



It is a basic task of high voltage engineering, to keep tangential electrical stresses at surfaces low. This can be achieved with different technologies for **field grading** or **potential grading**, which are explained for the example of cable entrance fittings [464], Figure 2.4-36:

(1) For **geometric field grading**, the ground potential of the cable shield is guided outwards by the geometric contour of the *conductive deflector*, Figure 2.4-36 (1). Its funnel-shaped curvature guarantees that the field strength decreases significantly along the contour from the cable surface outwards, similar to a Rogowski profile. For *cable entrance fittings*, (Section 7.1.1.4) the deflector is located in a conical body made of an elastomeric material (grading cone). The field strengths at the surfaces have to be reduced so much that there is no danger. For the edges of parallel-plate capacitors, the electrodes are often bent (Rogowski and Borda profiles, curvature according to Eq. 2.3-43). The geometric field grading requires huge volumes, therefore leading to designs with large dimensions and diameters.

(2) A **capacitive field grading** is achieved by conductive *grading layers* between ground and high voltage potential, Figure 2.4-36 (2). The capacitances between these layers guarantee the desired potential distribution. The shift of the layers in axial direction imposes the potential of the layers on the surface of the insulation arrangement. Capacitive grading is most effective, i.e. a completely linear distribution in the axial direction can be achieved, together with very small diameters and insulation thicknesses. Capacitive grading is therefore used for high voltage *bushings* up to the highest voltages, Section 7.1.2.

Note: For HV and MV cable entrance fittings and cable joints *geometric grading* is mostly applied nowadays, because of production and assembly reasons. Refractive, resistive and non-linear grading are also used for medium voltage applications.

(3) **Refractive field grading** can be performed by a tube made of a *high-permittivity material*,

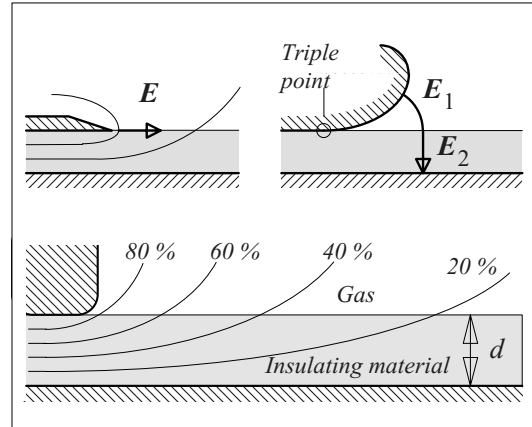


Figure 2.4-35: Triggering of surface discharges by tangential and normal field components (top left and right). Creepage surface with high tangential stress and thin insulation material (bottom, the refraction of equipotential lines is neglected).

which is shrunk or mounted onto the surface of the cable insulation and connected to the end of the cable shield, Figure 2.4-36 (3). By means of field refraction and field displacement, the field lines are displaced away from the edge of the cable shield. Thus the field is equalized and maximum field strength at the edge of cable shield is reduced.

(4) For **resistive field grading**, a *semi-conductive* (i.e. *semi-resistive*) coating is applied to the surface of the cable insulation, Figure 2.4-36 (4). The axial resistances and the radial capacitances relative to the inner conductor form a *RC* lattice network, which grades the potential along the coated insulation in the case of an AC stress.

(5) A **non-linear field grading** makes use of materials that have high insulating resistances at low field strengths and significantly increasing conductivities with increasing field strengths, Figure 2.4-36 (5). Thereby, locally enhanced conductivities displace the electric field and reduce the field magnitude at points with highest field strengths. *Non-linear materials* like zinc oxide (ZnO), silicon carbide (SiC) and iron oxide (FeO) are embedded in a polymeric matrix material and act as so-called *microvaristors*.

Stator winding insulation in generators and in big motors is the traditional application of resistive and non-linear field grading, Figure 7.1.6-4. The coatings are very thin and can therefore be applied to the insulated conductors, which are in close proximity to each other in winding heads.

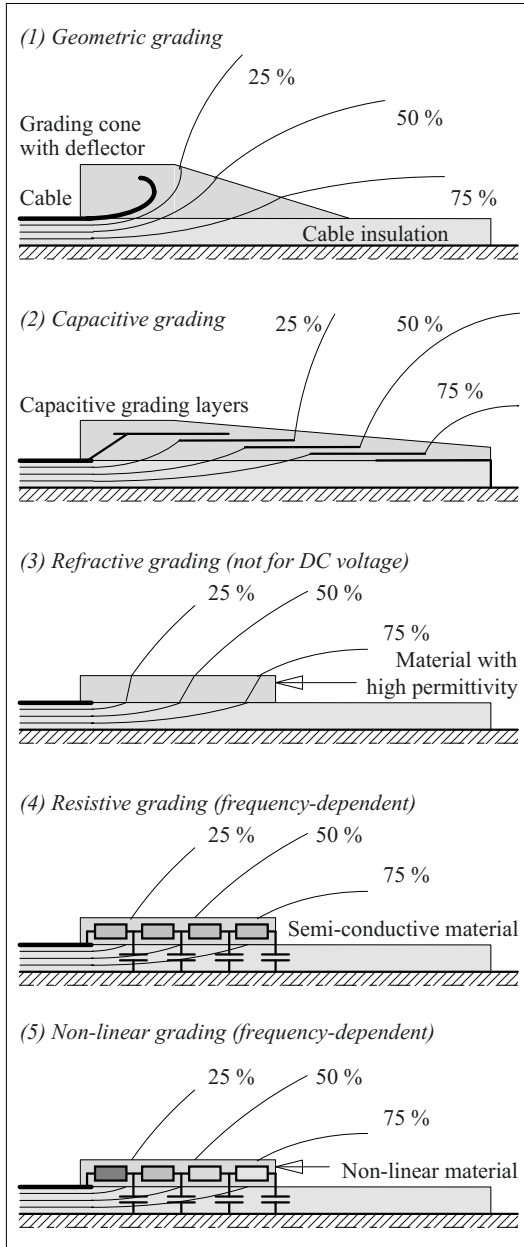


Figure 2.4-36: Technologies for field and potential grading on typical creepage surfaces, rotationally symmetric surfaces of cable insulation (example).

Refractive field grading can only be used for time-varying voltages (*AC or impulse*). **Resistive** and **non-linear gradings** are strongly *dependent on the frequency* of the applied voltage (frequency sensitivity).

All the field and potential grading methods have in common that they reduce the tangential field strengths at **interfaces and surfaces**. Nevertheless, these interfaces and surfaces are still normally *highly stressed* and have to be *treated with particular care*. This means that contaminations, pollutions, deposits, access of water, air-inclusions, voids and other defects have to be precluded with high reliability during the production process and service operation.

2.5 Numerical Field Calculation

Numerical field calculation is one of the most important tools for high voltage engineers in design, development and research. Sufficiently accurate field strength magnitudes for complex insulation arrangements can only be determined by numerical calculation.

Nevertheless, numerical calculations must not replace the intellectual analysis of the given kind of stress. Both a thorough preparation of the calculation and a thorough analysis/ discussion of the results are necessary, in order to avoid mistakes or wrong and too far-reaching conclusions.

It is recommended therefore, always to make a qualitative *visualization* by *field mapping* (Section 2.3.3) and to try an *analytical estimation* for a simplified insulation model. Based on these approximations, numerical results can be checked for plausibility.

2.5.1 Overview

For this introduction, the discussion is limited to electric **potential fields**, i.e to *static, quasi-static and steady-state* (capacitive) displace-

ment and conduction fields, which can be described by means of Poisson's or Laplace's Differential Equation (2.3-31), i.e. as so-called **Poissonian** or **Laplacian fields**.

Note: Considering the **vector potential**, fast changing and non-stationary fields, i.e. fields with *eddy currents* and *electromagnetic waves* can also be calculated [394].

The calculation of potential fields can be performed with different numeric methods. Three basically different solution approaches can be distinguished [34]:

a) Integral Equation Methods (IEM) superimpose field or potential quantities that are related to charges, currents and dipole moments, by means of summation or integration of the relevant contributions. If both vector and scalar potentials are defined [2], [3], the fast changing electromagnetic field can also be calculated.

The traditional method for the calculation of slowly changing fields is the *Charge Simulation Method* (CSM), which was already used for analytical calculations in Section 2.3.5. The charge simulation method can be extended to surface charges (SCSM: *Surface Charge Simulation Method*, BEM: *Boundary Element Method*). Fast changing fields can be treated with the *Method of Moments* (MOM).

It is an *advantage* of integral equation methods that spatially unlimited three-dimensional arrangements can be treated with a limited number of elements (charges, etc.). In high voltage engineering it is also advantageous that the influence of *space charges* can easily be considered. On the other hand, it is difficult to calculate fields in arrangements with many *different materials*, since the influence of interfaces has to be taken into account by many additional elements (charges, etc.)

b) The Finite Difference Method (FDM) and the Finite Element Method (FEM) discretize the whole field volume and set up differential equation systems based on the discretized Poisson's Equation (FDM) or on the extreme

value determination of a specific energy-functional (FEM). The treatment of non-stationary fast changing fields is possible with the FEM.

The *advantage* of these methods is a very simple treatment of many different *materials and interfaces*. Therefore, insulation systems with a complex multi-layer arrangement and with *non-linear materials* can also be calculated. The method of finite elements has therefore prevailed in the calculation of magnetic fields because (non-linear) ferromagnetic materials have to be considered there.

It is a *disadvantage* that the *whole field volume* has to be discretized. A huge number of elements is thereby created, especially for three-dimensional arrangements. The method of finite elements allows one to adapt the fineness of the discretization to the different field regions, whereby the number of elements is further reduced.

Note: In other fields of engineering, the **Finite Element Method** has already been established for a long time, it provides mathematically analogous calculations of *mechanical, thermal and magnetic fields*. Therefore the method is very popular in many *industrial applications* because development and design of technical apparatus normally include thermal and mechanical stresses. Modern field calculation programs take the geometric data directly from CAD programs and they can calculate **combined stresses** of a mechanical, thermal, magnetic and electric nature (*multi-physics*).

c) The Monte Carlo Method (MCM) is based on the mean value theorem of the potential theory. It states that the potential $\phi(P)$ at the center point P of a sphere equals the average potential on the surface of the sphere. From a point under consideration, random walks are started, which reach the electrodes of given potential with a specific frequency. In this way, a mean value is determined from the hit electrode potentials. It is a statistical estimate of the potential $\phi(P)$, whose quality can be determined from the statistical dispersion [16].

The Monte Carlo Method is considered to be an adequate method for the calculation of single potential values in a part of the field volume which is of minor interest [34].

Hereafter, the Charge Simulation Method (Section 2.5.2), the Finite Difference Method (Section 2.5.3) and the Finite Element Method (Section 2.5.4) are discussed in more detail.

2.5.2 Charge Simulation Method

For the Charge Simulation Method, the fields of individual *equivalent charges* are superimposed at a reference point A_k by a *summation* of their individual contributions to the resulting *potential* φ_k . Figure 2.5.1:

$$\varphi_k = \varphi_{k1} + \varphi_{k2} + \varphi_{k3} + \dots + \varphi_{kn} \quad (2.5-1)$$

This idea was already used for the analytical field calculation in Section 2.3.5, Eq. (2.3-45).

Point charges (Q_1), *straight line charges* (Q_2) and *toroidal line (ring) charges* (Q_3) are successfully used as equivalent charges. The contributions φ_{kj} of the individual charges Q_j to the potential φ_k are described by *potential coefficients* p_{kj} :

$$\varphi_{kj} = p_{kj} \cdot Q_j \quad (2.5-2)$$

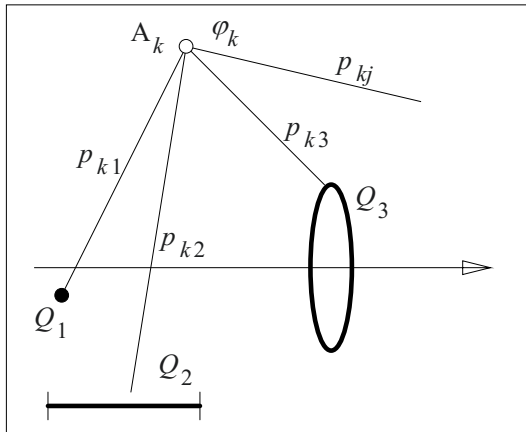


Figure 2.5-1: Point charge, line charge and toroidal line (ring) charge as basic equivalent charges of the Charge Simulation Method; contribution of a number of charges to the potential in reference point k .

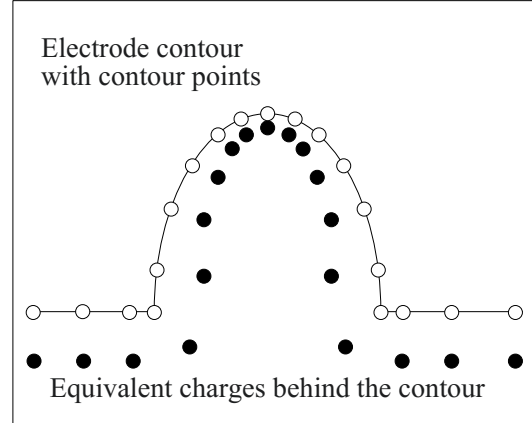


Figure 2.5-2: Setting of equivalent charges and contour points following the given contour of an electrode.

In the case of *point charges*, the potential coefficient according to Eq. (2.3-44) is given as $p_{kj} = 1/(4\pi\epsilon \cdot r_{kj})$, i.e. as a function of the distances r_{kj} .

Equivalent relations can also be given for the straight and toroidal line charges [16].

The electrode surfaces are approximated by *equipotential surfaces*. This means that discrete equivalent charges have to be set *behind* the desired electrode contour, in order to get finite potential values on the contour. The potential at the location of the equivalent charge itself is infinite.

In a *first* step, n *equivalent charges* of unknown magnitude *are set*, based on the given electrode contour, Figure 2.5-2.

In a *second* step, an equal number of n *contour points* are chosen on the electrode contour, with the given electrode potentials φ_{E1} till φ_{En} .

In a *third* step, a system of equations can be formed using the *potential coefficients*. It gives the known potentials φ_{Ek} of the contour points as a function of the unknown *equivalent charges* Q_j . These *magnitudes of the equivalent charges* can be determined by solving the system of equations:

$$\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_n \end{bmatrix} = \begin{bmatrix} \varphi_{E1} \\ \varphi_{E2} \\ \cdots \\ \varphi_{En} \end{bmatrix} \quad (2.5-3)$$

Now the equivalent charges are known, and the *potential of any point* in the field volume can be calculated according to Eq. (2.5-1) and (2.5-2).

