

Chapter 2

Modeling of AHWR and Control by Static Output Feedback

2.1 Introduction

The physical dimensions of Advanced Heavy Water Reactor (AHWR) are comparatively larger than the neutron migration length in the reactor core. As a result, a severe condition called flux tilt may take place in AHWR. Additionally, circumstances such as online refueling might cause momentary variations in flux shape from the equilibrium flux shape. For analysis of such conditions, it is required to develop an appropriate space-time kinetics model of AHWR. Moreover, the models used for detailed core physics calculations, thermal hydraulics analysis or burnup optimization are usually of very large order and are not readily suited to control studies. A simplified model for representing the space-time kinetics phenomena in a Pressurized Heavy Water Reactor has been derived in [15] based on finite difference approximation of multigroup diffusion equations. In a similar manner, mathematical model of AHWR is obtained in [11, 12, 14] and the same has been used for the study carried out in this monograph.

Designing a robust spatial control technique for a nuclear reactor heavily depends on thorough understanding of system/plant dynamics. These dynamics are nothing but the interactions among the various state variables. To explore reactor dynamics and investigate spatial control strategies, a suitable reactor model, capturing important features and reasonable in its complexity, is required. So, simulation studies can be carried out in the form of offline computer programs to study the performance of nuclear reactor in various predicted accident conditions [1, 7, 10, 16].

This chapter provides mathematical modeling of AHWR. This includes modeling of core neutronics and thermal hydraulics behaviors along with internal reactivity feedbacks. Subsequently, model is linearized and design of static output feedback control (SOFC) is investigated. This chapter also serves as basis for subsequent chapters in this monograph.

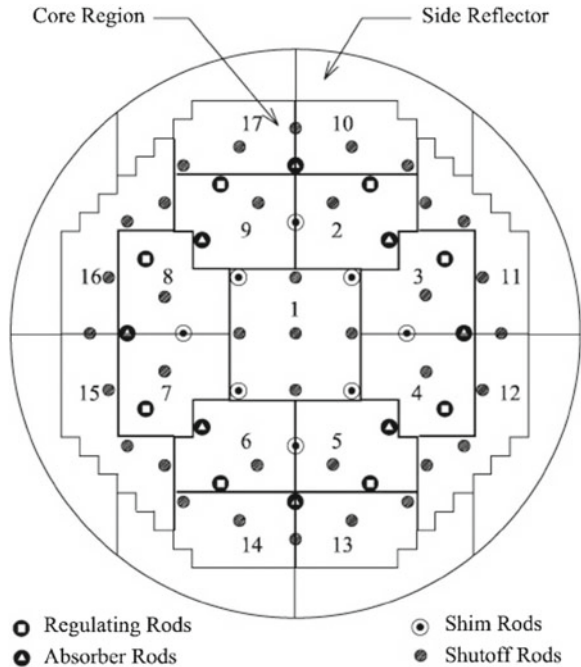
2.2 Mathematical Modeling of AHWR

Complete AHWR model is developed by combining models of core neutronics behavior and thermal hydraulics of main heat transport system.

2.2.1 Core Neutronics Modeling

The simplified core neutronics model is obtained by the nodal approach, based on finite difference approximation of the two group neutron diffusion equations and the associated equation for an effective single group of delayed neutron precursor's concentration. In this, for decoupling the space and time dependence, the AHWR core is divided into 17 nodes as depicted in Fig. 2.1. This 17 nodes division is obtained by considering a central region (node 1) in the core and concentric radial regions. The inner radial region consists of nodes 2–9 which also contain individual regulating rods (RRs). The outer radial region consists of nodes 10–17. The top and bottom reflector regions are divided into 17 nodes in an identical pattern, whereas the side reflector region is divided into 8 nodes. Within each node in the reactor core, the neutron fluxes and other neutronic parameters are represented by the respective average values integrated over its volume. The nodes 2, 4, 6, and 8 contain RRs under

Fig. 2.1 17 nodes AHWR scheme



automatic control [13]. For convenience, the RRs under automatic control are numbered according to the nodes containing them as RR2, RR4, RR6, and RR8. These nodes are considered as 17 small cores, each of which is coupled to its neighboring nodes through neutron diffusion. The following nonlinear equations constitute the neutronics model of the reactor core without internal reactivity feedback.

