Chapter 2
Chinese Mathematics Education System and Mathematics Education Tradition

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2.1 Understanding Traditional Chinese Mathematics Education

Mathematics education in ancient China has a long history of over 3000 years, during which a lot of outstanding mathematicians have been cultivated; the first institute of higher education for mathematics in the world, named the *Ming Suan Ke* (Computation Department), was established in the Tang Dynasty, and splendid methods to promote mathematical thinking development. However, current mathematics education researchers and teachers in primary and secondary schools in China have not fully understood and applied the thoughts created in ancient China. Even though, in recent years, there has been some work linking mathematics education with mathematics culture and history, more emphasis has been placed on the history and culture of Western mathematics, which focuses on "concept games" instead of practical operability. The work which covers Chinese mathematics history and culture is limited to some typical cases—"Wu Bu Zhi Shu (Amount Calculation)," "Gou-Gu Ding Li (the Pythagorean theorem)," and some stories about well-known Chinese mathematicians such as Li Yan, Qian Baocong and Li Di. Admittedly, some works, like *Zhong Guo Gu Suan Jie Qu* (*Interesting Computation Cases in Ancient China*) by Yu Zuquan and *Shu Xue Liaozhai* (*Mathematical Stories*) by Wang Shuhe are very impressive, but they cannot be used in classroom teaching because of their limited function in rigorous exams. In a word, few educators can search the rich historical materials of Chinese mathematics for inspiration and apply it to mathematics education.

Benedetto Croce (1866–1952), a well-known Italian historian, said, "When there is a need with the development of life, the dead history will be resurrected and

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something from the past will become current…. Hence, most parts of the history viewed currently as chronicles and silent historical documents will be lit up in turn by new life and work well again" (Croce, 2005, p. IV). "From the perspective of chronology, no matter how long the history of a fact is, it is still a history related to current needs and forms, and those facts are continuously astonishing" (Croce, 2005, p. 6). The thought of "gaining new insights through reviewing old material" has particularly been advocated and practiced since ancient times in China, and importance has been attached to grasping new knowledge on the basis of reviewing old materials in individual learning. In the development of the nation, taking history as a mirror, Chinese people apply the knowledge, experience, and thinking methods gained in the past to current practice, which also serves the purpose of gaining new insights.

In recent years, through reviewing journal articles on mathematics education, masters' theses from normal universities, and other work in this field, I have found that a lot of young researchers present ideas that, directly or indirectly, deny traditional Chinese mathematics education. As Zhang Dianzhou said, "In recent years, the so-called traditional educational methods are almost the synonyms of 'backward' and 'obsolescent'" (Zhang & Zhao, 2012). I wonder how much these researchers know about traditional Chinese mathematics education. In my opinion, the tradition, the present, and the future are inseparable; "innovation" does not mean the inevitable separation from "tradition," but can be considered as a prerequisite to surpass and violate traditions, and there are some points between them where their values are compatible. As Ye Xiushan, a famous Chinese philosopher, said:

> History consists of the past, the present and the future. 'The past' results in 'the present', and 'the future' influences 'the present' as well. 'The past' and 'the future' are both included in 'the present'. 'The present' is not a geometrical 'point', but a 'side'. People live and work under the principles of 'the past' and the attraction of 'the future'. 'The past' has not gone, and 'the future' can be pursued. 'Values' and 'meanings' are not fragments, but extensions. (Ye, 2010, p. 149)

In the continuum of past, present, and future, "tradition" and "innovation" are two sides of the dialectical movement of culture development. We are expected to achieve a connection and balance between inheritance and development on the basis of traditional education so as to avoid the extremes. Without historical facts, any wise conclusions are not convincing. Therefore, it is necessary for those who intend to deny traditional Chinese mathematics education to make a careful study of it first.

Traditions are the sources of cultural memory of a nation, the driving force for innovation, and the stimulators for further growth. These traditions are the social factors passed from generation to generation; distinctive features from the past are integrated, directly or indirectly, with present features, and continue to play a role. No matter how sweeping the social changes are, the traditions of thoughts cannot be altered thoroughly. While the contents of school mathematics education in China are from the West, the education views and teaching and learning approaches follow the Chinese traditions of "honoring teachers and esteeming truth," "teaching benefitting teachers as well as students," and "teaching only the essential elements and ensuring plenty of practice," which display an inseparable connection between the past and the present.

Zhang Dianzhou said "Education in China has its own 'beauty,' and we need national confidence. Teachers in China are the watchmen of excellent educational traditions of China" (Zhang & Zhao, 2012). In my opinion, education in China has its own "truth" and "goodness" as well as "beauty," and its "beauty" is involved with its "truth" and "goodness"; furthermore, its "truth" and "goodness" are demonstrated in the form of "beauty." In short, abundant excellent content in the traditional education of China has vitality and integrates truth, goodness, and beauty, and this applies to mathematics education in China.

2.2 Brilliant Ancient Chinese Mathematics and Education Culture

China was not only a great civilization in ancient times, but also renowned for its mathematical genius. Ancient Chinese mathematicians made world-acclaimed achievements. Over its long history of development, a unique algorithm-centered and practical-oriented system has emerged with induction as the main method and problem collection as the primary model. With thousands of years of development behind it, Chinese mathematics education has accumulated rich experiences that provide a rich heritage for current education.

2.2.1 The Beginnings of Ancient Chinese Mathematics Education

Chinese mathematics education dates back more than 3000 years to the Shang Dynasty (1600–1048 BC). Over its long development, there has emerged a large number of outstanding mathematicians who dealt with the practical needs of everyday life, and who also contributed to the development of mathematics around the world. For example, traditional Chinese mathematics is the origin of Japanese mathematics: *Suanjing Shi Shu* (*Ten Mathematical Classics*) was once used as a textbook in Japan and Korea. In addition, ancient Chinese mathematics education created some world firsts. For example, the mathematics school built in the Tang Dynasty (AD 618–907) was the first advanced mathematics school in the world. The "Program for Learning Mathematics" in *Chengchu Tongbian Benmo* (*Multiplication and Division Developed from the Basics*), written in 1274 by Yang Hui, a mathematician and mathematics educator during the Southern Song Dynasty (AD 1127–1279), was the earliest mathematics syllabus in the world.

Mathematics education already existed in China during the Xia and Shang dynasties. The *Shuo Wen Jie Zi* (*Interpretation of Characters*), written by Xu Shen in AD 121, stated that "mathematics means calculation." In the Xi Zhou Dynasty (1046–256 BC), the "six arts" for national learning—rites, music, archery, riding, writing, and arithmetic—were formulated. "Rites" referred to political education, including moral norms, protocol, and the learning of religious and legal institutions in the slave societies. "Music" referred to arts education, including the learning of music, poetry, and dancing. "Archery" and "riding" were military training programs, and "writing" and "arithmetic" were part of the basic curriculum. "Six arts [techniques]" education aimed to combine general knowledge and military ability with a focus on rites, and was the highest education level attainable by China's slave society. Writing and arithmetic were minor arts, taught in primary schools, while rites, music, archery and riding, were major arts, taught in colleges. The idea of teaching mathematics as an art was a concept unique to ancient China. The inclusion of mathematics in the "six arts" education shows clearly that mathematics learning was an indispensable part of an official's education. Mathematics education was included as early as in the Xi Zhou Dynasty, which pointed the way for its future development. The mathematics education system was set up in China during the Qin Dynasty (221–206 BC).

