

Chapter 2

Generalized Array Imaging on Rupture Processes of Earthquakes: Principle and Theoretical Tests

The rupture process of an earthquake is composed of nucleation of the earthquake at the hypocenter, subsequent rupture propagation on the fault, and stopping phase of the rupture. The rupture front usually proceeds at a speed less than the P wave velocity nearby the causative fault, accompanying with seismic energy releasing, thermal energy releasing, and strain energy consumed by the crack expansion. The ratio of the three kinds of energy depends on fault geometry, stress distribution, frictional property, media of the footwall and hanging wall, involvement of fluid, and tectonic stress. Among the three kinds of energy, only the seismic energy can be estimated with seismograms. The seismograms are convolution of the earthquake source, the structure of the earth's interior traveled through by the rays, and instrumental responses of seismometers. Typically, the instrumental responses are known. The structure of the earthquake from the earthquake to the seismometers can be expressed as Green's function. Given a velocity model like AK135 (Kennett et al. 1995), the Green's function can be calculated. Otherwise, if there are one-order less magnitude aftershocks with a similar focal mechanism to the mainshock, the waveforms of the aftershocks are suitable as empirical Green's function (EGF, Hartzell 1978). The slip models of the earthquakes can be obtained by fitting the observations on the basis of the known instrumental responses and Green's function using linear or nonlinear inversion algorithms. These slip models can provide very important references for post-earthquake rescuing and tsunami warning and show more rupture properties of the earthquakes. Thus, the investigation of the rupture processes of earthquakes plays a significant role in anti-seismic hazard and studying earthquake sources.

Rupture models of earthquakes can be derived by inverting the seismic body waves or surface waves. To stabilize the inversion and make computation effective, several constraints such as smoothing, minimum seismic moment, and non-reverse slip are placed during the inversion. Moreover, the solution of the inversion strongly depends on the initial model. Additionally, the velocity model used in the calculation of the Green's function deviates greatly from the structure of the earth; this leads to systematic or subjective errors in the resultant rupture model.

Besides the seismic data, other kinds of data such as water height of tsunami and displacement of the surface after an earthquake surveyed by GPS or Interferometric Satellite Aperture Radar (InSAR) can be incorporated into the inversion of the rupture model. This can greatly improve the accuracy of the rupture model since multiple datasets provide much more tight constraints for the inversion. Nevertheless, since the global seismic data can be fetched shortly after an earthquake, most of the rupture models are derived from the seismic data.

Also, there is another way, namely generalized array techniques such as back-projection method (BPM, Ishii et al. 2005), to quickly obtain the rupture processes of the earthquakes. The back-projection method can be applied to regional arrays and first used by Ishii et al. (2005) to study the spatial-temporal rupture of the December 26, 2004, Mw 9.2 Sumatra–Andaman earthquake. Actually, the rupture front is tracked by the BPM since the high-frequency seismic waveforms radiated from the rupture front (Madariaga 1977, 1983) are used by the BPM. Because the locations and rupture times of the rupture front can be traced, the rupture dimensions as well as rupture velocity can be accurately estimated.

Compared with the rupture models derived by the inversions, the back-projection imaging has advantages as follows. First, the higher-frequency data were used. Thus, the back-projection imaging can provide more rupture details in theory. Second, less known information is required to carry out the imaging procedure. Third, relative back-projection imaging depends less on the velocity model, since all the subevents are imaged relative to the epicenter. Given the advantages listed above and the easy operation, the back-projection method has been widely used to study the rupture processes of large earthquakes such as the 2004 Mw 9.2 Sumatra–Andaman earthquake (Ishii et al. 2005, 2007; Kruger and Ohrnberger, 2005; Du et al. 2009; Liu et al. 2007), the 2010 Mw 8.8 Chilean earthquake (Kiser and Ishii, 2011; Wang and Mori 2011; Liu et al. 2010), the 2011 Mw 9.0 Tohoku, Japan, earthquake (Ishii 2011; Kiser and Ishii 2012; Koper et al. 2011a, b; Maercklin et al. 2012; Meng et al. 2012a; Wang and Mori 2011; Roten et al. 2012; Yagi et al. 2012; Yao et al. 2011, 2012; Zhang et al. 2011), and the 2012 Mw 8.6 Sumatra offshore earthquake (Meng et al. 2012c; Satriano et al. 2012; Yue et al. 2012; Zhang et al. 2012). The researches on these large earthquakes dramatically renew one's conventional understanding on earthquake sources, such as frequency-dependent rupture, segmentation, and multiple-fault rupture, which will be discussed in the following chapters.

The advantages of the BPM make it complementary to the finite fault modeling in the rupture of earthquakes. For the inversion of the slip model, rupture velocity is strongly coupled with slip rates of fault patches (Spudich and Frazer 1984). To address this issue, the rupture velocity can be accurately estimated by the BPM. Thus, providing the rupture velocity derived by the BPM can lead to a better estimate to the slip models of the earthquakes using finite fault modeling.

Although the back-projection method has been prevalently applied to image the rupture processes of the earthquakes, there still exist several issues in the back-projection imaging. The most prominent issue is the imaging artifacts, which could lead to an interpretation of fake rupture. The artifacts mainly have two

sources: One is the subevents releasing more high-frequency energy, which results in imaging “swimming,” and the other one is the multiples or depth phases from earlier subevents. To suppress the first kind of artifacts, a relative back-projection method (RBPM) is proposed in this book. For the second kind of artifacts, a combination of back-projection imaging from multiple arrays can be used to mitigate this kind of artifacts (Zhang et al. 2016a, 2017; Zhang and Ge 2016a, b).

To demonstrate the effectiveness of the RBPM to suppress “swimming” artifacts, the principle of the beamforming used in the back-projection imaging is first deduced. Next, five of the most used generalized array techniques are introduced. More importantly, the procedure of the RBPM is elaborated. Finally, using the synthesized waveforms, the temporal and spatial resolutions of the RBPM are explored and the imaging results derived by the RBPM are compared with those from the traditional BPM.

2.1 Principle of Beamforming

Large earthquakes have a duration of 100 s up to 500 s and a rupture length of several hundred kilometers up to 1300 km (Ammon et al. 2005; Lay et al. 2005; Ishii et al. 2005). The rupture process of a large earthquake can be simulated by a series of subevents rupturing in distinct locations at different rupture times (Ge and Chen 2008). All the subevents can be tracked by the back-projection method, and the rupture process of the earthquake is thus imaged. The key technique used in the back-projection method is slant stacking. To demonstrate the effectiveness of the BPM, the principle of the slant stacking is elaborated as follows.

Assuming that a large earthquake occurred, the seismic waves from the earthquake are recorded by an array composed of M stations $\{\vec{x}_k, k = 1, \dots, M\}$ as shown in Fig. 2.1. According to the displacement representation theorem by Aki

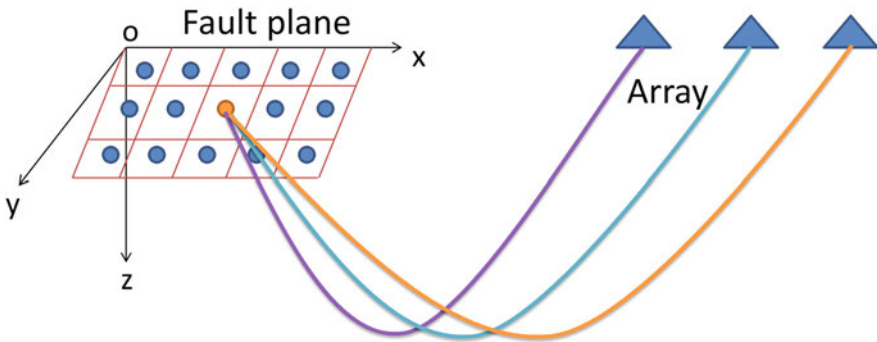


Fig. 2.1 Discretization of a fault plane. When some subfault (orange dot) ruptures, the radiated waves are recorded by an array composed of multiple seismometers

and Richards (2002), dislocation on the fault plane $\sum$ generates the displacement recorded at the k th station $\vec{x}_k$ as

$$u_n(\vec{x}_k, t) = \int_{-\infty}^{+\infty} d\tau \iint_{\sum} \left[u_i(\vec{\xi}, \tau) \right] c_{ijpq} v_j \frac{\partial}{\partial \xi_q} G_{np}(\vec{x}_k, t - \tau; \vec{\xi}, 0) d\sum, \quad (2.1)$$

where $u_n(\vec{x}_k, t)$ is the displacement along the n direction recorded at the station $\vec{x}_k$, $u_i(\vec{\xi}, \tau)$ is dislocation at $\vec{\xi}$ along the direction j , c_{ijpq} is the fourth-order elastic tensor, v_j is the projection to the direction j of the unit normal vector of the fault, and $G_{np}(\vec{x}_k, t - \tau; \vec{\xi}, 0)$ is the Green's function, which is the displacement along the direction n at the station $\vec{x}_k$ at the moment $t - \tau$ generated by the point force exerting at $\vec{\xi}$ on the fault along the direction p at 0 s.

Provided that Green's function is independent of time, the representation theorem can be expressed as

$$u_n(\vec{x}_k, t) = \int_{-\infty}^{+\infty} d\tau \iint_{\sum} \left[u_i(\vec{\xi}, \tau) \right] c_{ijpq} v_j \frac{\partial}{\partial \xi_q} G_{np}(\vec{x}_k, t; \vec{\xi}, \tau) d\sum, \quad (2.2)$$

where Green's function $G_{np}(\vec{x}_k, t; \vec{\xi}, \tau)$ is the displacement along the direction n at the station $\vec{x}_k$ at the moment t generated by the point force exerting at $\vec{\xi}$ on the fault along the direction p at the moment τ .

There are two end-members to describe heterogeneous dynamic rupture on the fault. One is the barrier model proposed by Das and Aki (1977), Aki (1984), and the other one is the asperity model proposed by Lay and Kanamori (1981), Rudnicki and Kanamori (1981). Both models show that the faults ruptured by earthquakes are not totally a flat plane, and elastic coefficients as well as fault strength are unevenly distributed on the fault. Thus, elastic coefficient tensor and unit normal vector of the fault plane are related to $\vec{\xi}$ as $c_{ijpq}(\vec{\xi})$ and $v_j(\vec{\xi})$. After substituting the moment density tensor $m_{pq}(\vec{\xi}, t) = u_i(\vec{\xi}, \tau) c_{ijpq} v_j$ (Aki and Richards 2002), the displacement at the station $\vec{x}_k$ can be expressed as

$$u_n(\vec{x}_k, t) = \iint_{\sum} m_{pq}(\vec{\xi}, t) * G_{np,q}(\vec{x}_k, t; \vec{\xi}) d\sum, \quad (2.3)$$

where $G_{np,q}(\vec{x}_k, t; \vec{\xi}) = \frac{\partial}{\partial \xi_q} G_{np}(\vec{x}_k, t; \vec{\xi})$.

The data used in back-projection are velocity seismograms, which have more high-frequency component relative to displacement. The high-frequency component

of the seismic data is what be used in the back-projection. The velocity record at the station $\vec{x}_k$ is a derivative of displacement as

$$v_n(\vec{x}_k, t) = \iint \sum \frac{d}{dt} \{m_{pq}(\vec{\xi}, t) * G_{np,q}(\vec{x}_k, t; \vec{\xi})\} d\sum, \quad (2.4)$$

where $v_n(\vec{x}_k, t) = \dot{u}_n(\vec{x}_k, t)$.

Since the convolution has a differentiating property (Xu et al. 2005) as

$$\frac{d}{dt} [f_1(t) * f_2(t)] = \frac{df_1(t)}{dt} * f_2(t),$$

where $f_1(t)$ and $f_2(t)$ are as a function of time, the velocity can be rewritten as

$$v_n(\vec{x}_k, t) = \iint \sum \dot{m}_{pq}(\vec{\xi}, t) * G_{np,q}(\vec{x}_k, t; \vec{\xi}) d\sum, \quad (2.5)$$

where $\dot{m}_{pq}(\vec{\xi}, t) = \frac{d}{dt} \{m_{pq}(\vec{\xi}, t)\}$.

The rupture of large earthquakes can be simulated by slip evolving with time on a series of subfaults (Ge and Chen 2008). The discretization of the fault plane into multiple subfaults is the typical strategy of the seismic inversion for the rupture models (Kikuchi and Kanamori 1982; Olsen and Apsel 1982; Hartzell and Heaton 1983). Similarly, the back-projection is also taking the same strategy to process the source region. Thus, the fault plane $\sum$ is discretized into N subfaults with an area of ΔA . Thus, the velocity at the station $\vec{x}_k$ can be expressed as

$$v_n(\vec{x}_k, t) = \sum_{i=1}^N \dot{m}_{pq}(\vec{\xi}_i, t) * G_{np,q}(\vec{x}_k, t; \vec{\xi}_i) \Delta A. \quad (2.6)$$

The moment tensor of the i th subfault is expressed as

$$M_{pq}(\vec{\xi}_i, t) = m_{pq}(\vec{\xi}_i, t) \Delta A.$$

The expression of the velocity can be changed as

$$v_n(\vec{x}_k, t) = \sum_{i=1}^N \dot{M}_{pq}(\vec{\xi}_i, t) * G_{np,q}(\vec{x}_k, t; \vec{\xi}_i). \quad (2.7)$$

Another seismic approach to study discontinuities in the earth's interior is teleseismic P wave receiver function. For example, the P wave receiver function has been used to clearly resolve the crustal structure of the North America Mid-Continent Rift (Zhang et al. 2016b). The principle of the receiver function

indicates the teleseismic direct P wave can be thought as an impulse source for its multiples and the Green's function can be derived by deconvolving the radial component of the waveforms with the vertical component (Langston 1976); this indicates that the Green's function of the direct P waves can be expressed with Dirac- δ function. The spatial derivative of the Green's function can be decomposed to be summation of direct P waves and multiples (Yao et al. 2012) as

$$G_{np,q}(\vec{x}_k, t; \vec{\xi}_i) = G_{np,q}^d(\vec{x}_k; \vec{\xi}_i) \delta(t - t_{ik}) + G_{np,q}^m(\vec{x}_k, t; \vec{\xi}_i),$$

where $G_{np,q}^d(\vec{x}_k, t; \vec{\xi}_i)$ and $G_{np,q}^m(\vec{x}_k, t; \vec{\xi}_i)$ are the spatial derivatives of the direct P waves and multiples, respectively, and t_{ik} is the travel time from the source $\vec{\xi}_i$ to the station $\vec{x}_k$.

Replacing the Green's function in Eq. 2.7 with the above equation, the velocity can be expressed as

$$v_n(\vec{x}_k, t) = \sum_{i=1}^N \left\{ G_{np,q}^d(\vec{x}_k; \vec{\xi}_i) \dot{M}_{pq}(\vec{\xi}_i, t - t_{ik}) + v_n^m(\vec{x}_k, t) \right\}, \quad (2.8)$$

where $v_n^m(\vec{x}_k, t)$ is the waveforms without the direct P waves.