In a *fourth* step, the quality of the equivalent charge positioning is first tested. Additional *test points* are chosen on the given contour, and the given potentials are compared with the calculated values. If the differences cannot be tolerated, the positioning of the equivalent charges has to be improved.

The contour of the calculated equipotential surface will never be identical with the given electrode surface. Both surfaces touch each other at the contour points according to the initial condition. For the test points in between, the deviation of the calculated potential on the electrode surface is determined.

There are some proven rules for the *positioning of equivalent charges*, Figure 2.5-2:

- An equivalent charge and a contour point shall be set as a pair with a close spatial relationship.
- The distance between charge and contour point shall be comparable to the distance to the adjacent charges.
- Depending on the curvature of the electrode contour, the charges shall be set closer or further apart.
- Variations of charge and contour point distances shall be performed gradually, without major variations of the adjacent distances.

The *simulation quality* is increased with the number and density of the equivalent charges. Nevertheless, the number of charges must not be too great, in order to avoid numerical prob-

lems during the solving of the system of Equations (2.5-3).

The influence of *dielectric interfaces* can be taken into account by surface charges or by equivalent charges on both sides of the interfaces [16]. In arrangements with many different materials, a significant additional effort is required; therefore, another calculation method might often then be better.

An electrode at *free potential* can be simulated, if equivalent charges are set behind the electrode with the charge-sum zero: $\sum Q_i = 0$.

Additional equations are thereby defined, which are necessary for the determination of the electrode potential [16].

A further development of the Charge Simulation Method makes use of *surface charges*, which approximate the physical charge distribution on the electrode surfaces and the interfaces (Surface Charge Simulation Method, Boundary Element Method).

Note: The *Charge Simulation Method* has a close relation to the *physical origin of electrostatic fields*. Therefore the CSM can easily be understood and it is one of the first numerical simulation methods used in HV engineering. Nevertheless, the position and distribution of the equivalent charges are not identical with the position and distribution of the physical charges, but the magnitudes are equal.

2.5.3 Finite Difference Method

For field calculation with the Finite Difference Method, the field volume is discretized with a grid of regular meshes, Figure 2.5-3. The potential φ_0 on a node "0" can then be given as a function of the potentials $\varphi_1, \varphi_2, \varphi_3, \dots$ on the adjacent nodes, if the potential $\varphi(x,y,z)$ in the directions x, y and z is approximated by a Taylor series.

In Cartesian coordinates, a variation either in x -, y - or z - direction, without any variation in the other two directions, is

$$\begin{aligned}\varphi(x) &= \varphi_0 + \Delta x \cdot \partial\varphi/\partial x + (\Delta x^2/2) \cdot \partial^2\varphi/\partial x^2 + \dots, \\ \varphi(y) &= \varphi_0 + \Delta y \cdot \partial\varphi/\partial y + (\Delta y^2/2) \cdot \partial^2\varphi/\partial y^2 + \dots, \\ \varphi(z) &= \varphi_0 + \Delta z \cdot \partial\varphi/\partial z + (\Delta z^2/2) \cdot \partial^2\varphi/\partial z^2 + \dots\end{aligned}$$

For a plane **two-dimensional** arrangement according to Figure 2.5-3, the neighbor potentials of φ_0 are estimated from the Taylor series that is cut off after the second order term:

$$\begin{aligned}\varphi_1 &\approx \varphi_0 + h \cdot \partial\varphi/\partial x + (h^2/2) \cdot \partial^2\varphi/\partial x^2 \\ \varphi_3 &\approx \varphi_0 - h \cdot \partial\varphi/\partial x + (h^2/2) \cdot \partial^2\varphi/\partial x^2 \\ \varphi_2 &\approx \varphi_0 + h \cdot \partial\varphi/\partial y + (h^2/2) \cdot \partial^2\varphi/\partial y^2 \\ \varphi_4 &\approx \varphi_0 - h \cdot \partial\varphi/\partial y + (h^2/2) \cdot \partial^2\varphi/\partial y^2\end{aligned}$$

The cut off of the series after the second-order term means that the step width h , i.e. the mesh width h , must be chosen small enough, so that the potential variation can be described by a second order polynomial with sufficient accuracy. In very non-uniform fields, very small mesh elements and a very large number of nodes is required.

If the sum of the four neighbor potentials is calculated, the first order terms compensate each other because of opposite signs. According to Laplace's Equation (2.3-32) for the case without space charge, the sum of the second-order terms is also zero:

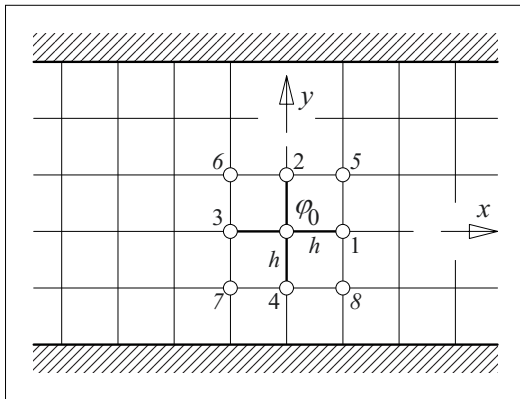


Figure 2.5-3: Discretization of a 2D field region by means of a grid with square meshes for the field calculation with the Finite Difference Method.

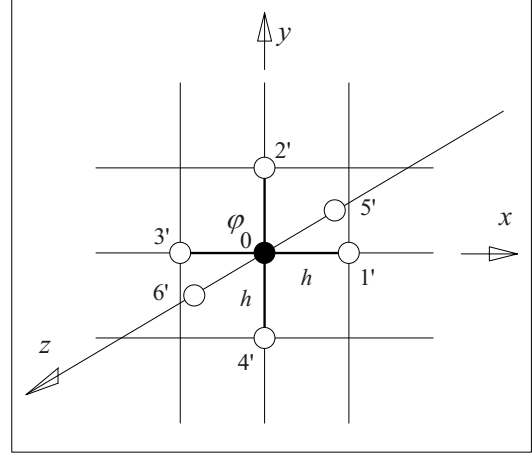


Figure 2.5-4: Discretization of the field volume for the three-dimensional field calculation with the Finite Difference Method.

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 = 4 \cdot \varphi_0. \quad (2.5-4)$$

Therefore, the potential φ_0 is the average of the four neighbor potentials ("square algorithm"):

$$\varphi_0 = (\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4)/4 \quad (2.5-5)$$

In the same way, φ_0 can be calculated from the potentials in the diagonally adjacent nodes ("diagonal algorithm")

$$\varphi_0 = (\varphi_5 + \varphi_6 + \varphi_7 + \varphi_8)/4 \quad (2.5-6)$$

Analogously, the equivalent equations for **three-dimensional** fields are determined. Then φ_0 is calculated from the potentials of six adjacent nodes ("cube-algorithm"), Figure 2.5-4:

$$\varphi_0 = (\varphi_{1'} + \varphi_{2'} + \varphi_{3'} + \varphi_{4'} + \varphi_{5'} + \varphi_{6'})/6 \quad (2.5-7)$$

The finite difference method also allows one to calculate **multi-dielectrics insulation systems**. Within the materials, the above-mentioned equations are valid. For points on an interface, there are adjacent points on both sides of the interface, Figure 2.5-5. The summation of the potentials comparable to Eq. (2.5-4) has to consider that the boundary con-

ditions Eq. (2.4-13) to (-16) are satisfied. Therefore, the different potentials have to be weighted differently. Then the potential φ_0 on the interface is given by a new “square algorithm” comparable to Eq. (2.5-5) [16]:

$$\varphi_0 = \frac{1}{4} \left\{ \varphi_1 + \varphi_3 + 2 \cdot \frac{\varepsilon_{r1} \varphi_2 + \varepsilon_{r2} \varphi_4}{\varepsilon_{r1} + \varepsilon_{r2}} \right\} \quad (2.5-8)$$

If the above-mentioned equations are applied to all points of the grid, a *system of linear equations* results. In order to solve the equations, *knowledge of the boundary potentials* is required.

Programming the finite difference algorithms, e.g. for iterative solution of the system of equations, *requires little effort*. The FDM was one of the first numerical methods in practical use [4].

Example: Foil edge in a capacitor dielectric

The potential distribution in a capacitor dielectric close to the edge of a metallic foil shall be calculated by means of the Finite Difference Method. Figure 2.5-6 shows a cross section through the plane insulation arrangement (compare to Figure 2.4-30). In order to get a clear and comprehensible calculation, a rather large mesh width of the grid is chosen.

For the calculation, knowledge of *all* boundary potentials is necessary. The electrode potentials are therefore set to $\varphi/V = 1$ and $\varphi/V = 0$. As the arrangement is open on the left and right hand sides, there are additional boundary points, whose potentials also have to be defined. Because of the uniformity of the field in the outer regions, the boundary potentials are set to $\varphi/V = 0$ (left) and $\varphi/V = 0.5$ (right).

Because of the symmetry of the arrangement, the calculation is limited to the potentials φ_1 to φ_6 in the lower (or upper) half of the arrangement. The equation solution based on the “square algorithm” Eq. (2.5-5) is performed by iteration and starts with assumed initial values $\varphi_i = 0$.

The iterative re-calculation of the potentials according to Eq. (2.5-5) progresses from φ_6 (right) to φ_1 (left). In the next iteration step, the direction of the iteration is reversed. In this way a fast convergence is achieved.

In the following, the iteration results are given:

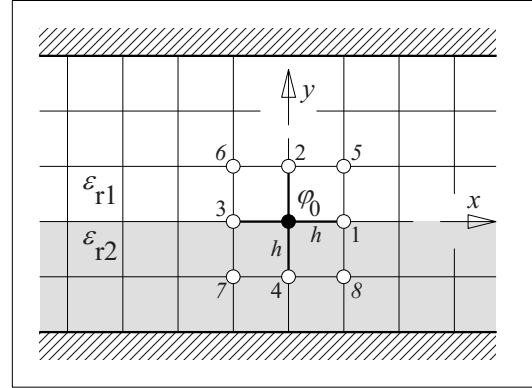


Figure 2.5-5: Calculation of multi-layer dielectrics with the Finite Difference Method.

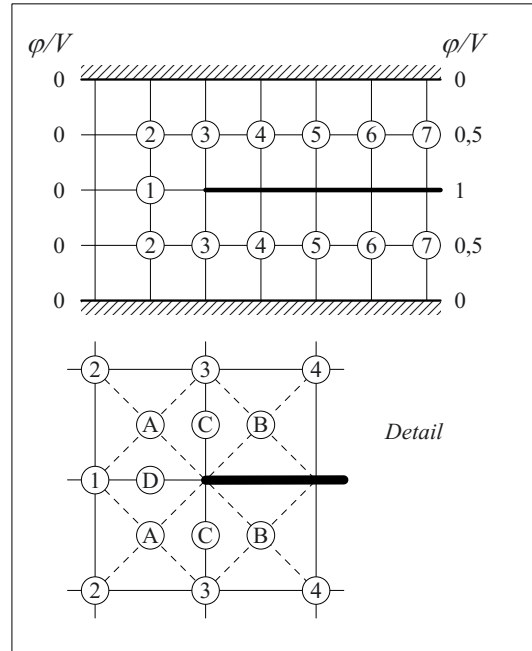


Figure 2.5-6: Calculation of a capacitor dielectric with the help of the Finite Difference Method.

	φ_1/V (Direction)	φ_2/V	φ_3/V	φ_4/V	φ_5/V	φ_6/V	φ_7/V (Boundary)
Start:	0	0	0	0	0	0	0.5
←	0.292	0.084	0.334	0.336	0.344	0.375	0.5
→		0.156	0.373	0.429	0.451	0.488	0.5
←	0.337	0.147	0.405	0.463	0.479	0.488	0.5
→		0.186	0.412	0.473	0.490	0.498	0.5
←	0.344	0.188	0.416	0.476	0.493	0.498	0.5
→		0.190	0.417	0.477	0.494	0.499	0.5
←	0.345	0.190	0.417	0.478	0.494	0.499	0.5

The iteration is finished, if the potential values only change within a predefined limit from iteration step to iteration step (here $\Delta\phi/V < 0.001$).

By means of the “diagonal algorithm” Eq. (2.5-6), potentials at intermediate points can be calculated by *interpolation*, e.g. at A, B, C and D, Figure 2.5-6 (detail):

$$\phi_A = 0.25 \cdot (\phi_1 + \phi_2 + \phi_3 + V) = 0.488 V$$

$$\phi_B = 0.25 \cdot (\phi_3 + \phi_4 + V + V) = 0.724 V$$

$$\phi_C = 0.25 \cdot (\phi_3 + \phi_A + \phi_B + V) = 0.658 V$$

$$\phi_D = 0.25 \cdot (\phi_1 + \phi_A + \phi_A + V) = 0.580 V$$

For the drawing of *field or equipotential patterns*, equipotential lines have to be determined by interpolation between the calculated potentials of the fixed nodes in the net. For this purpose, the grid in the given example is far too coarse.

Nevertheless, the result shows clearly that there is a *concentration of field and equipotential lines* close to the edge of the metallic foil. However, the calculated values are not very accurate, because of the coarse grids and the sharp edge of the metallic foil.

The Finite Difference Method has some **disadvantages**, and other methods are better suited in many cases:

1. The fixed regular grid cannot describe *curved electrodes and interface contours* with sufficient accuracy. The Charge Simulation Method and Finite Element Method are much more flexible.

Note: Sometimes the approximation of electrode and interface contours can be improved to a certain extent by a local distortion of the grid. In this case, Eqs. (2.5-4) to (-8) have to be modified.

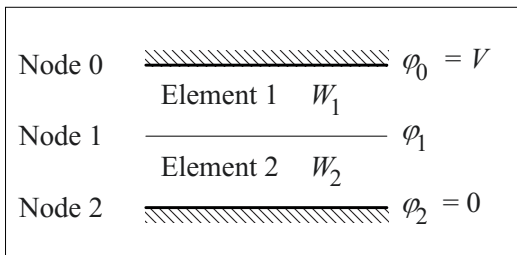


Figure 2.5-7: Calculation of potentials in a parallel-plate capacitor by minimizing the field energy (normally triangular elements are chosen).

2. The regular grid cannot be adapted to the non-uniformity of the field. A *finer discretization* in the non-uniform regions of the field would unequivocally be extended to the whole field volume. Therefore, unreasonable computing resources and times would be required. This is especially serious for three-dimensional fields. Often, the Charge Simulation Method is the simpler method.

2.5.4 Finite Element Method

The Finite Element Method (FEM) is based on the discretization of the field volume by a grid of triangles. For the calculation of **Laplacian fields**, the potentials of the *nodes* in the grid are determined in such a way that the whole *field energy is minimized*.

