Chapter 2
Precision of a Non-driven Mechanical Gyroscope with Negative Velocity Feedback

2.1 Measurement Precision of a Constant Angular Velocity Rotating Around The Horizontal Axis

The output signal is connected with the angular velocity by the scale factor: $U_{\text{out,amplitude}} = K_{\text{SF}}\Omega$. So the study of the measurement precision of a constant angular velocity rotating around the horizontal axis of the aircraft is ultimately attributed to the stability of the calibration scale factor of the instrument. In this chapter, the precision of the gyroscope is studied based on the dynamic characteristics of the instrument. Therefore, the closed-loop transfer function expressions (1–143), (1–146) and (1–147) are used, and nT_{amplify} replaces T_{amplify}:

$$\Phi_0(s) = \frac{K}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{s}{T_0^2 nT_{\text{amplify}}s^3 + \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)s^2 + \left(nT_{\text{amplify}} + 2\xi_0 T_0 + K\right)s + 1} \tag{2.1}$$

$$\Phi_1(s) = \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{1}{1 + T_{\text{integrator}}s} \times \frac{s}{T_0^2 nT_{\text{amplify}}s^3 + \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)s^2 + \left(nT_{\text{amplify}} + 2\xi_0 T_0 + K\right)s + 1} \tag{2.2}$$

$$\Phi_2(s) = \frac{KK_{\text{integrator2}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{T_1 s + 1}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1} \times \frac{s}{T_0^2 nT_{\text{amplify}}s^3 + \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)s^2 + \left(nT_{\text{amplify}} + 2\xi_0 T_0 + K\right)s + 1} \tag{2.3}$$

F. Zhang et al., *Non-driven Micromechanical Gyroscopes and Their Applications*,
https://doi.org/10.1007/978-3-662-54045-9_2

Because $U_{\text{out}}(s) = \Phi(s) M_{\text{inertia}}(s)$ and $M_{\text{inertia}}(s)$ changes by the harmonic law, U_{out} is determined as

$$U_{\text{out}} = \left|\Phi(j\omega)\right|_{\omega=\dot{\varphi}_0} H_0 \Omega \sin\left\{ \dot{\varphi}_0 t + \arg\left[\Phi(j\omega)_{\omega=\dot{\varphi}_0} \right] \right\} \tag{2.4}$$

$$\begin{cases} U_{\text{amplitude}} = H_0 \Omega \left|\Phi(j\omega)\right|_{\omega=\dot{\varphi}_0} = (C_1 - A_1 + B_1)\dot{\varphi}_0 \Omega \left|\Phi(j\omega)\right|_{\omega=\dot{\varphi}_0} \\ \chi = \arg \Phi(j\omega)_{\omega=\dot{\varphi}_0} \end{cases} \tag{2.5}$$

From Eqs. (2.1), (2.2) and (2.3) the amplitude of the output signal is

$$U_{\text{amplitude0}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{1}{\sqrt{\left\{ \left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi) \right] \dot{\varphi}_0 - T_0^2 n T_{\text{amplify}} \dot{\varphi}_0^3 \right\}^2 + \left[1 - \left(T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}} \right) \dot{\varphi}_0^2 \right]^2}} \tag{2.6}$$

$$U_{\text{amplitude1}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{1}{\sqrt{1 + T_{\text{integrator}}^2 \dot{\varphi}_0^2}} \times \frac{1}{\sqrt{\left\{ \left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi) \right] \dot{\varphi}_0 - T_0^2 n T_{\text{amplify}} \dot{\varphi}_0^3 \right\}^2 + \left[1 - \left(T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}} \right) \dot{\varphi}_0^2 \right]^2}} \tag{2.7}$$

$$U_{\text{amplitude2}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K K_{\text{integrator2}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{\sqrt{1 + T_1^2 \dot{\varphi}_0^2}}{\sqrt{\left(1 - T_2^2 \dot{\varphi}_0^2\right)^2 + \left(2\xi_{\text{integrator}} T_2 \dot{\varphi}_0\right)^2}} \times \frac{1}{\sqrt{\left\{ \left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi) \right] \dot{\varphi}_0 - T_0^2 n T_{\text{amplify}} \dot{\varphi}_0^3 \right\}^2 + \left[1 - \left(T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}} \right) \dot{\varphi}_0^2 \right]^2}} \tag{2.8}$$

When the integrator is ideal ($T_{\text{integrator}}$ is very large) and the ideal compensation of the influence of the mutual inductance ($n = 0$) is considered, because nT_{amplify} and ξ_0 are small and can therefore be ignored, from Eq. (2.7) we can obtain

$$U_{\text{amplitude1}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{1}{T_{\text{integrator}} \dot{\varphi}_0 \sqrt{\left(1 - T_0^2 \dot{\varphi}_0^2\right)^2 + \left(2\xi T_0 \dot{\varphi}_0\right)^2}} \tag{2.9}$$

When the instrument is operating at close to the resonant point, $T_0^2\dot{\varphi}_0^2 \approx 1$ and $C_1 - A_1 = B_1$.

$$\begin{aligned} U_{\text{amplitude1}} &= \frac{2B_1 \Omega K K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}} T_{\text{integrator}} 2\xi T_0} \\ &= \frac{2B_1 \Omega K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}} T_{\text{integrator}}} (\text{in the close to ideal condition}) \end{aligned} \tag{2.10}$$

In the close to ideal condition, the output signals of the instrument are determined only by the transfer coefficients of the torque winding of the torque converter, a current amplifier and the integrator. The measurement result of the stability of the above parameters is a percentage, and can satisfy the high precision requirement of the angular velocity measurement of the rotating flight carrier.

When the integrator with the transfer function (1–130) is used, then where this function is in the ideal integral condition, T_1 and T_2 are relatively large. Assume that $n = 0$ and $\xi_0 = 0$, there holds

$$\begin{aligned} U_{\text{amplitude2}} &= (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K K_{\text{integrator2}}}{K_{\text{moment}} K_{\text{electric current}}} \\ &\quad \times \frac{T_1 \dot{\varphi}_0}{T_2^2 \dot{\varphi}_0^2 \sqrt{\left(1 - T_0^2 \dot{\varphi}_0^2\right)^2 + (2\xi T_0 \dot{\varphi}_0)^2}} \end{aligned} \tag{2.11}$$

But in the case of the resonance, there holds

$$U_{\text{amplitude2}} = \frac{2B_1 K K_{\text{integrator2}} T_1}{K_{\text{moment}} K_{\text{electric current}} T_2^2 2\xi T_0} \Omega = \frac{2B_1 K_{\text{integrator2}} T_1}{K_{\text{moment}} K_{\text{electric current}} T_2^2} \Omega \tag{2.12}$$

When $\frac{T_2^2}{T_1} = T_{\text{integrator}}$, Eq. (2.12) corresponds to Eq. (2.10).

When there is no integrator and no assumption ($n = 0$, $\xi_0 = 0$,) at the output end of the instrument, from Eq. (2.6), we can obtain

$$\begin{aligned} U_{\text{amplitude0}} &= (C_1 - A_1 + B_1)\dot{\varphi}_0^2 \Omega \frac{K}{K_{\text{moment}} K_{\text{electric current}}} \\ &\quad \times \frac{1}{\sqrt{\left(1 - T_0^2 \dot{\varphi}_0^2\right)^2 + (2\xi T_0 \dot{\varphi}_0)^2}} \end{aligned} \tag{2.13}$$

In the case of resonance, there holds

$$U_{\text{amplitude0}} = \frac{2B_1 K \dot{\varphi}_0}{K_{\text{moment}} K_{\text{electric current}} 2\xi T_0} \Omega = \frac{2B_1 \dot{\varphi}_0}{K_{\text{moment}} K_{\text{electric current}}} \Omega \tag{2.14}$$

Comparing Eqs. (2.10), (2.12) and (2.14) shows that if there is no integrator at the output end of the instrument, the output signal of the instrument will be unstable. This is because there is a linear relationship between the stability of the output signal and the angular velocity rotating around the longitudinal axis of the aircraft. The integrator at the output end of the instrument, as shown in Eqs. (2.10) and (2.12) can eliminate the dependency between the output signal and the rotational angular velocity around the longitudinal axis of the aircraft. Therefore, the integrator can significantly improve the stability of the output signal.