Besides the multiples, there are background or man-made noises in the waveforms. This means that noises need to be included in the waveforms as

$$v_n^m(\vec{x}_k, t) = \sum_{i=1}^N \left\{ G_{np,q}^m(\vec{x}_k, t; \vec{\xi}_i) * \dot{M}_{pq}(\vec{\xi}_i, t) \right\} + n(\vec{x}_k, t),$$

where $n(\vec{x}_k, t)$ is the noise at the k th station. Thus, even if incorporating the noise term, the expression of the velocity still stands.

The focal mechanism on the subfault is constant when the dimensions of the subfault are small enough. Under this assumption, the derivative of moment tensor relative to time can be expressed as

$$\dot{M}_{pq}(\vec{\xi}_i, t) = M_{pq}(\vec{\xi}_i) \dot{s}(\vec{\xi}_i, t), \quad (2.9)$$

where $\dot{s}(\vec{\xi}_i, t)$ is the rate of the source time function at the subfault $\vec{\xi}_i$.

Chen and Gu (2008) provide an expression of the source time function as

$$s(t) = \begin{cases} 0, & t < 0, \\ 1, & t \rightarrow \infty. \end{cases}$$

For a subfault $\vec{\xi}_i$, the rise time of the rupture T_s^i is limited. Different subfaults have distinct initial rupture time t_i^0 . Without loss of generality, the source function can be set to a ramp function as

$$s(\vec{\xi}_i, t) = \begin{cases} 0, & t < t_i^0, \\ \frac{t}{T_s^i}, & t_i^0 \leq t \leq t_i^0 + T_s^i, \\ 1, & t > t_i^0 + T_s^i. \end{cases}$$

The derivative of the source time function with respect to time is derived as

$$\dot{s}(\vec{\xi}_i, t) = \begin{cases} 0, & t < t_i^0, \\ \frac{1}{T_s^i}, & t_i^0 \leq t \leq t_i^0 + T_s^i, \\ 0, & t > t_i^0 + T_s^i. \end{cases}$$

Substituting the Heaviside function into the rate of source time function, the above equation can be changed to

$$\dot{s}(\vec{\xi}_i, t) = \frac{H(t - t_i^0)H(-t + t_i^0 + T_s^i)}{T_s^i}, \quad (2.10) \quad (2.10)$$

where $H(t)$ is Heaviside function as

$$H(t) = \begin{cases} 0, & t < 0, \\ 1, & t \geq 0. \end{cases}$$

Substituting the Eq. 2.10 into the Eq. 2.9, the time derivative of the moment tensor can be expressed as

$$\dot{M}_{pq}(\vec{\xi}_i, t) = \frac{M_{pq}(\vec{\xi}_i)H(t - t_i^0)H(-t + t_i^0 + T_s^i)}{T_s^i}. \quad (2.11)$$

After incorporating the Eq. 2.11 into the Eq. 2.8, the velocity at the station $\vec{x}_k$ can be deduced to

$$v_n(\vec{x}_k, t) = \sum_{i=1}^N \{A(\vec{x}_k, \vec{\xi}_i)H(t - t_{ik} - t_i^0)H(-t + t_{ik} + t_i^0 + T_s^i)\} + v_n^m(\vec{x}_k, t), \quad (2.12)$$

where $A(\vec{x}_k, \vec{\xi}_i) = \frac{G_{np,q}^d(\vec{x}_k; \vec{\xi}_i)M_{pq}(\vec{\xi}_i)}{T_s^i}$, which is the amplitude of the direct P waves.

Before conducting beamforming or slant stacking, the velocity record for each station is normalized with the maximum amplitude $A_{\max}(\vec{x}_k)$. The normalized velocity records can be expressed as

$$\bar{v}_n(\vec{x}_k, t) = \sum_{i=1}^N \{ \bar{A}(\vec{x}_k, \vec{\xi}_i) H(t - t_{ik} - t_i^0) H(-t + t_{ik} + t_i^0 + T_s^i) \} + \bar{v}_n^m(\vec{x}_k, t), \quad (2.13)$$

where $\bar{v}_n(\vec{x}_k, t) = v_n(\vec{x}_k, t)/A_{\max}(\vec{x}_k)$, $\bar{A}(\vec{x}_k, \vec{\xi}_i) = A(\vec{x}_k, \vec{\xi}_i)/A_{\max}(\vec{x}_k)$, and $\bar{v}_n^m(\vec{x}_k, t) = v_n^m(\vec{x}_k, t)/A_{\max}(\vec{x}_k)$. For the subfault l , the seismograms at M stations are back-projected and stacked up. The beam thus is formed as

$$B(\vec{\xi}_l, t) = \frac{1}{M} \sum_{k=1}^M \sum_{i=1}^N \{ \bar{A}(\vec{x}_k, \vec{\xi}_i) H(t + t_{lk} - t_{ik} - t_i^0) H(-t - t_{lk} + t_{ik} + t_i^0 + T_s^i) \} + \frac{1}{M} \sum_{k=1}^M \bar{v}_n^m(\vec{x}_k, t + t_{lk}), \quad (2.14)$$

where t_{lk} is the travel time from the l th subfault to the k th station, and $B(\vec{\xi}_l, t)$ is the beam back-projected at the l th subfault $\vec{\xi}_l$.

The difference between the travel times of the direct P waves from the i th subfault $\vec{\xi}_i$ and the i th subfault $\vec{\xi}_i$ to the station $\vec{x}_k$ is distinct. Back-projection is to reverse the time of the waveforms back to the initial rupture time of the subfault $\vec{\xi}_l$. The waveforms at different stations would be completely coherent when the l th subfault $\vec{\xi}_l$ is just the i th subfault $\vec{\xi}_i$. This back-projection operation significantly suppresses the beam of the multiples; this leads to negligibility of the second term of the Eq. 2.14. In this case, the beam of the velocity records can be expressed as

$$\begin{aligned} B(\vec{\xi}_l, t) &= \frac{1}{M} \sum_{k=1}^M \sum_{i=1}^N \{ \bar{A}(\vec{x}_k, \vec{\xi}_i) H(t + t_{lk} - t_{ik} - t_i^0) H(-t - t_{lk} + t_{ik} + t_i^0 + T_s^i) \} \delta_{il}, \\ &= \left\{ \frac{1}{M} \sum_{k=1}^M \bar{A}(\vec{x}_k, \vec{\xi}_l) \right\} H(t - t_l^0) H(-t + t_l^0 + T_s^l), \\ &= \left\{ \frac{1}{M} \sum_{k=1}^M \frac{G_{np,q}^d(\vec{x}_k; \vec{\xi}_l) M_{pq}(\vec{\xi}_l)}{T_s^l A_{\max}(\vec{x}_k)} \right\} H(t - t_l^0) H(-t + t_l^0 + T_s^l), \\ &= \left\{ \frac{1}{M} \sum_{k=1}^M \frac{G_{np,q}^d(\vec{x}_k; \vec{\xi}_l) M_{pq}(\vec{\xi}_l)}{A_{\max}(\vec{x}_k)} \right\} \frac{H(t - t_l^0) H(-t + t_l^0 + T_s^l)}{T_s^l}, \\ &= \bar{A}^d(\vec{\xi}_l) \dot{s}(\vec{\xi}_l, t), \end{aligned} \quad (2.15)$$

where δ_{il} is Kronecker delta function, and $\bar{A}^d(\vec{\xi}_l)$ is a scale factor associated with the generalized array at the l th subfault $\vec{\xi}_l$. This proves that the beam of the waveforms is the product of the derivative of the source time function and the array scale factor. Thus, the subfault can be effectively tracked by the back-projection method, which has been proved above in theory.

2.2 Typical Generalized Array Approaches

The theoretical proof of the beamforming described above confirms the reliability of these array techniques as beamforming is fundamental to the generalized array techniques. Most of the data used by the generalized array techniques are the direct P waves because the P waves arrive earlier at some station. The earlier arrivals facilitate picking-up of the arrival time of the first P waves; this is significantly important for calibrating the travel times through one-dimensional velocity model to the heterogeneous three-dimensional earth's interior.

Generalized array techniques can be classified into several categories on the basis of the wave phases or the domain in which the techniques are exploited. From the view of wave phase, teleseismic direct P waves were first used to study the 2004 Mw 9.2 Sumatra–Andaman earthquake (Ishii et al. 2005; Krüger and Ohrnberger 2005) and were the most kind of data used until now (Walter et al. 2005; Xu et al. 2009; Zhang and Ge 2010; Kiser and Ishii 2011; Koper et al. 2011a, b; Wang and Mori 2011; Meng et al. 2011; Yao et al. 2012; Zhang and Ge 2014). Besides the direct P waves, the direct S or Pn waves were used to investigate the local or regional earthquakes (Allmann and Shearer 2007; Meng et al. 2012b). Additionally, the fundamental surface waves were employed to image the rupture process of the earthquakes (Yue et al. 2012; Roten et al. 2012), though the resolutions of surface waves are less than those of the body waves. Meanwhile, the generalized array techniques can also be grouped into time-domain type (Ishii et al. 2005; Walter et al. 2005; Xu et al. 2009; Zhang and Ge 2010; Roten et al. 2012; Yagi et al. 2012) and frequency-domain type (Goldstein and Archuleta 1991; Meng et al. 2011; Yao et al. 2011).

Each kind of generalized array techniques has their own advantages. Five techniques including traditional back-projection method, cross-correlation back-projection, surface-wave back-projection method, multiple signal classification method, and compression sensing will be introduced as follows.

2.2.1 Traditional Back-Projection Method

Ishii et al. (2005) were the first ones to apply the traditional back-projection method to study the rupture process of the 2004 Mw 9.2 Sumatra–Andaman earthquake using teleseismic P waves, though the array techniques had been used in the research of rupture of local and regional earthquakes (Bolt et al. 1982; Spudich and Cranswick 1984; Goldstein and Archuleta 1991; Huang 2001). The traditional back-projection method (TBPM) is based on beamforming, but with a significant improvement to eliminate the assumption of plane wave impingement, this leads to usage of the TBPM for large teleseismic events instead of local or regional earthquakes. The procedure of the TBPM was elaborated by Ishii et al. (2007) and will be briefly introduced below.

At first, the source region of an earthquake is gridded into N potential subevents. The travel time of P waves from a potential subevent to some station $\{\vec{x}_k, k = 1, \dots, M\}$ can be calculated with a one-dimensional velocity model such as AK135 (Kennett et al. 1995). However, the real earth's interior is heterogeneous. A time correction term has to be added to compensate this deviation of earth's structure. The first P arrivals generated by the subevent at the hypocenter can be accurately picked up. Thus, taken the hypocenter as a reference, the time calibration at the station $\vec{x}_k$ can be estimated as

$$\delta t_k = t_k^o - t_{ek}^c,$$

where t_k^o is the observed P travel time of the hypocenter to the station $\vec{x}_k$, and t_{ek}^c is the travel time of the hypocenter to the station $\vec{x}_k$ calculated with a given velocity model. For each potential subevent, the travel times to the stations can be calculated. After the time calibration being incorporated, the direct P waves are back-projected by converting the recording time to the rupture time. For each rupture time, the waveforms are summed up to obtain the beams. Thus, the beam generated by the i th potential subevent can be expressed as

$$B(\vec{\zeta}_i, t) = \frac{1}{M} \sum_{k=1}^M \alpha_k v(\vec{x}_k, t + t_{ik}^c + \delta t_k), \quad (2.16)$$

where $\alpha_k = \frac{p_k}{A_k^{\max}} w_k$, p_k is the polarity of the P waves at the station $\vec{x}_k$, $A_k^{\max}$ is the maximum absolute amplitude of the k th waveform, and w_k is the weight of the k th waveform such as signal-to-noise ratio (SNR), $v(\vec{x}_k, t)$ is the velocity at the station $\vec{x}_k$, and t_{ik}^c is the calculated travel time of the i th potential subevent to the station $\vec{x}_k$.

It is inappropriate to only use the initial time calibration to compensate the one-dimensional velocity model to the real earth, when the studied earthquakes have very large rupture dimensions such as the 2004 Mw 9.2 Sumatra-Andaman earthquake (Ammon et al. 2005; Ishii et al. 2005; Lay et al. 2005). Relative to the large mainshock, the aftershocks have much shorter durations and may have sharper first arrivals. Since the aftershocks are typically distributed all over the fault of the mainshock, the arrivals of the aftershocks can be used as the time calibrations when there are enough aftershocks. Thus, given L aftershocks, the time calibrations of the i th potential subevent to the station $\vec{x}_k$ (Ishii et al. 2007) can be estimated as

$$\delta t_{ik} = \frac{\sum_{j=1}^L \{w_j \delta t_{jk} / \Delta_{ij}\}}{\sum_{j=1}^L \{w_j / \Delta_{ij}\}},$$

where w_j is the weight of the j th aftershock, Δ_{ij} is the distance of the j th aftershock to the i th potential subevent, and $\delta t_{jk} = t_{jk}^o - t_{jk}^c$ is the time calibration of the j th aftershock to the station $\vec{x}_k$, t_{jk}^o and t_{jk}^c are the observed and calculated travel times of the direct P waves generated from the j th aftershock to the station $\vec{x}_k$, respectively.

Using the more accurate time calibrations, the beam at the i th potential subevent is formed as

$$B(\vec{\xi}_i, t) = \frac{1}{M} \sum_{k=1}^M \alpha_k v(\vec{x}_k, t + t_{ik}^c + \delta t_{ik}) = \frac{1}{M} \sum_{k=1}^M \bar{v}_{ik}(t), \quad (2.17)$$

where $\bar{v}_{ik}(t)$ is weighted, normalized velocity record at the station $\vec{x}_k$.

To reduce the interference of coda of the earlier-ruptured subevents, the N th root stacking technique (Muirhead 1968; Kanasewih et al. 1973) was applied by Xu et al. (2009). The N th root stacking can be expressed as

$$B(\vec{\xi}_i, t) = |B'(\vec{\xi}_i, t)|^{1/N} \cdot \text{sign}\{B'(\vec{\xi}_i, t)\}, \quad (2.18)$$

where $B'(\vec{\xi}_i, t) = \frac{1}{M} \sum_{k=1}^M |\bar{v}_{ik}(t)|^{1/N} \cdot \text{sign}\{\bar{v}_{ik}(t)\}$, $\text{sign}\{\cdot\}$ is sign function, and N is an integer. When $N = 1$, Eq. 2.18 is the same as Eq. 2.17; this indicates the linear slant stacking is a special case of the N th root stacking.

A large earthquake can be thought as a series of subevents (Ge and Chen 2008). The power of the beam at the i th potential subevent in the l th time window with a length of T_w can be expressed as

$$P_{il} = \frac{1}{T_w} \int_{T_l}^{T_l + T_w} |B(\vec{\xi}_i, t)|^2 dt, \quad (2.19)$$

where $T_l = (l - 1)T_s$, and T_s is the step of the moving time window. For each step, the potential subevent with the maximum power is identified as the real subevent. Thus, the rupture front evolving with rupture time is obtained.

2.2.2 Cross-Correlation Back-Projection Method

Cross-correlation is prevalent in signal processing such as communication, industrial seismology, and signal recognition and relocation (Shear 1997). This technique can be understood in both time and frequency domains (Anstey 1964). In time domain, the cross-correlation function indicates the similarity of one waveform to another one. In frequency time, the cross-correlation function of one waveform is equivalent to a filter with the same amplitude spectra, but the opposite phase spectra to the reference waveform.

