

Komplexe Zahlen

Polardarstellung komplexer Zahlen:

$$z = x + iy = |z| e^{i\varphi}, \quad |z| = \sqrt{x^2 + y^2}, \quad \tan \varphi = \frac{y}{x}$$

Euler'sche Formel:

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

Vektorrechnung

Dreidimensional

$$\begin{aligned} \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) \\ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \\ (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) &= (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) \\ (\mathbf{a} \times \mathbf{b})^2 &= a^2 b^2 - (\mathbf{a} \cdot \mathbf{b})^2 \end{aligned}$$

Kronecker-Symbol:

$$\delta_{ij} = \begin{cases} 1 & \text{wenn } i = j \\ 0 & \text{wenn } i \neq j \end{cases}$$

Levi-Civita-Symbol:

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{wenn } ijk \text{ gerade Permutation von } 123 \\ -1 & \text{wenn } ijk \text{ ungerade Permutation von } 123 \\ 0 & \text{sonst} \end{cases}$$
$$\varepsilon_{ijk} \varepsilon_{lmk} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}, \quad \varepsilon_{ijk} \varepsilon_{ijk} = 2\delta_{il}$$

Einstein'sche Summenkonvention (kartesische Koordinaten):

$$\mathbf{a} \cdot \mathbf{b} = a_i b_i \equiv \sum_{i=1}^3 a_i b_i, \quad (\mathbf{a} \times \mathbf{b})_i = \varepsilon_{ijk} a_j b_k$$

Ableitungen von skalaren Feldern $\phi(\mathbf{r})$, Vektorfeldern $\mathbf{X}(\mathbf{r})$:

$$\text{Gradient: } \mathbf{grad} \phi = \nabla \phi \quad \text{bzw.} \quad (\nabla \phi)_i = \partial_i \phi$$

$$\text{Divergenz: } \operatorname{div} \mathbf{X} = \nabla \cdot \mathbf{X} = \partial_i X_i$$

$$\text{Rotation: } \mathbf{rot} \mathbf{X} = \nabla \times \mathbf{X} \quad \text{bzw.} \quad (\mathbf{rot} \mathbf{X})_i = \varepsilon_{ijk} \partial_j X_k$$

Zweite Ableitungen:

$$\operatorname{div} \mathbf{grad} \phi = \nabla \cdot \nabla \phi = \Delta \phi$$

$$\mathbf{grad} \operatorname{div} \mathbf{X} = \nabla(\nabla \cdot \mathbf{X})$$

$$\operatorname{div} \mathbf{rot} \mathbf{X} = \nabla \cdot (\nabla \times \mathbf{X}) = 0$$

$$\mathbf{rot} \mathbf{grad} \phi = \nabla \times (\nabla \phi) = \mathbf{0}$$

$$\mathbf{rot} \mathbf{rot} \mathbf{X} = \nabla \times (\nabla \times \mathbf{X}) = \nabla(\nabla \cdot \mathbf{X}) - \Delta \mathbf{X}$$

Vierdimensional

Kronecker-Symbol:

$$\delta_{\nu}^{\mu} = \begin{cases} 1 & \text{wenn } \mu = \nu \\ 0 & \text{wenn } \mu \neq \nu \end{cases}$$

Minkowski-Metrik:

$$(\eta_{\mu\nu}) = (\eta^{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \eta^{\mu\lambda} \eta_{\lambda\nu} = \delta_{\nu}^{\mu}$$

Lorentz-invariantes Skalarprodukt (mit Summenkonvention):

$$a_{\mu} b^{\mu} = \eta_{\mu\nu} a^{\nu} b^{\mu} = a_0 b^0 + a_i b^i = a^0 b^0 - a^i b^i$$

Viererortsvektor:

$$(x^{\mu}) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \equiv \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}, \quad \mu \in \{0, 1, 2, 3\}$$

Vierergradient:

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}, \quad (\partial_{\mu}) = \left(\frac{1}{c} \partial_t \right), \quad (\partial^{\mu}) = (\eta^{\mu\nu} \partial_{\nu}) = \left(\frac{1}{c} \partial_t, -\nabla \right)$$

Wellen-, d'Alembert- oder Quabla-Operator:

$$\partial_{\mu} \partial^{\mu} \equiv \partial^2 \equiv \square = \frac{1}{c^2} \partial_t^2 - \nabla^2 \equiv \frac{1}{c^2} \partial_t^2 - \Delta$$