As has been discussed before, in the case of non-precise tuning, for example in the case of $\mu = 0.9$, there is instability between the amplitude of the output signal and the angular velocity rotating around the longitudinal axis of the aircraft. When T_{damping} is smaller, for these loops, $\xi = \frac{D_{\text{feedback}}}{2\dot{\varphi}_0 \mu B_1}$ can be substituted into Eq. (2.11) and $\dot{\varphi}_0$ is replaced by $\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0$.

$$\begin{aligned} U_{\text{amplitude2}} &= \frac{(\mu^2+1) D_{\text{feedback}} K_{\text{integrator2}} T_1 \Omega}{\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) K_{\text{moment}} K_{\text{electric current}} T_2^2 \sqrt{(\mu^2-1)^2 + \left[\frac{D_{\text{feedback}}}{\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) B_1}\right]^2}} \\ &= \frac{(\mu^2+1) B_1 D_{\text{feedback}} K_{\text{integrator2}} T_1 \Omega}{K_{\text{moment}} K_{\text{electric current}} T_2^2 \sqrt{\left[\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) B_1 (\mu^2-1)\right]^2 + D_{\text{feedback}}^2}} \end{aligned} \tag{2.15}$$

In order to obtain the general result, the effect of the instability of the angular velocity rotating around the longitudinal axis on the amplitude of the output signal is in the form of a relative value:

$$\frac{U_{\text{amplitude}}}{U_{\text{amplitude}}(\Delta\dot{\varphi}_0 = 0)} = \sqrt{\frac{\left[\dot{\varphi}_0^0 B_1 (\mu^2-1)\right]^2 + D_{\text{feedback}}^2}{\left[\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) B_1 (\mu^2-1)\right]^2 + D_{\text{feedback}}^2}} \tag{2.16}$$

The coefficient K is introduced to denote the ratio of the unstable value of the angular velocity rotating around the longitudinal axis of the aircraft to the average value of this angular velocity, i.e., $\triangle\dot{\varphi}_0 = K\varphi_0^0$. In this case Eq. (2–16) has the following form:

$$\frac{U_{\text{amplitude}}}{U_{\text{amplitude}}(\Delta\dot{\varphi}_0 = 0)} = \sqrt{\frac{(\mu^2-1)^2 + \left(2\xi^0\mu\right)^2}{(1 \pm K)^2 (\mu^2-1)^2 + \left(2\xi^0\mu\right)^2}} \tag{2.17}$$

where $\xi^0 = \frac{D_{\text{feedback}}}{2\dot{\varphi}_0^0 \mu B_1}$ is the attenuation coefficient, which has nothing to do with the instability of the angular velocity rotating around the longitudinal axis of the aircraft.

The curve corresponding to Eq. (2.17) is shown in Fig. 2.1.

Obviously, in the case of the precise tuning the amplitude ratio in Eq. (2.17) is equal to 1 and has no relationship with $\Delta\dot{\varphi}_0$ which is confirmed by Fig. 2.1a. It is worth noting that the enhanced damping effect of the instrument will lead to the

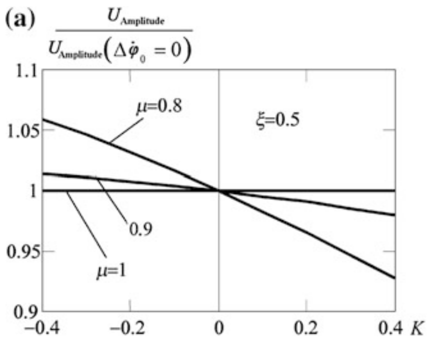

Fig. 2.1 Effect of instability of the angular velocity rotating around the longitudinal axis of the aircraft on the output signal amplitude of a non-driven mechanical gyroscope, **a** Variation of the detuning coefficient; **b** Variation of the damping coefficient

decrease of the dependency in Eq. (2.17), which is proved by Fig. 2.1b. When the angular velocity rotating around the longitudinal axis of the aircraft varies in the whole range (10–20 Hz) as well as in the case of $\mu = 0.9$ and $\xi = 0.5$, the relative error of the amplitude of the output signal value is 3.8%, that is, the ratio associated with the average amplitude is ±1.9%.

Because the instantaneous value of the output signal is used in the instrument, the phase of the output signal and the phase stability should be noted accurately.

Corresponding to Eq. (2.5) there holds

$$\chi_0 = \frac{\pi}{2} - \arctan\left(\frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2}\right) \tag{2.18}$$

$$\begin{aligned}\chi_1 = {} & \frac{\pi}{2} - \arctan\left(T_{\text{integrator}}\dot{\varphi}_0\right) \\ & - \arctan\left\{\frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2}\right\}\end{aligned} \tag{2.19}$$

$$\begin{aligned}\chi_2 = {} & \frac{\pi}{2} + \arctan\left(T_1\dot{\varphi}_0\right) - \arctan\left(\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1 - T_2^2\dot{\varphi}_0^2}\right) \\ & - \arctan\left\{\frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2}\right\}\end{aligned} \tag{2.20}$$

Under the above conditions, $n = 0$, $\xi_0 = 0$, T_1, T_2 and $T_{\text{integrator}}$ are very large.

$$\chi_0 = \frac{\pi}{2} - \arctan\left(\frac{2\xi T_0 \dot{\varphi}_0}{1 - T_0^2 \dot{\varphi}_0^2}\right) \tag{2.21}$$

$$\chi_1 = -\arctan\left(\frac{2\xi T_0 \dot{\varphi}_0}{1 - T_0^2 \dot{\varphi}_0^2}\right) \tag{2.22}$$

$$\chi_2 = -\arctan\left(\frac{2\xi T_0 \dot{\varphi}_0}{1 - T_0^2 \dot{\varphi}_0^2}\right) \tag{2.23}$$

It can be seen that under the condition of using the circuit with the different integrators, the phase shifts are equal and the phase difference between the circuit with the different integrators and the circuit without the integrators is 90°.

In order to determine the stability of these values, Eqs. (2.21), (2.22) and (2.23) have the following form:

$$\chi_0 = \Delta\chi_0$$

$$\chi_1 = -\frac{\pi}{2} - \Delta\chi_1$$

$$\chi_2 = -\frac{\pi}{2} - \Delta\chi_2$$

Because χ_1, χ_0 and χ_2 can be described by the same method, only χ_1 is studied and obtained as

$$\tan\chi_1 = \tan\left(-\Delta\chi_1 - \frac{\pi}{2}\right) = \cot\Delta\chi_1 = \frac{1}{\tan\Delta\chi_1}$$

$$\tan\Delta\chi_1 = \frac{1}{\tan\chi_1} = \frac{T_0^2 \dot{\varphi}_0^2 - 1}{2\xi T_0 \dot{\varphi}_0} \tag{2.24}$$

In this case, substituting Eqs. (1.124), (1.125) and (1.134) obtains

$$\tan\Delta\chi_1 = \frac{\frac{B_1}{C_1 - A_1} - 1}{\frac{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}{(C_1 - A_1)\dot{\varphi}_0}} = \frac{[B_1 - (C_1 - A_1)]\dot{\varphi}_0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} \tag{2.25}$$

and there holds

$$\chi_1 = -\frac{\pi}{2} - \arctan\left\{\frac{[B_1 - (C_1 - A_1)]\dot{\varphi}_0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}\right\} \tag{2.26}$$

Because the tuning factor $\mu < 1$, there holds $(C_1 - A_1) < B_1$ and the symbol term "arctan" in the expression is positive.

Equation (2.26) shows that because the signal phase does not exceed $-\frac{\pi}{2}$, the instrument works outside the resonance point. And because

$$\dot{\varphi}_0 = \dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0 \tag{2.27}$$

where $\dot{\varphi}_0^0$ is the average angular velocity rotating around the longitudinal axis of the aircraft (here it is 15 Hz) and $\Delta\dot{\varphi}_0$ is the unstable value of the angular velocity rotating around the longitudinal axis of the aircraft (here it is the maximum value $\Delta\dot{\varphi}_0$ = 5 Hz).

Combining this with Eq. (2.26), it is known that the phase shift of the instrument is unstable because it has a dependent relationship with the unstable angular velocity rotating around the longitudinal axis of the aircraft.

Substituting Eq. (2.27) into Eq. (2.26) obtains

$$\chi_1 = -\frac{\pi}{2} - \arctan\left\{\frac{[B_1 - (C_1 - A_1)]\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}\right\} \tag{2.28}$$

Because the value of the expression "arctan" is small, there holds

$$\chi_1 = -\frac{\pi}{2} - \arctan\left\{\frac{[B_1 - (C_1 - A_1)]\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}\right\} = \chi_{10} \pm \Delta\chi_{10} \tag{2.29}$$

where

$$\chi_{10} = -\frac{\pi}{2} - \frac{[B_1 - (C_1 - A_1)]\dot{\varphi}_0^0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} \tag{2.30}$$

When the instrument is mounted to the machine body, the angle is compensated by the method of rotating the instrument or when the output signal of the instrument is furtherly processed, the angle is compensated in the aircraft. χ_{10} is a constant and does not have a dependent relationship with the unstable value $\Delta\dot{\varphi}_0$.