Fletcher et al. (2006) are the first ones to apply the cross-correlation back-projection method to image the rupture propagation of the 2004 Parkfield

earthquake using a local, small-aperture array nearby the earthquake. For the small-aperture array, the impinging waves can be thought to be plane waves (Schweitzer et al. 2002). The delay time of the plane wave impinging into the array between two stations in the array can be expressed as

$$\tau_{ij} = \vec{s} \cdot \vec{r}_{ij} + \delta t_i - \delta t_j, \quad (2.20)$$

where $\vec{s} = (s_E, s_N, s_z)$ is slowness vector in a Cartesian coordinate, $\|\vec{s}\| = 1/c$, c is velocity of the phase used, $\vec{r}_{ij}$ is the distance vector between stations i and j , and δt_i and δt_j are the travel time calibrations of the stations i and j . The cross-correlation between the waveforms at the two stations can be written as

$$cc_{ij} = \frac{\sum_t [v(\vec{x}_i, t) - \bar{v}(\vec{x}_i)] [v(\vec{x}_j, t) - \bar{v}(\vec{x}_j)]}{\sqrt{\sum_t [v(\vec{x}_i, t) - \bar{v}(\vec{x}_i)]^2 [v(\vec{x}_j, t) - \bar{v}(\vec{x}_j)]^2}}, \quad (2.21) \quad (2.21)$$

where $\bar{v}(\vec{x}_i)$ and $\bar{v}(\vec{x}_j)$ are the means of the waveforms at the stations i and j , respectively.

For a horizontal slowness vector (s_E^p, s_N^q) , the vertical slowness can be obtained as

$$s_z = \sqrt{\frac{1}{c^2} - s_E^p \cdot s_E^p - s_N^q \cdot s_N^q}.$$

For each station, the delay time can be estimated with the Eq. 2.20 and the cross-correlation coefficient can be also derived with the Eq. 2.21. The cross-correlation coefficient of the whole array can be obtained by averaging those of all the stations as

$$\overline{cc}_{pq} = \frac{1}{M(M-1)} \sum_{k=1}^M \sum_{j=1}^M cc_{kj}(s_E^p, s_N^q), \quad i \neq j. \quad (2.22)$$

At each time step, the plane wave with the horizontal slowness makes the array cross-correlation coefficient maximum stem from the real subevent of the earthquake. According to the azimuth of the subevent and apparent velocity of the plane wave, the location of the subevent can be obtained. Thus, the subevents of the earthquake evolving with time can be derived.

Besides the cross-correlation between the waveforms at different stations, the theoretical Green's function can also be used to calculate the cross-correlation coefficient with the waveforms at the stations (Yagi et al. 2012). This technique can eliminate the interference of depth phases (pP and sP) if the Green's function is correct. However, it is really hard to obtain correct high-frequency Green's function because it requires a high-resolution velocity model.

For the Green's function, the cross-correlation coefficient with the observations can be expressed as

$$cc(\vec{x}_i, \vec{\xi}_j, t) = \frac{\int_{t_w} v(\vec{x}_i, \tau + t) g(\vec{x}_j, \vec{\xi}_i, \tau) d\tau}{V_i \sqrt{\int_{t_w} g^2(\vec{x}_j, \vec{\xi}_i, \tau) d\tau}}, \quad (2.23)$$

where $V_i = \max\left(\sqrt{\int_w v^2(\vec{x}_i, \tau + t)}, 0 \leq t \leq t_w\right)$, t_w is the time length of the waveforms, and $g(\vec{x}_j, \vec{\xi}_i, \tau)$ is Green's function. The beam of the cross-correction functions can be derived as

$$B(\vec{\xi}_i, t) = |B'(\vec{\xi}_i, t)|^{1/N} \cdot \text{sign}\{B'(\vec{\xi}_i, t)\}, \quad (2.24)$$

where $B'(\vec{\xi}_i, t) = \frac{1}{M} \sum_{k=1}^M |cc(\vec{x}_i, \vec{\xi}_j, t)|^{1/N} \cdot \text{sign}\{cc(\vec{x}_i, \vec{\xi}_j, t)\}$. The power of the beam can be expressed the same as the Eq. 2.31. Similarly, the subevent at each time step is identified with the maximum power. Thus, the rupture of the earthquake proceeding with time can be imaged.

2.2.3 Multiple Signal Classification

Multiple signal classification (MUSIC) was proposed by Schmidt (1986) to discriminate the sources with different direction of arrivals (DOA). Goldstein and Archuleta (1991) applied this approach in frequency-wavenumber domains to study the earthquake sources using a local, small-aperture array. As the number of global stations increases, the MUSIC has been used to study the rupture processes of large earthquakes (Meng et al. 2011, 2012c).

The imaging procedure of the MUSIC is demonstrated as follows. First of all, the waveforms (M) in the first time window (with a length of t_w) are aligned to the first direct P arrivals in a manually pick way or using a multiple-channel cross-correlation approach (Vandecar and Crosson 1990). The covariance matrix of the M waveforms in the l th time window can be expressed as

$$R_{pq}^l = \left\langle v_l(\vec{x}_p, t) v_l^\dagger(\vec{x}_p, t) \right\rangle_t,$$

where $\langle \cdot \rangle_t$ indicates an operator averaging time and $\dagger$ is the Hermitian conjugate operator. Next, the covariance matrix R^l will be solved to obtain the eigenvectors.

It is reasonable to assume that the R^l has Q eigenvectors ($Q < M$) and λ_q is the q th eigenvector. If the wave field is decomposed into plane waves, the eigenvectors of the seismic covariance matrix R^l become the propagation vectors of the plane waves (Goldstein and Archuleta 1987). Thus, with respect to the hypocenter, the plane waves generated by the q th subevent have a phase shift as

$$\mathbf{u}(\hat{\xi}_q) = \left[e^{i\omega(t_{q1}-t_{e1})}, \dots, e^{i\omega(t_{qM}-t_{eM})} \right]^T,$$

where ω is circular frequency, T is transpose of a matrix, and t_{qk} and t_{ek} are travel times of the k th station to the hypocenter ξ_e and the q th subevent $\hat{\xi}_q$.

Assuming that the background noise is a random variable normally distributed with a variance of σ^2 , the seismic covariance matrix $\mathbf{R}^l$ can be decomposed as

$$\mathbf{R}^l = \mathbf{U}\mathbf{S}\mathbf{U}^\dagger + \sigma^2\mathbf{I},$$

where $\mathbf{U} = (\mathbf{u}(\hat{\xi}_1)^T \dots \mathbf{u}(\hat{\xi}_Q)^T 0 \dots 0)$ is a $M \times M$ matrix, $\mathbf{I}$ is a unit matrix,

$$\mathbf{S} = \begin{pmatrix} \lambda_1^2 & 0 & \dots & \dots & \dots & 0 \\ 0 & \ddots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \lambda_Q^2 & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & 0 & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 \end{pmatrix}.$$

The Q eigenvectors are composed of the signal subspace. The left $M - Q$ eigenvectors represent the noise subspace. The signal eigenvectors are orthogonal to the noise eigenvectors as

$$\mathbf{e}_p \cdot \mathbf{u}(\hat{\xi}_q) = 0, p = Q + 1, \dots, M; q = 1, \dots, Q.$$

The seismic covariance matrix $\mathbf{R}^l$ can thus be decomposed to the summation of the signal and the noise as

$$\mathbf{R}^l = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^\dagger + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^\dagger,$$

where $\mathbf{E}_s = (\mathbf{u}(\hat{\xi}_1)^T \mathbf{u}(\hat{\xi}_2)^T \dots \mathbf{u}(\hat{\xi}_Q)^T)$ is a $M \times Q$ matrix,

$$\mathbf{\Lambda}_s = \begin{pmatrix} \lambda_1^2 & 0 & \dots & \dots & 0 \\ 0 & \ddots & \dots & \dots & \vdots \\ \vdots & \vdots & \lambda_p^2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \lambda_Q^2 \end{pmatrix}.$$

The control vector from the i th subevent (M subevents in total) can be expressed as

$$\mathbf{a}(\hat{\xi}_i) = [e^{i\omega(t_{i1}-t_{i1})}, \dots, e^{i\omega(t_{iM}-t_{iM})}]^T.$$

To resolve the arrival direction of the impinging signal, a pseudo-spectrum can be derived with signal directional vectors and noise vectors (Meng et al. 2011) as

$$P_m^l(\xi_i) = \left| \frac{\mathbf{a}^\dagger(\xi_i)\mathbf{a}(\xi_i)}{\mathbf{a}^\dagger(\xi_i)\mathbf{E}_n\mathbf{E}_n^\dagger\mathbf{a}(\xi_i)} \right|.$$

The pseudo-spectrum reaches the maximum when the $\mathbf{a}(\xi_i)$ is the directional vector of the real signal. After the azimuth of the signal is determined, the power spectrum can be derived through the signal subspace of the covariance matrix (Mestre et al. 2007) as

$$P_m^l(\xi_i) = \left| \frac{\mathbf{a}^\dagger(\xi_i)\mathbf{a}(\xi_i)}{\mathbf{a}^\dagger(\xi_i)\mathbf{E}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{E}_s^\dagger\mathbf{a}(\xi_i)} \right|. \quad (2.25) \quad (2.25)$$

For each time step, the location and power of the subevent relative to the hypocenter can be resolved as described above. Thus, the rupture process of the earthquake evolving with time is imaged.

2.2.4 Compressive Sensing

Compressive sensing is a recent technique processing images on the basis of resampling or sampling of data (Donoho 2006; Candes et al. 2006). The principle of the technique is to use data points as few as possible to recover the original signal; this is realized with l_1 norm having a special property of sparsity. Before the compressive sensing, $l_p(p \leq 1)$ norm regularization was already widely used to resolve the arrival directions of sparse signals using antenna arrays (Gorodnitsky and Rao 1997; Fuchs 2001; Malioutov et al. 2005). Only limited subfaults slip at some moment during the rupture of an earthquake. This indicates there exists sparsity in the rupture process of the earthquake, which suffices the requirement of compressive sensing. The 2011 Tohoku earthquake is the first one studied by Yao et al. (2011) using compressive sensing.

The procedure to apply the compressive sensing is described below. First of all, the source region is gridded into N subfaults $(\xi_1, \dots, \xi_N)$. The waveforms radiated from an earthquake are recorded at M stations $(\mathbf{x}_1, \dots, \mathbf{x}_N)$. Next, the waveforms in the first time window are aligned to let the epicenter be imagined as the first

subevent. The waveforms can be decomposed into plane waves in the frequency. The velocity records at the k th station can be expressed as

$$v(\mathbf{x}_k, \omega) = A(\mathbf{x}_k, \xi_i, \omega)s(\xi_i, \omega) + n(\mathbf{x}_k, \omega),$$

where $v(\mathbf{x}_k, \omega)$ is the Fourier transformation of the velocity records at the k th station in frequency domain, $A(\mathbf{x}_k, \xi_i, \omega) = e^{i\omega\Delta t_{ik}}$ is phase offset of the waveforms back-projected to the i th subfault relative to the hypocenter ξ_e , $\Delta t_{ik} = t_{ik} - t_{ek}$ is the difference between the travel times of the k th station to the i th subfault and the hypocenter, $s(\xi_i, \omega)$ is the source time function of the i th subfault ξ_i , and $n(\mathbf{x}_k, \omega)$ is background noise.

The Eq. 2.50 can be expressed in a vector way as

$$\mathbf{v}(\omega) = \mathbf{A}(\omega)\mathbf{s}(\omega) + \mathbf{n}(\omega),$$

where $\mathbf{v}(\omega) = [v(\mathbf{x}_1, \omega), \dots, v(\mathbf{x}_M, \omega)]^T$, $\mathbf{s}(\omega) = [s(\mathbf{x}_1, \omega), \dots, s(\mathbf{x}_N, \omega)]^T$, and

$$\mathbf{A}(\omega) = \begin{pmatrix} e^{i\omega\Delta t_{11}} & e^{i\omega\Delta t_{12}} & \dots & e^{i\omega\Delta t_{1N}} \\ e^{i\omega\Delta t_{21}} & e^{i\omega\Delta t_{22}} & \dots & e^{i\omega\Delta t_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{i\omega\Delta t_{M1}} & \dots & \dots & e^{i\omega\Delta t_{MN}} \end{pmatrix}.$$

The number of subfaults is usually larger than the number of stations, namely $N > M$. This indicates the equation is under-determined and should have infinite solutions. Since there is sparsity in the earthquake source, l_1 norm of the model can be used as regularization. Moreover, the optimal problem with the regularization of the l_1 norm is convex and always has global optimal solutions (Malioutov et al. 2005). Providing that the solutions are sparse enough and the basis is over-complete, the solutions to the optimal problem are stable, even if there are significant noises contaminating the signals. The optimal problem can be constructed as

$$\min(\|\mathbf{d}(\omega) - \mathbf{A}(\omega)\mathbf{s}(\omega)\|_2 + \lambda\|\mathbf{s}(\omega)\|_1),$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ represent the l_1 and l_2 norms, respectively, and λ is regularization factor. The solution to the aforementioned optimal problem can be expressed as

$$\hat{\mathbf{s}}(\omega) = \arg[\min(\|\mathbf{d}(\omega) - \mathbf{A}(\omega)\mathbf{s}(\omega)\|_2 + \lambda\|\mathbf{s}(\omega)\|_1)].$$

Since it is a second-order convex problem, the optimal problem can be solved using interior point solvers (IPS, Boyd and Vandenberghe 2004). The aligned waveforms can be divided into L segments, and each segment has a length of T_w . For each segment, the location and rupture time of the subevent can be obtained by

solving the optimal problem described above. Thus, the rupture process of the earthquakes can be imaged.

2.2.5 Surface-Wave Back-Projection Method

Most of the back-projection studies are conducted using high-frequency body waves. For some large earthquakes or local earthquakes, long period surface waves can also be used to image the rupture processes (Yue et al. 2012; Roten et al. 2012). For local or regional earthquakes, the head waves may arrive earlier than the direct waves, and the direct S waves are followed by the large-amplitude Lg waves. Using the typical direct P waves, the imaging results would be significantly influenced by the other phases. However, surface waves have much larger amplitudes than the body waves. The imaging from the surface waves would not be affected by the preceding waves, though the resolution of the surface-wave imaging is very low.

Surface waves are dispersed, and the surface waves with different frequencies have distinct phase velocities. Wavelet transformation can be applied to extract the surface wave at a certain frequency f_s . The wavelet transformation of the waveforms at the k th station can be expressed as

$$wt(\mathbf{x}_k, s, \tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} v(\mathbf{x}_k, t) \psi^* \left(\frac{t - \tau}{s} \right) dt,$$

where $\psi^* \left(\frac{t - \tau}{s} \right)$ is the conjugate of the wavelet and s is the scaling factor.