$$\frac{dQ_i}{dt} = (\rho_i - \alpha_{ii} - \beta) \frac{Q_i}{\ell} + \sum_{j=1}^{17} \alpha_{ji} \frac{Q_j}{\ell} + \lambda C_i, \quad (2.1)$$

$$\frac{dC_i}{dt} = \frac{\beta}{\ell} Q_i - \lambda C_i, \quad i = 1, 2, \dots, 17, \quad (2.2)$$

where α_{ji} and α_{ii} denote the coupling coefficients between j th and i th nodes and self-coupling coefficients of i th node, respectively, β and λ , respectively, are effective one group delayed neutron yield and decay constant, ℓ is the neutron lifetime. Q_i and C_i are the nodal power level and the effective one group delayed neutron precursor concentration of i th node, respectively.

The most important fission product poison is xenon because of its exceptionally large capture cross section for thermal neutrons and half-life of 9.2 h. Main proportion of this isotope in a reactor originates from radioactive decay of iodine with half-life of 6.7 h [4, 5]. To formulate xenon reactivity feedback, iodine and xenon dynamics in each node are represented as

$$\frac{dI_i}{dt} = \gamma_I \Sigma_{f_i} Q_i - \lambda_I I_i, \quad (2.3)$$

$$\frac{dX_i}{dt} = \gamma_X \Sigma_{f_i} Q_i + \lambda_I I_i - (\lambda_X + \bar{\sigma}_{X_i} Q_i) X_i, \quad (2.4)$$

where γ_I and γ_X are fission yields of iodine and xenon respectively, λ_I and λ_X are, respectively, decay constants of iodine and xenon and $\bar{\sigma}_{X_i} = \sigma_{X_i} / E_{eff} \Sigma_{f_i} V_i$; I_i denotes iodine concentration and X_i the xenon concentration of i th node. Also, E_{eff} is energy liberated in each fission, V_i is the node volume, and Σ_{f_i} is thermal neutron fission cross section of i th node.

Regulating rods are driven by the respective reversible variable speed-type three-phase induction motor and static frequency converter. The speed of RR is directly proportional to the voltage applied to the drive motor and it is given by

$$\frac{dH_k}{dt} = \kappa v_k, \quad k = 2, 4, 6, 8; \quad (2.5)$$

where v_k is control signal applied to the RR drives in the range of ± 1 volt, κ is a constant having value 0.56, and H_k is ‘% in’ position of RR of k th node. Differential equations (2.1)–(2.5) characterize the nodal model of AHWR core neutronics. The neutronic parameters, coupling coefficients, nodal volumes, and cross sections are given in Tables 2.1, 2.2, and 2.3, respectively.

Table 2.1 Neutronic parameters

Parameter	Corresponding value
β	2.643×10^{-3}
λ	$6.4568 \times 10^{-2} \text{ s}^{-1}$
ℓ	$3.6694 \times 10^{-4} \text{ s}$
λ_I	$2.878 \times 10^{-5} \text{ s}^{-1}$
λ_X	$2.1 \times 10^{-5} \text{ s}^{-1}$
γ_I	5.7×10^{-2}
γ_X	1.1×10^{-2}
σ_X	$1.8 \times 10^{-22} \text{ cm}^{-1}$
E_{eff}	$3.2 \times 10^{-11} \text{ J}$

