Preface

The seminal work by Lorenz in 1963 [264], and later by May in 1976 [273–275], has led scientists and engineers to recognize that nonlinear systems can exhibit a rich variety of dynamic behavior. From simple systems, such as the evolution of single species [314], an electronic or biological oscillator [423, 424], to more complex systems, such as chemical reactions [33], climate patterns [153], bursting behavior by a single neuron cell [208], and flocking of birds [333, 393], *Dynamical Systems* theory provides quantitative and qualitative (geometrical) techniques to study these and many other complex systems that evolve in space and/or time. Regardless of the origins of a system, i.e., Biology, Chemistry, Engineering, Physics, or even the Social Sciences, dynamical systems theory seeks to explain the most intriguing and fundamental features of spatio-temporal phenomena.

In recent years, systems made up of individual units coupled together, either weakly or tightly, have gained considerable attention. For instance, the dynamics of arrays of Josephson junctions [18, 99, 100, 160], central pattern generators in biological systems [85, 226, 227], coupled laser systems [328, 419], synchronization of chaotic oscillators [315, 426], collective behavior of bubbles in fluidization [163], the flocking of birds [393], and synchronization among interconnected biological and electronic nonlinear oscillators. These are only a few representative examples of a new class of *complex dynamical systems* or *complex networks*. The complexity arises from the fact that individual units cannot exhibit on their own the collective behavior of the entire network. In other words, the collective behavior is the exclusive result of the mutual interaction that takes place when multiple units are interconnected in some fashion.

In most cases, three factors are normally considered when studying the collective behavior of a complex system. Mainly, the internal dynamics of each individual unit or cell, the topology of cell connections, i.e., which cells communicate with each other, and the type of coupling. More recently, a fourth factor has gained further attention—symmetry. It is well-known that symmetry alone can restrict the type of solutions of systems of ordinary– and partial differential equations, which often serve as models of complex systems. So it is reasonable to expect that certain aspects of the collective behavior of a complex system can be inferred from the presence of symmetry alone. In fact, the work by Golubitsky [145, 146, 149] lays down the theoretical foundations for a model-independent analysis to understand, and predict, the behavior of a dynamical system using, mainly, the underlying symmetries of the system while separating the fine details of the model. While this approach has been widely successful in explaining computer simulations and experimental observations of many different spatio-temporal phenomena, it has found limited use in the conceptualization and development of nonlinear devices even though many of those systems are, inherently, symmetric. And while many works have been dedicated to study the symmetry-preserving phenomenon of *synchronization* [25, 315, 317, 382, 426], significantly less is known about how one can exploit the rich variety of collective patterns that can emerge via symmetry-breaking bifurcations, such as *heteroclinic cycles* [63].

Over the past 17 years, we and other colleagues and students have been attempting to bridge the current gap between the theory of symmetry-based dynamics, *equivariant bifurcation theory*, and its application to developing nonlinear devices. At the beginning, around the year 2000, we were interested in developing new methods to manipulate frequency in arrays of nonlinear oscillators for antenna devices. Collaborators from the U.S. Navy had already shown [166] that small frequency perturbations applied to the end points of a chain of nonlinear oscillators can lead to a change in the direction of the radiation pattern. That is, they demonstrated that beam steering was possible without mechanically rotating an antenna. The next puzzle that we had to solve was to manipulate the collective frequency of the array over a broad band without changing the internal frequency of each individual oscillator. But just when we were about to solve this problem, we were steered, no pun intended, into developing a new class of highly sensitive, low-power and low-cost, magnetic- and electric field sensors. Theoretical work for this new class of sensors started around 2002. The fundamental principles were twofold: to exploit coupling-induced oscillations in symmetric networks to generate self-induced oscillations, thus reducing power consumption; and to exploit symmetry-breaking effects of heteroclinic cycles to enhance sensitivity. As a starting point, we chose fluxgate magnetometers as individual units, because their behavior is governed by a one-dimensional autonomous differential equation. Consequently, based on the fundamental theory of ODEs, it follows that in the absence of any forcing term the one-dimensional dynamics of the individual units cannot produce oscillations. But when the fluxgates are coupled then the network can, under certain conditions that depend on the coupling strength, oscillate. This configuration could demonstrate to skeptics that self-induced oscillations can indeed be engineered. In practice, the network would still need, of course, a minimum amount of energy to kick it off of its trivial equilibrium state and get the oscillations going. Overall, we were able to show that, under certain conditions, the sensitivity response of an array of weakly coupled fluxgate sensors can increase by four orders of magnitude while their cost could be simultaneously reduced to a fraction of that of an individual fluxgate sensor. This technology matured around 2005 with design, fabrication and deployment.