Without loss of generality, frequency B-spline is used as the wavelet (Teolis 1988):

$$\psi_B(\tau) = \sqrt{f_b} \left[\sin c \left(\frac{f_b t}{p} \right) \right]^p e^{i2\pi f_c t},$$

where f_b is band of the frequency, f_c is the central frequency of the wavelet, p is an integer greater than or equal to 2, and $\sin c(x) = \frac{\sin(x)}{x}$, $x \in R$. The scaling factor can be obtained with the following equation

$$s = \frac{f_c}{f_s}.$$

For the i th subfault, the waveforms after wavelet transferred can be back-projected to the source region and the beam is formed as

$$B(\xi_i, s, \tau, f_s) = \left| \sum_{k=1}^M w_{ik} wt(\mathbf{x}_k, s, \tau + t_{ik}(f_s)) \right|^2,$$

where τ is rupture time, w_{ik} is the weight in terms of the i th subfault and k th station, and $t_{ik}(f_s)$ is travel time of the surface wave with a frequency of f_s . For each time step, the location of the subevent can be obtained by solving the optimal problem as follows:

$$\hat{s}(f_s) = \arg[\max(B(\xi_i, s, \tau, f_s), i = 1, \dots, N)].$$

Thus, the rupture processes of the earthquakes can be imaged.

2.3 Relative Back-Projection Method

The back-projection methods introduced above have their own features. The first two are similar to the traditional back-projection method, and the second two can be used to identify directions of the signals. The traditional back-projection method is simple, fast, and efficient, but the imaging results severely suffer from the artifacts like “swimming.” The latter two approaches are more robust, but may consume more computational resources.

To mitigate the “swimming” artifacts, we modified the traditional back-projection to add a reference station. This is equivalent to add a reference window for the stacking (Meng et al. 2011) and facilitate to suppress the “swimming” artifacts caused by the trade-off between the location and rupture times. The effect of mitigating artifacts will be demonstrated with several synthetic tests in the following section. The improved method is called relative back-projection method (RBPM) due to the reference station used in the method. Moreover, the P th root stacking technique having more power in suppressing noise was incorporated into the RBPM (Zhang and Ge 2010).

The workflow of the RBPM with the P th root stacking is presented as follows (Fig. 2.2). First of all, the source region is gridded into N potential subevents. The reference station $\mathbf{x}_r$ was used to calculate the difference between the travel times of the i th potential subevent ξ_i to the k th station $\mathbf{x}_k$ and to the reference station

$$\Delta t_{ik}^r = t_{ik}^c - t_{ir}^c,$$

where t_{ik}^c and t_{ir}^c are the travel times of the potential subevent ξ_i to the k th station $\mathbf{x}_k$ and to the reference station $\mathbf{x}_r$, respectively. The time calibration of the k th station $\mathbf{x}_k$ is used to compensate the derivation of the one-dimensional velocity model from the heterogeneous earth and is expressed as

$$\delta t_k^r = t_k^0 - t_r^0 - t_{ek}^c + t_{er}^c,$$

where t_k^0 and t_r^0 are the observed arrival times at the k th station $\mathbf{x}_k$ and at the reference station $\mathbf{x}_r$, respectively, t_{ek}^c and t_{er}^c are the theoretical travel times of the epicenter to the k th station $\mathbf{x}_k$ and to the reference station $\mathbf{x}_r$, respectively. The

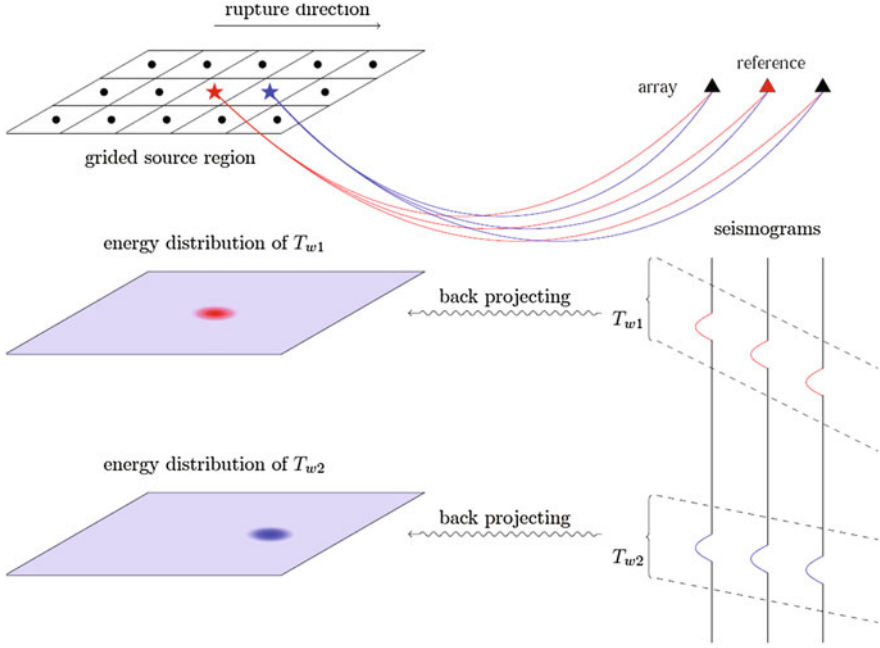


Fig. 2.2 Cartoon showing the back-projection imaging. First, the source region is gridded into smaller blocks (black dots). The red and blue stars indicate two sources at two moments, respectively. The signals generated by the two sources are recorded at the array with a reference station (red triangle). Next, the seismograms are back-projected to the gridded source region with the theoretical travel times. Thus, the power distributions at the two moments can be derived. The powers at the two sources are the maximum (red and blue smearing areas) at the moments when the sources occurred, respectively

velocity records were back-projected and stacked with the P th root stacking to obtain the beam by

$$B(\xi_i, t) = |B'(\xi_i, t)|^{1/N} \cdot \text{sign}\{B'(\xi_i, t)\},$$

where $B'(\xi_i, t) = \frac{1}{M} \sum_{k=1}^M |\bar{v}_{ik}^r(t)|^{1/N} \cdot \text{sign}\{\bar{v}_{ik}^r(t)\}$, $\bar{v}_{ik}^r(t) = \alpha_k v(\mathbf{x}_k, t + \Delta t_{ik}^r + \delta t_k^r)$, and α_k is the normalization factor of the k th station $\mathbf{x}_k$.

Assuming that a large earthquake is composed of L subevents with a duration of T_w , the l th subevent can be imaged based on the subevent having the maximum power. For the l th segment, the power of the beam can be expressed as

$$P_{il} = \frac{1}{T_w} \int_{T_l}^{T_l + T_w} |B(\xi_i, t)|^2 dt,$$

where $T_l = t_r^0 + (l-1)T_s$, and T_s is the time step. The l th subevent makes the power being the maximum

$$P_l^s = \max\{P_{il}, i = 1, \dots, N\}.$$

The location of the l th subevent can be identified. The rupture time of the subevent can be obtained using the following equation

$$t_k^{rup} = (l-1)T_s + t_{er}^c - t_{lr}^c.$$

Thus, after the locations and rupture times of all the subevents are obtained, the rupture process of the earthquake is imaged.

2.4 Advantages of the Relative Back-Projection Method

Compared with the traditional back-projection method, the relative back-projection method can be used to mitigate the “swimming” artifacts using a reference station. Moreover, the locations and rupture times of the subevents are resolved sequentially. The improvement of the RBPM can be demonstrated by the rupture model in Fig. 2.3. The rupture model is composed of two sources (S1 and S2). The rupture initiates at the S1 and extends to the S2 at 12 s.

The source causing “swimming” artifacts can be clearly demonstrated in left bottom panel of Fig. 2.3. The three potential subevents (S1–S3) in the rupture model are recorded at the same array with different travel time curves. Using the traditional back-projection method, the waveforms are back-projected to generate very distinct beams at the three potential subevents. Since the sources S1 and S2 are real, the beams back-projected at the two sources have the very large amplitudes due to the coherence of the waveforms as shown in Fig. 2.3b. However, when the waveforms are back-projected to the potential source 3, the beam at 12 s has very large amplitude resulted from the S1, which is larger than that of the beam formed at the potential subevent S2. On the basis of the rule of the maximum power to identify the subevent at each time step, the imaged subevent is the S3 instead of the real subevent S2. This clearly demonstrates the “swimming” artifacts caused by the subevents releasing more high-frequency seismic energy and usually move along the ray path of the array to the sources of the earthquake.

The “swimming” artifacts are caused by the larger-energy subevents during the earthquake. To overcome the influence of these subevents to the smaller subevents, adding a reference station can back-project the waveforms from the larger-energy subevents to all the potential subevents around their real rupture times as shown in Fig. 2.3b. Relative to the traditional BPM, the RBPM uses a reference time window to conduct the back-projection. The waveforms composed of signals from S1 and S2 will be back-projected to the potential subevents S1–S3. Due to high similarity of the waveforms from the same subevents, the beam at the potential subevent S1

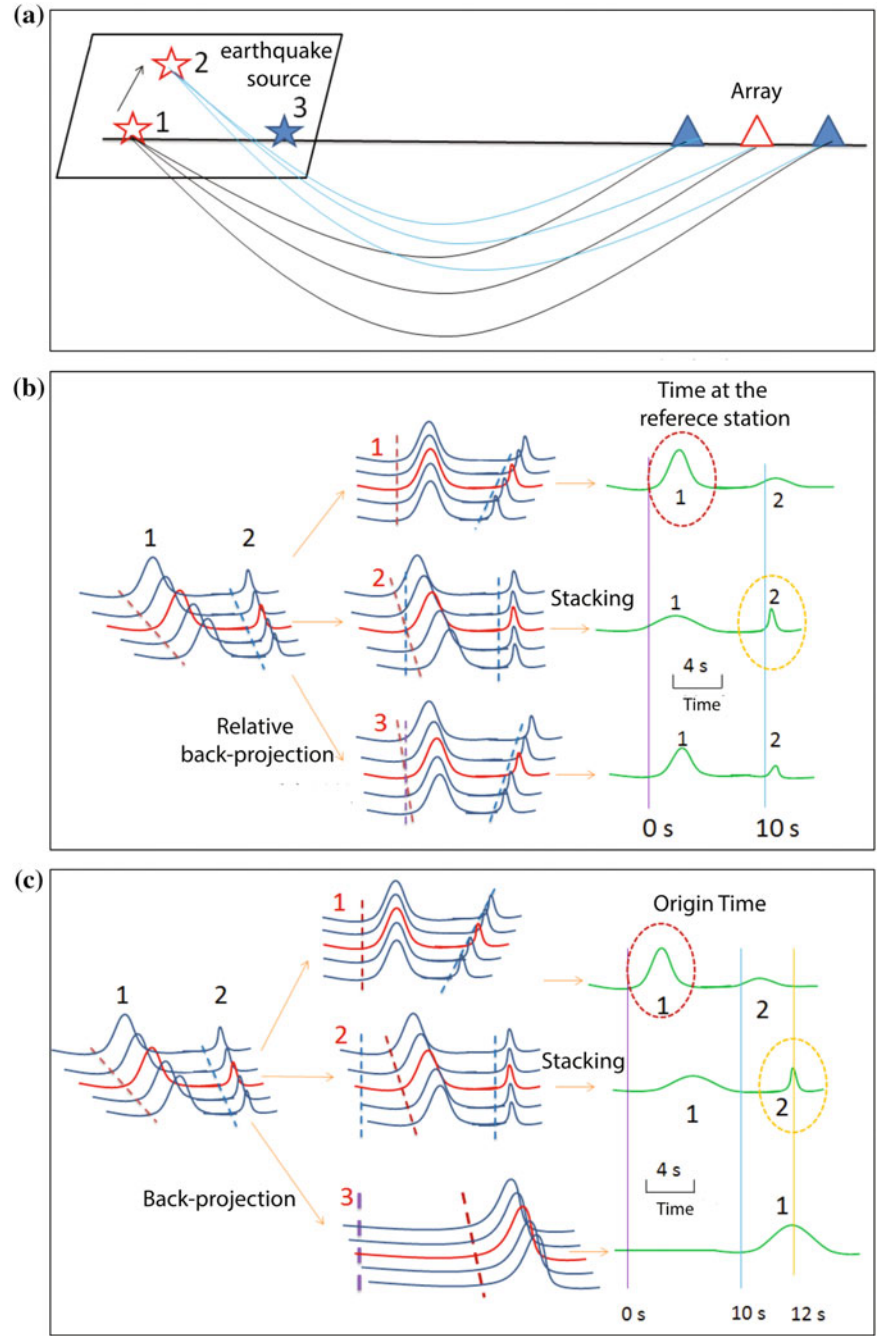


Fig. 2.3 Cartoon showing the difference between the relative back-projection (RBP) and traditional back-projection (BP) imaging. **a** Two source models. One is composed of sources 1 and 2, and the other one is composed of sources 1 and 3. **b** The imaging procedure of the RBP method. **c** The imaging procedure of the traditional BP method

has the maximum amplitude at 0 s, and the beam at the S2 has the maximum amplitude at 12 s. This means that both S1 and S2 at 0 s and 12 s, respectively can be correctly imaged by the RBPM and evidently demonstrate the effectiveness of the RBPM to suppress the “swimming” artifacts relative to the traditional BPM.

2.5 Synthetic Tests and Comparison

Imaging results by the relative back-projection method (RBPM) have less “swimming” artifacts compared to the traditional back-projection. To verify this point, a series of theoretical tests are conducted as follows. Moreover, the tests directly demonstrate the effective scope of the RBPM and sensitivity of parameters. Specifically, a linear array is used to test the resolutions in the range and azimuth directions. Meanwhile, we also test the influences from reference station, rupture velocity, and filter band. Finally, the rupture imaging results derived by the RBPM are compared to those derived by the traditional BPM.

2.5.1 *Unilateral Rupture Tests*

The unilateral rupture of an earthquake is simulated with a series of equal-strength explosion point sources for being easy to analyze the results. To obtain the resolutions in the range and azimuth directions, two simulated earthquakes are designed as shown in Fig. 2.4. The first one commences at the eastern end and ruptured westwards. The second one initiates at the southern end and propagates northwards. The receiver array, orientated in a N-S trend, is composed of 201 stations with a spacing of 0.25° , 60° away from the epicenter.

2.5.1.1 The Westward Rupture Earthquake

The earthquake is simulated with eleven explosion sources at a focal depth 0 km, which rupture westwards sequentially and radiate the same seismic moment to each other. The delay time between two neighboring sources 0.05° apart is set according to the rupture velocity of 1 km/s. The specific parameters of these explosion sources as well as the travel times of the direct P waves to the reference station are presented in Table 2.1.