Before the Sui and Tang dynasties, the mathematics education system was outlined by the following steps (Li, 1954, p. 253) which, despite their simplicity, laid a solid foundation for mathematics education in later ages.

- At six, a child learns numbers and the names of cardinal points. When he is ten, he goes to boarding school to get an education, learning writing and calculation. (*The Book of Rites*)
- A child is ready for education at eight when he has adult teeth, and he must go to school to learn writing and calculation. (*Bai Hu Tong*, by Ban Gu, AD 32–92)
- The sixth [of the six arts that must be taught to children] is the set of nine skills in mathematics. (*Zhou Li, Rites of Zhou*)
- At eight, a child goes to school to learn the Heavenly Stems and Earthly Branches, names of the cardinal points, as well as writing and calculation. (*Han Shu*, by Ban Gu, AD 32–92)
- In ancient times, a child went to school at eight to learn the Heavenly Stems and Earthly Branches, the names of the five directions, as well as writing and calculation. (*Ru Li Lun*, by Wang Can, AD 177–217)

2.2.2 *Ming Suan Ke: The First Mathematics School in the World*

Nine Chapters on Mathematics Arts (hereafter referred to as *Nine Chapters*) was used as a textbook for 500 years in private mathematics education in China. During this Period, there was no such thing as public mathematics education provided by the government. Despite its short-lived supremacy (AD 581–618), the Sui Dynasty had a profound influence upon the mathematics education of China. In AD 589, after the country was untied, the government of Sui enacted many laws and regulations. It restored national education and, for the first time in Chinese history, it started mathematics education. As well, it set up an education system for mathematics, recruited teachers, and enrolled students. “There were two mathematics masters, two assistants, and eighty students in the Imperial College” (Li, 1997). This marked the birth of state-run mathematics education in ancient China.

After the establishment of the Tang Dynasty, the Ming Suan Ke mathematics school was set up in the Imperial College in AD 656 after dozens of years of painstaking preparation. The curriculum, examination methods, and textbooks were all decided at the same time. In the school, there were usually about 30 students, 10 being the minimum. The students came from the families of eighth- or ninth-grade officials and common people’s families. The mathematics masters were ninth-grade officials, the lowest grade in the Tang Dynasty (Li, 1997, p. 305). Ming Suan Ke was run intermittently all the way through the Five Dynasties. It was the first mathematics school in the world, and Emperor Gaozong of the Tang Dynasty ordered that *Suanjing Shi Shu* (*Ten Mathematical Classics*) be the official textbook. People who learned mathematics and passed entrance examinations could get jobs in the government.

The culture of the Sui and Tang dynasties was extremely appealing to Japan and Korea at that time, especially to Japanese rulers and officials. They imitated China as extensively as they could, from its political system to its literature, arts, architecture, clothes, food, and written characters. Their mathematics education system was imitated in its entirety from that of the Tang Dynasty.

2.2.3 The Peak of Ancient Chinese Mathematics Culture

In the Song Dynasty (AD 960–1279) and the Yuan Dynasty (AD 1279–1368), the development of Chinese mathematics reached its peak and many world-class achievements were made. Thanks to the high level of mathematics education at that time, many famous mathematicians, such as Jia Xian, Qin Jiushao, Li Ye, Yang Hui, and Zhu Shijie, emerged.

In the 300-year history of the Northern and Southern Song dynasties, national mathematics education existed only for a short Period, without any significant developments. The government of the Northern Song Dynasty did not provide mathematics education until late in the dynasty, and mathematics education existed sporadically from 1083 to 1120. The achievements in this Period were not outstanding in any way, but two points are worth mentioning. First, in 1084, woodblock printing was adopted in printing some mathematics textbooks passed down from the Tang Dynasty. There is historical proof that *Zhou Bi Suanjing, Nine Chapters, Sun Zi Suanjing, Wu Cao Sunnjing, Zhang Qiujian Suanjing, Xiahou Yang Suanjing, Haidao Suanjing, and Ji Gu Suanjing* were printed using this method; however, it is not certain whether *Wu Jing Suanshu* and *Shu Shu Ji Yi* (*Memoir on Some Traditions of the Mathematical Art*) were printed (Li, 1955, p. 276). *Zhui Shu* was not printed, as it had been lost before then (Li, 1958). Second, the *Mathematics Education Guidelines* were formulated during the Yuanfeng Period (AD 1078–1085) and were amended and issued as an imperial order in 1107. These have been passed down from generation to generation until today.

During the 152 years of the Southern Song Dynasty from 1127 to 1279, the government made no effort to revive mathematics education. However, two facts deserve some attention. First, a scholar named Bao Huanzhi made every effort to collect the mathematics manuals printed during the Yuanfeng Period, and he found and copied *Shu Shu Ji Yi* in a temple in Hangzhou. He brought these copies to Changting County in Fujian Province when he went there to assume the duty of sheriff. He then reissued these faithful replicas in 1212–1213. Five-and-a-half of these copies still exist: *Zhou Bi Suanjing, Nine Chapters, Sun Zi Suanjing, Zhang Qiujian Suanjing, Wu Cao Suanjing*, and *Shu Shu Ji Yi*. Second, the “Program for Learning Mathematics” proposed by Yang Hui during the Southern Song Dynasty was a very precious work in Chinese mathematics education. It closely resembles the teaching syllabus that is currently being used.

The Jin Dynasty was established by Nüzhen (Jurchen), an ancient nationality in China. It challenged the Southern Song Dynasty in 1127 and was later destroyed by Mongols in 1234. There was no national mathematics education during the Jin Dynasty, but private mathematics education flourished all the way into the Yuan Dynasty.

From the twelfth to the thirteenth centuries, mathematics research and study were very popular in some areas of northern China, such as Shanxi, Hebei, and Shandong. This brought about an important academic achievement—Tianyuan shu, or the Tianyuan method, a method for the numerical solution of high-degree polynomial equations with one unknown. At that time, there was an intellectual group led by Liu Bingzhong in Wúan County, Hebei Province. Zhang Wenqian, Guo Shoujing, and Wang Xun were members of this group. They primarily studied science, including mathematics. Mathematician Li Ye set up the Fenglong Academy in Yuanshi County, Hebei Province, where he enrolled students and taught them mathematics as well as the humanities.

Zhu Shijie was a mathematician who lived during the thirteenth and fourteenth centuries. He made a living by teaching mathematics and traveled around the country for dozens of years to teach students. In 1299, Zhao Yuanzhen sponsored the publication of Zhu Shijie’s *Suanxue Qimeng* (*Introduction to Mathematical Sciences*) in support of Zhu’s teaching. Another of Zhu’s masterpieces, *Si Yuan Yu Jian* (*Jade Mirror of Four Unknowns*), was published in 1303.

The Mongol emperors of the Yuan Dynasty, Kubilai Khan and Mogul Khan, both paid considerable attention to mathematics. Mogul Khan studied Euclid’s *Elements*, and Kubilai Khan hired mathematician Wang Xun to teach the crown prince. Kubilai Khan also required officials’ children to learn mathematics. During the Yuan Dynasty, lower-rank officials were also required to master mathematics so that they could meet the demands of their jobs.

2.2.4 Transition from Traditional to Western Mathematics Education

Hongwu, the first emperor of the Ming Dynasty, set mathematics as a subject for education in 1369, just after the establishment of the dynasty in 1368. In February 1392, the government reiterated the mathematics syllabus, and examination content: "*Nine Chapters* must be learned and mastered proficiently, and should be tested" (Li & Dai, 2000, p. 95).