This principle of minimizing an *energy function* has been known in other fields of engineering and science for a long time [16], [36], [37], [38]. It is a great advantage of the Finite Element Method that common program systems can calculate *mechanical, thermal, magnetic and electric fields* at the same time (*multi-physics*). Especially for industrial applications, high voltage apparatus must be designed with respect to many different stresses.

Note: This approach can also be applied to **linearly superimposed fields** (e.g. conduction and displacement fields) and to **time-harmonic** or **transient fields**. In these cases, *different contributions to the energy* have to be summed, and their variation with time has to be considered by an additional discretization in time and by a calculation in consecutive time steps [282].

Note: With the idea of minimizing the energy, **non-stationary fields** and *electromagnetic waves* can also be treated, if *all energy contributions* are considered in the volume under consideration. This includes the stored electric energy, the stored magnetic energy, the energy fed and dissipated by conduction currents, the energy of a space charge density, the eddy current loss energy and an additional term allowing the so-called *Coulomb gauge* of the vector potential [394].

The basic calculation method for the potential determination by minimizing the field energy shall be explained for the example of a parallel-plate capacitor, Figure 2.5-7.

Example:**Minimum field energy in a parallel-plate capacitor**

In the uniform field of a parallel-plate capacitor, the potential midway between the two electrodes shall be determined numerically.

The electrodes are treated as “nodes” with the given potentials $\varphi_0 = V$ and $\varphi_2 = 0$. The equipotential surface midway in-between is considered as “node” 1 with the unknown potential φ_1 . Figure 2.5-7.

With Eq. (2.1-11), the field energies of the elements 1 and 2 between the nodes are

$$\begin{aligned} W_1 &= \frac{1}{2} \cdot C_1 \cdot (V - \varphi_1)^2 \\ \text{and} \\ W_2 &= \frac{1}{2} \cdot C_2 \cdot (\varphi_1 - 0)^2. \end{aligned} \quad (2.5-9)$$

Now, the total field energy

$$W = \sum W_i = W_1 + W_2 \quad (2.5-10)$$

is to be minimized by variation φ_1 . As φ_1 is the only variable potential, the *minimum field energy* is determined from the condition

$$\partial W / \partial \varphi = \partial W_1 / \partial \varphi_1 + \partial W_2 / \partial \varphi_1 = 0.$$

With Eq. (2.5-9) we get

$$\begin{aligned} \frac{1}{2} \cdot C_1 \cdot 2 \cdot (V - \varphi_1) \cdot (-1) + \frac{1}{2} \cdot C_2 \cdot 2 \cdot (\varphi_1 - 0) &= 0 \\ \text{and} \\ \varphi_1 &= V \cdot C_1 / (C_1 + C_2). \end{aligned} \quad (2.5-11)$$

For $C_1 = C_2$, the *expected solution* is

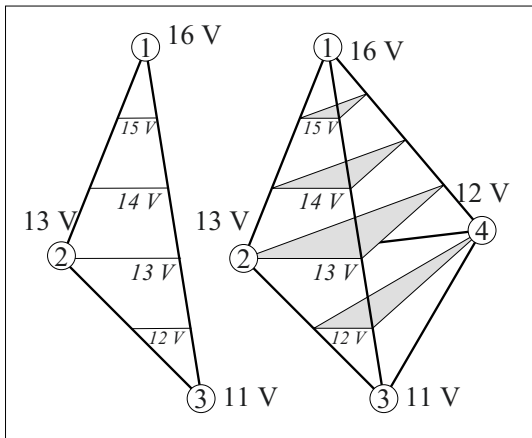


Figure 2.5-8: Triangles and tetrahedrons as (finite) elements for two- and three-dimensional fields with linear interpolation of the potential.

$$\varphi_1 = V/2 \quad (2.5-12)$$

Note: It can easily be shown that the calculated extreme value gives a *minimum of energy*. For the other extreme values $\varphi_1 = V$ and $\varphi_1 = 0$, there is a maximum of energy in each case.

By means of that example it can also be shown how easily *field regions with different materials* can be treated. The field energies of the considered elements are just calculated with different permittivities:

Example:**Minimum field energy in a multi-layer dielectric**

If in the arrangement according to Figure 2.5-7 different permittivities $\epsilon_{r1} = 1$ and $\epsilon_{r2} = 2$ are assumed, the capacitances are $C_2 = 2 \cdot C_1$ at equal distances. The expected solution follows directly from Eq. (2.5-11):

$$\varphi_1 = V/3 \quad (2.5-13)$$

For a realistic field calculation, the field volume has to be *discretized* by a mesh without regard to the usually unknown shape of the equipotential surfaces. Therefore, two-dimensional (plane or rotationally symmetric) problems are mostly calculated with *triangular* and three-dimensional problems with *tetrahedral elements*, Figure 2.5-8. The *corners* of these elements are the nodes of the mesh.

It is necessary to determine the field or the *potential distribution* throughout an element in order to calculate its field energy. This distribution can be *interpolated from the node potentials*. In Figure 2.5-8 a linear interpolation of the potential between the calculated or given node potentials is shown. In such a case, the field strength is assumed to be constant throughout the considered element.

The **discretization** of the whole field volume with triangular or tetrahedral elements has a significant *advantage* in comparison to the Finite Difference Method: The elements can be adjusted to the *shape of electrodes and interfaces*, and field regions of minor importance can be represented with a coarse mesh. In this way, despite high resolution in the regions of

interest, the *number of nodes and elements* is limited, Figure 2.5-9.

Electrodes with given potentials, e.g. grading layers of bushings, can be included by defining fixed node potentials. If the potentials are completely unknown, they can be simulated by regions of extremely enhanced permittivity. Then the field is displaced into the adjacent materials, and the field emanates approximately perpendicularly from the interface (comparable to a metallic electrode).

Normally, the mesh is generated by an automatic *mesh generator*, which follows the given contours. Afterwards it is often possible to optimize the mesh manually, i.e. by displacing, deleting and setting of nodes. It is very important to ensure that the dimensions of the elements are small enough for the assumption of uniform field strength within the considered element (for linear interpolation only).

For **minimizing the field energy**, the energy of the individual elements (1, 2, 3, ..., k , ..., n) is to be determined by volume integration of the field energy density:

$$W_k = \int_{V_k} \frac{1}{2} \varepsilon E^2 dV \quad (2.5-14)$$

Attention: V is the *volume* here and not the voltage!

The energy of an individual element can be expressed as a function of the respective three (or four) node potentials and node coordinates:

$$W_k = f(\varphi_p, \varphi_q, \varphi_r, \varphi_s; x_p, x_q, x_r, x_s) \quad (2.5-15)$$

The total energy is the sum of the energies of all elements:

$$W = \sum_{k=1}^n W_k \quad (2.5-16)$$

The minimum energy is determined if the partial derivatives with respect to all node potentials (φ_1 to φ_m) are set equal to zero, thus resulting in a **system of equations**:

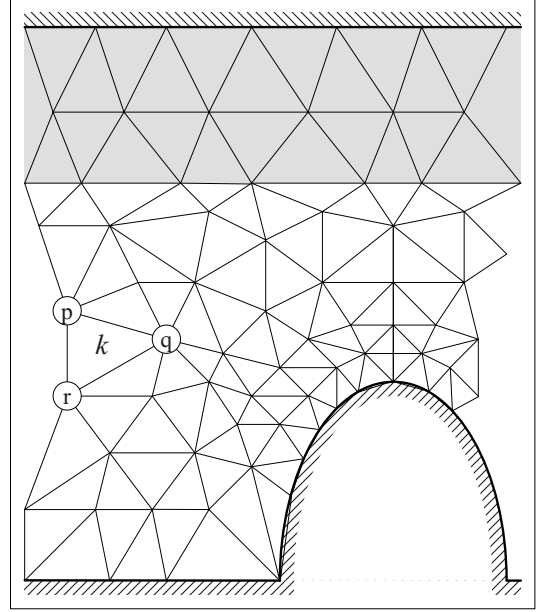


Figure 2.5-9: Discretization of a two dimensional field region with finite elements, which can be adapted to the electrode contours, to the interfaces and to the degree of field homogeneity (detail).

$$\begin{aligned} \frac{\partial W}{\partial \varphi_1} &= 0 \\ \frac{\partial W}{\partial \varphi_2} &= 0 \\ \dots\dots\dots & \\ \frac{\partial W}{\partial \varphi_j} &= 0 \\ \dots\dots\dots & \\ \frac{\partial W}{\partial \varphi_m} &= 0 \end{aligned} \quad (2.5-17)$$

The node potentials φ_1 to φ_m are given by the *solution of the System of Equations* (2.5-17). Instead of a direct solution, an *iterative approach* is frequently chosen, which can also solve non-linear cases (e.g. in magnetic circuits or at very high electric field strengths).

Generally, the knowledge of **boundary potentials** at the borders of the field volume is necessary (i.e. on the electrodes). *Border lines* with unknown potentials (e.g. between electrodes) are regarded as field lines rectangular to the equipotential surfaces. This must be taken into account during the selection of the field volume for calculation. If necessary, the area to be calculated is to extend well beyond the region of interest in order to keep the influence of field distortions at the borders low.

Another source of inaccuracy is the *interpolation* of the potential within the individual elements if the elements are so large that the potential distribution is no longer described accurately enough by an interpolation or approximation function. The inaccuracy can be especially high for the linear interpolation, see Figure 2.5-8. Therefore, **polynomials of higher order** are used, which allow steady transitions of gradients and curvatures of the equipotential lines at the boundaries of the elements. Both the *accuracy* and the *computational time* together with the danger of *numerical instabilities* increase significantly with the order of the interpolation polynomials.

Example: Comparison between analytical and numerical solution

Similar to the example in Figure 2.5-7, the field of a cylindrical capacitor ($R_i = r_0 = 2$ cm, $R_a = r_4 = 10$ cm) shall be determined by numerical iteration for $V = 20$ kV and compared with an exact analytical solution, Figure 2.5-10.

The equipotential surfaces at $r_0 = 2$ cm ($\varphi_0 = V = 20$ kV), $r_1 = 4$ cm (φ_1), $r_2 = 6$ cm (φ_2), $r_3 = 8$ cm (φ_3) and $r_4 = 10$ cm ($\varphi_4 = 0$) are chosen as “nodes”. By analogy with Eq. (2.5-9) and (-10) and by means of the capacitances C_{01} , C_{12} , C_{23} and C_{34} , the field energy can be expressed as a function of the unknown potentials φ_1 , φ_2 and φ_3 . With the condition for extreme values according to Eq. (2.5-17) a *system of equations* is established for the unknown potentials by setting the partial derivatives to zero:

$$\varphi_1(C_{01} + C_{12}) + \varphi_2(-C_{12}) = C_{01}V$$

$$\varphi_1(-C_{12}) + \varphi_2(C_{12} + C_{23}) + \varphi_3(-C_{23}) = 0$$

$$+ \varphi_2(-C_{23}) + \varphi_3(C_{23} + C_{34}) = 0$$

For a cylinder length of 1 m, the capacitances are $C_{01} = 80.2$ pF, $C_{12} = 137.1$ pF, $C_{23} = 193.3$ pF and $C_{34} = 249.2$ pF according to Eq. (2.3-20). The *potentials* are determined as

$$\varphi_1 = 11.38 \text{ kV}, \varphi_2 = 6.34 \text{ kV} \text{ and } \varphi_3 = 2.77 \text{ kV}$$

by iteratively or directly solving the equation system. A linear approach is chosen for determination of the *field strengths*:

$$E_{01} = (\varphi_0 - \varphi_1)/(r_1 - r_0) = 4.3 \text{ kV/cm}$$

$$E_{12} = (\varphi_1 - \varphi_2)/(r_2 - r_1) = 2.5 \text{ kV/cm}$$

$$E_{23} = (\varphi_2 - \varphi_3)/(r_3 - r_2) = 1.8 \text{ kV/cm}$$

$$E_{34} = (\varphi_3 - \varphi_4)/(r_4 - r_3) = 1.4 \text{ kV/cm}$$

The *comparison* with an *analytical* solution according to Section 2.3.1.3 shows that the node potentials were calculated correctly. This can be explained by the capacitance values, which were exact values and not numerical approximations. Nevertheless, field strengths are determined with substantial errors when using linear interpolation. The analytical calculation gives considerable differences, especially in the non-uniform region of the field:

$$E_0 = 6.2 \text{ kV/cm}$$

$$E_1 = 3.1 \text{ kV/cm}$$

$$E_2 = 2.1 \text{ kV/cm}$$

$$E_3 = 1.5 \text{ kV/cm}$$

$$E_4 = 1.2 \text{ kV/cm}$$

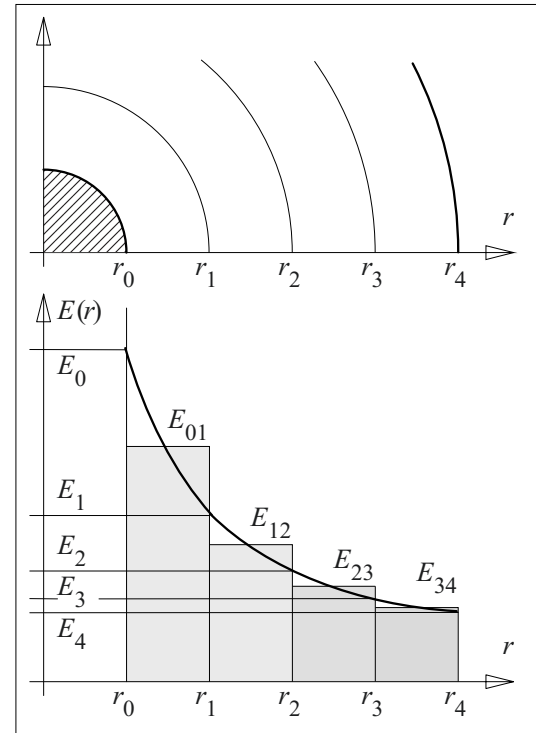


Figure 2.5-10: Comparison between analytical and numerical solution for the example of a cylindrical capacitor.

The numerically calculated field strengths can be considered as *mean field strengths* within the elements, Figure 2.5-10. Maximum values are higher, minimum values are lower. These deviations can be significantly reduced by selecting smaller elements and interpolation polynomials of higher order.

Program systems for field calculations with the Finite Element Method have user interfaces that guide the **user** through the calculation process and perform many steps automatically, Fig. 2.5-11. For this the user has particularly to learn program-specific features, which, depending on the complexity and degree of development of the program system, can involve quite considerable effort. The physical problem, understanding the problem and the basic calculation processes are often dangerously neglected by the user.