Given the parameters listed in Table 2.1, the seismograms recorded by the linear array can be synthesized using the reflectivity method (Wang 1999) as shown in

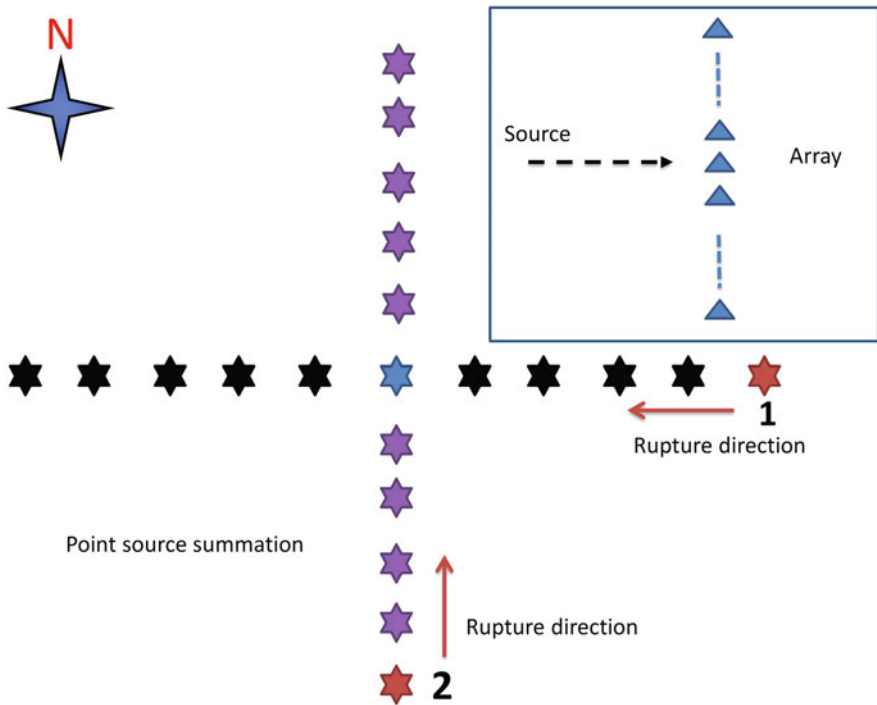


Fig. 2.4 Cartoon of two earthquakes simulated with a series of point sources. The first source model is composed of 11 point sources (black stars) rupturing from east to west, and the second source model is composed of 11 point sources (purple stars) rupturing from south to north. The red stars indicate the epicenters of the two earthquakes. Inset, distribution of a linear N-S striking array, comprising 201 stations (blue triangles)

Fig. 2.5. These vertical-component seismograms are filtered in a frequency band of 0.5–2.5 Hz and aligned with the first *P* arrivals (Fig. 2.5a). The moveouts of the *P* waves generated by the eleven sources are very clear and basically parallel. However, the moveout curve lines from the later ten sources are distorted due to the spatial difference of the ten sources with the first one.

The stacked waveform of the array can be obtained as shown in Fig. 2.5b. It is understandable that the amplitudes of *P* waves from the first source are the largest one close to 1. But the amplitudes of *P* waves from the other sources are all greater than 0.6, though the waveforms are aligned with the *P* waves generated by the first source. This could indicate spatial differences among the point sources may be not very large for the linear array.

The first step of the RBPM is to grid the source region into multiple blocks. The source region centered at the first source is gridded into 101×101 blocks with an interval of 0.01° at both the latitude and longitude. The grid points are the potential subevents. The waveforms in a 4-second-long time window are back-projected to these potential subevents with respect to the center station of the linear array every

Table 2.1 The source model for the simulated earthquake, which ruptures westward

Source #	Latitude (°)	Longitude (°)	Origin time (s)	Time (test 1)	Time (test 2)
1	0.00	0.25	0.00	0.00	0.00
2	0.00	0.20	5.57	5.90	5.90
3	0.00	0.15	11.12	11.80	11.70
4	0.00	0.10	16.68	17.70	17.50
5	0.00	0.05	22.24	23.60	23.40
6	0.00	0.00	27.79	29.50	29.30
7	0.00	−0.05	33.35	35.40	35.10
8	0.00	−0.10	38.91	41.30	41.00
9	0.00	−0.15	44.47	47.20	46.80
10	0.00	−0.20	50.03	53.10	52.70
11	0.00	−0.25	55.59	59.00	58.50

Note The parameters of the eleven point sources composing the simulated earthquake 1. The focal depths of all the point sources are set to 0 km. The fifth and sixth columns are the travel times of the sources to the central and southernmost stations, respectively

one second, and the powers are calculated. After the locations of the subevents are determined, their rupture times can be identified accordingly. Thus, the back-projection imaging results are obtained as shown in Fig. 2.6.

Typically, the area outlined by the 80% contour of the maximum power is thought to be the imaging uncertainty (Kruger and Ohrnberger 2005). All the eleven sources are imaged inside their uncertainty areas. The imaging results show that real sources 1–3 are imaged in a diamond area of the maximum power centered at source 1 (Fig. 2.6a, b, c). For source 4, the imaging area with the maximum power extends westward and includes the location of source 4 (Fig. 2.6d). The imaging area of source 4 is composed of two portions. The first one is similar to that of source 4, and the second one is a small area with the location of source 5 (Fig. 2.6e). The imaging area of source 6 mainly focuses on the given location of source 6 (Fig. 2.6f); this indicates the less interference of the waveforms generated by other sources. For sources 7 and 8, they are imaged inside of the diamond areas with the maximum power as well (Fig. 2.6g, h), but the imaging results are affected relatively more by the waveforms from other sources. The maximum imaging area of both sources 9 and 10 includes the real locations of the two sources, but the imaged source 11 is located at edge of the maximum imaging area (Fig. 2.6i, j, k).

These imaging results provide an estimate on the resolution of 22–28 km in the range direction for the linear array 60° away from the epicenter.

The imaging bias in Fig. 2.6 could be caused by the interference from other sources except for source 1. This is the reason why source 1 is imaged correctly. But the multiples, coda waves, or even direct *P* waves generated by the preceding sources could affect the imaging of the latter sources.

The reference station is necessary for the RMPM. In the preceding test, the center station of the linear array is chosen as the reference station. Now, the southernmost station is chosen as the reference station. Again, the same

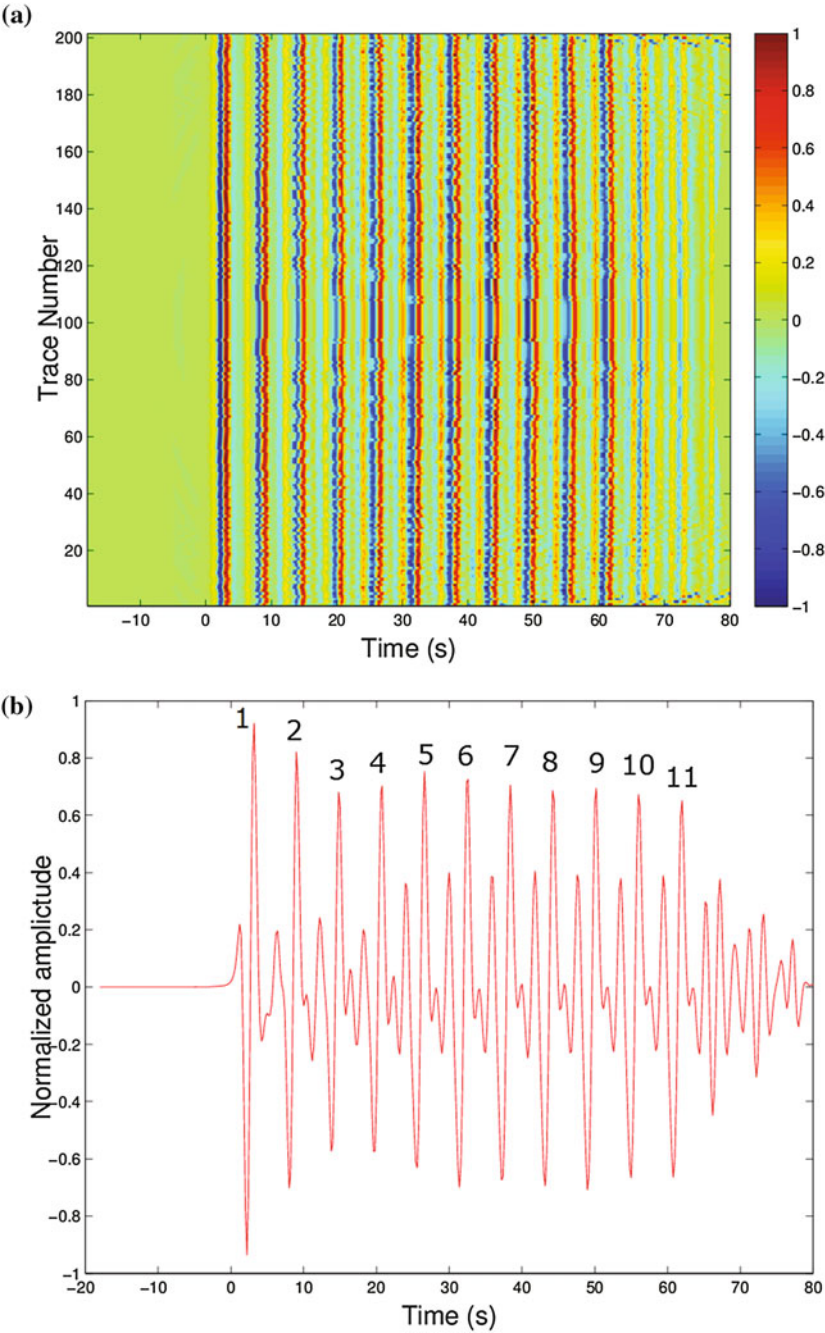


Fig. 2.5 **a** Vertical-component waveforms synthesized for the simulated earthquake 1 and filtered in a frequency range of 0.5–2.5 Hz. The warm color indicates positive amplitudes, whereas the cool color indicates negative amplitude. **b** The beam formed by stacking the waveforms aligned with the first *P* arrivals. The numbers represent the signals from the eleven sources composing the simulated earthquake 1

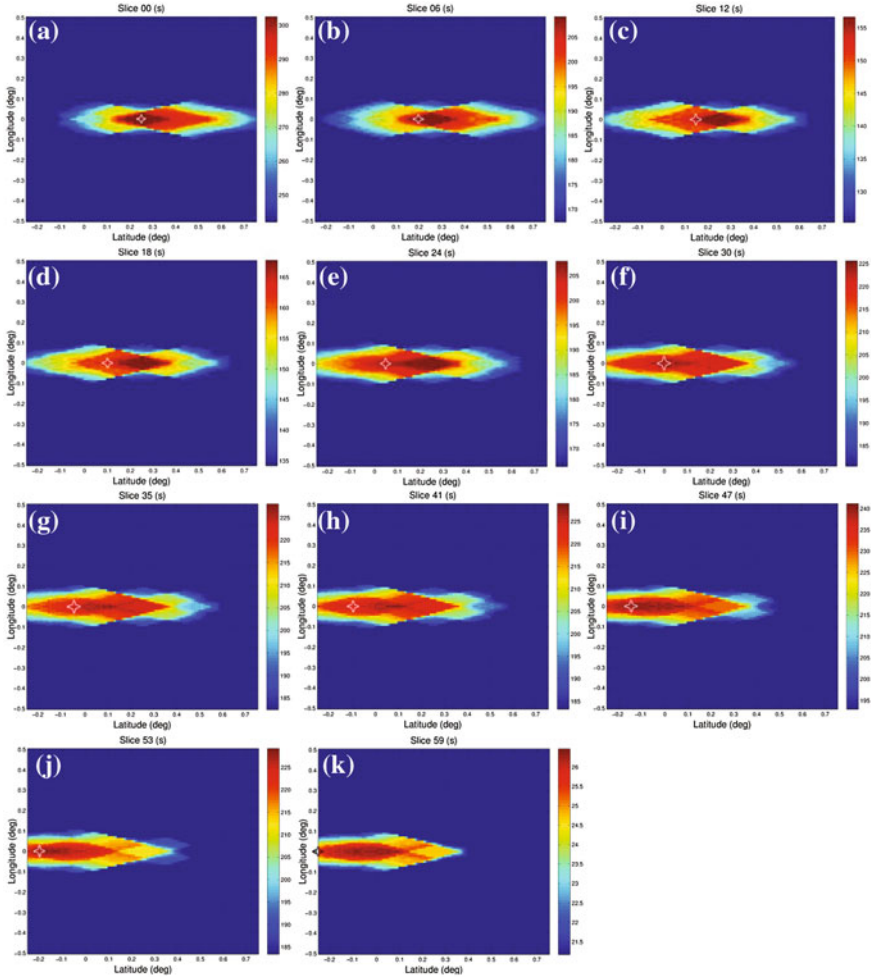


Fig. 2.6 Power distributions derived by the relative back-projection method at the rupture times of the eleven sources composing the simulated earthquake 1. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

back-projection procedure to the preceding test is conducted, and the imaging results are obtained as shown in Fig. 2.7. Thus, we can do the comparison between the two tests with different reference stations.

In the second test, sources 1–3 are imaged in the maximum imaging areas, and sources 4–6 are imaged just outside the maximum imaging areas. For sources 7–10, they are imaged inside the maximum power area. But source 11 is totally

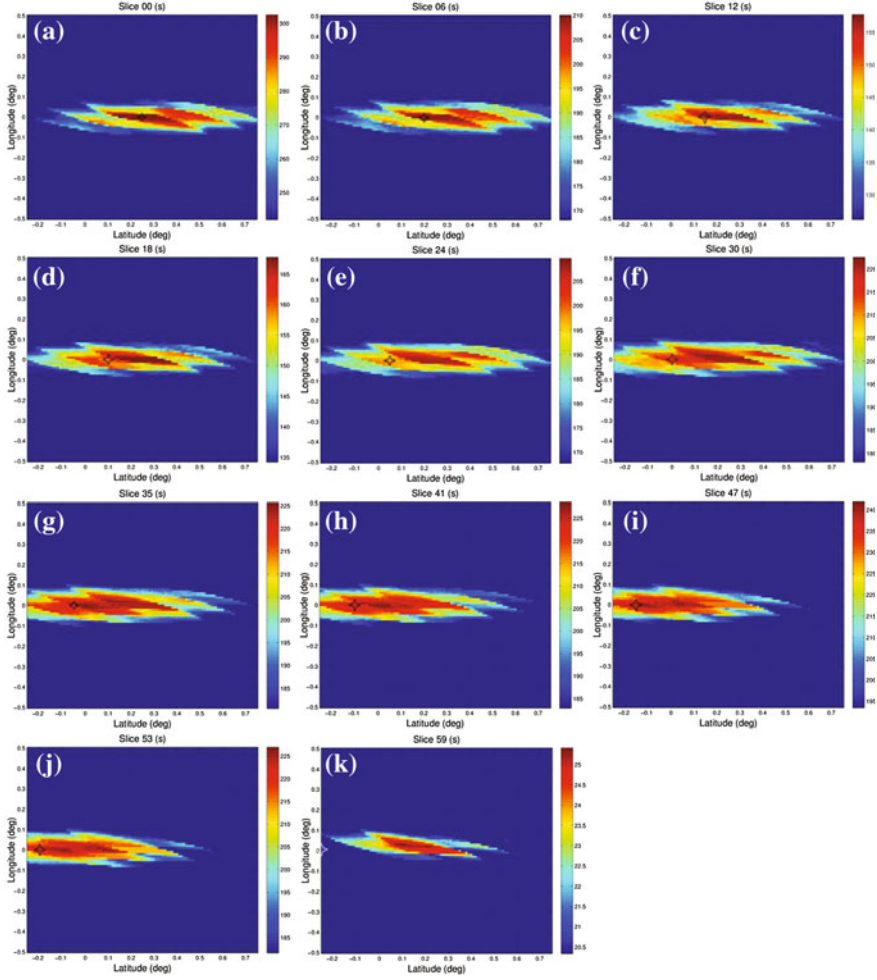


Fig. 2.7 Power distributions derived by the relative back-projection method after changing the southernmost station to the reference one. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

mis-imaged to an area nearby the locations of sources 1–3. Moreover, the imaging area is elongated in the range direction, along which the spatial resolution is 33 km. This strongly suggests the selection of the reference station does affect the imaging results and the center station is a better choice.