Private mathematics education in the Ming Period was very active. Cheng Da Wei wrote *Suan Fa Tong Zong,* and this became an important textbook for private education, significantly influencing other countries, like Japan.

During the 300 years from the end of the Ming Dynasty to the end of the Qing Dynasty, Western mathematics was introduced into China, which brought about a transition from the traditional mathematics education system to the modern Western mathematics education system. This transition was completed by the end of the Qing Dynasty. This Period of mathematics education had its own features; therefore, it could be seen from the traditional mathematics education to the transition Period of Western mathematics education.

Although, in the Sui, Tang, and Yuan Periods, some mathematical knowledge was introduced from foreign countries, it had only a trace of impact. From the end of the Ming Dynasty, China began to translate a large number of foreign mathematics books. It was very different from the past.

Toward the end of the Ming Period, Western European mathematics entered into China. All six volumes of Euclid's *Elements* were translated and calculations with figures and mathematical tools were imported. They had a wide influence and shocked the intellectuals of the age greatly.

At first, scholars had differing attitudes toward the introduction of Western elementary mathematics. Some of them, such as Xu Guangqi (also known as Seu KwangKe) and Li Zhizhao, studied it eagerly. However, most scholars, such as Li Dupei, either ignored it or integrated it with Chinese mathematics. A few objected to the idea openly. Over time, the number of people who learned and researched Western mathematics grew steadily. Some scholars wrote work based on the new knowledge. One well-known example was Mei Wending and his family (Li & Guo, 1988). His basic approach was to combine Chinese and Western knowledge. He composed work on geometry, trigonometry, and algorithms, among others.

Kang Xi, an emperor in the Qing Period, had a great interest in mathematics and left some reports after learning Eastern European mathematics and surveying. He edited *Shu Li Jing Yun* (3 volumes, 1721), and this literature eventually became the mathematics textbook in the late Qing Period.

By the end of the Qing Dynasty, the new mathematics education system had taken on its final form after nearly 50 years of preparation. The introduction of Western scientific knowledge marked the beginning of contemporary Chinese mathematics education. After that, Western mathematics and mathematics education gradually took over and transformed traditional Chinese mathematics education ideas and methods thoroughly.

In 1857, Li Shanlan (1811–1887) and A. Wylie (1836–1887) translated the latter nine volumes of *Elements*, *Algebra* written by Augustus De Morgan (1806–1872) and *Daiweiji Shiji*. One of the textbooks for mathematics education in churches was *Shu Xue Qi Meng* written by A. Wylie. This textbook also played a positive role in China's accepting modern mathematics. In the 1870s, Hua Hengfang (1833–1902) and British missionary John Fryer (1839–1929) co-translated many mathematical documents about algebra, trigonometry, calculus, and probability.

In 1862, Tong Wen Guan was established in Beijing and Suan Xue Guan was founded. Li Shanlan became the principal. The education system of Suan Xue Guan was of eight-year duration and was in use for 30 years.

At the end of the nineteenth century, several mathematics magazines were published. *Suan Xue Bao* was published in 1897 by Huang Qingcheng. In 1899, Zhu Xianzhang published this magazine. In 1900, Du Yaquan issued *Zhong Wai Suan Bao* in Shanghai. The founding of these magazines also promoted the popularization of mathematical knowledge and the development of mathematics education.

In 1902, the Qing government promulgated the *Ren Yin education system*. This system enacted an mathematics education system that was similar to the present one. The *Gui Mao education system* was established in 1904 again. The *Ren Yin education system* and *Gui Mao education system* imitated Japan's *Implementation Regulations for the Secondary School Order* (Meiji 32). Almost all of the mathematics textbooks of this time were translations of Japanese mathematics textbooks.

2.2.5 *The Characteristic of Ancient Chinese Mathematics Culture: The Cultural Characteristics of* Nine Chapters

Nine Chapters is a classic of traditional Chinese mathematics. It dominated the development of the ancient traditional Chinese mathematics culture. It had a profound impact on the development of both ancient Chinese mathematics and world mathematics with its structure, form, and content. *Nine Chapters* was a comprehensive masterpiece of mathematics that dated back to before the Han Dynasty (206 BC–AD 220) and brought together most of the mathematics achievements at that time. It was an encyclopedia-like mathematics masterpiece. *Nine Chapters* showed that ancient Chinese mathematics had already reached a very high level during the early Christian era. It was innovation in many aspects and advanced even in world terms. For example, the place-value system notation described in *Nine Chapters* did not appear in India until the end of the sixth century. Calculations with fractions were relatively advanced in China during the *Nine Chapters* era, but were not seen in India until the seventh century. Square root algorithms, which were developed in

Western countries only at the end of the fourth century, and cube root algorithms, which were yet to be developed elsewhere, were discussed in *Nine Chapters.* The book also covered other topics such as positive and negative numbers, some calculation rules, and the systems of linear equations and quadratic equations, which did not appear in Western countries until much later.

It is still uncertain who wrote *Nine Chapters*. Experts believe that it was not written independently by a single author, but compiled by several mathematicians. Before Liu Hui annotated *Nine Chapters* during the Wei and Jin Period (in the mid-third century), it already had an unshakable authoritative position in traditional Chinese mathematics. Liu Hui's annotations helped people to achieve a better understanding of *Nine Chapters.*

Nine Chapters comprising this work were Surveying of the Land, Millet and Rice, Distribution by Progression, Diminishing Breadth, Consultation on Engineering Works, Imperial Taxation, Excess and Deficiency, Calculating by Tabulation, and Gou-Gu (right-angled triangles), with a collection of 246 mathematical questions. Most chapters started by posing a question, which was followed by the answer and the technique for solving the problem; some of these techniques were mathematics theorems or formulae. In total, 202 techniques were presented in the book, 69 of which were of general importance. These techniques were the foundation for traditional mathematical theories in China.

The fundamental characteristic of traditional Chinese mathematics, with *Nine Chapters* as representative, can be summarized as practical applicability and algorithm-centered calculability.

2.2.5.1 The Characteristic of Practical Applicability

The practical applicability of *Nine Chapters* determined the features of traditional Chinese mathematics, which was also a reflection of traditional Chinese philosophy. In other words, the practical applicability of traditional Chinese mathematics was rooted in traditional Chinese philosophy.

First, the practical needs of society at that time determined the emergence and development of *Nine Chapters*. *Nine Chapters* was a practical mathematics masterpiece, meticulously compiled based on the research and collation of ancient mathematical materials. Its content was related closely to practical calculation issues in people's daily lives, such as the calculation of land area, food exchange, goods distribution, taxation, penalties, attendance entry, or civil engineering. Most of the selected 246 questions related to the practical problems of the era. Some were everyday mathematical puzzles, and about 190 questions dealt with economic activities. These questions recorded some important historical materials of the Period's social economy. The chapters had parallel content, and the content and calculation methods in each chapter progressed from simple to complex.

Mathematical knowledge was regarded as valuable, was subsequently developed insofar as it met social needs, and was helpful in solving everyday problems; otherwise, little heed would have been paid to it, and it might even have been abandoned. For example, although *Nine Chapters* was an encyclopedia of the mathematics achievements since the Qin Dynasty, its authors did not include the geometrical knowledge of the Mohist School (a school of thought in the Spring and Autumn and Warring State Periods, 770–221 BC) into their own mathematics system, since the geometry of this school was abstract mathematical knowledge about the concepts of points, lines, and surfaces and their logical relationships, and this knowledge has no direct applicability to practical problems.