Normally, a **numerical field calculation** consists of a number of steps:

1. The user has to enter the *geometric data* of the electrode contours and areas to be calculated, either directly or by means of CAD data sets. An **adequate simplification of the geometry** is the most important preparation that determines the quality of the calculation: On the one hand, the uncritical import of *all* design details causes extremely high computational effort. On the other hand, simplifications must not be *too coarse*, in order to avoid wrong conclusions.

This is a question of *engineering feeling*, which requires high analytical abilities and a sophisticated *physical understanding of field geometries*, especially in times of complex and “user friendly” program systems.

Therefore, a *qualitative feeling* for the expected result should already exist before a calculation is started.

2. The **discretization** of the geometry is performed by an automatic *mesh generator*. The user has to give parameters defining the size and the quality of the generated mesh as a function of geometry data (e.g. radii of curvature). This can be done, for example, by de-

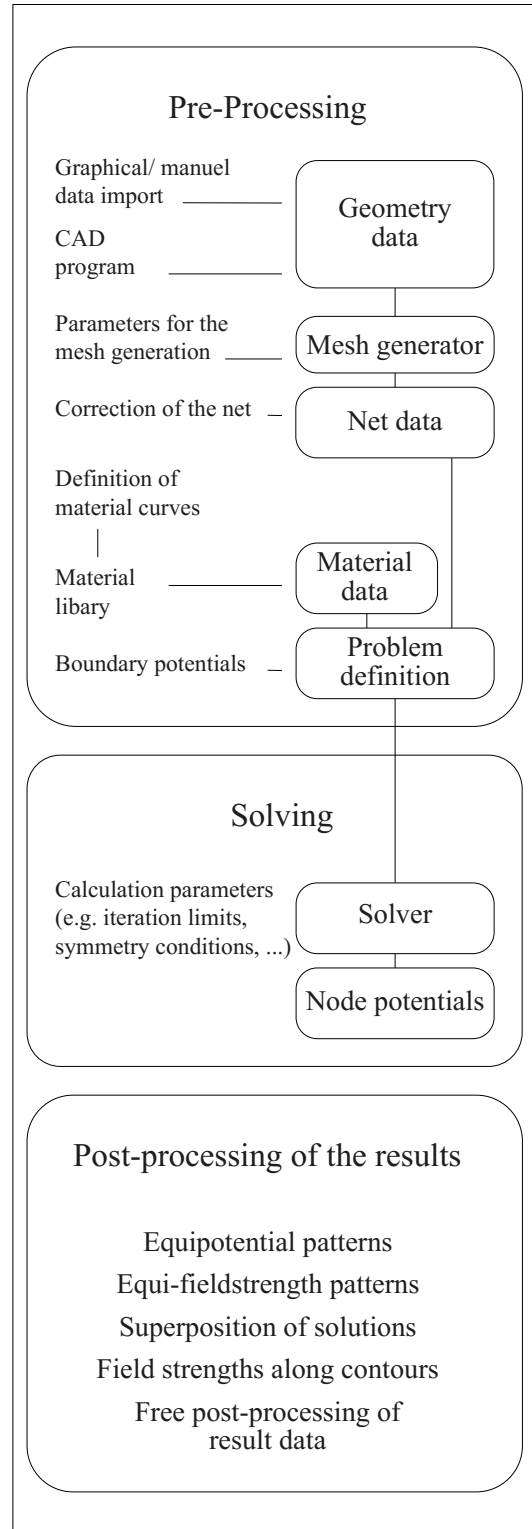


Figure 2.5-11: Structure of a Finite Element Program system for numerical field calculations.

fining node positions on the electrodes or along contours. Normally, the automatically generated mesh can be checked and manually *refined* afterwards, e.g. in order to achieve finer discretization for important field regions, or to subdivide triangles with unfavorable aspect ratios.

Again, these tasks require a qualitative feel for the expected results, as mentioned above.

3. Then, *material values* (or non-linear *material curves*) have to be allocated to the generated network elements and *potential values* have to be allocated to the electrodes and edge contours. For such a **definition of a calculation problem**, pre-defined materials can often be taken from a *material library*.

4. After this pre-processing, the **system of equations** is solved with an appropriate mathematical *solver*. Again, the user can define iteration limits, step widths and the order of the interpolation polynomials. He has to find a good compromise between accuracy of the calculation, computational time and numerical stability. The *result* is given as a list of *node potentials*. Potentials in the field volume are derived from interpolation and approximation polynomials.

Note: The solution is often performed in a number of *steps*, in which the discretization is improved based on a preceding result for the following iteration.

Note: Additionally, routines can be programmed sometimes, allowing the variation of geometry characteristics as a function of calculation results. In this way, electrode contours can be *optimized iteratively* in a number of calculation steps in order to avoid field strength increases.

5. Subsequent to the calculation, a **post-processing of the solution** or *the data* can be performed by any kind of mathematical operations.

Equipotential patterns or patterns with lines of equal field strengths are generated for visualization of the results. They can also contain field vectors in order to visualize the direction and magnitude of the field strength. Normally,

equipotential patterns are *colored* in order to visualize the *magnitude of the field strength*.

Complex processes can be calculated by *superposition* of different solutions. For example, the field after the polarity reversal of a DC voltage is given by the superposition of the initial steady-state conduction field and the dielectric displacement field associated with the voltage step during the polarity reversal [7], [10].

For the evaluation of electrical stresses it is often necessary to show the magnitudes of field strengths along given *contours*. Normally, field strength magnitudes, tangential components and normal components can be distinguished.

6. Finally, the user has the important task of **checking the plausibility of the results**. Unfortunately, the complexity of modern field calculation systems leads to input data errors remaining undiscovered and often only able to be detected from a non-plausible field pattern. Such an evaluation requires a very good physical understanding and imagination as mentioned above. In addition, for example, numerically calculated field strength values can be analytically recalculated in sub-regions with simple field geometry.

2.6 Rapidly Changing Fields and Traveling Waves

For the slowly varying fields that have been discussed previously it was assumed that propagation time effects can be neglected within the observed spatial dimensions. Field changes are so slow that they can approximately be considered as simultaneous in the relevant field volume. This condition was already formulated with Eqs. (2.1-36) to (-38) in Section 2.1.4.4.

If the conditions of the quasi-static field approximation are no longer fulfilled, Maxwell's Equations have to be used in their complete form for the description of the non-stationary

electromagnetic wave field, see Section 2.1.4.4.

In relation to lightning impulse voltages and fast transients, there are a number of rapidly changing stresses in high voltage engineering. Basically, they can be described as *guided traveling waves*, see Section 2.2.4 and 2.2.5.

Note: Electromagnetic waves that propagate in free space, i.e. that do not propagate in a transmission line, do not belong to the typical stresses of high voltage engineering. Only in the hopefully hypothetical case of a nuclear explosion outside the atmosphere, it is feared that the nuclear electromagnetic pulse (NEMP) will endanger electric power systems.

Nevertheless, free electromagnetic waves can couple into measuring systems and affect the *measuring signals* in impulse and partial discharge measuring circuits [5], [18], [19]. Moreover, the electromagnetic radiation field can purposefully be used for *diagnostics*, especially for partial discharge detection.

In the following, guided traveling waves (Section 2.6.1), reflections (Section 2.6.2) and examples (Section 2.6.3) are discussed.

2.6.1 Guided TEM Wave

A rapidly increasing electric field strength between the conductors of a line is associated with a displacement current between the conductors, Figure 2.6-1. It continues as an axial conduction current in the conductor's surface. The magnetic field lines associated with the current, are closed lines around the upper conductor.

In the wave front, the *electromagnetic wave* has orthogonal vectors $\mathbf{E}$ (electric field strength), $\mathbf{H}$ (magnetic field strength) and $\mathbf{u}$ (phase velocity). The time-varying field quantities $\mathbf{E}$ and $\mathbf{H}$ are mutually dependent according to Faraday's and Ampere's law, see also Figure 2.1-17. As both electric and magnetic field vectors are transverse (orthogonal) to the direction of wave propagation, the propagation mode is called TEM mode (or TEM wave).

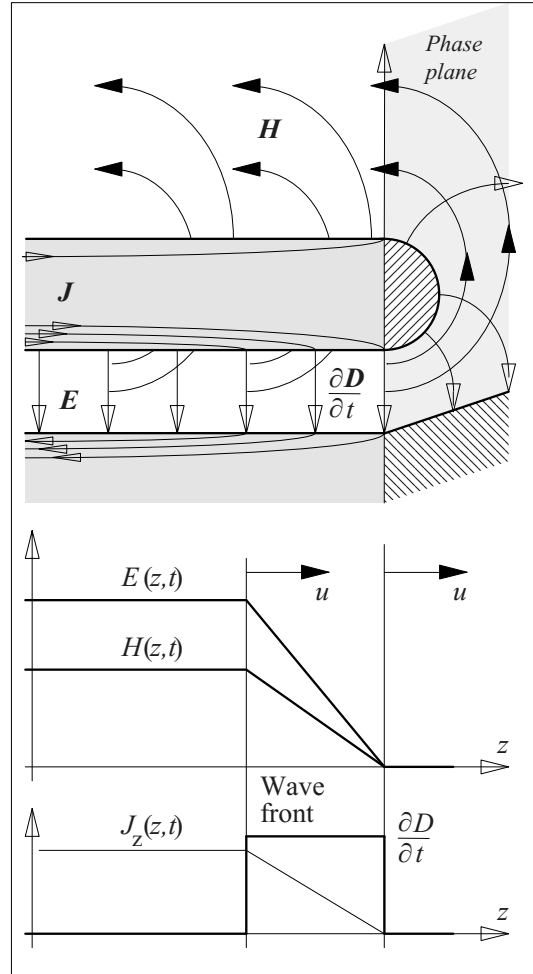


Figure 2.6-1: Propagation of a guided traveling wave on a transmission line, TEM wave with linearly rising electric and magnetic field strengths.

Note: Apart from this basic propagation mode, there are further modes which can propagate between the two conductors, but they are of minor importance for the electrical stress.

A significant simplification of the analysis is achieved, if *infinitesimal small line sections* Δz with *quasi-static field conditions* are considered, Figure 2.6-2. Instead of the field quantities $\mathbf{E}$ and $\mathbf{H}$, it is possible to perform calculations with the **integral quantities** *voltage* v and **current** i .

At first, a **lossless line** with $R' = 0$ and $G' = 0$ shall be considered. According to the equiva-

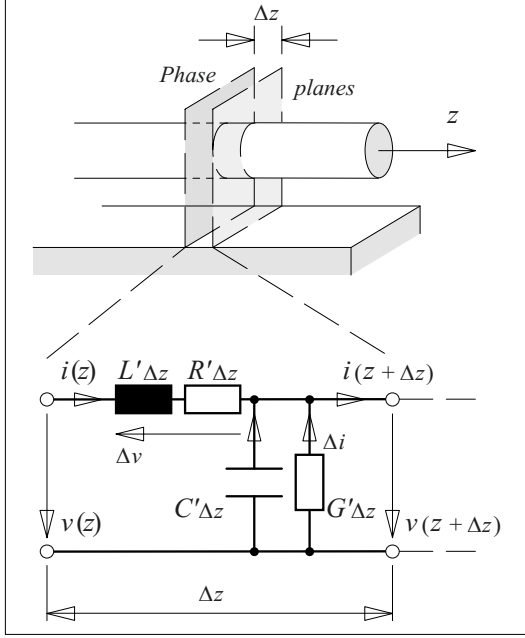


Figure 2.6-2: Description of a short line segment by an "electrically short" equivalent circuit.

lent circuit, the difference of the voltages at the positions z and $z+\Delta z$ (Kirchhoff's voltage law) is

$$v(z+\Delta z) - v(z) = \Delta v = -L'\Delta z \cdot \partial i / \partial t$$

and the difference of the currents (Kirchhoff's current law) is

$$i(z+\Delta z) - i(z) = \Delta i = -C'\Delta z \cdot \partial v / \partial t.$$

By the transition to infinitesimal small sections $\Delta z \rightarrow 0$, two partial differential equations are established for the dependence of the quantities $v(z,t)$ and $i(z,t)$ on time t and position z :

$$\partial v / \partial z = -L' \cdot \partial i / \partial t \quad (2.6-1)$$

and

$$\partial i / \partial z = -C' \cdot \partial v / \partial t \quad (2.6-2)$$

In order to insert these equations into each other, the first equation is differentiated with respect to the path z and the second equation with respect to time t :

$$\partial^2 v / \partial z^2 = -L' \cdot \partial^2 i / (\partial t \partial z) \quad (2.6-3)$$

$$\partial^2 i / (\partial z \partial t) = -C' \cdot \partial^2 v / \partial t^2 \quad (2.6-4)$$

By insertion of Eq. (2.6-4) into Eq. (2.6-3), the differential equation for the voltage is established:

$$\partial^2 v / \partial z^2 = L' C' \cdot \partial^2 v / \partial t^2 \quad (2.6-5)$$

The analog differential equation for the current is derived, if Eq. (2.6-1) is differentiated with respect to t and Eq. (2.6-2) with respect to z :

$$\partial^2 i / \partial z^2 = L' C' \cdot \partial^2 i / \partial t^2 \quad (2.6-6)$$

The differential Equations (2.6-5) and (-6) are called **transmission line wave equations**. The general solution for the voltage is

$$v(z,t) = V \cdot \{ f(z-ut) + g(z+ut) \}. \quad (2.6-7)$$

After differentiating it twice with respect to z and t , and by insertion, it can be shown that the solution satisfies the differential equation, if $u^2 = 1/(L'C')$. Thus, the **phase velocity** u is a function of the transmission line parameters L' and C' :

$$u = \frac{1}{\sqrt{L' C'}}. \quad (2.6-8)$$

Eq. (2.6.7) describes a traveling voltage-wave consisting of two components. The first term $f(z-ut)$ corresponds to a traveling wave in $+z$ -direction, because its argument doesn't change if z increases to the same extent as ut . The second term $g(z+ut)$ correspondingly describes a traveling wave in $-z$ -direction, Figure 2.6-3.