$\Delta\chi_{10}$ is caused by the instability of the angular velocity rotating around the longitudinal axis of the aircraft and is the unstable phase value of the output signal of the instrument. It should be noted that $\Delta\dot{\varphi}_0$ is quite large, so the phase error is relatively large.

$$\Delta\chi_{10} = \frac{[B_1 - (C_1 - A_1)]\Delta\dot{\varphi}_0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} \tag{2.31}$$

Similar to Eq. (2.29), for the circuit without the integrators and the circuit with the integrators, the expressions of the output signal phase are obtained as follows:

$$\chi_0 = \frac{\pi}{2} - \arctan\left\{\frac{[B_1 - (C_1 - A_1)]\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}\right\} = \chi_{00} \pm \Delta\chi_{00} \tag{2.32}$$

$$\chi_2 = -\frac{\pi}{2} - \arctan\left\{\frac{[B_1 - (C_1 - A_1)]\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}\right\} = \chi_{20} \pm \Delta\chi_{20} \tag{2.33}$$

We will estimate how the unstable phase value of the output signal affects its instantaneous value. As the output signal of the instrument is in the form of

$$U_{\text{out}} = U_{\text{amplitude}} \sin\left(\dot{\varphi}_0 t + \chi\right)$$

then

$$U_{\text{out}} = U_{\text{amplitude}} \sin\left\{(\dot{\varphi}_0 \pm \Delta\dot{\varphi}_0)t \mp \frac{[B_1 - (C_1 - A_1)]\Delta\dot{\varphi}_0}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}\right\} \tag{2.34}$$

The angle χ_0 in Eq. (2.30) is considered as a compensation in Eq. (2.34). $(\dot{\varphi}_0 \pm \Delta\dot{\varphi}_0)t$ will no longer calculate the unstable value of the angular velocity rotating around the longitudinal axis of the rotating flight carrier. Therefore, as this unstable value affects the steering engine of the rotating flight carrier and does not affect the processing precision of the input signal, then

$$U_{\text{out}} = U_{\text{amplitude}} \sin\left\{\dot{\varphi}_0 t \mp \frac{[B_1 - (C_1 - A_1)]\Delta\dot{\varphi}_0}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}\right\} \tag{2.35}$$

When the output signal is the maximum or minimum value, the sensitive axis of the instrument is coincident with the vector direction of the measured angular velocity. Thus

$$\dot{\varphi}_0 t = \pm\frac{\pi}{2} \pm \pi k$$

The error of the instrument is small because Eq. (2.35) has the following form:

$$U_{\text{out}} = \pm U_{\text{amplitude}} \cos\left\{\frac{[B_1 - (C_1 - A_1)]\Delta\dot{\varphi}_0}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}\right\} \tag{2.36}$$

Equation (2.36) shows that in the case of resonance, $\Delta\chi_{10} = 0$ in Eq. (2.31), then $U_{\text{out}} = \pm U_{\text{amplitude}}$, where the positive and negative symbols will determine the maximum or minimum value of the signal.

When the instrument is not in the resonant state but it is directly close to the resonant state, then because $\triangle\chi_{10}$ is determined and is small, the unstable value of the angular velocity rotating around the longitudinal axis of the aircraft will not significantly affect the maximum or minimum value of the instrument's output signal.

In this case, the output of the instrument is zero, which indicates that the instrument's sensitive axis is perpendicular to the vector direction of the measured angular velocity. Thus

$$\dot{\varphi}_0 t = 0 \pm \pi k$$

The form of the instrument's output signal is

$$U_{\text{out}} = \mp U_{\text{amplitude}} \sin\left\{\frac{[B_1 - (C_1 - A_1)]\Delta\dot{\varphi}_0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}\right\} \tag{2.37}$$

In this case, because the output signal is proportional to the sinusoidal value of $\Delta\chi_{10}$, its image is different. In the case of resonance, the output signal should be equal to zero but in the case of imprecise adjustment of the resonance because $\Delta\chi_{10}$ is very small, the output signal will be

$$U_{\text{out}} = \mp U_{\text{amplitude}} \left\{\frac{[B_1 - (C_1 - A_1)]\triangle\dot{\varphi}_0}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}}\right\} \tag{2.38}$$

Thus, in the above moment the signal error is, in essence, the cross error of the instrument and is proportional to the unstable value of the angular velocity rotating around the longitudinal axis of the aircraft. It should be pointed out that the size of $\Delta\chi_{10}$ can be up to a few degrees.

Under the condition that the integral time constant of the integral loop is not large enough, the influence degree of the integral loop at the output signal end on the phase stability of the output signal is measured. In Eqs. (2.19) and (2.20), assume that $n = 0$ and $\xi_0 = 0$, then

$$\chi_1 = \frac{\pi}{2} - \arctan\left(T_{\text{integrator}}\dot{\varphi}_0\right) - \arctan\left(\frac{2T_0\xi\dot{\varphi}_0}{1 - T_0^2\dot{\varphi}_0^2}\right) \tag{2.39}$$

$$\chi_2 = \frac{\pi}{2} + \arctan\left(T_1\dot{\varphi}_0\right) - \arctan\left(\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1 - T_2^2\dot{\varphi}_0^2}\right) - \arctan\left(\frac{2T_0\xi\dot{\varphi}_0}{1 - T_0^2\dot{\varphi}_0^2}\right) \tag{2.40}$$

According to the above calculations and obtains

$$
\begin{aligned}
\chi_1 &= -\frac{\pi}{2} + \Delta\chi_{\text{integrator1}} - \Delta\chi_1 \\
\chi_2 &= -\frac{\pi}{2} - \Delta\chi_{\text{integrator21}} + \Delta\chi_{\text{integrator22}} - \Delta\chi_2 \\
\tan\chi_1 &= \tan\left(-\Delta\chi_1 + \Delta\chi_{\text{integrator1}} - \frac{\pi}{2}\right) = \cot\left(\Delta\chi_1 - \Delta\chi_{\text{integrator1}}\right) \\
&= \frac{1}{\tan\left(\Delta\chi_1 - \Delta\chi_{\text{integrator1}}\right)} \\
\tan\chi_2 &= \tan\left(-\Delta\chi_2 - \Delta\chi_{\text{integrator21}} + \Delta\chi_{\text{integrator 22}} - \frac{\pi}{2}\right) \\
&= \frac{1}{\tan\left(\Delta\chi_2 + \Delta\chi_{\text{integrator21}} - \Delta\chi_{\text{integrator22}}\right)} \\
\tan\left(\Delta\chi_1 - \Delta\chi_{\text{integrator1}}\right) &= \frac{1}{\tan\chi_1} = \tan\left[\arctan\left(T_{\text{integrator}}\dot{\varphi}_0\right) + \arctan\left(\frac{2T_0\xi\dot{\varphi}_0}{1 - T_0^2\dot{\varphi}_0^2}\right)\right] \\
&= \frac{T_{\text{integrator}}\dot{\varphi}_0 - T_{\text{integrator}}T_0^2\dot{\varphi}_0^3 + 2T_0\xi\dot{\varphi}_0}{1 - T_0^2\dot{\varphi}_0^2 - 2T_{\text{integrator}}T_0\xi\dot{\varphi}_0^2} \\
&= \frac{T_{\text{integrator}}\dot{\varphi}_0^2(C_1 - A_1 - B_1) + K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}{\dot{\varphi}_0(C_1 - A_1 - B_1) - T_{\text{integrator}}\dot{\varphi}_0K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}}}
\end{aligned}
\tag{2.41}
$$

$$
\begin{gathered}
\tan\left(\Delta\chi_2 + \Delta\chi_{\text{integrator21}} - \Delta\chi_{\text{integrator22}}\right) = \tfrac{1}{\tan\chi_2} = \\
\tan\left[-\arctan(T_1\dot{\varphi}_0) + \arctan\left(\tfrac{2T_2\xi_{\text{integrator}}\dot{\varphi}_0}{1-T_2^2\dot{\varphi}_0^2}\right) + \arctan\left(\tfrac{2T_0\xi\dot{\varphi}_0}{1-T_0^2\dot{\varphi}_0^2}\right)\right] \\
\tan\left(\Delta\chi_2 + \Delta\chi_{\text{integrator21}} - \Delta\chi_{\text{integrator22}}\right) = \\
\tfrac{2\dot{\varphi}_0\left(T_2\xi_{\text{integrator}} + T_0\xi\right) - 2T_2T_0\dot{\varphi}_0^3\left(T_0\xi_{\text{integrator}} + T_2\xi\right) - T_1\dot{\varphi}_0 +}{1-\dot{\varphi}_0^2\left(T_2^2 + T_0^2\right) + T_0^2T_2^2\dot{\varphi}_0^4 - 4T_0T_2\xi_{\text{integrator}}\xi\dot{\varphi}_0^2 +} \rightarrow \\
\tfrac{T_1\dot{\varphi}_0^3\left(T_2^2 + T_0^2\right) - T_0^2T_1T_2^2\dot{\varphi}_0^5 + 4T_0T_1T_2\xi_{\text{integrator}}\xi\dot{\varphi}_0^3}{2T_1\dot{\varphi}_0^2\left(T_2\xi_{\text{integrator}} + T_0\xi\right) - 2T_2T_1T_0\dot{\varphi}_0^4\left(T_0\xi_{\text{integrator}} + T_2\xi\right)}
\end{gathered}
\tag{2.42}
$$