Second, to meet social needs and evaluation standards for applicability, the rationale and methods for compiling *Nine Chapters,* some of which are still in use today, were not defined or explained in the book; that is, the nature of these concepts was not disclosed. The logical relationships among these concepts were not so clear and, therefore, they seemed parallel to one another. People understood them intuitively when learning and using mathematics, based on their mathematical experience. Ancient Chinese mathematicians were more interested in the applicability of these concepts than in disclosing the relationships among them, nor were they aware of the importance of making clear these relationships to learners.

Third, there was no basic mathematical knowledge, such as the introduction of counting rods or the nine-by-nine multiplication table (in rhyme), in the book. This could be attributed to the mathematical level of either the learners at that time or the authors of the book. It is likely that the users of *Nine Chapters* had already understood the above-mentioned basic mathematical knowledge before they read the book or that they could grasp these concepts while learning and using the calculation methods. Moreover, such an arrangement was determined by the practical nature of the book; there was no need for a person to understand these terms, basic concepts, and common calculation methods when solving daily problems. The fact that the mathematical concepts and judgments in the book were presented with any logical reasoning shows that it was not a general textbook, but one for senior mathematics scholars or a practical mathematics manual for government officials. As mathematics historian Li Di said, "In the process of its composition, *Nine Chapters* was never independent of the government's economic department (it may have also been kept in the national library). It was a very practical book whose purpose was to serve the economy of the time" (Li, 1997, p. 109).

Fourth, the origins of the terms in *Nine Chapters* also reflected its practical characteristics. Most of the content and terms used in the book were related directly to social production. These were no abstract concepts, but a reflection of real aspects of existence. This was entirely different from *Elements* of Euclid, in which none of the issues discussed were practical; rather, they were all about the relationships among concepts. It is widely known that "geometry deals with points, lines, planes, angles, circles, triangles, and the like. For Euclid and for the Greeks whose work Euclid was presenting, those terms represented not physical objects themselves, but abstract concepts derived from physical objects. Actually, only a few properties of the physical objects are reflected in the mathematical abstractions to which they give rise.… To be precise about what his abstract terms included, Euclid began with some definitions" (Kline, 1995, p. 42). However, for *Nine Chapters*, there was no need to define self-evident ideas as doing so did not have any practical application, but reflected the historical background from which they originated as well. For instance, the shi and fa used in writing fractions were both related to real life. “In ancient China, the dividend was called shi and the divisor fa. The dividends in ancient mathematics were all real items, such as grain or silk, which is what shi means in Chinese. The divisor was actually a certain standard, which is what fa means in Chinese”. That is why the divisor was called fa (Gu, 1963, p. 5). Almost all of the techniques in the book are linked to items used in daily life.

Fifth, the primary aim of ancient Chinese mathematics education was practicality. This suggests that mathematical knowledge was indispensable to real life and that people needed to learn it to complete real-life tasks successfully. “In a word, the aim of mathematics education was to train a certain skill and apply it in real life. It was not aimed at training the scientific spirit or methods to improve the quality of scholarship” (Li, 2005, p. 24). With such a goal, people did not need to carry out in-depth research on mathematics; rather, they only needed to grasp sufficient mathematical knowledge to meet their needs. All ancient Chinese mathematicians discussed, to a greater or lesser extent, the practicability of mathematics in their books. Cheng Dawei described the function of mathematics in *Suan Fa Tong Zong* (*Complete Collection of Algorithms*): “Mathematics is something that an intelligent child is able to understand while a foolish elder fails to comprehend. Mathematics tops all the skills in the human world. To be well educated but ignorant about mathematics is like staying in a dark room” (Mei & Li, 1990, p. 63).

Sixth, the social status and administrative influence of mathematicians also played a key role in making traditional mathematics practical. One important reason accounting for the difference between traditional Chinese mathematics and ancient Greek mathematics was that traditional Chinese mathematics texts were written by economic officials, while Greek mathematics researchers were scholars.

The scholars of ancient Greece who researched and compiled mathematics knowledge attempted to use it to describe the world and train the minds of certain talented people. They did not care about the application of mathematics in real life. Since scholars like Aristotle were also logicians, they applied logic methodology in mathematics research and produced deductive mathematical models, which were typically represented in Euclid’s *Elements*. Some people in ancient China also saw the possibility of formulating deductive mathematical models, but the economic officials made it impossible. They were in charge of compiling mathematical knowledge and gathered mathematics problems into collections of questions such as *Suanshu Shu* (*Writings on Reckoning*). Their purpose of compiling was for everyday use. *Nine Chapters* is a typical representative of their work and well represents the oriental mathematical model (Li, 2000).

The influence of technology and administration held back the minds of mathematics researchers in the development of traditional Chinese mathematics for over 2000 years. Many world-class achievements were made during this process but, unfortunately, the model of *Nine Chapters* remained unchanged.

The applicability of *Nine Chapters* was also a fundamental characteristic of traditional Chinese mathematics. It was radically different from the deductive feature of ancient Greek mathematics. Generally speaking, ancient Chinese people always thought highly of realism and were good at discovering and abstracting questions from reality and, hence, analyzing and solving them. They set up ancient Chinese mathematics based on extensive real-life practice. This was fundamentally different from Greek geometry, which was divorced from reality, and which adopted formalism that followed the thought of pure logical deduction.

So far, the practicability of *Nine Chapters* has been discussed from many different perspectives; however, not everything in the book was practical. Some puzzles in the book had no practical use, which inspired the writing of *Ce Yuan Hai Jing* (Sea Mirror of the Circle Measurements). Some puzzles in ancient Chinese mathematics were of global significance. For example, the "problem of unknown quantities" in *Sun Zi Suanjing* is equivalent to the Chinese Remainder Theorem, or Sun Zi (Sun Tzu) Theorem, in modern mathematics.

2.2.5.2 The Fundamental Characteristics of Algorithm-Centered Calculability

Algorithm-centered calculability is another important feature of traditional Chinese mathematics. In general, in the history of China, science has always had a strong relationship to technique, and it was in fact difficult to differentiate the two. If we had to do so, the difference might be that technique, compared to science, was stronger in its combination of local customs and production. Therefore, it had a firmer and more real hold on people's minds. In China, science was not approached through theoretical exposition. Rather, people acquired knowledge through life experiences. As a result, the knowledge acquired in this way was not necessarily sufficiently logical, and the core of it is calculation. The main task of astronomy and mathematics was to develop calculation techniques.

Practicability generates the need for calculation. In other words, the impetus of the advanced calculation level in ancient China was in the daily needs of real life.

Counting rods played a crucial role in the development of ancient Chinese mathematics and mathematics education. Counting rods were a peculiar counting and calculation tool of traditional Chinese mathematics, used to denote numbers in mathematics and many other scientific areas, and they acquired this name because people used small bamboo or wood strips to perform calculations. This kind of calculation tool was unique to China. It appeared in China no later than the Spring and Autumn Period (770–476 BC), and it was recorded in *Lau Tzu* that "people good at calculating can calculate without using rods."

Denoting numbers with counting rods was straightforward and efficient. The use of counting rods boosted the development of the decimal system in China. "Ancient Chinese mathematics made great achievements in numerical calculation. This should be attributed to the system of numeration, which used counting rods with place value" (Qian, 1992, p. 9).