Note: The expression "voltage" must only be used with great care in relation to rapidly changing processes: The definition of a voltage (scalar potential) is only possible if integration of the electric field strength along a path gives a value that is *independent* of the chosen path, the ring integral of $E d\mathbf{x}$ must be zero. This means a closed integration path must not enclose any magnetic flux that varies with time. For the given TEM wave, this condition is only fulfilled in the so-called *phase planes* orthogonal to the line direction z . These planes are not penetrated by the magnetic field, Figure 2.6-1 and -2. The specification of

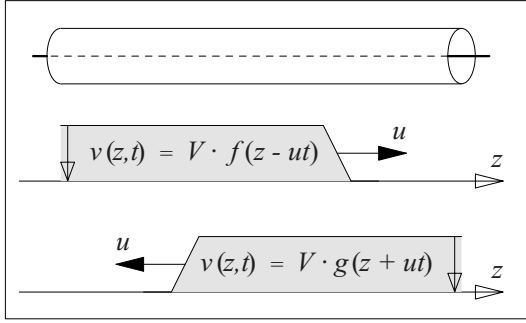


Figure 2.6-3: Traveling voltage waves in $+z$ - and $-z$ -direction as solutions of the transmission line wave equations.

a voltage thus only describes the electric field in a phase plane. The specification of a voltage between points that do not have the same z -coordinate is no longer allowed.

The traveling voltage-waves are accompanied by *related traveling current-waves*, which can be derived from the above-mentioned solution for the voltage waves: Eq. (2.6-7) is differentiated with respect to time and inserted into Eq. (2.6-2), which then can be used to determine the desired currents by integration. With

$$\partial v / \partial t = V \cdot \{(-u) \cdot f'(z-ut) + v \cdot g'(z+ut)\}$$

Eq. (2.6-2) is

$$\partial i / \partial z = V \cdot u \cdot C' \cdot \{f'(z-ut) - g'(z+ut)\}.$$

The current is determined by integration along the coordinate z :

$$i(z,t) = V \cdot u \cdot C' \cdot \{f(z-ut) - g(z+ut)\} \quad (2.6-9)$$

Again, there are traveling current-waves in $+z$ - and $-z$ -directions. The two traveling current waves have opposite polarities. This means that the current direction is reversed together with the wave propagation direction, if the voltage between the conductors does not change, Figure 2.6-4.

Traveling voltage and current waves with the same propagation direction always *belong together*. They are two different mathematical representations of the same physical process: This means that voltage and current are de-

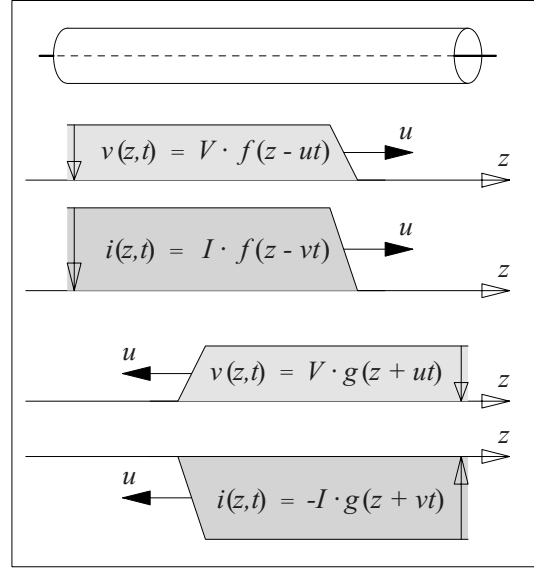


Figure 2.6-4: Related voltage and current waves in $+z$ - and $-z$ -direction (traveling waves).

rived from the electric and the magnetic field of the *same* traveling wave.

According to Eq. (2.6-9), the amplitudes of the related voltage and current waves have a constant ratio, the **characteristic (line) impedance** Z_L :

$$Z_L = v/i = 1/(uC')$$

For the lossless line it follows from Eq. (2.6-8) that

$$Z_L = \frac{v}{i} = \sqrt{\frac{L'}{C'}}. \quad (2.6-10)$$

The line parameters L' and C' (inductance and capacitance per unit length) and the characteristic line impedance Z_L depend on the kind of the line and on the geometric dimensions, Figure 2.6-5. Generally, it can be stated that the characteristic impedance increases with increasing distance between the conductors and that it decreases with increasing conductor area.

Characteristic impedances of *overhead transmission lines* ($C' \approx 10$ nF/km, $L' \approx 1$ mH/km) are somewhere in the region above 300Ω . For

a *gas-insulated coaxial line* with tubular conductors and the radius ratio $e:1$, the characteristic impedance is $Z_L = 60 \Omega$. A *high voltage cable* with $\varepsilon_r = 2.2$ and $r_a/r_i = e$ has $Z_L = 40 \Omega$. Even lower values are given for *medium voltage cables* with large inner conductor cross sections. Coaxial *signal cables* have a characteristic line impedance $Z_L = 50 \Omega$ normally. For *transformer windings* characteristic impedances are in the range of $10^2 \Omega$ to $10^4 \Omega$, because of the high inductances [45]. These values increase with the rated voltage and decrease with the rated power.

For the basic arrangements in Figure 2.6-5, the *phase velocity* u is independent from the geo-

metric dimensions. In vacuum and (by approximation also) in gases, u is equal to the *speed of light* $u_0 = 300,000 \text{ km/s} = 3 \cdot 10^8 \text{ m/s} = 0.3 \text{ m/ns}$. In dielectrics with higher permittivity, the phase velocity is reduced by the factor $\varepsilon_r^{-0.5}$, see Eq. (2.6-8).

Example:

Lightning strike into an overhead line

During a lightning strike an impulse current $\hat{I} = 10 \text{ kA}$ is fed into the phase conductor of an overhead line. The magnitude of the overvoltage shall be estimated.

The lightning stroke current is split up into two current waves that propagate in both directions from the point of the strike. With $Z_L = 300 \Omega$, a peak voltage $\hat{V} = 300 \Omega \cdot 5 \text{ kA} = 1.5 \text{ MV}$ is calculated from Eq. (2.6-10).

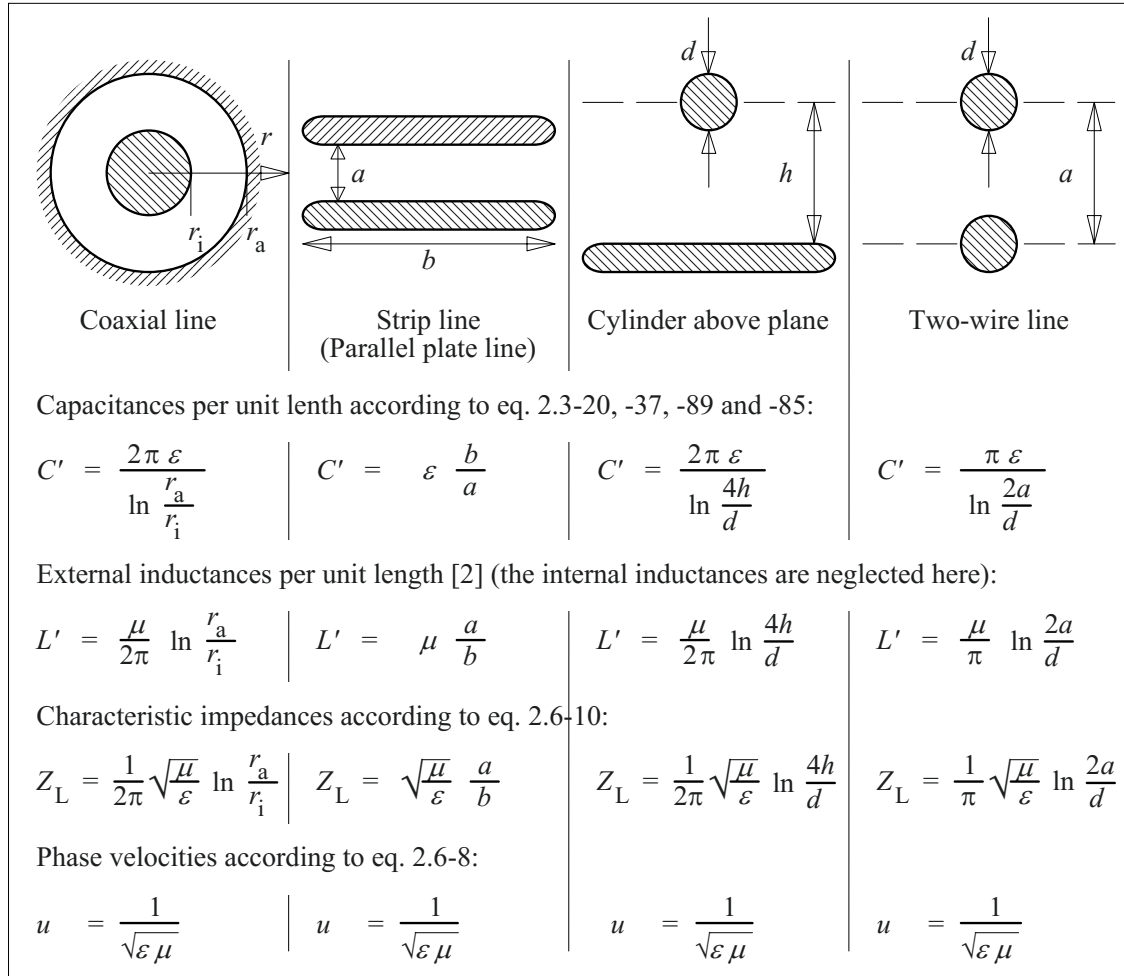


Figure 2.6-5: Line parameters, characteristic impedances and phase velocities for basic line arrangements.

Amplitude, shape and direction of a traveling wave are *not yet determined* by the transmission line wave equations or by their general solutions Eq. (2.6-7) and (-8). Furthermore, they depend on the *boundary conditions*, i.e. on the currents and voltages at both ends of the transmission line, see Section 2.6.2.

Analogous considerations also apply to the **field quantities** E and H in *dielectric media*. Based on Maxwell's Main Equations (2.1-14) and (-15), the relationships shown in Section 2.1.4.4 for the **intrinsic impedance** (*wave impedance*) Z , Eq. (2.1-43), and for the phase velocity, Eq. (2.1-42), are derived:

$$u = \frac{1}{\sqrt{\epsilon \mu}} \quad (2.6-11)$$

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}} \quad (2.6-12)$$

In vacuum and (by approximation also) in gases, the phase velocity is the *speed of light* and the intrinsic impedance is the *intrinsic impedance of free space* which is given by

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \, \Omega \quad (2.6-13)$$

Note: The Equations (2.6-11) to (-13) are not only valid for the field quantities of a conducted traveling wave on a line, but also for the field quantities of a *free electromagnetic TEM wave*, see Section 2.1.4.4.

Example: Field quantities in a strip line

The *characteristic (line) impedance* is $Z_L = v/i$. If voltage and current are expressed by the field quantities E and H , then because of the approximately uniform wave field between the electrodes, the characteristic impedance is

$$Z_L = \frac{v}{i} = \frac{\int E \, ds}{\oint H \, ds} = \frac{E \cdot a}{H \cdot b} \quad .$$

The comparison with the equations in Figure 2.6-5 shows that the ratio of the field quantities E and H gives the *intrinsic impedance* Z :

$$\frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}} = Z$$

The above considerations also apply to *lossless lines* (or *perfect dielectrics*), i.e. the line parameters R' and G' in Figure 2.6-2 are neglected. The **consideration of losses** owing to constant parameters R' and G' results in traveling wave amplitudes that are *decreasing exponentially* along the propagation path (*damped line or non-ideal dielectrics*).

If special conditions cannot be assumed (e.g. "distortion-free line"), the *shape of the traveling wave* also changes. This generally means that fast impulse fronts are *flattened* and traveling wave pulses are rounded. This can be explained by the *frequency dependence* of line parameters; especially R' increases strongly with frequency because of the skin effect, and high-frequency wave components are damped accordingly.

At very high voltages and field strengths, e.g. which can occur during a lightning strike into an overhead line, *corona discharges* can also occur. They must be accounted for by a voltage-dependent *leakage* (conductance) per unit length G' . This non-linear effect causes higher damping and distortion of the traveling wave.

Often, the damping of traveling waves is not considered because the *undamped wave* normally causes the highest stresses ("worst case").

2.6.2 Reflection Processes

2.6.2.1 Basics

Voltage and current at the end of a transmission line are determined by the termination, which can be a single circuit element, a network or another line.

If the ratios of voltage and current on the line and at the termination do not coincide, a *reflection* occurs, i.e. a wave traveling back, Figure 2.6-6.

Because of Kirchhoff's current and voltage laws, the superpositions of voltages and currents of the incident and the reflected waves (indices "i" and "r") give the voltage and the current at the termination. If the termination is another line, a *transmitted* or *refracted* traveling wave (index "t") propagates within it:

$$v_i + v_r = v_t \quad (2.6-14)$$

$$i_i + i_r = i_t \quad (2.6-15)$$

This idea can also be applied to a lumped termination, which absorbs the "transmitted" wave.

In the case of an **open circuit** ($Z_2 \gg Z_1$), there can be *no resulting current* at the reflection point, i.e. the currents of the transmitted and reflected waves are $i_t = 0$ and $i_r = -i_i$. According to the Eqs. (2.6-8) to (-10), the traveling current waves are accompanied by traveling voltage waves, which satisfy the condition $v_r = +v_i$ and $v_t = 2v_i$. Therefore, the *voltage is doubled* by the reflection, Figure 2.6-7 (left), and significant overvoltages can occur in the insulating systems.

In the case of a **short circuit** ($Z_2 \ll Z_1$), there can be *no resulting voltage* at the reflection point, i.e. the voltages of the transmitted and the reflected waves are $v_t = 0$ and $v_r = -v_i$. The traveling voltage waves are accompanied by traveling current waves with $i_r = +i_i$ and $i_t = 2i_i$. Therefore, the *current is doubled* by the reflection, Figure 2.6-7 (middle).

If the line is terminated by an ohmic resistance Z_2 , which is equal to the characteristic impedance Z_L of the line, voltages and currents are not changed when a traveling wave transits from the line to the terminating resistance. The energy of the incident traveling wave is absorbed by the terminating resistance without any reflection. This case is called "**impedance matching**", Figure 2.6-7 (right).

In the **general case**, the reflection coefficients and the transmission (refraction) coefficients have to be determined from Eq. (2.6-14) and

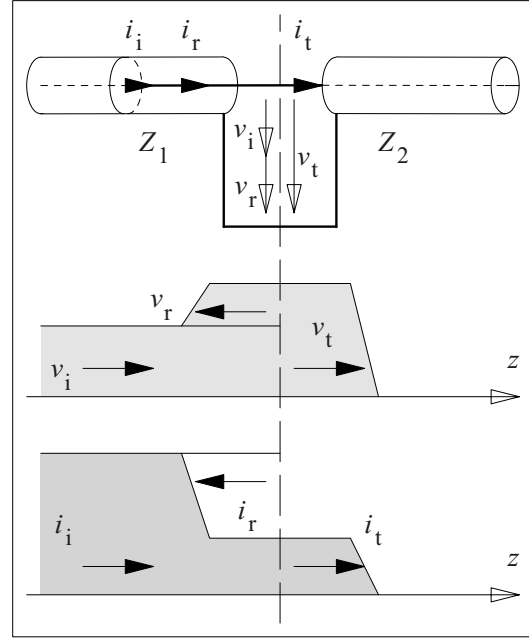


Figure 2.6-6: Reflection and transmission (refraction) of an incident traveling wave at a discontinuity of the characteristic line impedance.