In 2006, we extended the work on magnetic fields to electric field sensors. These sensors are also governed by one-dimensional, overdamped, bistable systems of equations. We conducted a complete bifurcation analysis that mirrors that of the fluxgate magnetometer and, eventually, translated the research work into a microcircuit implementation. This microcircuit was intended to be used for measuring minute voltage or current changes that may be injected into the system. The conceptualization of these sensors employs the model-independent approach of Golubitsky's theory for the study of dynamical systems with symmetry, while the development of laboratory prototypes takes into account the model-specific features of each device which, undoubtedly, may impose additional restrictions when we attempt to translate the theory into an actual experiment. For instance, a sensor device that measures magnetic flux, as oppose to electric field signals, may limit the type of coupling functions that can be realized in hardware. In other words, not every idealization of a network-based structure can be readily implemented in the laboratory. This and other similar restrictions need to be kept in mind while reading this book.

Around that same 2006 year, we started, in tandem, to the work on electric field sensors, theoretical studies of networks of Superconducting Quantum Interference Devices (SQUIDs). The work was suspended until 2009 when we returned to explore in greater detail the response of networks of non-uniform SQUID loops. The technology matured by 2012 with applications to antennas and communication systems. Around that same period, 2007–2009, we went back to the study of multi-frequency oscillations in arrays of nonlinear oscillators. In fact, we were able to develop a systematic way to manipulate collective frequency through cascade networks. The work matured in 2012 with the modeling, analysis, design and fabrication of the *nonlinear channelizer*. This is an integrated circuit made up of large parallel arrays of analog nonlinear oscillators, which, collectively, serve as a broad-spectrum analyzer with the ability to receive complex signals containing multiple frequencies and instantaneously lock-on or respond to a received signal in a few oscillation cycles. Again, the conceptualization of the nonlinear channelizer was based on the generation of internal oscillations in coupled nonlinear systems that do not normally oscillate in the absence of coupling. Between 2007–2011, we investigated various configurations of networks of coupled vibratory gyroscopes. The investigations showed that networks of vibratory gyroscopes can mitigate the negative effects of noise on phase drift. But the results were, mainly, computational and applicable only to small arrays. Finally, between 2012–2015, we developed the necessary mathematical approach to study networks of arbitrary size. This work showed the nature of the bifurcations that lead arrays of gyroscopes, connected bidirectionally, in and out of synchronization. The results were applicable to networks of arbitrary size.

In the past few years, previous works have led us into new topics. Networks of energy harvesters, which, interestingly, are governed by ODEs that resemble those of vibratory gyroscopes. This feature highlights again the model-independent nature of the analysis of differential equations with symmetry. In the year 2011, in particular, we started a new project to study the collective behavior of spin-torque nano-oscillators. The motivation for this work is a conjecture by the 2007 Nobel

Laureate, Prof. Albert Fert, about the possibility that synchronization of nano-oscillators could produce substantial amounts of microwave power for practical applications. Determining the regions of parameter space of stable synchronized solutions was a very challenging problem due to the nature (non-polynomial form) of the governing equations. Finally, this year we overcame the major difficulties by exploiting, again, equivariant bifurcation theory. And the most recent project that we started in 2016 is about networks of coupled oscillators for improving precision timing with inexpensive oscillators, as oppose to atomic clocks.

Along the way, several patents were approved by the U.S. Patent Office for the works related to these projects, including:

2007 **U.S. Patent # 7196590**.

Multi-Frequency Synthesis Using Symmetry Methods in Arrays of Coupled Nonlinear Oscillators.

2008 **U.S. Patent # 7420366**.

Coupled Nonlinear Sensor System.

2009 **U.S. Patent # 7528606**.

Coupled Nonlinear Sensor System for Sensing a Time-Dependent Target Signal and Method of Assembling the System.

2011 **U.S. Patent # 7898250**.

Coupled Fluxgate Magnetometers for DC and Time-Dependent (AC) Target Magnetic Field Detection.

2011 **U.S. Patent # 8049486**.

Coupled Electric Field Sensors for DC Target Electric Field Detection.

2012 **U.S. Patent # 8049570**.

Coupled Bistable Microcircuit for Ultra-Sensitive Electric and Magnetic Field Sensing.

2012 **U.S. Patent # 8212569**.

Coupled Bistable Circuit for Ultra-Sensitive Electric Field Sensing Utilizing Differential Transistors Pairs.

2015 **U.S. Patent # 8994461**.

Sensor Signal Processing Using Cascade Coupled Oscillators.

2015 **U.S. Patent # 9097751**.

Linear Voltage Response of Non-Uniform Arrays of Bi-SQUIDS.

2016 **Under review. Navy Case: 101427**.