In this case, even in the state of resonance, the phase of the output signal always maintains a complex relationship with the rotating angular velocity of the aircraft itself. The integral time constant is small and can be easily determined. The time constant causes the phase of the integral loop and the angular velocity rotating around the longitudinal axis of the aircraft to change, and make them keep very small dependency in the varying range of the above angular velocity. This condition is obtained by Eqs. (2.41) and (2.42), and the following two expressions are obtained, which show that the instrument is considered to be working in a resonant state.

$$
\tan\left(\Delta\chi_1 - \Delta\chi_{\text{integrator1}}\right) = \frac{1}{T_{\text{integrator}}\dot{\varphi}_0}
\tag{2.43}
$$

$$\tan\left(\Delta\chi_2 + \Delta\chi_{\text{integrator21}} - \Delta\chi_{\text{integrator22}}\right) = \frac{1 + \frac{2\xi_{\text{integrator}} T_2 T_1 \dot{\varphi}_0^2}{1 - T_2^2 \dot{\varphi}_0^2}}{T_1 \dot{\varphi}_0 - \frac{2\xi_{\text{integrator}} T_2 \dot{\varphi}_0}{1 - T_2^2 \dot{\varphi}_0^2}} \tag{2.44}$$

where

$$\chi_1 = -\frac{\pi}{2} - \arctan\left[\frac{1}{T_{\text{integrator}}\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}\right] \tag{2.45}$$

$$\chi_2 = -\frac{\pi}{2} - \arctan\left[\frac{1 + \frac{2\xi_{\text{integrator}} T_2 T_1 \left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2}{1 - T_2^2\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2}}{T_1\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) - \frac{2\xi_{\text{integrator}} T_2\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)}{1 - T_2^2\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2}}\right] \tag{2.46}$$

Equations (2.29), (2.30), (2.45) and (2.46) show that the instability of the rotating angular velocity signal has a huge influence on the stability of the phase and the instantaneous value of the instrument output signal. Therefore, various measures must be taken to reduce the phase instability. Introducing the differential circuit in the integrator can achieve this purpose. Then, Eqs. (1.129) and (1.130) have the following form by some transformations:

$$W_{\text{integrator1}}(s) = K_{\text{integrator1}} \frac{1 + T_{\text{damping}} s}{1 + T_{\text{integrator}} s} \tag{2.47}$$

$$W_{\text{integrator2}}(s) = K_{\text{integrator2}} \frac{\left(1 + T_{\text{damping}} s\right)\left(1 + T_{\text{integrator}} s\right)}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1} \tag{2.48}$$

The transfer function of the closed-loop system is described by the following expressions:

$$\begin{aligned}\Phi_1(s) = {} & \frac{K K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{1 + T_{\text{damping}} s}{1 + T_{\text{integrator}} s} \\ & \times \frac{s}{T_0^2 n T_{\text{amplify}} s^3 + \left(T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}}\right) s^2 + \left(n T_{\text{amplify}} + 2\xi_0 T_0 + K\right) s + 1}\end{aligned} \tag{2.49}$$

$$\begin{aligned}\Phi_2(s) = {} & \frac{K K_{\text{integrator2}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{\left(1 + T_1 s\right)\left(1 + T_{\text{damping}} s\right)}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1} \\ & \times \frac{s}{T_0^2 n T_{\text{amplify}} s^3 + \left(T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}}\right) s^2 + \left(n T_{\text{amplify}} + 2\xi_0 T_0 + K\right) s + 1}\end{aligned} \tag{2.50}$$

In this case, the amplitude and the phase of the output signal is also changed:

$$U_{\text{amplitude1}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2\Omega \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{\sqrt{1 + T_{\text{damping}}^2\dot{\varphi}_0^2}}{\sqrt{1 + T_{\text{integrator}}^2\dot{\varphi}_0^2}} \times \frac{1}{\sqrt{\left\{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3\right\}^2 + \left[1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2\right]^2}} \tag{2.51}$$

$$U_{\text{amplitude2}} = (C_1 - A_1 + B_1)\dot{\varphi}_0^2\Omega \frac{KK_{\text{integrator2}}}{K_{\text{moment}}K_{\text{electric current}}} \frac{\sqrt{\left(1 + T_1^2\dot{\varphi}_0^2\right)\left(1 + T_{\text{damping}}^2\dot{\varphi}_0^2\right)}}{\sqrt{\left(1 - T_2^2\dot{\varphi}_0^2\right)^2 + \left(2\xi_{\text{integrator}}T_2\dot{\varphi}_0\right)^2}} \times \frac{1}{\sqrt{\left\{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3\right\}^2 + \left[1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2\right]^2}} \tag{2.52}$$

Under the assumption used previously and the assumption that $\xi_0 = 0$, $T_{\text{integrator}}\dot{\varphi}_0 \geq 1$ and T_{damping} is very small ($T_{\text{damping}}\dot{\varphi}_0 \leq 1$), Eq. (2.51) is obtained as

$$U_{\text{amplitude1}} = \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \frac{(C_1 - A_1 + B_1)\dot{\varphi}_0^2\Omega}{T_{\text{integrator}}\dot{\varphi}_0\sqrt{\left[\left(nT_{\text{amplify}} + 2\xi T_0\right)\dot{\varphi}_0 - T_0^2 nT_{amplify}\dot{\varphi}_0^3\right]^2 + \left[1 - T_0^2\dot{\varphi}_0^2\right]^2}} \tag{2.53}$$

$$U_{\text{amplitude1}} = \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \frac{(C_1 - A_1 + B_1)\Omega}{2\xi T_0 T_{\text{integrator}}\sqrt{\left[1 + \frac{nT_{\text{amplify}}}{2\xi T_0}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]^2 + \left[\frac{1}{2\xi T_0}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]^2}}$$

Equation (2.53) shows that the differential loop whose time constant is small enough, is introduced into the integrator and will not affect the amplitude of the output signal. When the instrument is working in the resonant state and nT_{amplify} is very small, Eq. (2.53) is in accordance with Eq. (2.10).

Next we will analyze the situation when the instrument is not working at the resonance point, but is very close to the resonance point. This situation is more realistic. At this time, nT_{amplify} is a very small constant but is not equal to zero. Because nT_{amplify} is very small and $\dot{\varphi}_0$ approaches ω_0, $\frac{nT_{\text{amplify}}}{2\xi T_0}\left(1 - T_0^2\dot{\varphi}_0^2\right)$ is far less than 1 and its square value can be neglected when compared with 1. There holds

$$U_{\text{amplitude1}} = \frac{K_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \frac{(C_1 - A_1 + B_1)\Omega}{T_{\text{integrator}}\sqrt{\frac{1 - T_0^2\dot{\varphi}_0^2}{2\xi T_0}\left[\frac{1 - T_0^2\dot{\varphi}_0^2}{2\xi T_0\dot{\varphi}_0^2} + 2nT_{\text{amplify}}\right] + 1}} \tag{2.54}$$

Substituting Eqs. (1.124), (1.125) and (1.134) obtains

$$U_{\text{amplitude1}} = \frac{K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}}} \frac{(C_1 - A_1 + B_1)\Omega}{T_{\text{integrator}} \sqrt{\frac{(C_1 - A_1 - B_1)\dot{\varphi}_0^2}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} \left[\frac{(C_1 - A_1 - B_1)}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} + 2nT_{\text{amplify}} \right] + 1}} \tag{2.55}$$

When the influence of the mutual inductance effect is not completely compensated ($n \neq 0$) and precisely tuned, the signal amplitude at the output end of the instrument will have a complex dependency on the angular velocity rotating around the longitudinal axis of the aircraft.

Equations (2.54) and (2.55) show that in the process of approaching the resonant point, not only is the influence of the angular velocity rotating around the longitudinal axis of the aircraft on the amplitude of the output signal decreased, but its influence on the time constant nT_{amplify} is also decreased. The effect of these parameters is reduced under the condition that the feedback damping function is enhanced.