(-15). By inserting $v_i = i_i Z_1$, $v_r = -i_r Z_1$ and $v_t = i_t Z_2$, Eq. (2.6-14) gives

$$i_i Z_1 - i_r Z_1 = i_t Z_2$$

and

$$i_i - i_r = i_t Z_2 / Z_1.$$

With Eq. (2.6-15) we derive

$$2 \cdot i_i = i_t (1 + Z_2 / Z_1).$$

Then, the *transmission (refraction) coefficient for the current* is

$$\rho_i = \frac{i_t}{i_i} = \frac{2 \cdot Z_1}{Z_1 + Z_2}. \quad (2.6-16)$$

With $v_t = i_t Z_2$ and $v_i = i_i Z_1$ the *transmission (refraction) coefficient for the voltage* is given by

$$\rho_v = \frac{v_t}{v_i} = \frac{2 \cdot Z_2}{Z_1 + Z_2}. \quad (2.6-17)$$

From these equations, the *reflection coefficients for current and voltage* are determined by insertion of Eq. (2.6-14) and (-15):

$$r_i = \frac{i_r}{i_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad (2.6-18a)$$

$$r_v = \frac{v_r}{v_i} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad (2.6-18b)$$

The relationship of transmission (refraction) and reflection coefficient is in general

$$R_{v,i} = \rho_{u,i} - 1. \quad (2.6-19)$$

Figure 2.6-7 presents the coefficients according to the Equations (2.6-16) to (-19) for some special cases.

2.6.2.2 Equivalent transmission-line circuit

According to Eq. (2.6-17), the voltage at a junction between two lines or at a line with termination resistance Z_2 is given by

$$v_t = 2 \cdot v_i \cdot Z_2 / (Z_1 + Z_2).$$

Obviously, the voltage v_t can be described by an equivalent circuit, the so-called **transmission-line circuit**, Figure 2.6-8. This means that the source voltage $2 \cdot v_i$ is divided downwards to v_t by a *voltage divider* consisting of

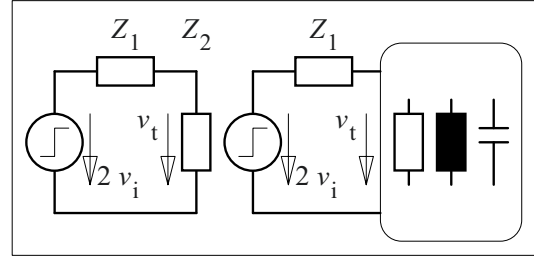


Figure 2.6-8: Transmission-line circuit with ohmic termination (left) and with a general R,L,C termination (right). The incident traveling and the line is described by the voltage source and the internal impedance.

the characteristic impedance Z_1 and the termination impedance Z_2 . The same circuit is derived, if line 1 is considered as a source with the *open-circuit voltage* $2 \cdot v_i$ and the *short-circuit current* $2 \cdot i_i$. This can be described by an *equivalent source* with the source voltage $2 \cdot v_i$ and the internal impedance $Z_i = (2 \cdot v_i) / (2 \cdot i_i) = Z_1$.

The significance of the *transmission-line circuit* is the ability to handle *any kind of line terminations* consisting of R, L, C-circuits [2]: By means of the time-dependent voltage $v_i(t, z_1)$ at the line discontinuity $z = z_1$, the time-depend-

Open circuit		Short circuit		Impedance matching	
<u>Reflection</u>		<u>Reflection</u>		<u>Reflection</u>	
$r_v = +1$		$r_v = -1$		$r_v = 0$	
$r_i = -1$		$r_i = +1$		$r_i = 0$	
<u>Transmission</u>		<u>Transmission</u>		<u>Transmission</u>	
$\rho_v = +2$		$\rho_v = 0$		$\rho_v = +1$	
$\rho_i = 0$		$\rho_i = +2$		$\rho_i = +1$	

Figure 2.6-7: Reflection and transmission (refraction) of an incident traveling wave at a discontinuity of the characteristic line impedance for the special cases of open circuit, short circuit and matching impedance.

ence of $v_t(t, z_1)$ is calculated. Then, the time-dependence of $v_r(t, z_1)$ is calculated from Eq. (2.6-14) as the difference of $v_t(t, z_1)$ and $v_i(t, z_1)$:

$$v_r(t, z_1) = v_t(t, z_1) - v_i(t, z_1) \quad (2.6-20)$$

The transmission-line circuit describes a *single reflection* only; it is *no* longer valid for multiple reflections.

Example: Reflection at a capacitance

A steep-fronted traveling wave with a step voltage amplitude V on a line with the characteristic impedance Z is reflected at a capacitance C , Figure 2.6-9.

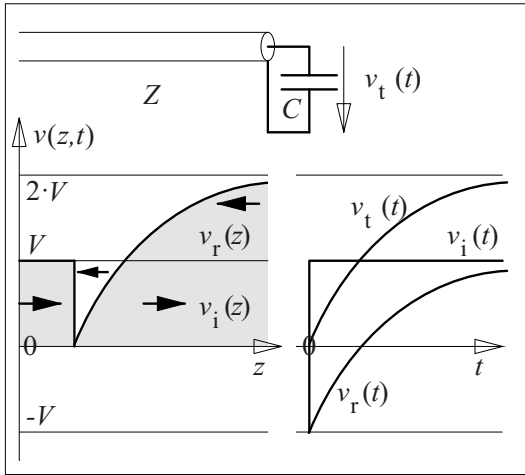


Figure 2.6-9: Reflection of a steep-fronted traveling wave at a capacitance.

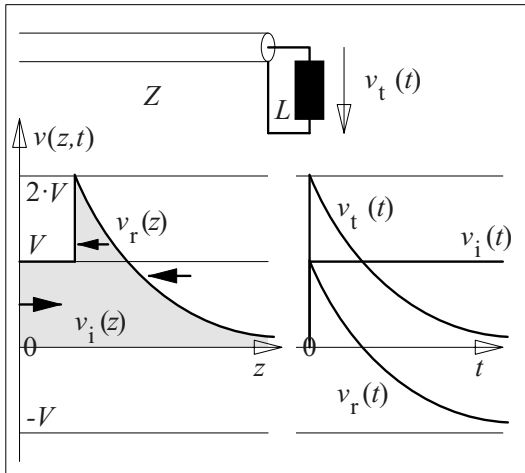


Figure 2.6-10: Reflection of a steep-fronted traveling wave at an inductance.

With respect to the transmission-line circuit, the voltage v_t increases exponentially from 0 to $2 \cdot V$ with the time constant ZC . According to Eq. (2.6-20), the voltage

$$v_r(t) = V \cdot \{2 \cdot [1 - e^{-t/(ZC)}] - 1\}$$

increases from $-V$ to $+V$. This means that the capacitive termination looks like a short circuit at first, i.e. while the capacitance is uncharged, and when the capacitance has been charged it looks like an open circuit. The reflected traveling wave is superimposed on the incident wave in such a way that the voltage in the wave front is compensated to zero at first and subsequently increases exponentially to $2 \cdot V$, Figure 2.6-9 (left).

Example: Reflection at an inductance

A steep-fronted wave with the step voltage amplitude V on a line with the characteristic impedance Z is reflected at an inductance L , Figure 2.6-10.

With respect to the transmission-line circuit, the voltage v_t decreases exponentially from $2 \cdot V$ to 0 with the time constant L/Z . According to Eq. (2.6-20), the voltage

$$v_r(t) = V \cdot \{2 \cdot e^{-t/(L/Z)} - 1\}$$

decreases from $+V$ to $-V$. This means that the inductive termination looks like an *open-circuit* at first, i.e. as long as there is no significant current through the inductance. If the current is increased the inductance looks like a *short circuit*. The reflected traveling wave is superimposed on the incident wave in such a way that the voltage in the wave front increases to $2 \cdot V$ at first and subsequently decreases to zero, Figure 2.6-10 (left).

2.6.2.3 Multiple Reflections

In systems with distributed parameters, normally there are not only single but also **multiple reflections**. This means that a reflected traveling wave can be reflected again and again at other discontinuities of the line and are superimposed on the original wave. Even with a few reflection points there are very complicated conditions for the spatiotemporal development of the resulting wave field.

Therefore, it is advisable to systematically visualize the propagation of the traveling waves by means of a “**traveling-wave road map**” (*Bewley lattice diagram* [39]). The propagation of the individual waves is depicted by so-called z, t propagation lines for every considered transmission line, Fig 2.6-11.

Wave components, which are fed in and which are reflected at the terminations, are depicted by their individual propagation lines. The magnitudes of the waves to be superimposed are calculated from the reflection and transmission coefficients.

The instantaneous value of the incident wave is regarded as the *incoming supply*. A clear description requires a *discretization* of the incident wave, i.e. only *selected wave points* are considered. Their propagation is traced by their individual propagation lines. In the case of a transition through a line discontinuity between line j and line k , the instantaneous value is multiplied by the transmission (re-

fraction) coefficient ρ_{jk} . In the case of a reflection on line j at the junction to line k , the instantaneous value is multiplied by the reflection coefficient r_{jk} .

Example: Transmission line between two cables

In Figure 2.6-11 numerical values are given for two cables 1 and 3 with $Z_1 = Z_3 = 40 \Omega$ and for an overhead transmission line 2 between them with $Z_2 = 360 \Omega$. The incident traveling wave with the amplitude V at $t = \tau$ was discretized in such a way that the propagation of four wave points with the magnitudes $v_i(t=0) = 0$, $v_i(t=\tau) = V$, $v_i(t=2\tau) = 0.5 \cdot V$ and $v_i(t=3\tau) = 0$ is traced by propagation lines.

The voltage magnitudes at the two discontinuities $z = z_a$

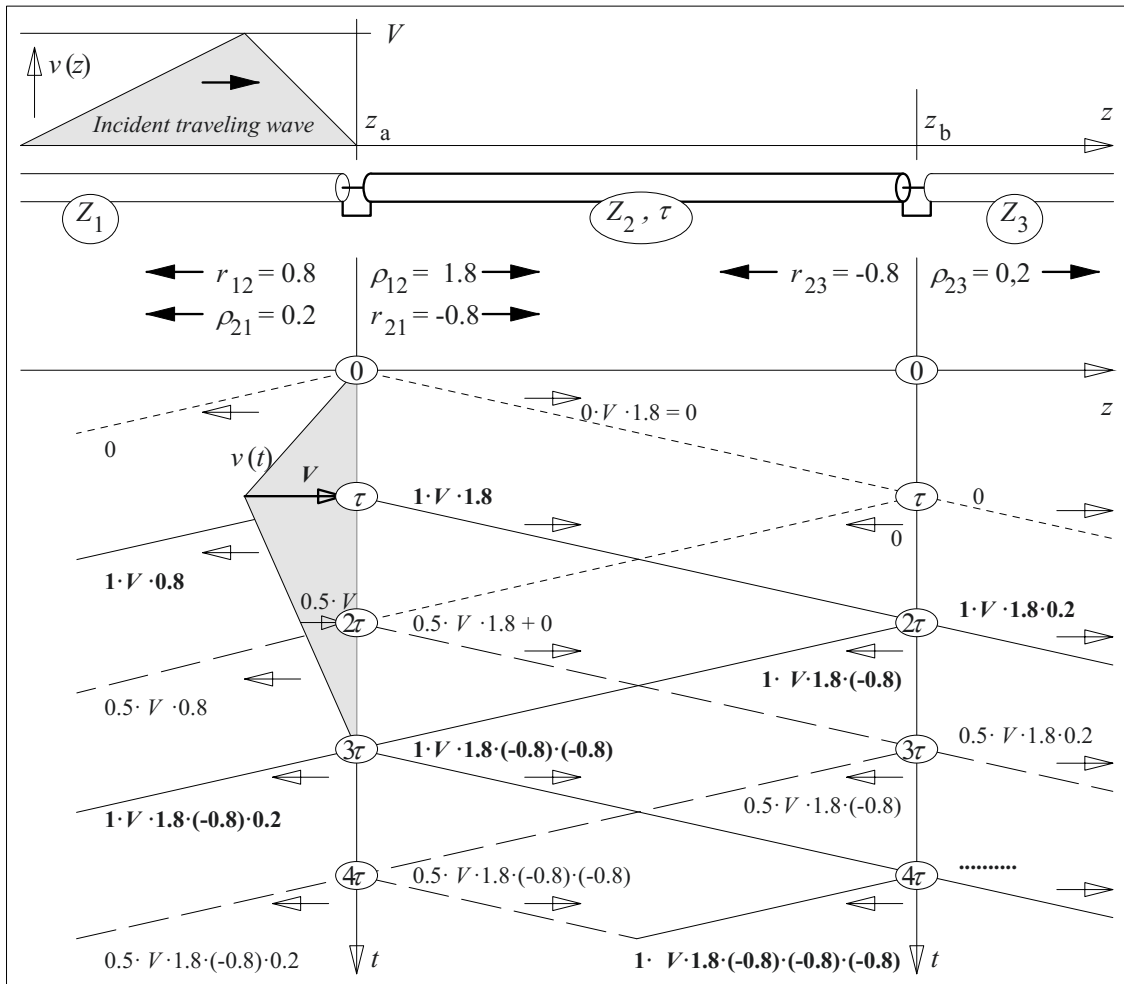


Figure 2.6-11: Description of the propagation, reflection and transmission (refraction) of traveling waves by means of a traveling-wave road map (Bewley lattice diagram) for an example.