Vierdimensionales Levi-Civita-Symbol:

$$\varepsilon^{\mu\nu\sigma\tau} := \begin{cases} +1 & \text{wenn } \mu\nu\sigma\tau \text{ gerade Permutation von } 0123 \\ -1 & \text{wenn } \mu\nu\sigma\tau \text{ ungerade Permutation von } 0123 \\ 0 & \text{sonst.} \end{cases}$$
$$\varepsilon^{0123} = +1, \quad \varepsilon_{0123} = \eta_{00} \eta_{11} \eta_{22} \eta_{33} \varepsilon^{0123} = -1$$

Integralsätze

Linienintegral:

$$\int_{\mathbf{x}_a}^{\mathbf{x}_b} \nabla \phi \cdot d\mathbf{r} = \phi(\mathbf{x}_b) - \phi(\mathbf{x}_a)$$
$$\int_{\mathbf{x}_a}^{\mathbf{x}_b} dx_i \partial_i (\dots) = (\dots)|_{\mathbf{x}_b} - (\dots)|_{\mathbf{x}_a}$$

Satz von Stokes:

$$\int_F d\mathbf{f} \cdot \mathbf{rot} \mathbf{X} = \oint_{\partial F} d\mathbf{r} \cdot \mathbf{X}$$
$$\int_F df_i \varepsilon_{ijk} \partial_j (\dots) = \oint_{\partial F} dx_k (\dots)$$

Satz von Gauß:

$$\int_V dV \operatorname{div} \mathbf{X} = \oint_{\partial V} d\mathbf{f} \cdot \mathbf{X}$$
$$\int_V dV \partial_i (\dots) = \oint_{\partial V} df_i (\dots)$$

Green'sche Integralsätze:

$$\int_V dV [(\nabla\phi) \cdot (\nabla\chi) + \phi \Delta\chi] = \oint_{\partial V} d\mathbf{f} \cdot \phi \nabla\chi$$

$$\int_V dV (\phi \Delta\chi - \chi \Delta\phi) = \oint_{\partial V} d\mathbf{f} \cdot (\phi \nabla\chi - \chi \nabla\phi)$$

Zylinderkoordinaten

$$x_1 \equiv x = \varrho \cos \varphi \quad \hat{\mathbf{e}}_\varrho = \cos \varphi \hat{\mathbf{e}}_1 + \sin \varphi \hat{\mathbf{e}}_2$$

$$x_2 \equiv y = \varrho \sin \varphi \quad \hat{\mathbf{e}}_\varphi = \cos \varphi \hat{\mathbf{e}}_2 - \sin \varphi \hat{\mathbf{e}}_1$$

$$x_3 \equiv z \quad \hat{\mathbf{e}}_z = \hat{\mathbf{e}}_3$$

$$dV = d^3x = \varrho d\varrho d\varphi dz$$

$$\nabla f = \hat{\mathbf{e}}_\varrho \frac{\partial f}{\partial \varrho} + \hat{\mathbf{e}}_\varphi \frac{1}{\varrho} \frac{\partial f}{\partial \varphi} + \hat{\mathbf{e}}_z \frac{\partial f}{\partial z}$$

$$\Delta f = \frac{1}{\varrho} \frac{\partial}{\partial \varrho} \left(\varrho \frac{\partial f}{\partial \varrho} \right) + \frac{1}{\varrho^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\frac{\partial^2 f}{\partial \varrho^2} + \frac{1}{\varrho} \frac{\partial f}{\partial \varrho}$$

Kugelkoordinaten (sphärische Polarkoordinaten)

$$x_1 \equiv x = r \sin \vartheta \cos \varphi$$

$$x_2 \equiv y = r \sin \vartheta \sin \varphi$$

$$x_3 \equiv z = r \cos \vartheta$$

$$dV = d^3x = r^2 dr d\Omega = r^2 dr \sin \vartheta d\vartheta d\varphi$$

$$\hat{\mathbf{e}}_r = \sin \vartheta \cos \varphi \hat{\mathbf{e}}_1 + \sin \vartheta \sin \varphi \hat{\mathbf{e}}_2 + \cos \vartheta \hat{\mathbf{e}}_3$$

$$\hat{\mathbf{e}}_\vartheta = \cos \vartheta \cos \varphi \hat{\mathbf{e}}_1 + \cos \vartheta \sin \varphi \hat{\mathbf{e}}_2 - \sin \vartheta \hat{\mathbf{e}}_3$$