Under the same assumptions, Eq. (2.52) can be obtained by analogy:

$$U_{\text{amplitude2}} = \frac{K_{\text{integrator2}}}{K_{\text{moment}} K_{\text{electric current}}} \frac{T_1 (C_1 - A_1 + B_1)\Omega}{T_2^2 \sqrt{\frac{(C_1 - A_1 - B_1)\dot{\varphi}_0^2}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} \left[\frac{(C_1 - A_1 - B_1)}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}}} + 2nT_{\text{amplify}} \right] + 1}} \tag{2.56}$$

When the integrator is introduced into the differential loop, the phase change of the output signal is obtained:

$$\begin{aligned} \chi_1 = {} & \frac{\pi}{2} + \arctan\left(T_{\text{damping}} \dot{\varphi}_0\right) - \arctan\left(T_{\text{integrator}} \dot{\varphi}_0\right) \\ & - \arctan\left\{ \frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}} \dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2} \right\} \end{aligned} \tag{2.57}$$

$$\begin{aligned} \chi_2 = {} & \frac{\pi}{2} + \arctan\left(T_{\text{damping}} \dot{\varphi}_0\right) + \arctan\left(T_1 \dot{\varphi}_0\right) - \arctan\left(\frac{2\xi_{\text{integrator}} T_2 \dot{\varphi}_2}{1 - T_2^2 \dot{\varphi}_0^2}\right) \\ & - \arctan\left\{ \frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}} \dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2} \right\} \end{aligned} \tag{2.58}$$

Because the damping of the air is small, and set $\xi_0 = 0$, there holds

$$\begin{aligned} \chi_1 = {} & \frac{\pi}{2} + \arctan\left(T_{\text{damping}} \dot{\varphi}_0\right) - \arctan\left(T_{\text{integrator}} \dot{\varphi}_0\right) \\ & - \arctan\left(\frac{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2 \dot{\varphi}_0^2\right)\right]}{1 - T_0^2 \dot{\varphi}_0^2}\right) \end{aligned} \tag{2.59}$$

$$\chi_2 = \frac{\pi}{2} + \arctan\left(T_{\text{damping}}\dot{\varphi}_0\right) + \arctan\left(T_1\dot{\varphi}_0\right) - \arctan\left(\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1 - T_2^2\dot{\varphi}_0^2}\right) - \arctan\left\{\frac{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]}{1 - T_0^2\dot{\varphi}_0^2}\right\} \tag{2.60}$$

Introducing the following symbols:

$$\chi_{\text{integrator}} = -\arctan\left(T_{\text{integrator}}\dot{\varphi}_0\right) \tag{2.61}$$

$$\chi_{\text{integrator1}} = \arctan\left(T_1\dot{\varphi}_0\right) \tag{2.62}$$

$$\chi_{\text{integrator2}} = -\arctan\left(\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1 - T_2^2\dot{\varphi}_0^2}\right) \tag{2.63}$$

$$\chi_{\text{integrator0}} = -\arctan\left\{\frac{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]}{1 - T_0^2\dot{\varphi}_0^2}\right\} \tag{2.64}$$

For larger values of $T_{integrator}$, T_1, T_1 and the working state of approaching the resonant point, from transformations (2.22) and (2.23), Eqs. (2.61), (2.62), (2.63) and (2.64) can be transformed and rewritten as

$$\chi_{\text{integrator}} = -\frac{\pi}{2} + \Delta\chi_{\text{integrator}}$$

$$\chi_{\text{integrator1}} = \frac{\pi}{2} + \Delta\chi_{\text{integrator1}}$$

$$\chi_{\text{integrator2}} = -\pi + \Delta\chi_{\text{integrator2}}$$

$$\chi_{\text{integrator0}} = -\frac{\pi}{2} + \Delta\chi_{\text{integrator0}}$$

Because $\Delta\chi_{\text{integrator}}$, $\Delta\chi_{\text{integrator1}}$, $\Delta\chi_{\text{integrator2}}$ and $\Delta\chi_{\text{integrator0}}$ are small, there holds

$$\Delta\chi_{\text{integrator}} = -\frac{1}{\tan\chi_{\text{integrator}}} = \frac{1}{T_{\text{integrator}}\dot{\varphi}_0} \tag{2.65}$$

$$\Delta\chi_{\text{integrator1}} = -\frac{1}{\tan\chi_{\text{integrator1}}} = -\frac{1}{T_1\dot{\varphi}_0} \tag{2.66}$$

$$\Delta\chi_{\text{integrator2}} = \tan\chi_{\text{integrator2}} = -\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1 - T_2^2\dot{\varphi}_0^2} = \frac{2\xi_{\text{integrator}}}{T_2\dot{\varphi}_0} \tag{2.67}$$

$$\Delta\chi_{\text{integrator0}} = -\frac{1}{\tan\chi_{\text{integrator0}}} = \frac{1 - T_0^2\dot{\varphi}_0^2}{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]} \tag{2.68}$$

For a small T_{damping}, $\arctan\left(T_{\text{damping}}\dot{\varphi}_0\right) = T_{\text{damping}}\dot{\varphi}_0$, then

$$\chi_1 = -\frac{\pi}{2} + T_{\text{damping}}\dot{\varphi}_0 + \frac{1}{T_{\text{integrator}}\dot{\varphi}_0} + \frac{1 - T_0^2\dot{\varphi}_0^2}{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]} \tag{2.69}$$

$$\chi_2 = -\frac{\pi}{2} + T_{damping}\dot{\varphi}_0 + \frac{2\xi_{integrator}}{T_2\dot{\varphi}_0} - \frac{1}{T_1\dot{\varphi}_0} + \frac{1 - T_0^2\dot{\varphi}_0^2}{\dot{\varphi}_0\left[2\xi T_0 + nT_{amplify}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]} \tag{2.70}$$

The augments of the third term and the fourth term in Eq. (2.70) were synthesized as:

$$\chi_2 = -\frac{\pi}{2} + T_{\text{damping}}\dot{\varphi}_0 + \frac{2\xi_{\text{integrator}}T_1 - T_2}{T_2T_1\dot{\varphi}_0} + \frac{1 - T_0^2\dot{\varphi}_0^2}{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]} \tag{2.71}$$

When $T_{\text{integrator}} = \frac{T_2T_1}{2\xi_{\text{integrator}}T_1 - T_2}$, Eq. (2.71) is completely consistent with Eq. (2.69).

Figure 2.2 shows the frequency characteristic curves of two integrator schemes that have the transfer functions in Eq. (1.130). The frequency characteristic curves with different $\xi_{\text{integrator}}$ are drawn.

Comparing Eqs. (2.10) and (2.12) with (2.69) and (2.71), obtains two conditions in Eqs. (2.72) and (2.73). Under the above two conditions, the frequency characteristic curves for the integrator with the transfer function in Eq. (1.130) are in accord with the frequency characteristic curves of the non-periodic loop.

$$\frac{T_2^2}{T_1} = T_{\text{integrator}} \tag{2.72}$$

$$T_{\text{integrator}} = \frac{T_2T_1}{2\xi_{\text{integrator}}T_1 - T_2} \tag{2.73}$$

Because the three parameters of the integrator are only related to two equations, there are many groups of parameter solutions that satisfy the two equations.

The parameters of the frequency characteristics in the integrator are determined by Eqs. (2.72) and (2.73). It is recommended that these parameters are selected in the following order:

(1) Select a sufficiently large time constant $T_{\text{integrator}}$.

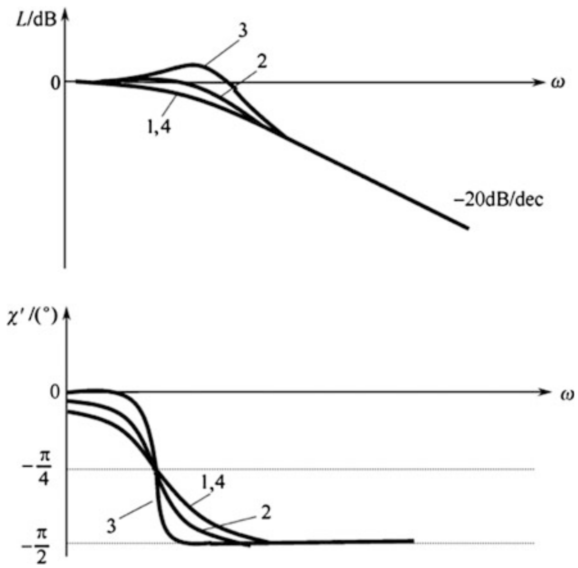

Fig. 2.2 Frequency characteristic curves of two integrators. 1—Integrator with the transfer function in Eq. (1.130), non-periodic loop; 2—Curve with $\xi_{\text{integrator}}$ = 0.8; 3—Curve with $\xi_{\text{integrator}}$ = 0.5; 4—Curve with $\xi_{\text{integrator}}$ = 1

(2) Select a sufficiently large time constant T_1.
(3) From (2.72), calculate the time constant T_2.
(4) From (2.73), determine the coefficient $\xi_{\text{integrator}}$.
Under the condition of these parameters, calculate the requirement that the frequency characteristic curve of the integrator is fully in accord with that of the non-periodic loop.
(5) As shown in Fig. 2.2, the coefficient $\xi_{\text{integrator}}$ is changed on the basis of the phase frequency characteristic curve. In the working frequency range, the required phase stability is obtained.

It should be noted that under the condition that the time constants T_1 and T_2 are large enough, then the change of $\xi_{\text{integrator}}$, within the working frequency range, does not actually affect the amplitude frequency characteristic curve.