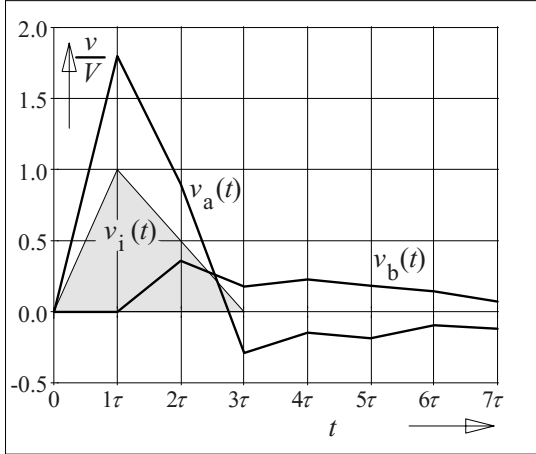


Figure 2.6-12: Voltage curves at the beginning (a) and at the end (b) of an overhead transmission line, determined with Bewley's lattice diagram, fig. 2.6-11.

and $z = z_b$ are given by transmission (refraction), reflection and superposition. They are determined by superposition of all back and forth traveling waves on one side of the discontinuity for the considered instant, Figure 2.6-12:

$T = 0$	$v(z_a) = 0$	$Vv(z_b) =$	$0 V$
τ	$1.800 V$		$0 V$
2τ	$0.900 V$		$0.360 V$
3τ	$-0.288 V$		$0.180 V$
4τ	$-0.144 V$		$0.230 V$
5τ	$-0.184 V$		$0.185 V$
6τ	$-0.092 V$		$0.147 V$
7τ	$-0.118 V$		$0.074 V$

At the *beginning* of the line 2 ($z = z_a$), the voltage $v_a(t)$ follows the voltage $v_i(t)$ of the incident traveling wave. Only after twice the propagation time for $t > 2\tau$, there are deviations because of reflections coming from the other end of line 2.

At the *end* of line 2 ($z = z_b$), the traveling wave does not arrive before the single propagation time $t = \tau$, and the voltage $v_b(t)$ follows the incident wave $v_i(t-\tau)$ with a time-shift τ for two further propagation times.

Note: This example shows that a traveling wave propagating from a cable to an overhead transmission line can cause *significant overvoltages* by reflections. This also applies to *fast transients in gas-insulated switchgear (GIS)* at the bushings, which are connected to the overhead lines. For a transition from a high characteristic impedance (overhead line) to a low characteristic im-

pedance (cable, GIS), the overvoltage is reduced by reflections.

The **Bergeron method** is another graphical method for the description of multiple reflections [39]: The voltages at the beginning (a) and at the end (b) of a transmission line are described by *resistance lines* in a v,i -diagram, Figure 2.6-13. The gradients are determined by the termination resistances R_1 and R_2 .

Furthermore, it can be shown by addition and subtraction of Eqs. (2.6-7) and (-9) that the propagation in the $+z$ and $-z$ -direction is equivalent to lines with different gradients. For a given wave point with a constant argument, we find for the propagation

$$\text{in } +z\text{-direction} \quad 2 \cdot f(z-ut) = v + i \cdot Z = \text{const.}$$

and

$$\text{in } -z\text{-direction} \quad 2 \cdot g(z+ut) = v - i \cdot Z = \text{const.}$$

(2.6-21)

The propagation of the traveling wave from one end of the line to the other is equivalent to

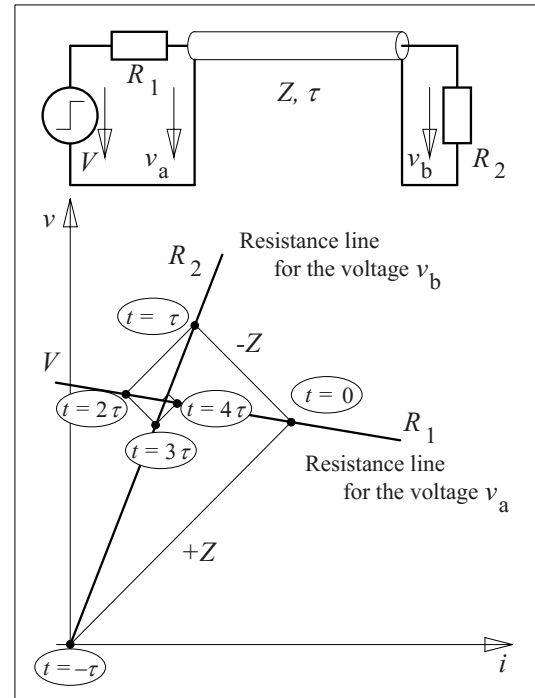


Figure 2.6-13: Description of the traveling wave propagation with Bergeron's method.

the transition from one resistance line to the other along the lines according to Eq. (2.6-21), i.e. along the “*Bergeron lines*” (thin lines in Figure 2.6-13). The gradient of the Bergeron line is $dv/di = Z$ or $dv/di = -Z$. It is advisable to choose the scales for v and i in such a way that the Bergeron lines lie at an angle of 45° to the axes and are thus mutually perpendicular. We start at a time instant $t = -\tau$ at the line end (b) with the voltage $v_b = 0$ and we reach the start of the line (a) at the time instant $t = 0$ with the starting voltage v_a , which is caused by the voltage step V . The voltages for multiples of the propagation time τ are found on the respective resistance line.

Graphical methods are often unsuitable for solving complex traveling-wave problems. In particular, problems with damped lines, non-ohmic terminations, frequency dependences and non-linearities can only be solved with *network-analysis programs*. Thereby, a number of electrically short equivalent line-elements can approximate long lines according to Figure 2.6-2. Another possibility is approximation with controlled voltage sources with delayed voltages [40].

2.6.3 Examples

Traveling-wave phenomena play a role in many high voltage applications. In the following, three examples are discussed, the disconnector circuits in gas-insulated switchgear (Section 2.6.3.1), the protection zone of a lightning arrester (Section 2.6.3.2) and impulse generation by traveling-wave generators (Section 2.6.3.3).

2.6.3.1 Gas-Insulated Switchgear (“Fast Transients”)

If a *disconnecter* between a de-energized overhead line and an energized busbar in a **gas-insulated switchgear** (GIS) is closed, there is a breakdown of the remaining clearance between the approaching contacts shortly before the contacts touch each other, Figure 2.6-14. Then, a very fast rising traveling wave is propagating on the coaxial line of the switchgear bay (1), and it is reflected at the bushing capacitance (4). These waves are the so-called *fast transients*, already mentioned in Section 2.2.5. They can propagate in the coaxial lines of a GIS with very low damping.

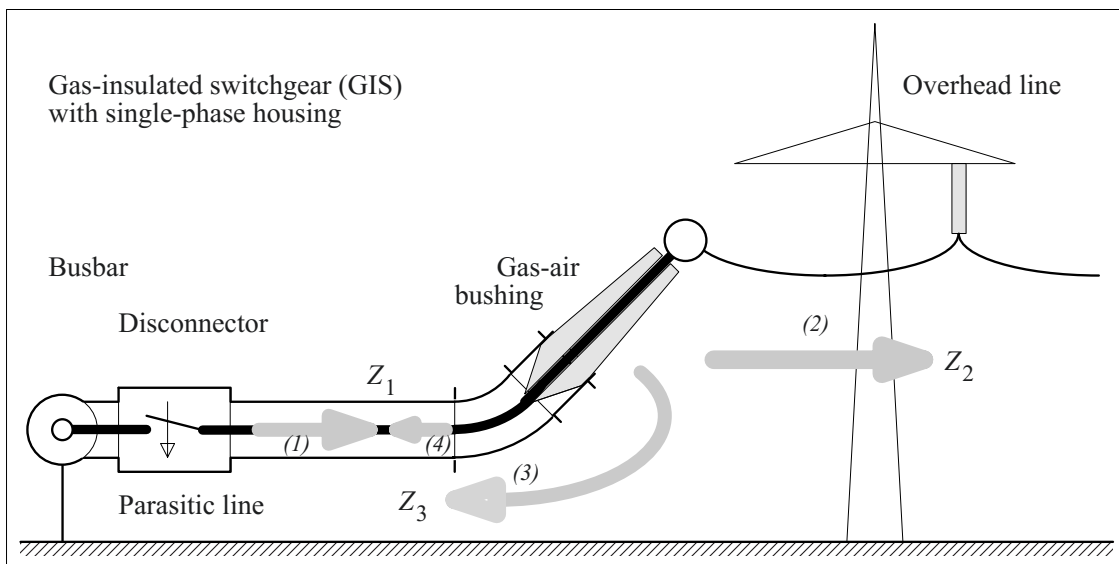


Figure 2.6-14: Generation of a traveling wave 1 by connection of a de-energized overhead line to an energized busbar. Wave 1 is reflected at the gas-air bushing (wave 4) and transmitted or refracted resp. (waves 2 and 3). The transmitted (refracted) waves propagate along the overhead line (wave 2) and along the parasitic line between GIS housing and conducting grounded structures of the building (wave 3).

The transmitted (refracted) wave is split into two waves that propagate along the overhead line and along a *parasitic line* between the enclosure of the GIS and the conducting parts of the GIS building (waves 2 and 3).

The amplitudes of the different traveling waves are determined by the characteristic (line) impedances Z_1 , Z_2 and Z_3 . Furthermore, the capacitance C of the bushing has to be considered in the first moment, because it must first be charged from the incident wave. See Figure 2.6-9.

According to the equivalent transmission-line circuit Figure 2.6-8, the voltage amplitude of the incident wave can be determined by two factors, i.e. by the *voltage difference* between the energized and de-energized lines at the instant of the clearance breakdown between the approaching contacts and by the characteristic (line) impedances of the adjacent lines on both sides of the disconnecter. Because of the reflection at the relatively high characteristic impedance of the overhead line (Z_2), a significant *voltage enhancement* can occur, which stresses the insulation of the bushing, the switchgear and the overhead line.

The traveling wave (3) occurring between the GIS enclosure and other conducting structures is especially critical. Because of the low characteristic (line) impedance Z_3 , the voltage amplitude is low. Nevertheless, this wave can cause significant damage in insufficiently protected *secondary equipment* (measurement systems, control equipment etc.) [41]. For instance, a momentary rise of the enclosure potential above ground potential can cause *back flashovers* into low-voltage circuits, e.g. into information technology circuits. Generally, rapidly changing electric and magnetic fields leaving the GIS can cause strong interference with neighboring circuits and systems. Therefore, ensuring electromagnetic compatibility (EMC) is of particular value in system design in order to avoid malfunction and damage.

Note: During the *closing of the disconnecter*, the described breakdown with the subsequent

transients is not the only process. After the equalization of the potential on both sides of the disconnecter, there is no longer any current and the discharge expires. Because of the sinusoidal voltage on the busbar, a voltage difference is re-established and the insulating gap between the still moving contacts breaks down again. Furthermore, a *higher number of re-ignitions* occur until the contacts are fully closed. Very steep voltage and current gradients are generated thereby.

Similar processes occur if the *disconnecter is opened*. With increasing distance of the contacts, the breakdown voltage increases and results in increasing amplitudes of the traveling voltage waves. The voltage enhancements caused by re-ignitions are superimposed onto voltage enhancements caused by slow switching-transients.

Note: In *large-scale gas-insulated switchgear*, reflection processes occur which are difficult to understand and yet still depend on the switching state of the substation. Insulation stresses caused by fast transients are often determined by measurements and by complex numerical simulations. For example, the direct connection of *transformers* to a gas-insulated switchgear requires a very careful analysis of the transients: Because of the high characteristic impedances of *transformer windings*, high voltage enhancements caused by reflections can be expected. Additionally, particularly in bigger installations, voltage enhancements can be caused by resonances and slower transients.

Note: Fast transients can cause stresses in insulation regions, which are without any stress in a static or quasi-static case. An example is given by a *bushing core*, Figure 2.6-15. At first, the incident wave is split up in relation to the characteristic impedances of the coaxial lines, which are formed by the concentric grading layers. Therefore, waves can also propagate in *parasitic lines* between the grounded flange and the outermost grading-foil and between the high-voltage conductor and the innermost grading-foil.

2.6.3.2 Protection Zone of a Lightning Arrester

Lightning arresters are non-linear components (resistors), which *limit overvoltages* and which carry a very small leakage current only at operating voltage. Principles and designs are discussed in Section 6.1.4.3.

For a *metal-oxide arrester*, the current increases very strongly if the voltage exceeds the rated voltage V_r , Figure 2.6-16. During a lightning impulse stress, the voltage-current characteristic of the arrester and the lightning current (which can be calculated in an equivalent transmission-line circuit according to Figure 2.6-8) determine the value of the so-called *residual voltage* V_{res} , which defines the lightning impulse **protection level** V_{pl} .

Note: For a lightning arrester consisting of a non-linear resistor *in series with a spark gap*, the protection level is defined by the *spark-over voltage* of the gap.

An arrester in the course of an overhead transmission line is now discussed, its location is point 1, Figure 2.6-17 (top). As long as the amplitude of the *incident traveling wave* is below the protection level V_{pl} , it is assumed for simplicity that the arrester remains highly resistive and that there are no reflections, Figure 2.6-17 (middle). If the amplitude of the traveling voltage wave exceeds the protection level, the arrester becomes low-resistive; there are *reflected and transmitted wave components* that reduce the voltage amplitudes before and

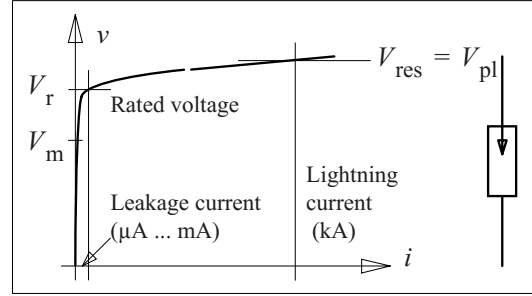


Figure 2.6-16: Ideal v,i -characteristic of a metal-oxide lightning arrester.

after the arrester, Figure 2.6-17 (bottom). The resulting voltage distribution is depicted for two different time points by bold lines.

The voltage drop Δv at the arrester causes two traveling waves with amplitudes $-\Delta v$, which are propagating in the opposite $+z$ and $-z$ directions. In the propagation direction of the incident wave ($+z$ direction), the voltage is limited to the protection level V_{pl} throughout the line. Furthermore, in front of the arrester there is a so-called **protection zone** L_p , where a given maximum voltage V_{max} is not exceeded. It can be seen from the two time points depicted in Figure 2.6-17 that the voltage limit V_{max} at point 2 is valid for any time point. This means that the increasing voltage of the incident wave is always compensated by the increasing voltage of the reflected wave within the protection zone L_p .