Decimal number denotation is the most outstanding and ingenious element of modern culture. It is as important as breathing, but the average person knows so little about it, just as he/she knows little about the chemical composition of air. It is a miracle that just ten digits can denote any number, no matter how large or small. Calculating with counting rods, or rod-arithmetic, was a distinctive way and system of calculating created by the Chinese people and was very different from calculating with a pen. It was convenient to add, subtract, multiply, and divide with rod-arithmetic, which was also simple to understand.

Counting rods exerted a great influence on the formation of the ancient Chinese mathematics model. As Li (1986) said, "Counting rods are not merely pure number operations, but a set of strip-style calculations. Chinese mathematicians not only used counting rods in different places to denote different values and invented the decimal notation, but also used strips in different positions to form specific mathematical patterns to describe certain types of practical problems."

The features of traditional Chinese mathematics and its great achievements in algebra were not accidental—they were actually related closely to the counting rods. When comparing Chinese and Indian mathematics, Japanese scholar Yoshio Mikami (1875–1950) said:

> The main contribution of Chinese mathematics is the development of algebra. Other than this, it is not distinctive in other areas. At three different points in history, Chinese algebra was top in the world. The first was the achievements of *Nine Chapters* in the Han Dynasty, *Ji Gu Suanjing (Continuation of Ancient Mathematics)* in the Tang Dynasty, and various mathematical masterpieces in the Song and Yuan Dynasty….The main reason was that the Chinese have used counting rods since ancient times, and counting rods have a great deal to do with the development of Chinese mathematics. Only when we consider the development of Chinese algebra together with the use of counting rods can we understand the relationship between them (Dai, 2003, pp. 74–207).

In ancient Chinese mathematics, the place-value system was not only used to denote the digits of a number, but also represented different numbers in calculating procedures, like the method of separation of variables in modern mathematics.

Despite their many advantages, counting rods also had some disadvantages that were hard to overcome. For instance, it was hard to avoid mistakes when denoting big numbers with them, and it was difficult to identify mistakes in long calculations. Therefore, they did not facilitate the abstraction of Chinese mathematics and the setting up of a strict, logical theoretical system.

2.3 The Soul of Traditional Mathematics Education Thoughts of China

As an inseparable part of Chinese traditional education, traditional mathematics education in China shared its guiding ideology. In other words, the thoughts of China's traditional education are the soul of its traditional mathematics education, which are embodied in the eternal themes of "honoring teachers and esteeming truth," "teaching benefiting teachers as well as students," and so on. In this respect, in Chapter One of my work *Mathematics Education in China: Tradition and*

Reality, mathematics education values of ancient China are expounded in great detail. In order to give a better understanding of the key thoughts of traditional mathematics education of China, some points are picked out here.

First, the traditional mathematics education of China advocates both the dominant role of students and the leading function of teachers in the teaching process. Since ancient China, teachers have been respectable and have held an ethically lofty social status. The following are some brilliant expositions of ancient Chinese philosophers in this aspect.

> Only if teachers are respected, truth and knowledge can be esteemed and acquired. (*Note of Learning in Book of Rites*)

> The country in which teachers are highly respected will thrive, and the country in which teachers are belittled will decline. (*A Conspectus to Xuncius*)

> A teacher in one day, is a father all the life. (*Taigong Family Education in Lost Ancient Books found in Mingsha Mountain*)

> For the academic students, nothing is more important than following the teacher's instruction. (*Revised Constitution of LiuYang Mathematics School*)

"Esteeming truth" means that knowledge is presented according to the reality of students, which can be realized only if the dominant role of students is guaranteed. However, some people misunderstand this as denying the principal status of students during the teaching process. Being strict with students cannot be confused with denying their dominant roles in education. Meanwhile, it should be reminded that an enjoyable classroom atmosphere is not equated with the realization of students' dominant roles.

Second, Confucius, a great educator and ideologist in ancient China, presented the idea of "teaching benefitting teachers as well as students" over 2000 years ago, which concisely indicated the teaching thought emphasizing both the dominant role of students and the leading function of teachers. In a word, the idea advocates that teaching and learning can promote each other; a mutual improvement can be achieved between teachers and students, and between teachers' teaching practice and their self-learning, which disapproves the dichotomy of "student-centered" or "teacher-centered" patterns and encourages the balance and harmony between teaching and learning. It has different expressions in various teaching settings and times. For instance, "Among any three people walking, I will find something to learn for sure" (*The Analects of Confucius*). "So pupils are not necessarily inferior to their teachers, nor teachers better than their pupils. Some learn the truth earlier than others, and some have special skills—that is all" (*On Teaching, Hanyu*). This concept is always in the practice and development of Chinese traditional mathematics education.

As for learning, Confucius valued the combination of learning and thinking: "Learning without thinking leads to confusion; thinking without learning ends in danger" (*The Analects of Confucius*). As an old Chinese saying says, "Have a master at home, train skills on your own." All of these encourage the learners' initiative of learners.

As for teaching, Confucius emphasized teachers' devotion. He advised teachers "to be insatiable in learning and to be tireless in teaching" (The Analects of Confucius). If a student cannot learn well, a teacher must take responsibility for this to some extent.

In mathematics teaching, a teacher's role is to lead the students through working out the teaching plan, fulfilling the teaching design, and implementing teaching access. In brief, mathematics education in China emphasizes both the leading function of teachers and the dominant role of students, striking a balance and harmony between teaching and learning, between teachers and students.

The idea of "teaching benefitting teachers as well as students" can be realized by the discussion and communication between teachers and students, which is similar to Socrates' elicitation teaching theory. Many typical cases of mathematics teaching in ancient China were in the form of conversation and discussion. The following case from *Zhou Bi Suan Jing* will illustrate this point vividly.

> In this case, Rong Fang asked Chen Zi about the solution to a mathematical problem, but Chen Zi instructed him to learn the methods of thinking instead of telling him the answer immediately. After Rong Fang's tirelessly repeated thinking, Chen Zi explained the causes of the problem and its solution.
>
> Once Rong Fang asked Chen Zi, "I have heard that you are so talented that you know the height and the size of the sun, the distance light can travel and a person can cover in a day; you even know the positions of all of the stars and how big the earth and the universe are, no matter whether they are close or far. Is that true?"
>
> Chen Zi said, "Yes."
>
> Rong Fang said, "Though I don't know about it, I hope you can tell me. Could you please teach me how to do it?"
>
> Chen Zi said, "Certainly. This is what mathematics can do. You are good enough at mathematics to know about it as long as you keep thinking."
>
> So Rong Fang went home and thought it over, but he couldn't get any answer. Then he interviewed Chen Zi again and asked, "I kept thinking about it but I failed, could you tell me now?"
>
> Chen Zi said, "You are thinking but not deeply pondering over it. This is also the skill to calculate the distance or the height, but you cannot get it because of your lack of analogy. Those who find it difficult to grasp the skill always learn but not extensively, or learn extensively but not intensively, or learn intensively but not analogically. It is the analogical skill that makes a difference between fools and wise men. In other words, only the person with analogical skills is qualified for intensive learning. As a result, there is no use explaining to you the abstruse problem now. Please go back again and ponder over it."
>
> Rong Fang went back again and pondered over it for many days without any solution. Then he visited Chen Zi and said, "I have thought it over but I am not intelligent enough to figure it out. Please tell me this time." Then Chen Zi instructed him how to calculate the distance between the sun and the earth with *Gou-Gu Ding Li* in the form of analogy.