According to the circuit shown in Fig. 1.18, the parameters of the integrator have the following forms:

$$K_{\text{integrator}} = \frac{R_5 + R_6}{R_4} \tag{2.74}$$

$$T_1 = \frac{C_2 R_5 R_6}{R_5 + R_6} \tag{2.75}$$

$$T_2 = \sqrt{C_1 C_2 R_5 R_6} \tag{2.76}$$

$$\xi_{\text{integrator}} = \frac{C_1(R_5 + R_6) + C_2R_5}{2\sqrt{C_1C_2R_2R_3}} \tag{2.77}$$

Review Eqs. (2.69) and (2.71), decompose $\frac{1}{T_{\text{integrator}}\dot{\varphi}_0}$ and $\frac{2\xi_{\text{integrator}}T_1 - T_2}{T_1T_2\dot{\varphi}_0}$ into Taylor series, reject the two and higher powers of $\left(\dot{\varphi}_0 - \dot{\varphi}_0^0\right)$ because of their small values, and substitute them into Eqs. (2.69) and (2.71)

$$\frac{1}{T_{\text{integrator}}\dot{\varphi}_0} = \frac{1}{T_{\text{integrator}}\dot{\varphi}_0^0} - \frac{\dot{\varphi}_0 - \dot{\varphi}_0^0}{T_{\text{integrator}}\left(\dot{\varphi}_0^0\right)^2} + \cdots \tag{2.78}$$

$$\frac{2\xi_{\text{integrator}}T_1 - T_2}{T_{\text{integrator}}\dot{\varphi}_0} = \frac{2\xi_{\text{integrator}}T_1 - T_2}{T_{\text{integrator}}\dot{\varphi}_0^0} - \frac{\left(2\xi_{\text{integrator}}T_1 - T_2\right)\left(\dot{\varphi}_0 - \dot{\varphi}_0^0\right)}{T_{\text{integrator}}\left(\dot{\varphi}_0^0\right)^2} + \cdots \tag{2.79}$$

Substituting Eqs. (1.124), (1.125) and (1.134), Eqs. (2.69) and (2.71) are transformed into

$$\begin{aligned}\chi_1 = -\frac{\pi}{2} + \frac{2}{T_{\text{integrator}}\dot{\varphi}_0^0} + \dot{\varphi}_0\Bigg[& T_{\text{damping}} - \frac{1}{T_{\text{integrator}}\left(\dot{\varphi}_0^0\right)^2} \\ & + \frac{C_1 - A_1 - B_1}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}} + nT_{\text{amplify}}\dot{\varphi}_0^2(C_1 - A_1 - B_1)}\Bigg]\end{aligned} \tag{2.80}$$

$$\begin{aligned}\chi_2 = -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1T_2\dot{\varphi}_0^0} + \dot{\varphi}_0\Bigg[& T_{\text{damping}} - \frac{2\xi_{\text{integrator}}T_1 - T_2}{T_1T_2\left(\dot{\varphi}_0^0\right)^2} \\ & + \frac{C_1 - A_1 - B_1}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}} + nT_{\text{amplify}}\dot{\varphi}_0^2(C_1 - A_1 - B_1)}\Bigg]\end{aligned} \tag{2.81}$$

The conclusion can be drawn from Eqs. (2.80) and (2.81) that the differential loop with the time constant T_{damping} is used in the integrator, which can be a part of the compensation for the instability of the self-rotating angular velocity of the aircraft.

For χ_1:

$$\begin{aligned}T_{\text{damping}} = & \frac{1}{T_{\text{integrator}}\left(\dot{\varphi}_0^0\right)^2} \\ & - \frac{C_1 - A_1 - B_1}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}} + nT_{\text{amplify}}\dot{\varphi}_0^2(C_1 - A_1 - B_1)}\end{aligned} \tag{2.82}$$

For χ_2:

$$T_{\text{damping}} = \frac{2\xi_{\text{integrator}} T_1 - T_2}{T_1 T_2 \left(\dot{\varphi}_0^0\right)^2} - \frac{C_1 - A_1 - B_1}{K_{\text{moment}} K_{\text{amplify}} K_{\text{measure}} K_{\text{electric current}} + n T_{\text{amplify}} \dot{\varphi}_0^2 (C_1 - A_1 - B_1)} \tag{2.83}$$

Obviously, because of the relationship between Eqs. (2.82) and (2.83) and the rotating angular velocity around the longitudinal axis of the rotating flight carrier, it is not possible to achieve full compensation. However, the instrument approaches the working state of the resonance point, and the size of T_{damping} can be determined as follows:

For χ_1:

$$T_{\text{damping}} = \frac{1}{T_{\text{integrator}} \left(\dot{\varphi}_0^0\right)^2} \tag{2.84}$$

For χ_2:

$$T_{\text{damping}} = \frac{2\xi_{\text{integrator}} T_1 - T_2}{T_1 T_2 \left(\dot{\varphi}_0^0\right)^2} \tag{2.85}$$

Equations (2.84) and (2.85) point out that a larger integral time constant is required for a better quality of integral. And as has been discussed previously, for a small value of T_{damping}, it is also required that the inconformity of the amplitude frequency characteristic does not produce a negative effect. For example, when $T_{\text{integrator}} = 0.1$, the effect of the differential loop of the integrator on the amplitude of the output signal reaches ±4%. However, if a larger $T_{\text{integrator}}$ is selected, it should be kept in mind that the increase of $T_{\text{integrator}}$ will lead to a decrease in the amplitude of the output signal and an increase in the phase lag of the output signal.

According to Eqs. (2.82) and (2.83), the effect evaluations of $\Delta\,\dot{\varphi}_0$ on χ_1 and χ_2 can be obtained. For χ_1, there holds

$$T_{\text{damping}} = \frac{1}{T_{\text{integrator}} \left(\dot{\varphi}_0^0\right)^2} - \frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + n T_{\text{amplify}} \left(\dot{\varphi}_0^0\right)^2 (C_1 - A_1 - B_1)} \tag{2.86}$$

However for χ_2, there holds

$$T_{\text{damping}} = \frac{2\xi_{\text{integrator}} T_1 - T_2}{T_1 T_2 \left(\dot{\varphi}_0^0\right)^2} - \frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + n T_{\text{amplify}} \left(\dot{\varphi}_0^0\right)^2 (C_1 - A_1 - B_1)} \tag{2.87}$$

Substituting Eqs. (2.86) and (2.87) into Eqs. (2.80) and (2.81) obtains

$$\chi_1 = -\frac{\pi}{2} + \frac{2}{T_{\text{integrator}}\dot{\varphi}_0^0} + \left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) \times \left[\frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + nT_{\text{amplify}}\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2 (C_1 - A_1 - B_1)} - \frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + nT_{\text{amplify}}\left(\dot{\varphi}_0^0\right)^2 (C_1 - A_1 - B_1)}\right] \tag{2.88}$$

$$\chi_2 = -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1 T_2 \dot{\varphi}_0^0} + \left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right) \times \left[\frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + nT_{\text{amplify}}\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2 (C_1 - A_1 - B_1)} - \frac{C_1 - A_1 - B_1}{D_{\text{feedback}} + nT_{\text{amplify}}\left(\dot{\varphi}_0^0\right)^2 (C_1 - A_1 - B_1)}\right] \tag{2.89}$$

Select $\xi_{\text{integrator}} = 1$ and $T_1 = T_2 = T_{\text{integrator}}$, χ_2 can be obtained as

$$\chi_2 = -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1 T_2 \dot{\varphi}_0^0} + \frac{\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)(C_1 - A_1 - B_1)^2 nT_{\text{amplify}}\left[\left(\dot{\varphi}_0^0\right)^2 - \left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2\right]}{D_{\text{feedback}}^2 + D_{\text{feedback}}^2 nT_{\text{amplify}}(C_1 - A_1 - B_1)\left[\left(\dot{\varphi}_0^0\right)^2 + \left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)^2\right] + \left[nT_{\text{amplify}}(C_1 - A_1 - B_1)\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)\dot{\varphi}_0^0\right]^2} \tag{2.90}$$

Because all the augments in the denominator are compared with D_{feedback}^2, they are very small and can be ignored. Then

$$\begin{aligned}
\chi_2 &= -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1 T_2 \dot{\varphi}_0^0} + \frac{(C_1 - A_1 - B_1)^2 nT_{\text{amplify}}}{D_{\text{feedback}}^2}\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)\left(\mp 2\dot{\varphi}_0^0\Delta\dot{\varphi}_0 - \Delta\dot{\varphi}_0^2\right) \\
&= -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1 T_2 \dot{\varphi}_0^0} - \frac{\left[B_1 - (C_1 - A_1)\right]^2 nT_{\text{amplify}}}{D_{\text{feedback}}^2}\left(\dot{\varphi}_0^0 \pm \Delta\dot{\varphi}_0\right)\left(\Delta\dot{\varphi}_0 + 2\dot{\varphi}_0^0\right)\Delta\dot{\varphi}_0 \\
&= -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1 T_2 \dot{\varphi}_0^0} - \frac{\left[B_1 - (C_1 - A_1)\right]^2 nT_{\text{amplify}}}{D_{\text{feedback}}^2}\left(3\dot{\varphi}_0^0\Delta\dot{\varphi}_0 \pm 2\left(\dot{\varphi}_0^0\right)^2 \pm \Delta\dot{\varphi}_0^2\right)\Delta\dot{\varphi}_0
\end{aligned} \tag{2.91}$$

From Eq. (2.91), it can be clearly seen that the sign of the unstable value of the angular velocity around the longitudinal axis of the rotating flight carrier is changed and the sign of the second augment in the final parentheses is also changed. But even so, the output signal has higher phase stability due to the small coefficient of the final bracket. When the coefficient is very small, the last term of Eq. (2.91) is comparable to the others and can be ignored.