2.5.1.2 The Northward Rupture Earthquake

The northward rupture earthquake is also simulated with eleven explosion source at a depth of 0 km but in a N-S trending orientation. Given a rupture velocity of 1 km/s, the specific parameters of the sources are listed in Table 2.2. With these sources, the vertical-component seismograms are synthesized and filtered in a frequency band of 0.5–2.5 Hz. After they are aligned with the first *P* arrivals of source 1, these waveforms are plotted as shown in Fig. 2.8a. The moveouts of the *P* waves from sources 2–10 are not parallel to that from source 1, which is different from the test on the westward rupture earthquake (Fig. 2.5). This difference is further confirmed by significant decrease in the amplitudes of the stacked waveforms for sources 7–11 (Fig. 2.8b). This indicates the N-S striking array has more spatial sensitive in the azimuth direction than in the range direction.

The *P* waves are back-projected with the same parameters as used in the preceding tests. The imaging results are obtained as shown in Fig. 2.9. The imaging results show that all the sources are imaged at or near the real locations of these sources. The imaging of the later sources is much less affected by the preceding sources relative to the westward rupture earthquake. This agrees with the aforementioned analysis to the synthesized waveforms. The spatial resolution of the N-S linear is about 5.5 km in the azimuth direction, which is much higher than that in the range direction.

2.5.1.3 Frequency-Dependent Test

Since the back-projection imaging has higher resolution for the N-S trending linear array, the northward rupture earthquake is used thereafter to carry out the left tests.

Table 2.2 The source model for the simulated earthquake, which ruptures northward

Source #	Latitude (°)	Longitude (°)	Origin time (s)	Arrival time (s)
1	−0.25	0.00	0.00	0.00
2	−0.20	0.00	5.52	5.50
3	−0.15	0.00	11.04	11.00
4	−0.10	0.00	16.56	16.60
5	−0.05	0.00	22.08	22.10
6	0.00	0.00	27.61	27.60
7	0.05	0.00	33.12	33.10
8	0.10	0.00	38.65	38.70
9	0.15	0.00	44.17	44.20
10	0.20	0.00	49.70	49.70
11	0.25	0.00	55.22	55.20

Note: The parameters of the eleven point sources composing the simulated earthquake 2. The focal depths of all the point sources are set to 0 km. The fifth column is the travel times of the sources to the central station

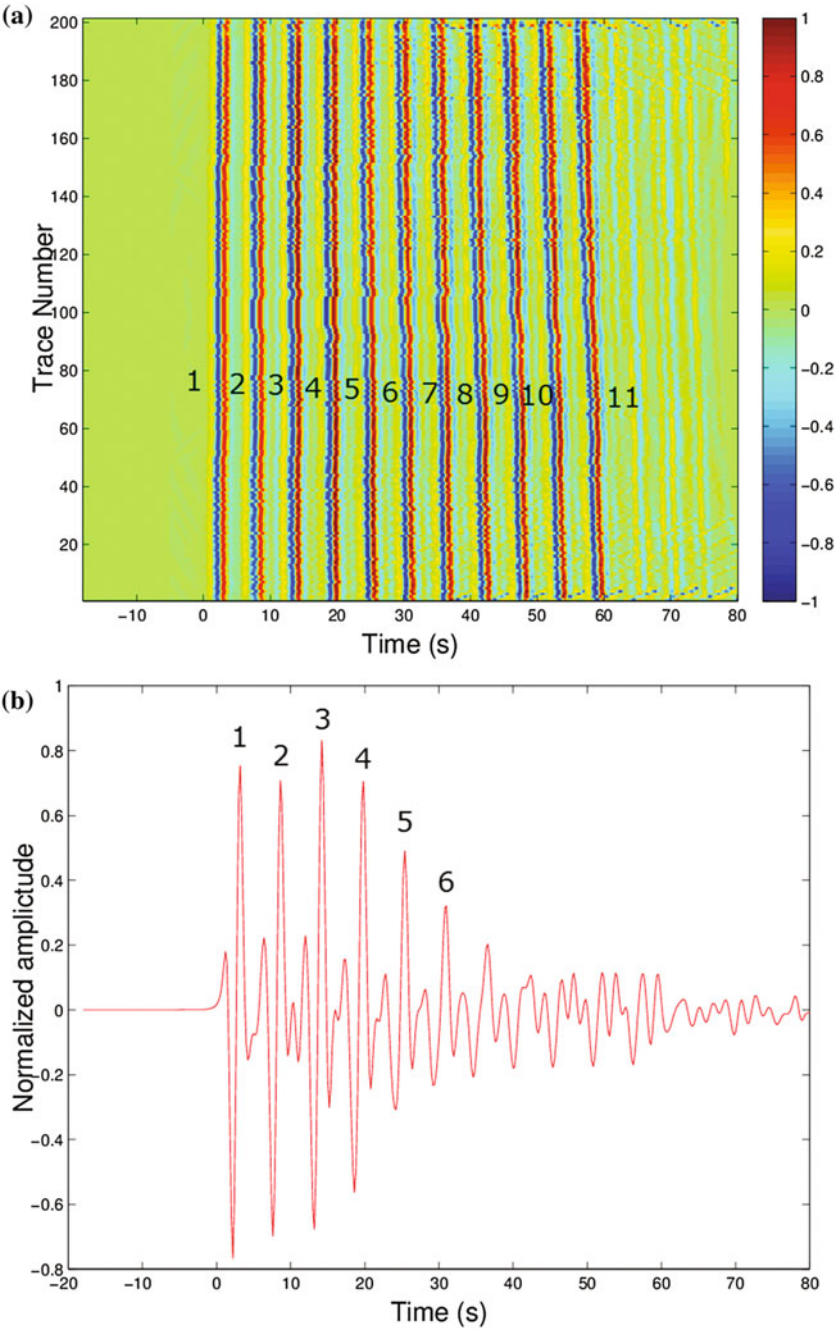


Fig. 2.8 **a** Vertical-component waveforms synthesized for the simulated earthquake 2 and filtered in a frequency range of 0.5–2.5 Hz. The warm color indicates positive amplitudes, whereas the cool color indicates negative amplitude. **b** The beam formed by stacking the waveforms aligned with the first *P* arrivals. The numbers represent the signals from the first six sources composing the simulated earthquake 2

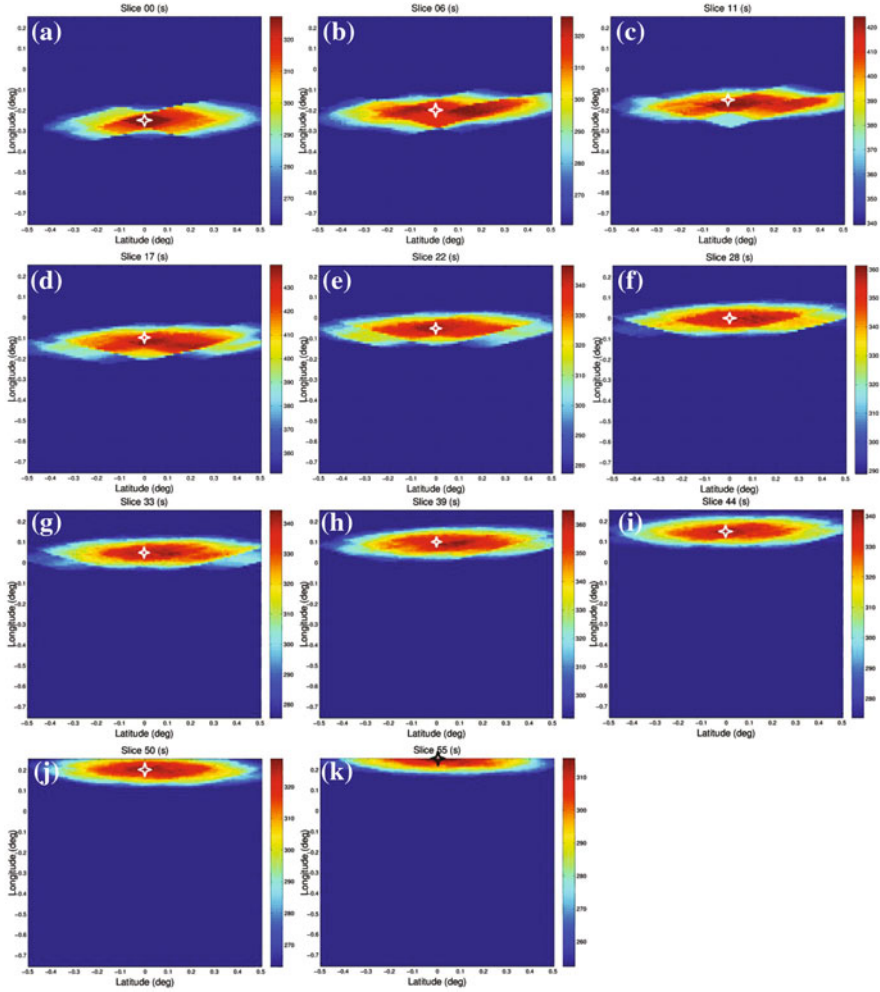


Fig. 2.9 Power distributions derived by the relative back-projection method at the rupture times of the eleven sources composing the simulated earthquake 2. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

Using the back-projection method, a new feature called frequency dependent has been discovered in the rupture of the subduction zone megathrust earthquakes (e.g., Kiser and Ishii 2011; Koper et al. 2011a; Lay et al. 2012; Yao et al. 2013). The frequency-dependent rupture indicates that the high-frequency sources are located deeper than or at the edges of the low-frequency rupture sources. To check whether the imaging results agree with the high-frequency ones, we image the northward rupture earthquake in a low-frequency band of 0.1–0.5 Hz. This facilitates judging whether this frequency-dependency feature is real or caused by the method itself.

The waveforms are simulated with the eleven sources rupturing from south to north at a speed of 1 km/s. The waveforms filtered in the frequency band of 0.1–0.5 Hz are aligned with the first *P* waves as shown in Fig. 2.10a. The longer period of the waveforms makes the interference of the earlier sources to the later sources stronger. Specifically, the amplitudes of the *P* waves generated by the eleven sources decay more gently than the higher-frequency case as shown in the stacked waveform (Figs. 1.8b and 1.10b). As demonstrated in the comparison of the back-projection between range and azimuth directions, this indicates the lower-frequency back-projection has lower resolution.

For the lower-frequency test, the parameters are the same as the higher-frequency back-projection expect for the moving time window with a length of 5 s. These waveforms are back-projected to the source region. The rupture process of the simulated earthquake is imaged as shown in Fig. 2.11.

The imaging results show that all the eleven sources are imaged in the maximum power areas. Specifically, sources 1–4 are imaged at the centers of the maximum power areas (Fig. 2.11a–d), which coincide with the real locations of these sources, respectively. This means these sources are imaged correctly without any bias. However, sources 5–11 are imaged at left edge of the maximum power area. This indicates imaging of these sources is affected by multiples or the direct *P* waves by the preceding sources. Interestingly, the imaging power is mainly elongated in E-W direction. Thus, the imaging of the sources in the N-S direction is not affected by the interference from other sources. This indicates the frequency band used to filter waveforms would not bias the location the sources if the arrays used have appropriate configurations and enough stations. This strongly supports the argument that the frequency-dependent feature discovered in the rupture of the megathrust subduction zone earthquakes is real (Lay et al. 2012; Yao et al. 2013).

An important thing is that the rise times of the explosion sources are 1 s, which agrees with the frequency band of the data used in the back-projection (0.1–2.5 Hz). For data with frequency beyond this scope, the results of the preceding tests should be applied to do analysis with caution.

2.5.1.4 Test on Rupture Velocity

Rupture velocity is a very important parameter for an earthquake and is typically in a range of 1.0–3.5 km/s. The preceding tests are based on a slow rupture velocity of 1.0 km/s. Next, we will carry out a back-projection test on the northward rupture

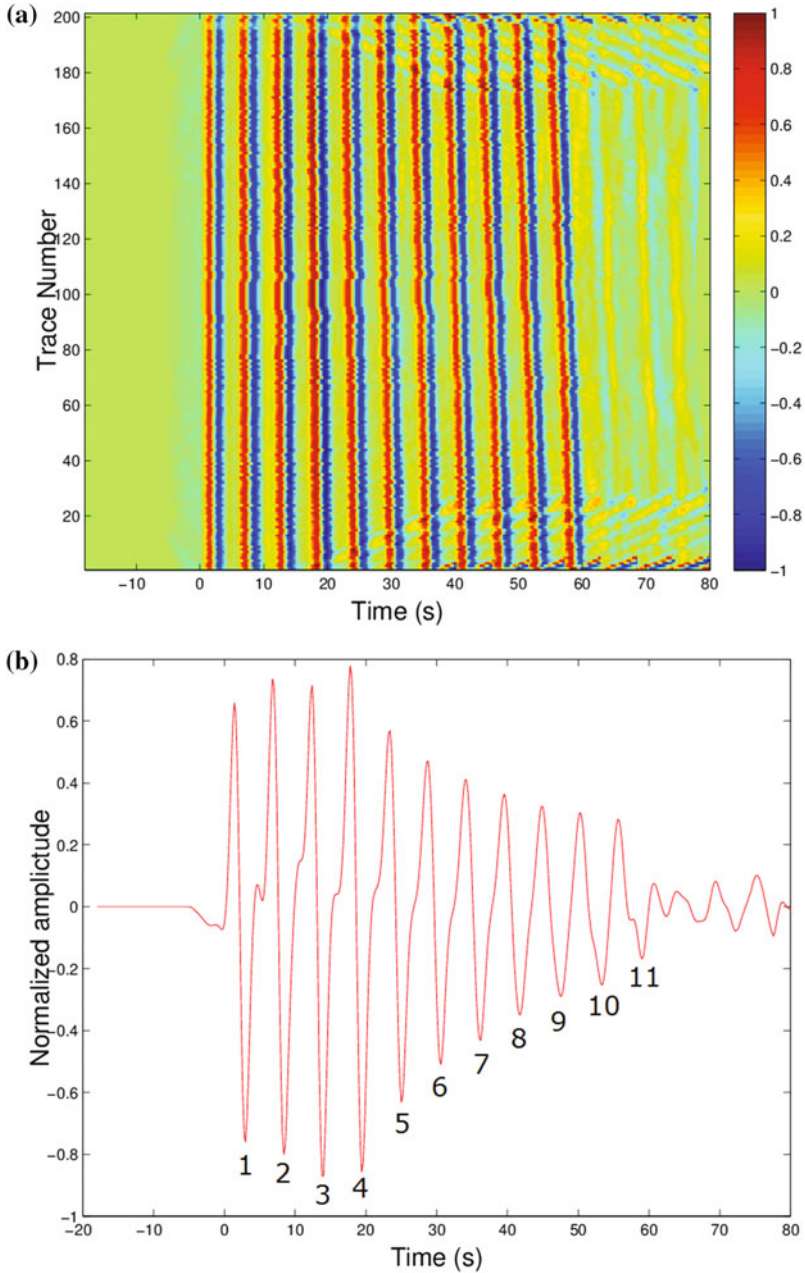


Fig. 2.10 **a** Vertical-component waveforms synthesized for the simulated earthquake 2 and filtered in a low-frequency range of 0.1–0.5 Hz. The warm color indicates positive amplitudes, whereas the cool color indicates negative amplitude. **b** The beam formed by stacking the waveforms aligned with the first *P* arrivals. The numbers represent the signals from the eleven sources composing the simulated earthquake 2