$$\hat{\mathbf{e}}_\varphi = \cos \varphi \hat{\mathbf{e}}_2 - \sin \varphi \hat{\mathbf{e}}_1$$

$$\nabla f = \hat{\mathbf{e}}_r \frac{\partial f}{\partial r} + \hat{\mathbf{e}}_\vartheta \frac{1}{r} \frac{\partial f}{\partial \vartheta} + \hat{\mathbf{e}}_\varphi \frac{1}{r \sin \vartheta} \frac{\partial f}{\partial \varphi}$$

$$\Delta f = \left(\Delta_r + \frac{1}{r^2} \Delta_\Omega \right) f$$

$$\Delta_r := \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \equiv \frac{1}{r} \frac{\partial^2}{\partial r^2} r \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}$$

$$\Delta_\Omega := \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \sin \vartheta \frac{\partial}{\partial \vartheta} + \frac{1}{\sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2}$$

Kugelflächenfunktionen

$$\Delta_\Omega Y_{\ell m}(\vartheta, \varphi) = -\ell(\ell+1) Y_{\ell m}(\vartheta, \varphi), \quad \ell \in \mathbb{N}_0, \quad m = -\ell, \dots, \ell$$

$$Y_{\ell m}(\vartheta, \varphi) = \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos \vartheta) e^{im\varphi}$$

$$P_\ell^m(u) = \frac{(-1)^{\ell+m}}{2^\ell \ell!} (1-u^2)^{m/2} \frac{d^{\ell+m}}{du^{\ell+m}} (1-u^2)^\ell, \quad P_\ell \equiv P_\ell^0$$

$Y_{\ell m}(\vartheta, \varphi)$	$\ell = 0$	$\ell = 1$	$\ell = 2$
$m = 0$	$\sqrt{\frac{1}{4\pi}}$	$\sqrt{\frac{3}{4\pi}} \cos \vartheta$	$\sqrt{\frac{5}{16\pi}} (3 \cos^2 \vartheta - 1)$
$m = 1$		$-\sqrt{\frac{3}{8\pi}} \sin \vartheta e^{i\varphi}$	$-\sqrt{\frac{15}{8\pi}} \sin \vartheta \cos \vartheta e^{i\varphi}$
$m = 2$			$\sqrt{\frac{15}{32\pi}} \sin^2 \vartheta e^{2i\varphi}$

$$Y_{\ell, -m} = (-1)^m Y_{\ell m}^*, \quad \int d\Omega Y_{\ell m}^*(\vartheta, \varphi) Y_{\ell' m'}(\vartheta, \varphi) = \delta_{\ell \ell'} \delta_{mm'}$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1} \frac{r_{<}^\ell}{r_{>^{\ell+1}}} Y_{\ell m}(\vartheta, \varphi) Y_{\ell m}^*(\vartheta', \varphi')$$

$$= \sum_{\ell=0}^{\infty} \frac{r_{<}^\ell}{r_{>^{\ell+1}}} P_\ell(\cos \alpha), \quad \cos \alpha \equiv \frac{\mathbf{r} \cdot \mathbf{r}'}{r r'}$$

$$r_{<} = \min(r, r'), \quad r_{>} = \max(r, r')$$

Fourier-Transformation

$$f(t, \mathbf{x}) = \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}} \int \frac{d^3k}{(2\pi)^{3/2}} \tilde{f}(\omega, \mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

$$\tilde{f}(\omega, \mathbf{k}) = \int_{-\infty}^{\infty} \frac{dt}{\sqrt{2\pi}} \int \frac{d^3x}{(2\pi)^{3/2}} f(t, \mathbf{x}) e^{-i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

Entwicklung nach einem vollständigen orthonormalen Funktionensystem f_j

$$f(x) = \sum_j a_j f_j(x), \quad a_j = \langle f_j, f \rangle, \quad \langle f, g \rangle = \int_I f^*(x) g(x) dx$$

$$\langle f_j, f_k \rangle = \delta_{jk}, \quad \sum_j f_j^*(x') f_j(x) = \delta(x' - x)$$