Regarding this case, Zhao Shuang commented in his notes that a teacher would not explain unless a student was desperately anxious and determined to learn and, after being given an instance, the student was expected to infer other things from it. This viewpoint is quite similar to Liu Hui's ideas put forward in his notes of Su Mi

(Millet) Chapter and Equation Chapter in *Nine Chapters on Mathematical Procedures*, which emphasize the importance of analogy in the mathematical application.

Although the cases mentioned above are special ones, they reflect the philosophy of mathematics learning methods. The following can be inferred. In the mathematics teaching of ancient China, heuristic teaching methods were applied through the give-and-take of conversation between teachers and students, and this interaction was highly valued. This was similar to Socrates' "elicitation teaching theory" used in his geometry teaching mentioned in *Menon* by Plato of ancient Greek; Chen Zi advocated independent study, and Rong Fang clarified the importance of reflection in mathematics learning. Chen Zi did not pass on his knowledge to Rong Fang until the latter pondered on it over and over again for a long time.

In brief, it was emphasized in *Zhou Bi Suan Jing* that, in mathematics learning, inferences about other cases should be drawn from one instance; in addition, the drill and cultivation of thinking ability as well as some basic knowledge are essential. The heuristic education in *Zhou Bi Suan Jing* is comparable to Socrates' "elicitation teaching theory," and both of them have played a positive role in mathematics education history in the East and West, then and now.

2.4 Typical Cases in Chinese Traditional Mathematics Teaching Practice

That "three non-collinear points determine a plane" is a self-evident truth. Similarly, Chinese conventional thinking, educational ideology, and unique thinking methods of traditional mathematics determine Chinese traditional mathematics education as a perfect whole due to their dynamic integration produces. The mutual promotion and interdependence of these factors in the teaching practice lead to the internal positive cycle and vitality of the whole. Nevertheless, the whole is not a closed system but one advancing with the times, which adopts new ideas and methods constantly to realize its self-improvement and self-transcendence.

In Chinese traditional mathematics education, the cases of intelligent solutions to practical problems are too numerous to mention one-by-one. The following are some cases to illustrate this.

2.4.1 Methodology of Wholeness

Case 1 The proof of Gou-Gu Ding Li (Pythagorean theorem) by Zhao Shuang.

In *Zhou Bi Suan Jing*, Gou-Gu Ding Li is recorded as the square of Gou plus the square of Gu, and after being extracted by the square root, the chord length is gained (Vol. II, Zhou Bi Suan Jing) (Jiang and Xie 1996).

Gou-Gu Ding Li was proved by Zhao Shuang with his “Gou-Gu Yuan Fang Tu” in his annotations on *Zhou Bi Suan Jing*; this was created after rotations and symmetry transformation of some times according to the out-in complementary principle. “The square of Gou plus the square of Gu is the square of the chord, then it extracts a root, and the chord length is the result. Further illustrations are given that, according to the chordal graph, Guo multiplies Gu and then multiplies two, plus the square of the difference between Gou and Gu, and the result is the square of the chord as well.”

【 朱:Zhu 股:Gu 勾:Gou 弦:Xian 黄:Huang】

In the chordal graph, the product of Gou multiplied by Gu is illustrated as a red rectangle named “Zhu Shi,” and four rectangles if doubled. The square of the difference between Gou and Gu is presented as a small yellow square in the middle called “Zhong Huang Shi.” Therefore, two “Zhu Shi” plus one “Zhong Huang Shi” is a “Xian Shi”—the square of the chord (Fig. 2.1).

The above proof can be illustrated by modern mathematical symbols.

If *a, b,* and *c,* respectively, represent Gou, Gu, and Xian in Gou-Gu graph (a right triangle), one “Zhu Shi” is $\frac{1}{2}ab$, four “Zhu Shi” is $2ab$, and “Huang Shi” is $(b-a)^2$. So $c^2 = 2ab + (b-a)^2 = a^2 + b^2$, i.e., $c^2 = a^2 + b^2$.

Case 2 The Proof of Guo Gu Ding Li by Liu Hui.

Liu Hui structured two different rectangles (as a whole) with the out-in complementary principle and dealt with the relationship between the base line, the height, and the size of a triangle from a global perspective, i.e., the size of a triangle is equal to the half size of the rectangle.

As shown in Fig. 2.2, if two right angle sides of a right-angled triangle are, respectively, *a* and *b*, and the hypotenuse is *c*, on each side of the right-angled triangle three squares can be structured counterclockwise with respective side lengths of *a*, *b,* and *c*, so the size of the square with the side length of *c* is c^2, i.e., $c^2 = \frac{1}{2}[(b-a)+b]\cdot b + \frac{1}{2}ab + a^2$, so $c^2 = a^2 + b^2$.

The proof methods of Gou-Gu Ding Li used by Zhao Shuang and Liu Hui are typical examples of geometrical proof methods in ancient China, which view the

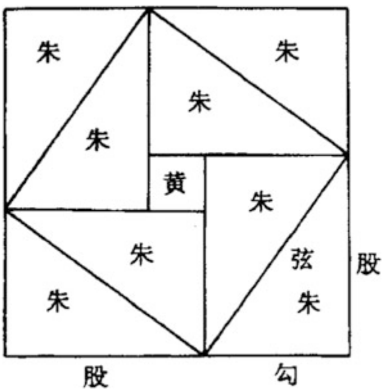
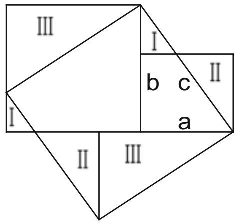


Fig. 2.1 Diagram proof by Zhao Shuang

Fig. 2.2 Diagram proof by Liu Hui



internal relationships between conditions and conclusion and tackle it as a whole with intuition, so that the proof procedure can be explained briefly. The method called "the out-in complementary principle" implies the feature of wholeness in Chinese traditional thought, and geometrical proof involved attempts to structure orderly wholeness. The principle has two traits. First, the wholeness ideology is applied. A new holonomic shape is structured according to the size and shape of the known geometric form, and then, a conclusion is drawn in terms of its size and volume. In other words, the features of parts are handled from the global perspective. Besides, a conclusion is drawn according to the relationship between the wholeness and parts with a quantity standard, which means that geometric transformation is conducted through the congruence relationship between geometric forms (or size and volume) in order to structure the wholeness. However, only the size and volume are taken into consideration, but other factors such as angles and relations between the locations of lines are not. In other words, Liu Hui and other mathematicians in ancient China tackled the relationships of these factors by direct viewing in the geometric transformations, which resulted in some seemingly ambiguous proof methods. This research approach is quite different from that used in ancient Greek. For example, Zhao Shuang structured a wholeness, a square, after rotations and moving transforms of several times, then proved Gou-Gu Ding Li on this basis, which is quite different from the method used in *Elements of Geometry* by Euclid in ancient Greek to prove the Pythagorean theorem.

Case 3 Problems of "meetings of wild ducks and wild geese" in *Nine Chapters on the Mathematical Art*.