When the working state of the instrument approaches the state of the resonant point, the integrity and accuracy influence of the compensation effect of $\triangle\dot{\varphi}_0$ increases. In the resonant state, the instability influence of the angular velocity rotating around the longitudinal axis of the aircraft does not exist.

But as has been mentioned earlier, in the state of the near-resonant point (when the detuning coefficient $\mu = 0.9$), the last term of Eq. (2.91) is negligible. At this time, Eq. (2.91) can be simplified and its expression without $\varDelta\ \dot{\varphi}_0$ is obtained as

$$\chi_2 = -\frac{\pi}{2} + \frac{2\left(2\xi_{\text{integrator}}T_1 - T_2\right)}{T_1T_2\dot{\varphi}_0^0} \tag{2.92}$$

And so on

$$\chi_1 = -\frac{\pi}{2} + \frac{2}{T_{\text{integrator}}\dot{\varphi}_0^0} \tag{2.93}$$

Equation (2.91) also shows that when the differential loop is introduced into the integrator, the influence of the angular velocity rotating around the longitudinal axis of the aircraft on the phase of the output signal decreases, and the damping effect guaranteed by the feedback increases and the compensation coefficient of the mutual inductance influence n decreases. Even when the resonant state is missed, but under the influence of a complete compensation of the mutual inductance winding (n = 0), and in the condition of using a non-periodic loop as the integrator, when Eq. (2.86) is implemented, or in the condition that the integrator with a transfer function (1–130) is used, when Eq. (2.87) is implemented, the phase has nothing to do with $\triangle\dot{\varphi}_0$.

At the output end of the instrument without an integrator, the output without an integrator, the impact of introducing the differentiator on the phase of the output signal would be determined.

$$\begin{aligned}\chi_0 = {} & \frac{\pi}{2} + \arctan\left(T_{\text{damping}}\dot{\varphi}_0\right) \\ & - \arctan\left(\frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0 + \xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3}{1 - \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2}\right)\end{aligned} \tag{2.94}$$

According to Eq. (2.69) the phase of the output signal of the instrument without an integrator can be written by the following expression:

$$\chi_0 = T_{\text{damping}}\dot{\varphi}_0 + \frac{1 - T_0^2\dot{\varphi}_0^2}{\dot{\varphi}_0\left[2\xi T_0 + nT_{\text{amplify}}\left(1 - T_0^2\dot{\varphi}_0^2\right)\right]} \tag{2.95}$$

Combining Eqs. (1.124), (1.125) and (1.134) obtains

$$\chi_0 = \dot{\varphi}_0\left[T_{\text{damping}} + \frac{C_1 - A_1 - B_1}{K_{\text{moment}}K_{\text{amplify}}K_{\text{measure}}K_{\text{electric current}} + nT_{\text{amplify}}\dot{\varphi}_0^2(C_1 - A_1 - B_1)}\right] \tag{2.96}$$

In this case, the time constant of the differential equation can be obtained by several other expressions. Unlike Eqs. (2.82) and (2.83), there is no added entry that determines the parameters of the integrator. The compensation accuracy is the same as that of the circuit with an integrator.

In order to ensure that the measurement accuracy of the angular velocity around the longitudinal axis of the aircraft is higher, two additional loops must be added to the velocity feedback loop to reduce the instability and mutual effect of the angular velocity around the longitudinal axis of the aircraft.

The basic formula for the amplitude of a rotor vibration gyroscope with negative feedback is as follows

$$\Phi(s) = \frac{KK_{\text{integrator2}}}{K_{\text{moment}}K_{\text{electric current}}}\frac{(1+T_1 s)(1+T_D s)}{T_2^2 s^2 + 2\xi_{\text{integrator}}T_2 s + 1} \times \frac{s}{T_0^2 nT_{\text{amplify}}s^3 + \left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)s^2 + \left(nT_{\text{amplify}} + 2\xi_0 T_0 + K\right)s + 1}$$

$$U_{\text{amplitude}} = \frac{(C_1 - A_1 + B_1)\dot{\varphi}_0^2\Omega KK_{\text{integrator2}}\sqrt{\left(1+T_1^2\dot{\varphi}_0^2\right)\left(1+T_D^2\dot{\varphi}_0^2\right)}}{K_{\text{moment}}K_{\text{electric current}}\sqrt{\left(1-T_2^2\dot{\varphi}_0^2\right)^2 + \left(2\xi_{\text{integrator}}T_2\dot{\varphi}_0\right)^2}} \times \frac{1}{\sqrt{\left\{\left[nT_{\text{amplify}} + 2T_0(\xi_0+\xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3\right\}^2 + \left[1-\left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2\right]^2}}$$

$$\chi = \frac{\pi}{2} + \arctan(T_D\dot{\varphi}_0) + \arctan(T_1\dot{\varphi}_0) - \arctan\left(\frac{2\xi_{\text{integrator}}T_2\dot{\varphi}_0}{1-T_2^2\dot{\varphi}_0^2}\right) - \arctan\left(\frac{\left[nT_{\text{amplify}} + 2T_0(\xi_0+\xi)\right]\dot{\varphi}_0 - T_0^2 nT_{\text{amplify}}\dot{\varphi}_0^3}{1-\left(T_0^2 + 2\xi_0 T_0 nT_{\text{amplify}}\right)\dot{\varphi}_0^2}\right)$$

When nT_{amplify}, $T_{\text{integrator}}$ and ξ_o are small, and T_1 and T_2 are large, the measured amplitude of the constant angular velocity is

$$U_{\text{amplitude}} = \frac{(C_1 - A_1 + B_1)T_1\Omega K_{\text{amplify}}K_{\text{integrator3}}K_{\text{integrator2}}}{T_2^2\sqrt{\left(K_{\text{integrator3}}K_{\text{electric current}}K_{\text{moment}}K_{\text{amplify}}\right)^2 + (C_1 - A_1 - B_1)^2\dot{\varphi}_0^2}}$$

$$\chi = -\arctan\left(\frac{2\xi T_0\dot{\varphi}_0}{1-T_0^2\dot{\varphi}_0^2}\right)$$

In the case of precise tuning, there holds

$$U_{\text{amplitude}} = \frac{2B_1 K_{\text{integrator2}} T_1}{K_{\text{moment}} K_{\text{electric current}} T_2^2} \Omega$$

$$\chi = -\frac{\pi}{2}$$

2.2 Regulation of a Non-driven Mechanical Gyroscope

Compare the regulation quality of three instrument loops with a velocity feedback, which include the no integrator type, the integrator type with transfer function (1.129), and the integrator type with transfer function (1.130) at the output end of the instrument.

Comparing Sect. 1.4, the analogy method can be used to determine the response of the instrument to the constant input force in the implicated coordinate system. Therefore, according to Eqs. (2.1), (2.2) and (2.3), the differential loop in the integrator is designated as

$$\Phi_0(s) = \frac{K}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{s(T_{\text{damping}} s + 1)}{T_0^2 n T_{\text{amplify}} s^3 + (T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}}) s^2 + (n T_{\text{amplify}} + 2\xi_0 T_0 + K) s + 1} \tag{2.97}$$

$$\Phi_1(s) = \frac{K K_{\text{integrator1}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{1}{1 + T_{\text{integrator}} s} \times \frac{s(T_{\text{damping}} s + 1)}{T_0^2 n T_{\text{amplify}} s^3 + (T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}}) s^2 + (n T_{\text{amplify}} + 2\xi_0 T_0 + K) s + 1} \tag{2.98}$$

$$\Phi_2(s) = \frac{K K_{\text{integrator2}}}{K_{\text{moment}} K_{\text{electric current}}} \times \frac{T_1 s + 1}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1} \times \frac{s(T_{\text{damping}} s + 1)}{T_0^2 n T_{\text{amplify}} s^3 + (T_0^2 + 2\xi_0 T_0 n T_{\text{amplify}}) s^2 + (n T_{\text{amplify}} + 2\xi_0 T_0 + K) s + 1} \tag{2.99}$$

Because nT_{amplify}, ξ_0 and T_{damping} are very small, Eqs. (2.97), (2.98) and (2.99) are rewritten as

$$\Phi_0(s) = \frac{K}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{s}{T_0^2 s^2 + Ks + 1} \tag{2.100}$$

$$\begin{aligned}\Phi_1(s) &= \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{1}{1 + T_{\text{integrator}} s} \\ &\times \frac{s}{T_0^2 s^2 + Ks + 1} = \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{\Phi_0(s)}{1 + T_{\text{integrator}} s}\end{aligned} \tag{2.101}$$

$$\begin{aligned}\Phi_2(s) &= \frac{KK_{\text{integrator2}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{T_1 s + 1}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1} \times \frac{s}{T_0^2 s^2 + Ks + 1} \\ &= \frac{KK_{\text{integrator2}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{(T_1 s + 1)\Phi_0(s)}{T_2^2 s^2 + 2\xi_{\text{integrator}} T_2 s + 1}\end{aligned} \tag{2.102}$$