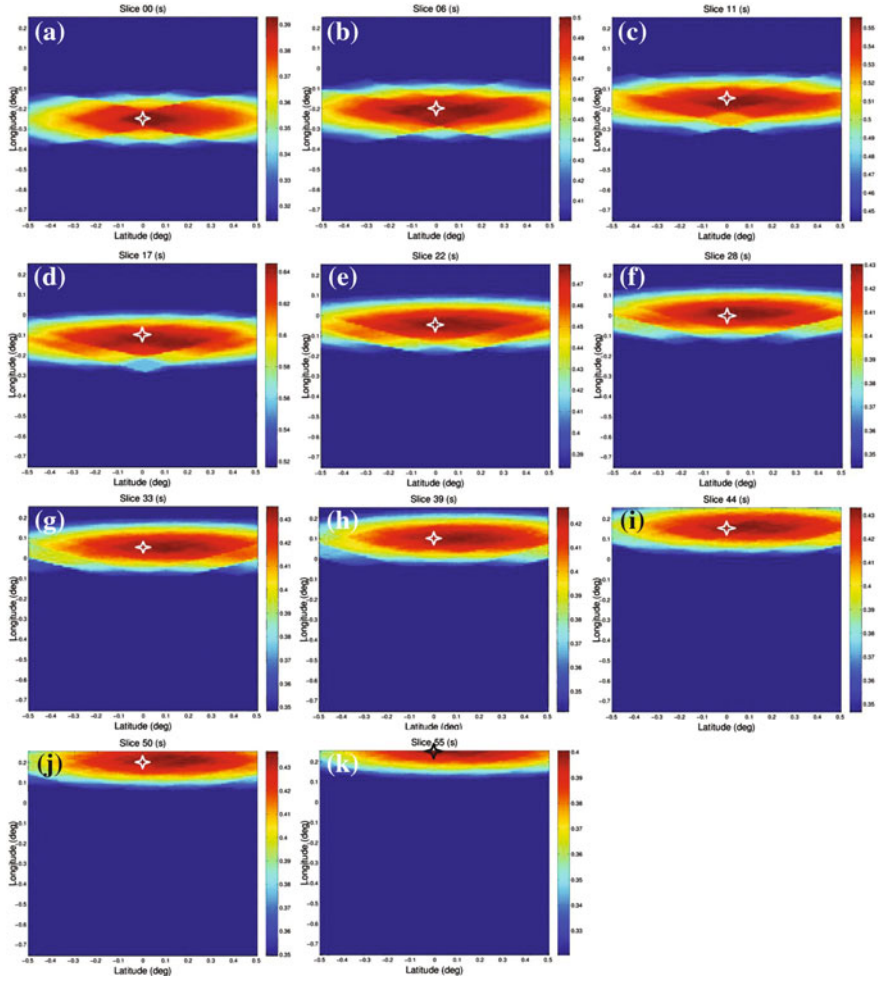


Fig. 2.11 Low-frequency power distributions derived by the relative back-projection method at the rupture times of the eleven sources composing the simulated earthquake 2. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

earthquake (Fig. 2.4) using a high rupture velocity of 3 km/s. Thus, the comparison between high- and low-velocity cases can test the effect of the rupture velocity on the back-projection imaging.

Using the rupture model comprising the eleven sources, the waveforms are synthesized and then filtered in a frequency band of 0.5–2.5 Hz. After they are aligned with the first time window, the stacked beams are obtained by summing up the aligned waveforms as shown in Fig. 2.12. The P arrivals from these sources have less time intervals and the interval becomes shorter and shorter from south to north. Moreover, the multiples have larger amplitudes. Since the time length of the filtered P waves is 2–3 s and the rupture time offset between two neighboring sources is ~ 2 s, the P waves from the two sources overlap with each other. The overlapping of the P waves could lead to the interference of the imaging.

Since the rupture velocity is faster, the time step is changed to 0.5 s. Other parameters are set to the same. After applying the relative back-projection procedure described above, the rupture of the simulated earthquake can be imagined as shown in Fig. 2.13.

The imaging results show that the P waves generated by the early-ruptured sources interfere the imaging of the later ruptured sources. The sources 1–4 are imaged outside of the maximum power areas (Fig. 2.13a–d). This is caused by the interference of the waves from the early-ruptured sources. However, the sources 5–9 except for source 8 are imaged inside their own maximum power areas (Fig. 2.13e–g, i), though the imaging is elongated to E-W direction. Sources 8 and 10 are correctly imaged at the real locations in the N-S direction with a minor westward offset less than 5.5 km (Fig. 2.13h, j). Nevertheless, source 11 is imaged at the location of source 10, which is caused by the interference of the waveforms from source 10.

Although there are larger imaging offsets in E-W direction, the imaging offsets of the sources are less than 5.5 km in the E-N direction. These offsets are resulted from the early-ruptured sources due to the overlapping of waveforms for two neighboring sources. The rupture velocity controls the time offset of waveforms from the two neighboring sources. This indicates the faster the rupture velocity is, the more imaging interferences from the early-ruptured sources are for the later ruptured sources. To avoid these interferences, an appropriate time window needs to be chosen.

2.5.2 Comparison Between the RBPM and Traditional BPM

To carry out the comparison, we exploit the traditional BPM to image the two simulated earthquakes as imaged by the RBPM. The data are filtered in the frequency band of 0.5–2.5 Hz, and the rupture velocity is set to 1.0 km/s.

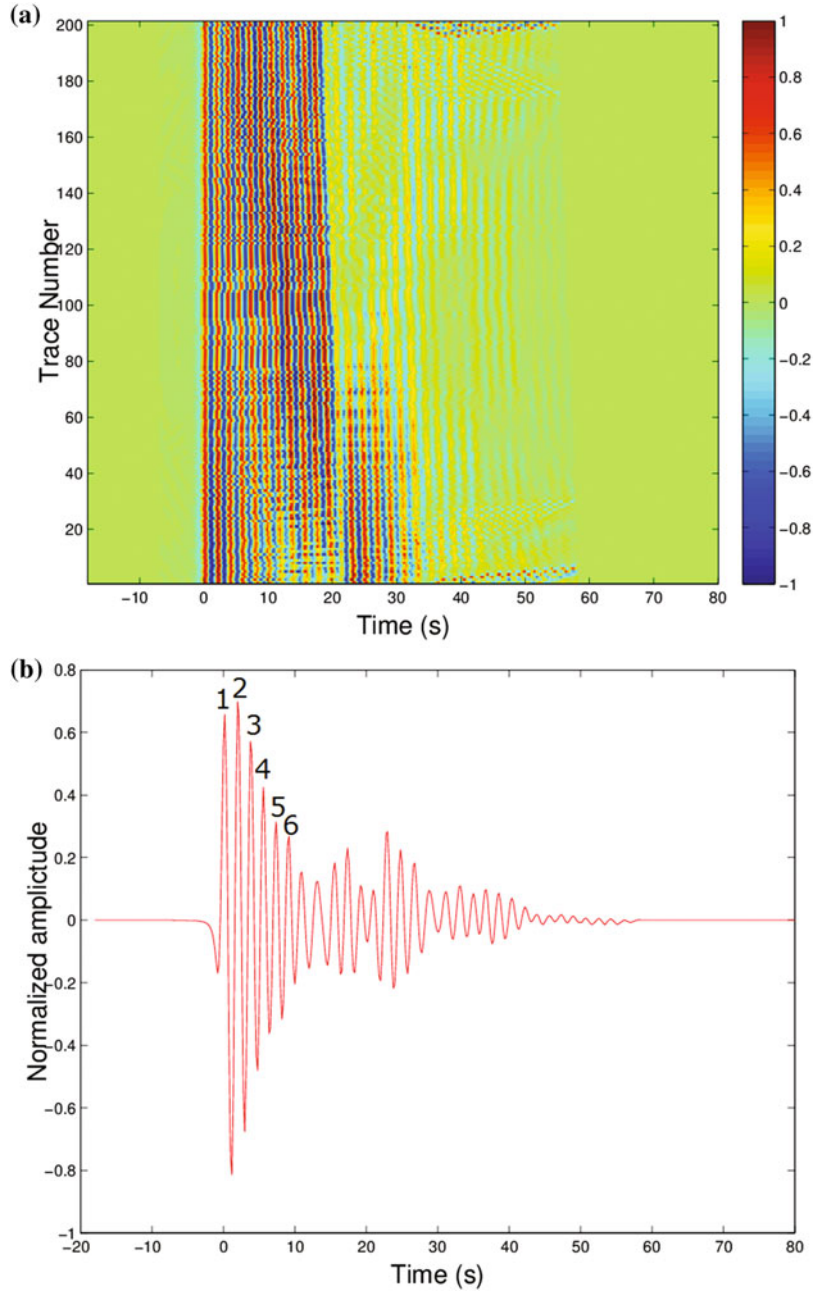


Fig. 2.12 **a** Vertical-component waveforms synthesized for the simulated earthquake 2 and filtered in a frequency range of 0.5–2.5 Hz. The rupture velocity of 3 km/s was used in the simulation of the earthquake. The warm color indicates positive amplitudes, whereas the cool color indicates negative amplitude. **b** The beam formed by stacking the waveforms aligned with the first *P* arrivals. The numbers represent the signals from the first six sources composing the simulated earthquake 2

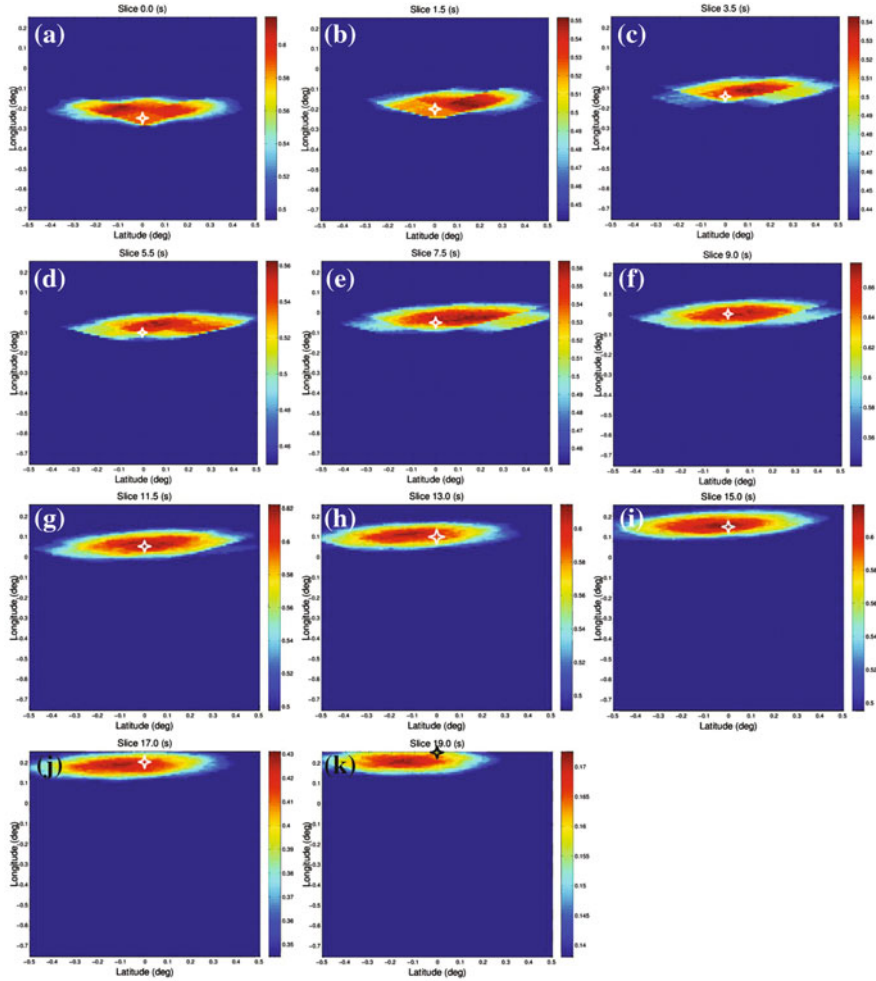


Fig. 2.13 Power distributions derived by the relative back-projection method at the rupture times of the eleven sources composing the earthquake 2 simulated at the rupture velocity of 3 km/s. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

2.5.2.1 Westward Rupture Earthquake Imaged by the Traditional BPM

According to the preceding analysis of the waveforms synthesized from the western rupture earthquake, the N-S orientated linear array has a lower resolution to image the earthquake relative to the following simulated earthquake. These waveforms are used to image the rupture of the earthquake using the traditional BPM with the same parameters to the RBPM. Thus, the earthquake is obtained as shown in Fig. 2.14.