Funktionentheorie

Cauchy-Riemann'sche Differenzialgleichungen:

$$f(z) = u(x + iy) + iv(x + iy): \quad u_x = v_y, \quad u_y = -v_x$$

Residuensatz:

$$\oint f(z) dz = 2\pi i \sum_{j=1}^n \text{Res}_{z_j} f(z)$$

Residuum bei einer einfachen Polstelle:

$$\text{Res}_{z_0} f(z) = \lim_{z \rightarrow z_0} (z - z_0) f(z)$$

Mechanik

Lagrange-Gleichungen erster Art:

$$m_i \ddot{\mathbf{x}}_i = \mathbf{F}_i + \sum_{a=1}^r \lambda_a \nabla_i f_a, \quad f_a(t, \mathbf{x}_1, \dots, \mathbf{x}_N) = 0$$

Lagrange-Gleichungen zweiter Art:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j, \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0, \quad L = T - V$$

Hamilton'sche kanonische Gleichungen:

$$\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i, \quad H = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

Poisson-Klammern:

$$\{F, G\} = \sum_i \left(\frac{\partial F}{\partial q_i} \frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial q_i} \right)$$

Elektrodynamik

Maxwell-Gleichungen (Gauß'sches System):

$$\begin{aligned} \operatorname{div} \mathbf{D} &= 4\pi \rho_f, & \mathbf{D} &\equiv \mathbf{E} + 4\pi \mathbf{P} \\ \operatorname{div} \mathbf{B} &= 0 \\ \operatorname{rot} \mathbf{E} &= -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B} \\ \operatorname{rot} \mathbf{H} &= \frac{4\pi}{c} \mathbf{j}_f + \frac{1}{c} \frac{\partial}{\partial t} \mathbf{D}, & \mathbf{H} &\equiv \mathbf{B} - 4\pi \mathbf{M} \end{aligned}$$

Potenziale:

$$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Lineare Medien:

$$\begin{aligned} \mathbf{P} &= \chi_e \mathbf{E}, & \mathbf{D} &= (1 + 4\pi \chi_e) \mathbf{E} \equiv \epsilon \mathbf{E}, \\ \mathbf{M} &= \chi_m \mathbf{H}, & \mathbf{B} &= (1 + 4\pi \chi_m) \mathbf{H} \equiv \mu \mathbf{H} \end{aligned}$$

Vakuum: $\mathbf{D} \rightarrow \mathbf{E}, \mathbf{H} \rightarrow \mathbf{B}, \rho_f \rightarrow \rho, \mathbf{j}_f \rightarrow \mathbf{j}$

Maxwell-Gleichungen (SI-System):

$$\begin{aligned} \operatorname{div} \mathbf{D}^{[\text{SI}]} &= \rho_f^{[\text{SI}]}, & \mathbf{D}^{[\text{SI}]} &\equiv \epsilon_0 \mathbf{E}^{[\text{SI}]} + \mathbf{P}^{[\text{SI}]} \\ \operatorname{div} \mathbf{B}^{[\text{SI}]} &= 0 \\ \operatorname{rot} \mathbf{E}^{[\text{SI}]} &= -\frac{\partial}{\partial t} \mathbf{B}^{[\text{SI}]} \\ \operatorname{rot} \mathbf{H}^{[\text{SI}]} &= \mathbf{j}_f^{[\text{SI}]} + \frac{\partial}{\partial t} \mathbf{D}^{[\text{SI}]}, & \mathbf{H}^{[\text{SI}]} &\equiv \frac{1}{\mu_0} \mathbf{B}^{[\text{SI}]} - \mathbf{M}^{[\text{SI}]} \\ \mu_0 &= 4\pi \cdot 10^{-7} \text{N/A}^2, & \epsilon_0 &\equiv \frac{1}{c^2 \mu_0} \end{aligned}$$

Potenziale:

$$\mathbf{E}^{[\text{SI}]} = -\nabla \phi^{[\text{SI}]} - \frac{\partial}{\partial t} \mathbf{A}^{[\text{SI}]}, \quad \mathbf{B}^{[\text{SI}]} = \nabla \times \mathbf{A}^{[\text{SI}]}$$