The incident voltage wave (highlighted in grey) is drawn for a time point at which the

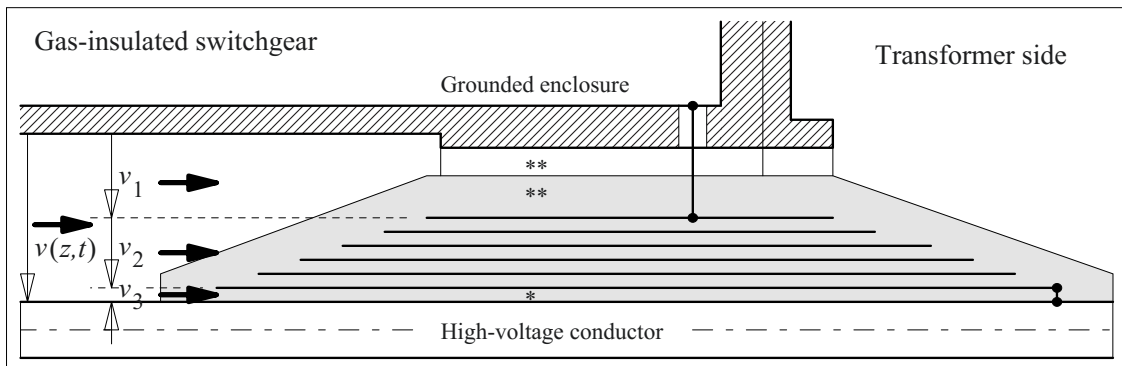


Figure 2.6-15: Transient field stresses in a bushing core above and beneath the grading layers at ground and high-voltage potential, i.e. in regions, which are without any field stress in a static or quasi-static case, (*) and (**).

permitted voltage $V_{\max}$ is reached at point 2. After that time point, the reflected wave limits the voltage magnitude to $V_{\max}$. The length of the protection zone L_p shall be derived from Figure 2.6-17. The (spatial) gradient of the wave front is

$$\begin{aligned}\Delta v/L_p &= \partial v/\partial z &= (\partial v/\partial t) \cdot (\partial z/\partial t)^{-1} \\ & &= (\partial v/\partial t) \cdot u^{-1}.\end{aligned}$$

With $2 \cdot \Delta v = V_{\max} - V_{pl}$ the protection zone is

$$L_p = \frac{1}{2} \cdot (V_{\max} - V_{pl}) \cdot u / (\partial v/\partial t). \quad (2.6-22)$$

Numerical example: A lightning arrester with $V_{pl} = 150$ kV shall limit a traveling wave rising on a three-phase line with a front gradient $\partial v/\partial t = 500$ kV/ μ s, so that, within the protection zone L_p , only 80 % of the lightning impulse voltage for the 123 kV voltage level shall be reached (this means that $V_{\max} = 0.8$

$\cdot 550$ kV = 440 kV). The phase velocity is $v = 300$ m/ μ s. According to Eq. (2.6-22) the relevant protection zone is $L_p = 87$ m.

Note: For the protection zone of a lightning arrester also

$$L_p/m \approx V_m/kV \quad (2.6-23)$$

is given as a rough guide in [22]. V_m is the maximum voltage for equipment (Section 6.1.4). More accurate calculation methods, which consider statistical error rates and acceptable error probabilities, normally give shorter protection zones [124].

Note: The calculation of a protection zone according to Eq. (2.6-22) is also valid for arrangements with an open-circuit or a high impedance at the end of the line (e.g. a *transformer winding*) [39]. The distance between arrester and line end or termination must not exceed L_p . The lightning arrester can be at the end of the line.

2.6.3.3 Traveling-Wave Generators (Transmission-Line Generators)

According to the principle of a so-called **cable generator**, the capacitively stored energy on a *charged transmission line* is converted by discharging into a very fast rising impulse in a matching impedance, Figure 2.6-18.

After the breakdown of the *switching spark gap*, a traveling wave with a voltage amplitude $V/2$ is propagated on the output line and is absorbed in a load impedance $R = Z$, which is matched with the characteristic impedance of the line. On the charged line (charging voltage V), a traveling wave with the voltage amplitude $-V/2$ is traveling in the $-z$ direction. After the reflection at the open-circuit line end, the reflected wave propagates with $-V/2$ in $+z$ direction and discharges the line completely. A *square-wave pulse* is thereby generated in the load with a voltage $V/2$ and a half-value width $t_H = 2 \cdot \tau_L$, which corresponds to twice the propagation time on the charged line.

Note: In practice, the (parasitic) inductance of the switching spark gap reduces the output voltage gradient $\partial v/\partial t$. Furthermore, mismatches and line damping cause impulse distortions.

Another principle is the discharging of two parallel lines in the so-called **Blumlein gen-**

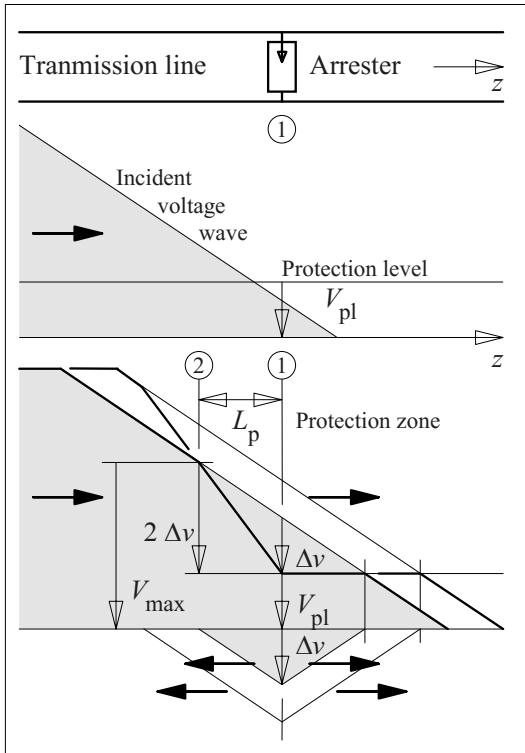


Figure 2.6-17: Protection zone of a lightning arrester with interference (i.e. compensation) of oppositely traveling voltage waves after the ignition of the lightning arrester (bottom).

erator, Figure 2.6-19. Both lines with the characteristic impedance Z are connected to their high-voltage conductors. The load $R = 2Z$ is connected to the two grounded conductors via an output line with the characteristic impedance $2Z$.

After the charging of the lines to the voltage V , the load is without any voltage, Figure 2.6-19 (top). After the ignition of the switching spark gap, the upper line is discharged by a traveling wave with the amplitude $-V$, see no. 1 in Figure 2.6-19 (middle). At the output end of the line, the characteristic impedance changes from Z to $2Z + Z = 3Z$. The reflection and transmission coefficients according to Eq. (2.6-19) and (-17) are $r_v = 1/2$ and $\rho_v = 3/2$. This means that the reflected wave travels backwards with the amplitude $-V/2$, see no. 2. The transmitted (refracted) wave with the am-

plitude $-3V/2$ is divided in the ratio of the characteristic impedances onto the output line to the load ($-V$) and onto the lower pulse-forming line ($-V/2$). The counting direction for the associated voltages is shown by reference arrows in the figure. At the load $R = 2Z$, which is matched with the output line, there is a voltage step $v_R(t) = V$ after the wave-front arrives.

The backwards traveling waves on the lines are reflected at the short-circuited spark gap (SC) on the upper line and at the open-circuit (OC) on the lower line, with and without polarity reversal, see no. 3. The reflected waves, which are transmitted into the output line, have the *same* polarity and (analogously to refraction no. 2) the amplitude $V/2$ both, see no. 4. Therefore, the field of the first wave, which was transmitted to the load, is completely compensated with a delay of $2 \cdot \tau_L$. The voltage

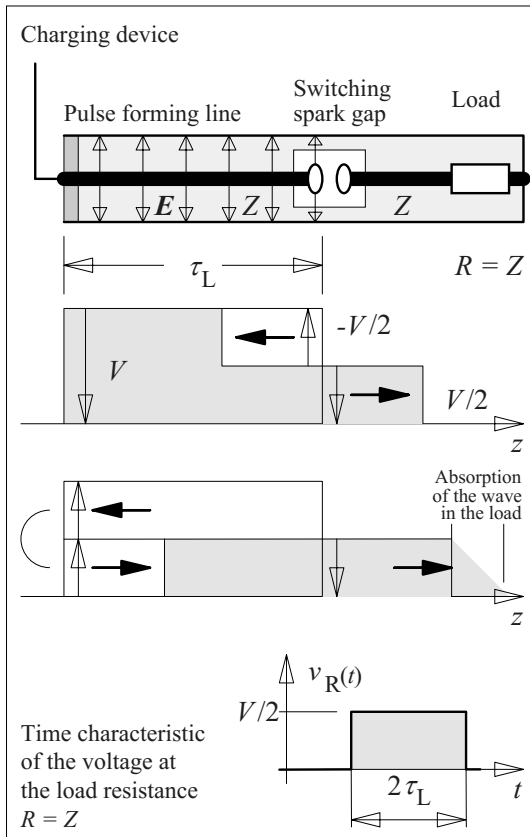


Figure 2.6-18: Generation of square-wave pulses by discharging of a pulse-forming line (traveling-wave generator).

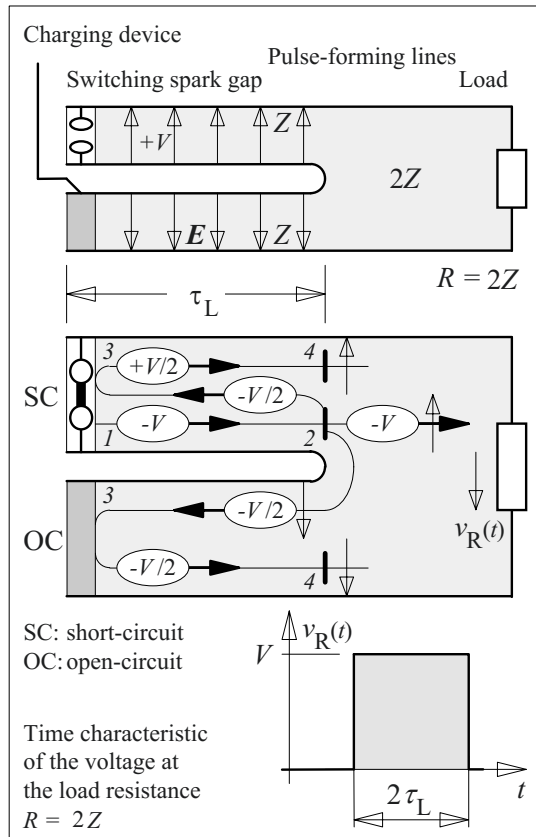


Figure 2.6-19: Generation of square-wave pulses by discharging of parallel pulse-forming lines (Blumlein generator).

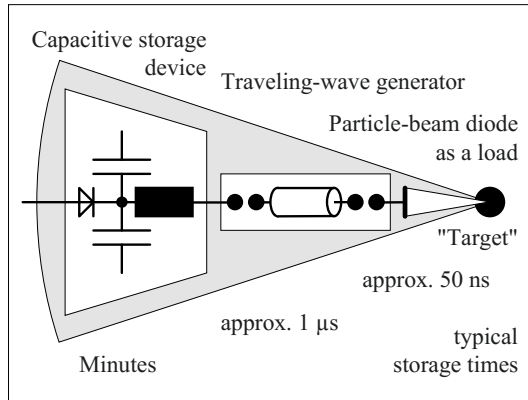


Figure 2.6-20: Module of a pulsed-power generator with spatial and temporal compression of the stored energy (schematic).

at the load decreases from V to zero. Further waves, which travel backwards into the line, compensate each other.

An important **application** of transmission-line generators is the generation of square-wave impulses for *step-response measurements* on measuring systems. Cable generators are mainly used for this purpose.

Another application is the **pulsed power technology** for the *spatial and temporal compression* of electromagnetic energy in an extremely powerful impulse [42]. For the generation of *extreme energy densities*, a number of generators are *circularly arranged around the target* as modules that are triggered simultaneously [14]. The traveling-wave generator arrangement can be a driver circuit for the acceleration of particles in basic research applications in physics, Figure 2.6-20. For example, matter could be brought into extreme conditions in order to ignite nuclear fusion reactions.

The principles of the *cable generator* or the *Blumlein generator* are chosen depending on voltage and load impedance. The generators are designed as coaxial lines or as parallel-plate lines [15]. Voltage enhancements can be achieved by multiple reflections at additional switching spark gaps, i.e. by so-called *double-bounce switching* [43]. *Water* is used as an insulating medium because of its very high relative permittivity $\epsilon_r = 81$ and because of its high dielectric breakdown strength. Thereby

high amounts of energy can be stored for short times. Furthermore, the phase velocity is reduced to $u = u_0/9 = 3 \text{ cm/ns}$ and the length of a line can be reduced by a factor of 9 in comparison with air, see Eq. (2.6-8). Because of the comparatively high conductivity of water, the energy can only be stored for a short time in a range of microseconds (μs). Therefore, it is necessary to charge the water-insulated line very rapidly from a conventional capacitor bank with approximately the same capacitance (see impulse generators, Section 6.2.3). The capacitor bank can store the energy for longer times, and rapid charge transfer is performed by oscillation. At the voltage maximum, the switching spark gap is ignited before significant *self-discharging* of the water-insulated capacitance can occur, Figure 2.6-20. The synchronous triggering of the switching spark gaps for the *parallel operation* of a number of modules places extreme demands on the triggering devices.

Example: Water-insulated impulse generator

A water-insulated traveling-wave generator according to Figure 2.6-18 shall be designed with coaxial lines for the generation of impulses with energies as high as possible. The peak-value of the voltage shall be $\hat{V} = 500 \text{ kV}$ and the half-value width $t_H = 50 \text{ ns}$. The maximum permissible field strength in water is $\hat{E}_{\text{max}} = 100 \text{ kV/cm}$.

According to Eq. (2.3-24) the maximum field energy of a coaxial line is given for $R_2/R_1 = e^{0.5} = 1.65$. With the charging voltage $V = 2\hat{V} = 1 \text{ MV}$ the radii $R_1 = 20 \text{ cm}$ and $R_2 = 33 \text{ cm}$ are calculated from Eq. (2.3-22). The length of the line is determined by the propagation time $\tau_L = t_H/2 = 25 \text{ ns}$ with $L = \tau_L \cdot v_0/\epsilon_r^{0.5} = 83 \text{ cm}$.

From the Equations in Figure 2.6-5, the capacitance $C = 7.5 \text{ nF}$ and the characteristic impedance $Z = 3.3 \Omega$ is derived. The current amplitude of the output impulse is $\hat{I} = \hat{V}/Z = 150 \text{ kA}$ and the power is $P = 75 \text{ GW}$.

The capacitively stored energy $W = \frac{1}{2} C \cdot V^2 = 3,75 \text{ kJ}$ is ideally completely transferred into impulse energy $W = \hat{I} \cdot \hat{I} \cdot t_H = 3,75 \text{ kJ}$. In practice, losses must of course be considered.

Other impulse circuits and many **applications** of **high-voltage impulse-technologies** are described in Section 7.3.2 and 7.4.2 [482].

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