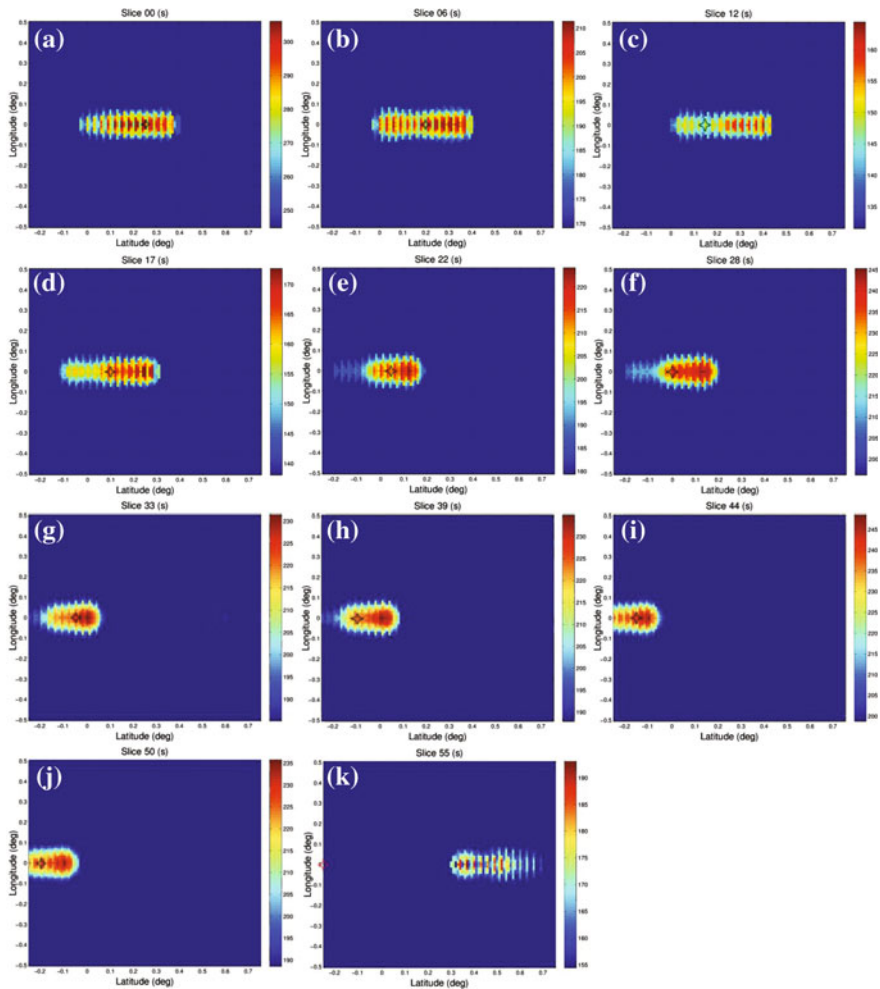


Fig. 2.14 Power distributions derived by the traditional back-projection method at the rupture times of the eleven sources composing the simulated earthquake 1. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

The imaging results show that all the sources are imaged inside 80% contour of the maximum power expect for source 11. Actually, only the imaging maximum power areas of sources 7 and 9 include their real locations, and other sources are outside the maximum power areas. This is the reason why 80% contour of the maximum power is treated as the uncertainty of a source (Kruger and Ohrnberger 2005). Thus, the traditional back-projection has a resolution of 28–44 km along the E-W direction. Moreover, the N-S trending imaging stripes are caused by the array geometry.

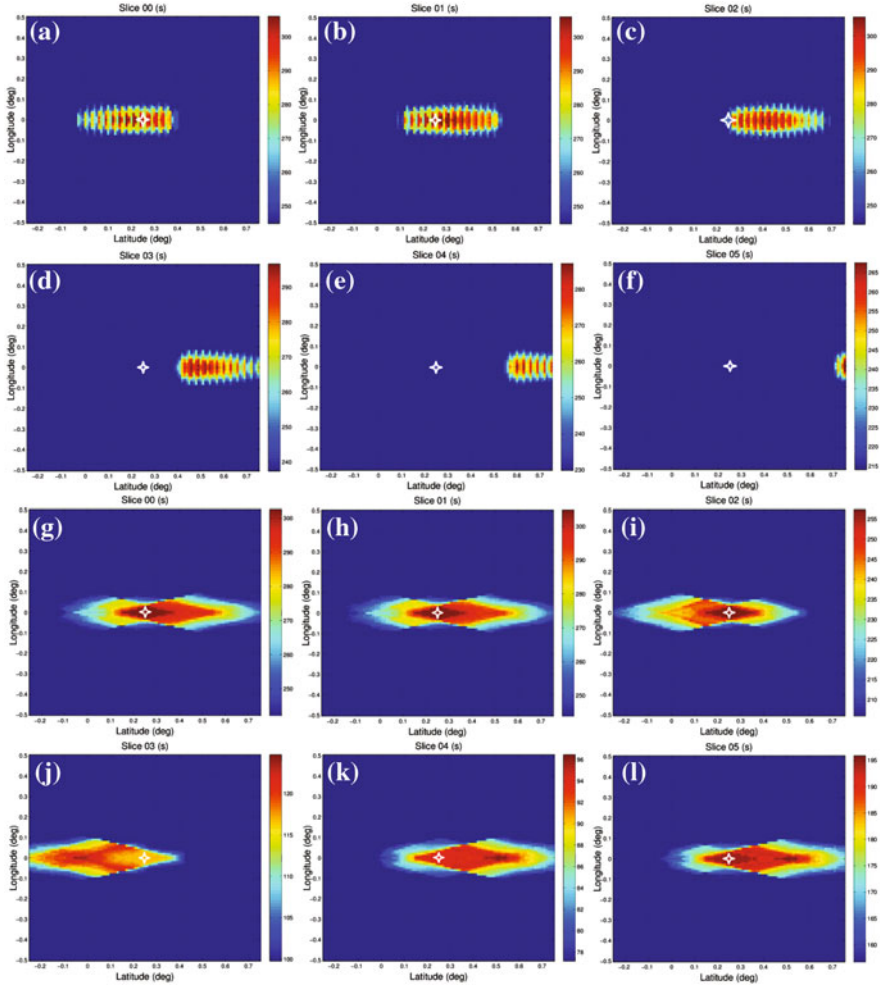


Fig. 2.15 “Swimming” artifacts in the imaging of the simulated earthquake 1 during the first 5 s. **a–f** The traditional back-projection imaging. **g–l** The relative back-projection imaging. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

To avoid missing sources, a time step enough short is used in the imaging. Usually, 1 s is a good choice for the simulated earthquake. However, the “swimming” artifacts clearly show up in the power distribution from 0 s through 5 s as illuminated in Fig. 2.15a–f. In this period, the high-intensity region moves from the center through the east with time. It is just like someone swimming, and this is the reason why this artifact is called “swimming” artifact. The imaging starts again at

the center at 6 s (Fig. 2.14b). Thus, the traditional back-projection imaging periodically commences at the real location and then moves toward the array.

For comparison, the relative back-projection imaging is located at the location of source 1 from 0 s through 2 s (Fig. 2.15g–h). However, the maximum power area moves westward at 3–4 s (Fig. 2.15j–k). Two power areas come out at 5 s (Fig. 2.15l): One is located at source 1, and the other one is located the east. Although imaging at 3–4 s deviates from the location of source 1, the maximum powers are less than half of those at 0–2 s (Fig. 2.15g–i). This is because the two neighboring sources are separated ~ 6 s (Table 2.1) and the time window is 4 s. Starting at 3 s, waveforms in the time window include signals from both sources 1 and 2; this causes the instability of imaging at 3–4 s and lowers down the maximum power due to the incoherency of the waveforms at different stations.

Obviously, the “swimming” artifacts do not exist in the relative back-projection imaging. All the sources are imaged inside 80% contour of the maximum power. As described above, the imaging results from 0 s through 5 s stem from the same source and thus have almost the same maximum powers (Fig. 2.15a–f). This indicates that the “swimming” artifact is a trade-off production of the location and rupture time. Through the comparison, one would know that the RBPM can be used to effectively reduce the “swimming” artifacts.

2.5.2.2 Northward Rupture Earthquake Imaged by the Traditional BPM

Similarly, the second earthquake at a speed of 1 km/s is also simulated (Fig. 2.4), and the waveforms are thus generated as shown in Fig. 2.8a. These waveforms are applied to image the rupture of the earthquake using the traditional BPM. The imaging results (Fig. 2.16) show that although the sources are imaged correctly in the N-S direction, there still exist “swimming” artifacts in the E-W direction.

Overall, after the comparison of the RBPM with the traditional BPM, one knows that the relative back-projection imaging has higher resolution in the range direction and does not have the “swimming” artifacts.

2.5.2.3 Comparison of the Maximum Power

To directly compare the imaging results between the RBPM and traditional BPM, we compile the deviation of the locations with the maximum power from the real locations of the sources for the tests described above. The deviations are obtained for both of the simulated earthquakes as shown in Fig. 2.17. For the first earthquake, all the relative back-projection imaging sources are located closer to the real locations except for sources 7–9 (Fig. 2.17a). For the second earthquake, all the

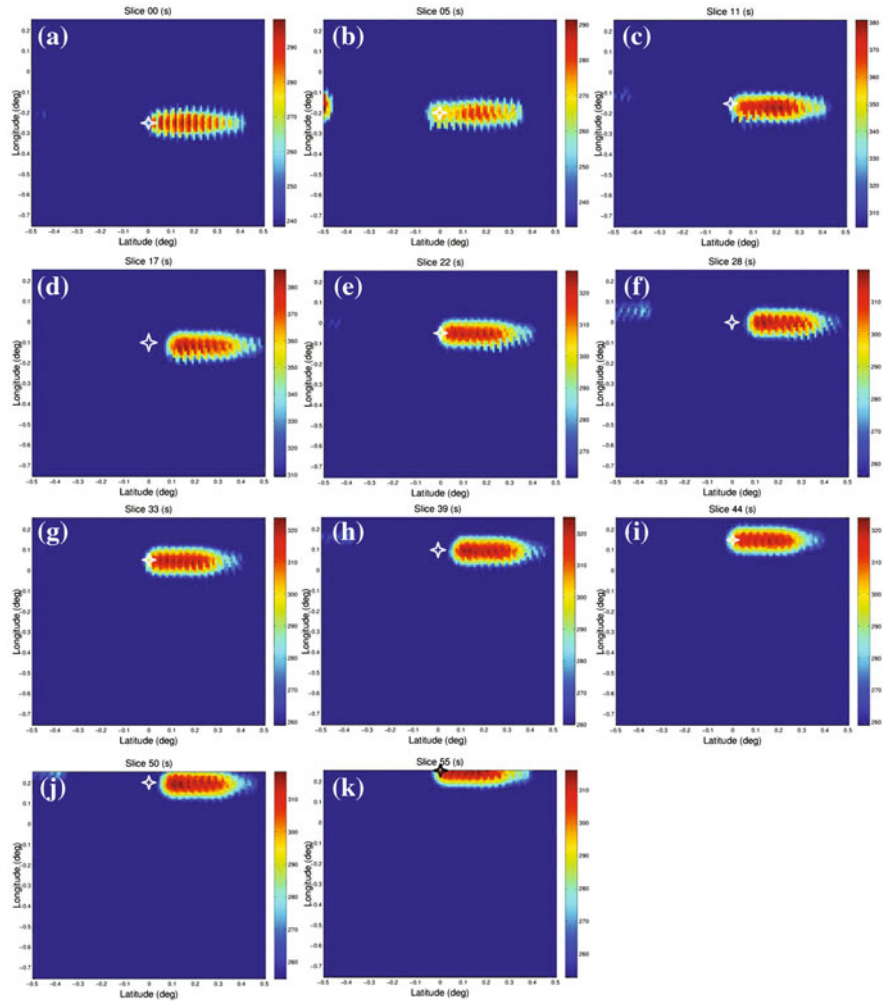


Fig. 2.16 Power distributions derived by the traditional back-projection method at the rupture times of the eleven sources composing the simulated earthquake 2. **a–k** Power distribution of the sources 1–11. The star indicates the source. The power is indicated by the color bar with 80% as the color changing indicator

relative back-projection imaging sources are located closer to the real locations. With the comparison, one can draw a conclusion that the RBPM is a better tool than the traditional BPM to accurately image the rupture of the earthquake with no “swimming” artifacts.

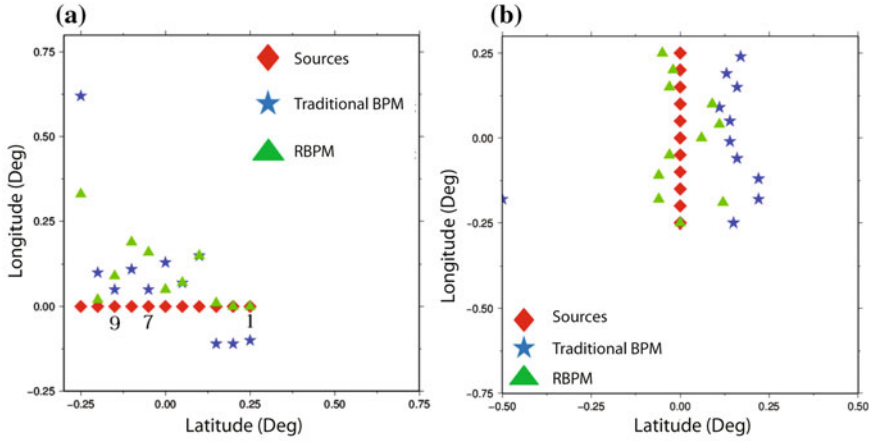


Fig. 2.17 Comparison of the subevents imaged by the relative back-projection and the traditional back-projection methods. **a** The imaging of the simulated earthquake 1. **b** The imaging of the simulated earthquake 2

2.6 Discussion and Summary

In this chapter, the principle of the generalized array techniques was deduced mathematically and it is proved to be effective and reliable. Five widely used generalized array techniques were then introduced. On basis of the traditional back-projection, an improved approach, namely relative back-projection method (RBPM), was briefly introduced and the workflow to use this method has been elaborated. To verify the reliability of the RBPM, a series of synthetic tests have been conducted. The tests reveal spatial resolutions in range and azimuth directions, and the effects to change important parameters, such as reference station, frequency band used to filter data, and rupture velocity. Finally, a comparison between the RBPM and traditional BPM was carried out to demonstrate the advantage of the RBPM to suppress the “swimming” artifacts in traditional back-projection results.

The principle of the RBPM has been proved above, though several assumptions have been provided. For example, a large earthquake can be approximated with a sequence of subevents and Green’s functions of teleseismic direct P waves are larger than multiple’s. The first assumption is always sufficed when the event is teleseismic, but the second one is violated when the actual reflection waves at surface have larger amplitudes than the direct P waves due to radiation pattern. Thus, these assumptions have to be considered first when the RBPM is applied to image the earthquakes.

The RBPM is based on beamforming or slant stacking. When some assumptions are violated, there would be the artifacts in the imaging results. For example, there are “swimming” artifacts in the traditional back-projection imaging results. As demonstrated above, the RBPM can mitigate this kind of artifacts to a great extent. Moreover, the good choice of the reference time and arrays can further improve the imaging.

Table 2.3 The source model for the simulated earthquake, which ruptures northward at a high rupture velocity of 3 km/s

Source #	Latitude (°)	Longitude (°)	Origin time (s)	Arrival time (s)
1	−0.25	0.00	0.00	0.00
2	−0.20	0.00	1.84	1.84
3	−0.15	0.00	3.68	3.68
4	−0.10	0.00	5.52	5.52
5	−0.05	0.00	7.36	7.36
6	0.00	0.00	9.20	9.20
7	0.05	0.00	11.04	11.04
8	0.10	0.00	12.88	12.88
9	0.15	0.00	14.72	14.72
10	0.20	0.00	16.57	16.57
11	0.25	0.00	18.41	18.41

Note The parameters of the eleven point sources composing the earthquake 2 simulated at the rupture speed of 3 km/s. The focal depths of all the point sources are set to 0 km. The fifth column is the travel times of the sources to the central station

The synthetic tests play a key role in understanding the RBPM. Importantly, we test the back-projection imaging using different frequency bands for a given earthquake. The test shows the back-projection results for the two frequency bands basically agree with each other. This confirms the frequency-dependent rupture feature discovered in megathrust subduction zone earthquake (Lay et al. 2012; Yao et al. 2013), indicating the friction varying with depth in the subduction zones. Meanwhile, for linear arrays, the spatial resolution is azimuthal dependent. Moreover, the resolution is also controlled by the rupture velocity and multiples. Overall, the RBPM has the advantage to suppress the “swimming” artifacts relative to the traditional BPM (Table 2.3).

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