Consider the expression (2.100) in Eq. (1.134), which is equivalent to the transfer function of the oscillation loop and is expressed by the angular velocity at the output end:

$$\Phi_0(s) = \frac{\alpha_0(s)}{M(s)}$$

where

$$\frac{\alpha_0(s)}{M(s)} = \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{1}{T_0^2 s^2 + Ks + 1}.$$

According to Eq. (1.91) and so on, when a constant force is acting on the instrument in the coordinate system which is connected to the aircraft, the transition process is expressed by the deflection angle of the sensitive element:

$$\alpha_0 = C_0 e^{-\xi\omega_0 t} \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t + \beta_0\right] + \frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{H_0}{B_0}\Omega \sin\gamma_1 \tag{2.103}$$

According to Eqs. (1.93) and (1.94), the integral constant has the form of

$$\beta_0 = \arctan\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \tag{2.104}$$

$$C_0 = -\frac{KK_{\text{integrator1}}}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{H_0\Omega \sin\gamma_1}{B_0\sqrt{1-\xi^2}} = -\frac{K_{\text{integrator1}}D_\alpha}{K_{\text{moment}}K_{\text{electric current}}} \times \frac{H_0\Omega \sin\gamma_1}{B_0^2\sqrt{1-\xi^2}} \tag{2.105}$$

It is obvious that there is no difference between the characteristics of the transition process of the gyroscope with a negative velocity feedback and the characteristics of the transition process of the gyroscope described in Eq. (1.95).

In order to obtain the impact of the different integrators on the transition process of the gyroscope, according to the oscillating angular velocity of the sensitive element, the transition process of the gyroscope without an integrator at the output end. When considering the complete expression of the integral constant, the transition process of the oscillating angular velocity of the sensitive element at the output of the preamplifier has the following form:

$$\begin{aligned}\dot{\alpha}_0 &= \frac{H_0 D_\alpha \sin\gamma_1 \omega_0 \mathrm{e}^{-\xi\omega_0 t}}{K_{\text{moment}} K_{\text{electric current}} B_0^2 \sqrt{1-\xi^2}} \Omega \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t\right] \\ &= \dot{\alpha}_0 \mathrm{e}^{-\xi\omega_0 t} \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t\right]\end{aligned} \tag{2.106}$$

The value of Eq. (2.106) is equal to zero, which is consistent with the value of the constant deviation angle of the sensitive element of the instrument determined by Eq. (1.106) in Sect. 1.4. Figure 2.3 shows the transition process of Eq. (2.106).

The effect of the integrator on the transition process is measured and the transition process of each integrator is added to the transient process of the sensitive element of a non-driven mechanical gyroscope used in a rotating state aircraft with negative velocity feedback. It can be seen that the transition process of each loop is the sum of the transient process of the sensitive element of a non-driven mechanical gyroscope used in a rotating state aircraft with a negative velocity feedback and the transient process of the integrator.

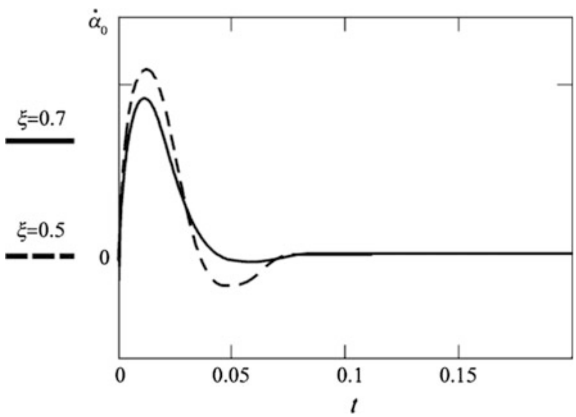

Fig. 2.3 Transition process of the sensitive element of a non-driven mechanical gyroscope used in a rotating state aircraft with velocity feedback (represented by the angular velocity)

$$\begin{aligned}\alpha_1 &= C' \mathrm{e}^{-\frac{1}{T_{\text{integrator}}}} + \frac{D_\alpha K_{\text{integrator1}} H \sin\gamma_1 \omega_0 \mathrm{e}^{-\xi\omega_0 t} \Omega \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t - \arctan\left(T_{\text{integrator}}\omega_0\sqrt{1-\xi^2}\right)\right]}{B_0^2\sqrt{1-\xi^2} K_{\text{moment}} K_{\text{electric current}}\sqrt{1+T_{\text{integrator}}^2\omega_0^2\left(1-\xi^2\right)}} \\ &= C' \mathrm{e}^{-\frac{1}{T_{\text{integrator}}}} + \frac{\dot{\alpha}_{0,\text{amplitude}}}{\sqrt{1+T_{\text{integrator}}^2\omega_0^2\left(1-\xi^2\right)}} \mathrm{e}^{-\xi\omega_0 t} \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t + \chi_{\text{integrator}}\right]\end{aligned} \tag{2.107}$$

Under the initial condition of the zero position, the integral constant is equal to

$$C' = -\frac{\dot{\alpha}_{0,\text{amplitude}}}{\sqrt{1+T_{\text{integrator}}^2\omega_0^2\left(1-\xi^2\right)}} \sin\left(\chi_{\text{integrator}}\right) \tag{2.108}$$

The transient process of the integrator has the following form:

$$\alpha_{\text{integrator}} = -\frac{\dot{\alpha}_{0,\text{amplitude}} \mathrm{e}^{-\frac{t}{T_b}}}{\sqrt{1+T_{\text{integrator}}^2\omega_0^2\left(1-\xi^2\right)}} \sin\left(\chi_{\text{integrator}}\right) \tag{2.109}$$

And the transition process of the whole gyroscope is

$$\begin{aligned}\alpha_1 = \frac{\dot{\alpha}_{0,\text{amplitude}}}{\sqrt{1+T_{\text{integrator}}^2\omega_0^2\left(1-\xi^2\right)}} &\left\{\mathrm{e}^{-\xi\omega_0 t} \sin\left[\left(\omega_0\sqrt{1-\xi^2}\right)t + \chi_{\text{integrator}}\right]\right. \\ &\left. - \mathrm{e}^{-\frac{1}{T_{\text{integrator}}}} \sin\left(\chi_{\text{integrator}}\right)\right\}\end{aligned} \tag{2.110}$$

Thus, the existence of the integrator changes the transition process of the instrument. Unlike the transition process studied in Sect. 1.4, in the implicated coordinate system of the aircraft the transition process of the sensitive element of the instrument is equal to zero under the action of a constant force. That is to say, the friction moment, the dynamic unbalanced moment of the sensitive element of the instrument and the moment produced by the centrifugal acceleration of the aircraft due to the constant angular deviation of the sensitive element would not cause the change of the output signal of the instrument.

As described by Eq. (2.106), and unlike the transition process expressed by the angular velocity of the oscillation framework, the time of the transition process expressed by the deviation angle of the instrument with the integrator that satisfies Eq. (2.110) and has the integrator with the transfer function (1.129), is mainly determined by the settling time of the integrator and $T_{\text{integrator}}$ can be seen in Fig. 2.4.

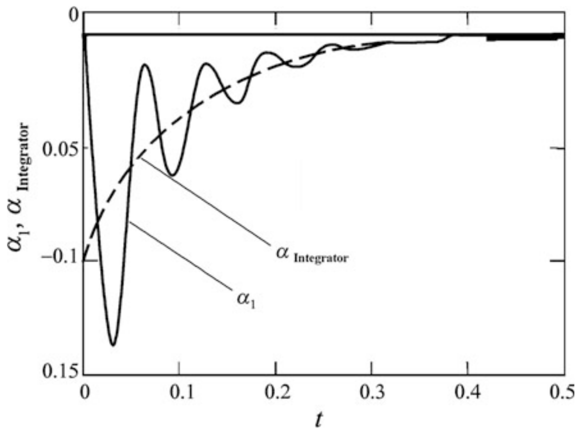

Fig. 2.4 Transition process of an integrator and the whole instrument

Because the time constant of an integrator increases, the settling time of the instrument also increases, and every input force instantly changes with a frequency greater than $1/(3\sim4)\ T_{\text{integrator}}$, the instrument cannot react.

Because every force acting on the instrument in flight has different characteristics, the integrator at the output end of the instrument is not a major detriment to the work of the instrument.

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http://www.springer.com/978-3-662-54043-5

Non-driven Micromechanical Gyroscopes and Their Applications
Zhang, F.; Zhang, W.; Wang, G.
2018, XVI, 361 p. 183 illus., 1 illus. in color., Hardcover
ISBN: 978-3-662-54043-5