> It takes seven days for wild ducks to fly from the South Sea to the North Sea, and nine days for wild geese from the North Sea to the South Sea. If they depart simultaneously from the South Sea and the North Sea, how long does it take for them to meet?
>
> The problem was solved in this way:
>
> The sum of days is used as the divisor, and the product of days multiplied as the dividend…. In this way, if it takes 7 days for wild ducks to arrive and 9 days for wild geese, it takes 63 days for wild ducks to arrive 9 times, and for wild geese 7 times. The day on which all of them arrive is their meeting day. So 63 days divided by 9 plus 7 is the number of days it takes for them to meet. This can be illustrated in the following numerical table:

$$\begin{array}{cc} & \begin{matrix}\text{凫} & \text{雁}\end{matrix} \\ \begin{matrix}\text{日}\\ \text{至}\end{matrix} & \begin{bmatrix}7 & 9\\ 1 & 1\end{bmatrix}\end{array} \xrightarrow[\text{齐其至}]{\text{同其日}} \begin{bmatrix}63 & 63\\ 9 & 7\end{bmatrix} \xrightarrow{\text{并齐至为共至}} \begin{bmatrix}63\\ 9+7\end{bmatrix}$$

[日: travel time 至: arrival time 凫: wild ducks 雁: wild geese

同其日: the same set days

齐其至: respective travel time

并齐至为共至: the sum of their respective travel time and the same set days]

i.e., they meet 16 times within 63 days, so the number of days it takes for them to meet once is:

$$\frac{63}{7+9} = \frac{63}{16} = 3\frac{15}{16} \text{ days.}$$

Another explanation given by Liu Hui is that the wild ducks cover 1/7 of the whole journey every day, and the wild geese 1/9. According to the requirement of "the set days and the respective travel time," they are written, respectively, as 9/63 and 7/63. Supposing it takes 63 days for them to travel from the south to the north, totally 16 times for both wild ducks and wild geese. So the number of days it takes to meet is $\frac{63}{16} = 3\frac{15}{16}$ days.

This question displays the basic fraction methodology used by mathematicians in ancient China, which is called "Qi Tong Shu," i.e., finding common denominators and equivalent fractions in order to carry out addition and subtraction. The solution to the question reflects the methodology of wholeness in handling practical problems as well.

2.4.2 Methodology of Symmetry

2.4.2.1 Yang Hui's Methodology of Symmetry

In the research on mathematics education methodology in ancient China, Yang Hui, a well-known mathematician and mathematics educator of the Southern Song Dynasty (1127–1279), should be introduced above all. One of his contributions to mathematics education was his work *Mathematics Teaching Program,* a vital document in the mathematics education history of ancient China, which was first published in another work of his, *Yang Hui Suan Fa* (*Yang Hui's Arithmetic*). It was also the first document on mathematics teaching discovered in the world, falling into the same category of mathematics teaching plans found later. It recorded a very complete mathematics knowledge hierarchy, definite learning process and goals, specific hierarchical analytical processes of textbooks and reference books, teaching methods involving intensive explanations and more practice; and emphasis on mathematics principles as the root of all methods.

Yang Hui's thoughts about mathematics education embodied those of the sages of the past, and some teaching ideas such as "gaining new insights through reviewing old materials," "following in proper order and improving gradually," and "teaching only the essential and ensuring plenty of practice" were fully incorporated into his teaching practice. In addition, he attached great importance to the combination of mathematics thought and the fostering of students' creative thinking skills. In conclusion, Yang Hui left later generations a legacy of ideas which integrate mathematics thoughts and Chinese traditional education methods.

Symmetry Approach in Computation Teaching

The aesthetic ideology of mathematics can be appreciated in Yang Hui's works, especially his symmetry approach.

Case 4 "The first group of numbers includes one, three, five, seven and nine, and the second group includes two, four, six, eight and ten. If the result is 55, what are the calculation processes? The method is as follows. The sum of each pair is 11, and if multiplied by 10, the product is 110, half of which is 55" (*Xu Guo Zhai Qi Suan Fa I*). Its illustration in the form of modern mathematics is as follows:

$$S = 1 + 2 + 3 + \cdots + 9 + 10,\ S = 10 + 9 + \cdots + 3 + 2 + 1$$

The addition of each symmetrical pair of numbers in each group from left to right is:

$$2S = (1 + 10) + (2 + 9) + \cdots + (2 + 9) + (1 + 10) = 11 \times 10 = 110$$

$$S = 110 \div 2 = 55$$

The arithmetic procedure was once recorded in detail in the biography of Gauss, the well-known mathematician: When he was in primary school, Gauss figured out this symmetrical calculation method without any hints from the teacher. Also, in the biography of Wu Zaiyuan, a famous mathematician in the Period of the Republic of China, it was recorded that he was very excited after he figured it out on his own as a child.

A new method was created by Yang Hui with this symmetrical principle (Dai, 2003, p. 204). It was clarified that the sum of each pair of numbers, which has the same distance to the first and the last number (Symmetry), is equal to the sum of the first and the last number (Uniformity). The discipline makes it convenient to carry out the calculation. As well, it is very inspiring for both mathematics teaching and research; for instance, the formula for the sum of the numbers in front of the arithmetic progression used in primary school education is deduced according to this discipline.

Symmetrical Methods of Area Calculation Teaching

Case 5 Symmetrical ideology is also applied to the deduction of the isosceles triangle area in *Nine Chapters on the Mathematical Art*, called the central symmetry principle. "Multiply half of the bottom side by the height. If half of the side is known, a rectangle can be created according to the out-in complementary principle. Also, it can be done with half of the height multiplied by the side" (Fig. 2.3).

Yang Hui gave more details about the deduction of the isosceles triangle area and its various supplementary conditions, which made it more comprehensive and flexible. "If the bottom side can be halved easily (i.e., a natural number), multiply half of it and the height. If the height can be halved easily, multiply half of it and the bottom side. If neither the bottom side nor the height can be halved easily, multiply the bottom line and the height and then reduce by half."

This is illustrated in the geometric figures, as shown in Figs. 2.3, 2.4 and 2.5. (Given ΔABC, D and E are, respectively, the midpoints of sides AB and AC.)

In the formula: $S = (\frac{1}{2}a)h$, $S = a(\frac{1}{2}h)$, and $S = \frac{1}{2}(ah)$ (S is the area of triangle, a is the bottom side, and h is the height).

In fact, while deducing the area formula for the geometric figure, the mathematicians of ancient China unconsciously adopted a simple center-transfer method of elementary geometry. Liu Hui and Yang Hui supplemented the incomplete figure to create a perfect integration—a rectangle, according to the out-in complementary principle. They based the area of a triangle on the concept of the rectangle.

Yang Hui's aesthetic ideology of symmetry is very useful in mathematics teaching. For instance, Fig. 2.3 is always used to teach the formula for the area of the triangle in primary schools in China. Some insightful teachers may use Fig. 2.3 first and then inspire the student with Figs. 2.4 and 2.5, which is more helpful for arousing students' interest and fostering their thinking ability.

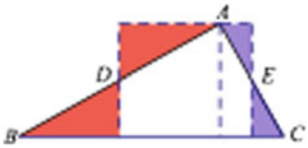


Fig. 2.3 Diagram derivation of the formula for area of triangle (1)

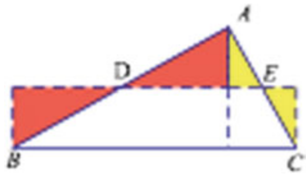


Fig. 2.4 Diagram derivation of the formula for area of triangle (2)

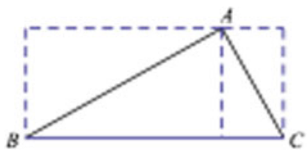


Fig. 2.5 Diagram derivation of the formula for area of triangle (3)

2.4.2.2 Zhen Luan's Problem-Solving Teaching

Case 6 Zhen Luan, a mathematician in the Northern and Southern dynasties, recorded some typical cases of problem-solving teaching using the symmetrical method in his notes of *Shu Shu Ji Yi*. "How can we know the width of a river with marks instead of any calculation?"