Enhanced Performance in Coupled Gyroscopes and Elimination of Biasing Signal in a Drive-free Gyroscope.

2016 **Under review. Navy Case: 101950**.

Arrays of Superconducting Quantum Interference Devices with Self Adjusting Transfer to Convert Electromagnetic Radiation into a Proportionate Electrical Signal to Avoid Saturation.

2016 **Under review. Navy Case: 102297**.

2D Arrays of Diamond Shaped Cells Having Multiple Josephson Junctions.

2016 **Under review. Navy Case: 103829**.

Network of Coupled Crystal Oscillators for Precision Timing.

None of these projects would have been possible without the active participation of students, joint work with collaborators, and the financial support from various sources. We would like to thank each of the students first: John Aven [21, 22], Jeremmy Banning [26], Katherine Beauvais [30], Susan Berggren [36, 159], Bernard Chan, Nathan Davies [91], Scott Gassner [132, 133], Mayra Hernandez [168], Habib Juarez, Tyler Levasseur, Patrick Longhini [261, 262], Daniel Lyons [266, 267], Antonio Matus [272], Derek Moore, Loni Olender, Steven Reeves [331] Norbert Renz [332], Richard Shaffer [359], Brian Sturgis-Jensen, James Turtle [398, 399], Huy Vu [407], Sarah Wang, Bing Zhu [433]. Special acknowledgement and thanks to Patrick Longhini, he was the first student that got involved in the work through his Master and, later on, Ph.D. thesis. He continues to be an extremely valuable asset to multiple ongoing projects. Collaborators include: Bruno Ando (Univ. of Catania), Marcio De Andrade (SPAWAR), Salvatore Baglio (Univ of Catania, Italy), Peter Blomgren (SDSU), Donald Bowling (NAWC), Pietro-Luciano Buono (Univ. of Ontario Institute of Technology, Canada), Adi Bulsara (SPAWAR), Lowell Burnett (QUASAR), Juan Carlos Chaves (HPTi), Ricardo Carretero (SDSU), Anna Leese de Escobar (SPAWAR), Jocirei Dias Ferreira (Federal Univ. of Mato Grosso, Brazil), Hugo Gonzalez-Hernadez (Instituto Tecnologico de Monterrey), Frank Gordon (SPAWAR), Takachi Hikihara (Kyoto Univ., Japan), Calvin Johnson (SDSU), Andy Kho (SPAWAR), Daniel Leung (SPAWAR), John F. Lindner (College of Wooster), Norman Liu (SPAWAR), Joseph M. Mahaffy (SDSU), LT Jerome McConnon (SPAWAR), Brian K. Meadows (SPAWAR), Oleg Mukhanov (HYPRES), Joseph Neff (SPAWAR), Suketu Naik (Weber State Univ.), Martin Nisenoff (M. Nisenoff Associates), Georgy Prokopenko (HYPRES), Wouter-Jan Rappel (UCSD), LT Sarah Rice (SPAWAR), Robert Romanofsky (NASA), Vincenzo Sacco (Univ. Catania), Benjamin Taylor (SPAWAR), Edmond Wong (SPAWAR), Yongming Zhang (QUASAR).

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The book is intended for a broad audience. For engineers who might be interested in applying ideas and methods from dynamical systems with symmetry and equivariant bifurcation theory to design and fabricate novel devices. For mathematicians and physicists who might be interested in translational research work to extrapolate fundamental research theorems into practical applications. And for scientists from many disciplines, viz. Biology, Chemistry, Computer Science, Geology, etc., who might be interested in the interplay between theory and real-life applications from the general field of nonlinear science.

The book is organized as follows. In Chap. 1 we present fundamental ideas of complex networks and bistability, which is a common feature of many sensor devices; and then we dedicate a few sections to introduce basic ideas, methods and examples in the analysis of differential equations (ODEs and PDEs) with symmetry. One particular class of solutions that rarely appears in generic versions of systems of differential equation are *heteroclinic cycles*. These types of solutions are, however, generic features of systems with symmetry. We exploit these cycles to enhance sensitivity and, thus, we dedicate a section to explain what they are and how they can be found. The book is then organized in two parts. Part I, Chap. 2 through Chap. 6 is dedicated to translational research work that already led to mature technologies. These technologies include networks of fluxgate magnetometers; arrays of micro-electronic electric field sensors; networks of SQUIDs; cascade arrays of nonlinear oscillators for multi-frequency generators; and a special chapter in honor of the theoretical work by Pietro-Luciano and Marty Golubitsky: a device realization of a Central Pattern Generator network of the animal gaits studied by them. Part II, Chap. 7 through Chap. 10 include, mainly, theoretical works that have not yet mature into actual device realizations. The technologies that may derive from these works are part of ongoing efforts.

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