Lineare Medien:

$$\begin{aligned} \mathbf{P}^{[\text{SI}]} &= \chi_e^{[\text{SI}]} \epsilon_0 \mathbf{E}^{[\text{SI}]}, & \mathbf{D}^{[\text{SI}]} &= (1 + \chi_e^{[\text{SI}]}) \epsilon_0 \mathbf{E}^{[\text{SI}]} \equiv \epsilon^{[\text{SI}]} \mathbf{E}^{[\text{SI}]}, \\ \mathbf{M}^{[\text{SI}]} &= \chi_m^{[\text{SI}]} \mathbf{H}^{[\text{SI}]}, & \mathbf{B}^{[\text{SI}]} &= (1 + \chi_m^{[\text{SI}]}) \mu_0 \mathbf{H}^{[\text{SI}]} \equiv \mu^{[\text{SI}]} \mathbf{H}^{[\text{SI}]} \end{aligned}$$

Umrechnung Gauß'sches und SI-System:

$$\begin{aligned} \mathbf{E} &= \sqrt{4\pi\epsilon_0} \mathbf{E}^{[\text{SI}]}, & \mathbf{B} &= \sqrt{\frac{4\pi}{\mu_0}} \mathbf{B}^{[\text{SI}]}, \\ \phi &= \sqrt{4\pi\epsilon_0} \phi^{[\text{SI}]}, & \mathbf{A} &= \sqrt{\frac{4\pi}{\mu_0}} \mathbf{A}^{[\text{SI}]}, \\ \rho &= \frac{1}{\sqrt{4\pi\epsilon_0}} \rho^{[\text{SI}]}, & \mathbf{j} &= \frac{1}{\sqrt{4\pi\epsilon_0}} \mathbf{j}^{[\text{SI}]} \end{aligned}$$

Kontinuitätsgleichung:

$$\nabla \cdot \mathbf{j} + \frac{\partial}{\partial t} \rho = 0$$

Lorentz-Kraft auf Punktladung $q = q^{[\text{SI}]} / \sqrt{4\pi\epsilon_0}$:

$$\mathbf{F} = q \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) = q^{[\text{SI}]} \left(\mathbf{E}^{[\text{SI}]} + \mathbf{v} \times \mathbf{B}^{[\text{SI}]} \right)$$

Quantenmechanik

Korrespondenzregeln im Ortsraum:

$$\mathbf{x} \mapsto \hat{\mathbf{x}} = \mathbf{x}, \quad \mathbf{p} \mapsto \hat{\mathbf{p}} = \frac{\hbar}{i} \nabla_{\mathbf{x}}$$

Zeitabhängige Schrödinger-Gleichung:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Formale Lösung für konservative Systeme:

$$|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle, \quad \hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

Stationäre Schrödinger-Gleichung für konservative Systeme:

$$\hat{H} |\psi\rangle = E |\psi\rangle, \quad |\psi(t)\rangle = e^{-iEt/\hbar} |\psi\rangle$$

Hamilton-Operator im Ortsraum für Teilchen ohne Spin:

$$\hat{H} = \frac{1}{2m} \left(\hat{\mathbf{p}} - \frac{q}{c} \mathbf{A}(t, \hat{\mathbf{x}}) \right)^2 + V(\hat{\mathbf{x}})$$

Entwicklung nach Eigenvektoren einer Observablen:

$$\hat{A} |a_n\rangle = a_n |a_n\rangle, \quad \langle a_m | a_n \rangle = \delta_{mn} \Rightarrow |\psi\rangle = \sum_n \langle a_n | \psi \rangle |a_n\rangle$$

Erwartungswert einer Observablen in einem Zustand:

$$\langle \hat{A} \rangle_\psi = \langle \psi | \hat{A} | \psi \rangle, \quad \langle \psi | \psi \rangle = 1$$

Unbestimmtheitsrelation für zwei Observablen:

$$\langle (\Delta \hat{A})^2 \rangle_\psi \langle (\Delta \hat{B})^2 \rangle_\psi \geq -\frac{1}{4} [\langle [\hat{A}, \hat{B}] \rangle_\psi]^2, \quad \Delta \hat{A} = \hat{A} - \langle \hat{A} \rangle_\psi$$

Übergang vom Schrödinger- zum Heisenberg-Bild:

$$|\psi_H\rangle = \hat{U}^{-1}(t) |\psi(t)\rangle, \quad \hat{A}_H(t) = \hat{U}^{-1}(t) \hat{A} \hat{U}(t)$$

Heisenberg-Gleichung für Observablen:

$$i\hbar \frac{d\hat{A}_H}{dt} = [\hat{A}_H, \hat{H}_H]$$

Harmonischer Oszillator:

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + 1/2 \right), \quad [\hat{a}, \hat{a}^\dagger] = 1, \quad E_n = \hbar\omega(n + 1/2)$$

Hermiteische Drehimpulsoperatoren:

$$[\hat{J}_i, \hat{J}_j] = i\hbar \varepsilon_{ijk} \hat{J}_k, \quad [\hat{J}^2, \hat{J}_i] = 0$$

Eigenzustände des Drehimpulses:

$$\hat{J}^2 |jm\rangle = \hbar^2 j(j+1) |jm\rangle, \quad \hat{J}_3 |jm\rangle = \hbar m |jm\rangle$$

Energien des Wasserstoffatoms:

$$E_n = -\frac{Z^2}{n^2} \text{Ry}, \quad \text{Ry} = \frac{m_e c^2}{2} \alpha^2 \approx 13,606 \text{ eV}$$

Änderung der Energie und des Zustandsvektors in erster Ordnung Störungstheorie:

$$E^{(1)} = \langle \psi_n^{(0)} | \hat{H}^{(1)} | \psi_n^{(0)} \rangle, \\ |\psi_n^{(1)}\rangle = \sum_{k \neq n} \frac{\langle \psi_k^{(0)} | \hat{H}^{(1)} | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}} |\psi_k^{(0)}\rangle$$

Thermodynamik

Einige wichtige thermodynamische Potenziale:

$U(S, V)$	innere Energie
$F(T, V) = U(S, V) - TS$	freie Energie
$H(S, P) = U(S, V) + PV$	Enthalpie
$G(T, P) = F(T, V) + PV$ $= U(S, V) - TS + PV$	freie Enthalpie

Vollständige Differenziale davon:

$$\begin{aligned} dU(S, V) &= T dS - P dV \\ dF(T, V) &= -S dT - P dV \\ dH(S, P) &= T dS + V dP \\ dG(T, P) &= -S dT + V dP \end{aligned}$$

Daraus abgeleitete Maxwell-Relationen:

$$\begin{aligned} \left(\frac{\partial T}{\partial V} \right)_S &= - \left(\frac{\partial P}{\partial S} \right)_V \\ \left(\frac{\partial S}{\partial V} \right)_T &= \left(\frac{\partial P}{\partial T} \right)_V \\ \left(\frac{\partial T}{\partial P} \right)_S &= \left(\frac{\partial V}{\partial S} \right)_P \\ \left(\frac{\partial S}{\partial P} \right)_T &= - \left(\frac{\partial V}{\partial T} \right)_P \end{aligned}$$

Großkanonische Zustandssummen für Maxwell-Boltzmann-, Fermi-Dirac- und Bose-Einstein-Systeme:

$$\begin{aligned} Z_{\text{gc}}^{\text{MB}} &= \sum_{N=0}^{\infty} \int d\Gamma(x) e^{-\beta(H(x) - \mu N)} \\ Z_{\text{gc}}^{\text{FD}} &= \prod_k \left(1 + e^{-\beta(\epsilon_k - \mu)} \right) \\ Z_{\text{gc}}^{\text{BE}} &= \prod_k \left(1 - e^{-\beta(\epsilon_k - \mu)} \right)^{-1} \end{aligned}$$

Großkanonische Verteilungsfunktionen dieser Systeme:

$$\begin{aligned} \rho_{\text{gc}}^{\text{MB}} &= \left(Z_{\text{gc}}^{\text{MB}} \right)^{-1} \sum_{N=0}^{\infty} e^{-\beta(H(x) - \mu N)} \\ \langle n_k^{\text{FD}} \rangle &= \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1} \\ \langle n_k^{\text{BE}} \rangle &= \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1} \end{aligned}$$

Beziehungen zwischen Zustandssummen und thermodynamischen Potenzialen:

$$\begin{aligned} S &= k_B \ln \mathcal{Q} \\ F &= -k_B T \ln Z_c \\ J &= -PV = -k_B T \ln Z_{\text{gc}} \end{aligned}$$

Gibbs-Duhem-Beziehung:

$$U - TS + PV = G = \sum_i \mu^{(i)} N^{(i)}$$

Theoretische Physik 1 | Mechanik

Bartelmann, M.; Feuerbacher, B.; Krüger, T.; Lüst, D.;

Rebhan, A.; Wipf, A.

2018, XXIV, 402 S. 178 Abb. in Farbe., Softcover

ISBN: 978-3-662-56114-0