The solution is as follows: "Supposing someone sets three parallel lines on the north side of the river, and the lines in the north and south have a distance of one Zhang (a length unit in ancient China). Standing in the north of the middle line he takes a straight look at the north bank (M) and marks the point (A) on the southern line, then takes a straight look at the south bank (N) and marks the point (B). The length between the two points is transferred to the counterparts (A′ and B′) on the northern line. Afterward, standing in the south of the middle line, he takes a straight look at the north side through the two points (A′ and B′), respectively, and gets the other two points (M′ and N′). So the distance between these two points is the width of the river."

In this case, an axial symmetry principle of geometry is used (Li, 1997, p. 263). The corresponding geometric figure is as follows (in Fig. 2.6A, A′, B, B′ are the marked points):

[北表:the north line 中表:the middle line 南表:the south line 记号:marked points 北岸:the north bank 南岸:the south bank 河宽:the river width].

2.4.3 *Formulae in Verse in Mathematics Teaching*

It was believed that a perfect memory is helpful for understanding. There is a variety of ways of developing a strong memory and the formula in verse is one of the favorites of the Chinese. In ancient China, some mathematics problems were adapted to formulae in verse for the convenience of mastery and instruction. Rhythm formulae orally display the connection between quantity and space form as well as the rules. The earliest rhythm formulae appeared in the preface of *Zhou Bi Suan Jing*, and many were found in the works of Yang Hui and Zhu Shijie during the Song and Yuan dynasties. In *Suan Fa Tong Zong* by Cheng Dawei of the Ming

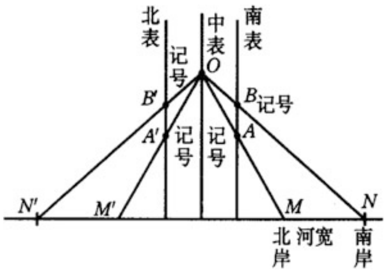


Fig. 2.6 Diagram of the measurement of river width by Zhen Luan

Dynasty, the abacus operation was accompanied by rhythm formulae as well as some mathematics questions.

These rhythm formulae are the organic integration of arts and mathematics, embodying the romance of ancient mathematics. Additionally, they played a role in mathematics learning and research, and business as well. Furthermore, they promoted the popularity of mathematics knowledge, although limited to some common sense.

Case 7 "Put a Ju (a measuring tool) flat to measure the horizontal and vertical level; set the Ju upright to measure the height; put it upside down to measure the depth; put it lying flat on the ground to measure the horizontal distance; put it encircled to get a circle; combine two Ju to get a rectangle" (Fig. 2.7).

The first four verses tell about the measuring methods with Ju, and the last two illustrate the formation of a circle and a rectangle. This theory is the same principle of similar triangles in plane geometry in secondary schools.

The measuring method of height is as follows: Put the side AC of the square flat and overlap the horizontal line of AE, keeping the side BC vertical. Look at the top of F, and the sight line of AF and BC intersect at D. So $\Delta ACD \backsim \Delta AEF$ $\therefore \frac{AC}{CD} = \frac{AE}{EF}$, height $EF = \frac{CD \times AE}{AC}$.

2.4.4 *Learning Mathematics Through Games*

In ancient China, people created plenty of mathematics games according to the characteristics of children's ages. Mathematics games help arouse children's interests and cultivate their thinking abilities. Generally, there are two kinds of games: number games and jigsaw (or take-apart) puzzles. "Qi Qiao Ban" (seven-piece puzzle) and "Yi Zhi Tu" (Puzzle Chart) are two popular jigsaw puzzles, which build graphs of the same size. The function of "Qi Qiao Ban" was recorded in the preface of *Qi Qian Ban He Bi* in 1803. "Nobody knows who created the seven-piece puzzle. The talented play it for entertainment."

Fig. 2.7 Diagram of Ju

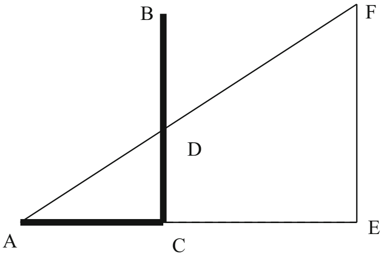


Fig. 2.8 Tangram shapes (1)

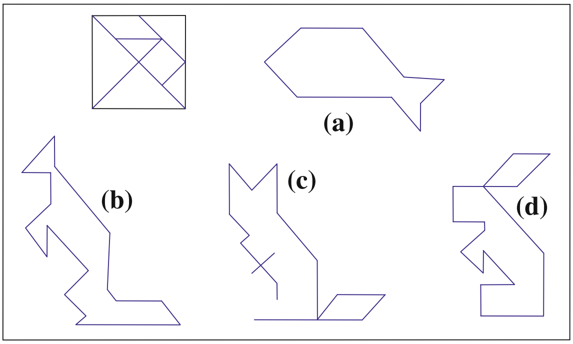
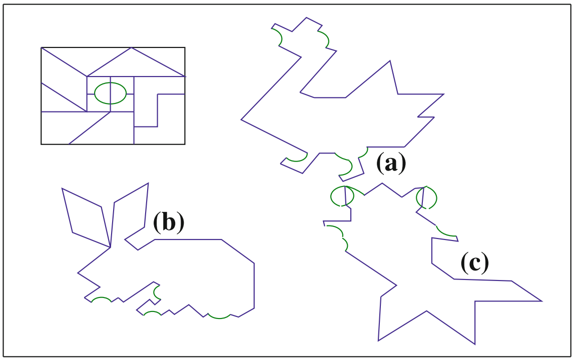


Fig. 2.9 Tangram shapes (2)



Case 8 Try to divide the figures (a, b, c, and d) as shown in Fig. 2.8 into the same parts in the square which consists of 5 kinds of figures, big-, small-, medium-sized, square, and inclined.

Case 9 Try to divide the three figures (a, b, and c) as shown in Fig. 2.9 into the same parts in the square.

2.5 Conclusion

In 1676, Newton said "If I have seen farther than others, it is because I was standing on the shoulders of giants." "The shoulders of giants" are the scientific traditions and achievements in Europe. Traditions are sources and the basis of new sources. Based on its own traditions, the development of mathematics education in China can still gain from the advanced experiences from other countries and positive elements of new ideas and thoughts. It is not perfect yet and its flaws should not be neglected.

From the perspectives of the past, the present, and the future, traditions and reformations should be integrated. The irrational factors in traditions must be sublated and new ideas and methods must be introduced gradually to avoid the waste of educational resources. It is helpful for the development of Chinese mathematics education to grasp traditional mathematics knowledge and understand traditional mathematics education deeply. As for traditional mathematics education, instead of denying it blindly or exaggerating it excessively, a mathematics educator is expected to have abilities of selection and judgment, like an excellent photographer who can take a good picture even with an old camera in contrast to an unskilled shutterbug who cannot do this even with an advanced one.

It is believed that a wonderful future for mathematics education can be achieved with a comprehensive and deep understanding of traditions, constant practice, and creativity.

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