Table 2.2 Coupling coefficients for the AHWR model

$\alpha_{1,1} = 3.1567 \times 10^{-2}$
$\alpha_{2,2} = \alpha_{5,5} = \alpha_{6,6} = \alpha_{9,9} = 5.4918 \times 10^{-2}$
$\alpha_{3,3} = \alpha_{4,4} = \alpha_{7,7} = \alpha_{8,8} = 6.2052 \times 10^{-2}$
$\alpha_{10,10} = \alpha_{13,13} = \alpha_{14,14} = \alpha_{17,17} = 3.8351 \times 10^{-2}$
$\alpha_{11,11} = \alpha_{12,12} = \alpha_{15,15} = \alpha_{16,16} = 4.3567 \times 10^{-2}$
$\alpha_{1,2} = \alpha_{1,5} = \alpha_{1,6} = \alpha_{1,9} = 6.5746 \times 10^{-3}$
$\alpha_{1,3} = \alpha_{1,4} = \alpha_{1,7} = \alpha_{1,8} = 6.5204 \times 10^{-3}$
$\alpha_{2,1} = \alpha_{5,1} = \alpha_{6,1} = \alpha_{9,1} = 4.5833 \times 10^{-3}$
$\alpha_{3,1} = \alpha_{4,1} = \alpha_{7,1} = \alpha_{8,1} = 4.3309 \times 10^{-3}$
$\alpha_{2,3} = \alpha_{5,4} = \alpha_{6,7} = \alpha_{9,8} = 1.3044 \times 10^{-2}$
$\alpha_{3,2} = \alpha_{4,5} = \alpha_{7,6} = \alpha_{8,9} = 1.2428 \times 10^{-2}$
$\alpha_{3,4} = \alpha_{4,3} = \alpha_{7,8} = \alpha_{8,7} = 1.6097 \times 10^{-2}$
$\alpha_{2,9} = \alpha_{5,6} = \alpha_{6,5} = \alpha_{9,2} = 1.0445 \times 10^{-2}$
$\alpha_{2,10} = \alpha_{5,13} = \alpha_{6,14} = \alpha_{9,17} = 2.3481 \times 10^{-2}$
$\alpha_{3,11} = \alpha_{4,12} = \alpha_{7,15} = \alpha_{8,16} = 2.7555 \times 10^{-2}$
$\alpha_{10,2} = \alpha_{13,5} = \alpha_{14,6} = \alpha_{17,9} = 1.9198 \times 10^{-2}$
$\alpha_{11,2} = \alpha_{12,5} = \alpha_{15,6} = \alpha_{16,9} = 5.6901 \times 10^{-3}$
$\alpha_{2,11} = \alpha_{5,12} = \alpha_{6,15} = \alpha_{9,16} = 5.4963 \times 10^{-3}$
$\alpha_{11,3} = \alpha_{12,4} = \alpha_{15,7} = \alpha_{16,8} = 2.9941 \times 10^{-2}$
$\alpha_{10,17} = \alpha_{17,10} = \alpha_{11,12} = \alpha_{12,11} = \alpha_{13,14} = \alpha_{14,13} = \alpha_{15,16} = \alpha_{16,15} = 9.9912 \times 10^{-3}$
Rest all $\alpha_{i,j} = 0$

2.2.2 Thermal Hydraulics Model

Thermal hydraulics model of main heat transport system of AHWR has been developed by evolving separate models for reactor core thermal hydraulics and for the steam drums, and afterwards clubbing them together [2, 12] as given below.

Table 2.3 Nodal volumes and cross sections

Node No.	Volume (m ³)	Σ_f (cm ⁻¹)	Σ_a (cm ⁻¹)
1	8.6822	2.6657×10^{-3}	6.9514×10^{-3}
2, 5, 6, 9	5.4042	2.3898×10^{-3}	6.6828×10^{-3}
3, 4, 7, 8	5.1384	2.5325×10^{-3}	6.7898×10^{-3}
10, 13, 14, 17	4.4297	2.5665×10^{-3}	6.8991×10^{-3}
11, 12, 15, 16	5.5814	2.5665×10^{-3}	6.8991×10^{-3}

2.2.2.1 Core Thermal Hydraulics

A thermal hydraulics model of the reactor core is obtained assuming an equivalent coolant channel for each node, ignoring the pressure drops in downcomers, feeders and tail pipes, and taking uniform distribution of nodal power along the flow axis. Also, the steam quality is considered to be uniformly increasing along the axial length in the channels after the point of onset of boiling. Now, applying mass and energy balance equations to the boiling section and solving them together, the core thermal hydraulics model is written as

$$e_{vp_i} \frac{dP}{dt} + e_{vx_i} \frac{dx_i}{dt} = Q_i - q_{d_i}(h_w - h_d) - x_i h_c q_{d_i}, \quad (2.6)$$

where P is drum pressure, h_w , h_d and h_c are water, downcomer, and condensation enthalpies, respectively, x_i is the nodal average exit quality, q_{d_i} is flow rate of the coolant entering the i th node through downcomer and e_{vp_i} and e_{vx_i} are constants of i th node.

2.2.2.2 Steam Drums

A simple lumped model of the steam drums is developed assuming (1) insignificant carry over and carry under effects; (2) a mixture of saturated water and steam enters the steam drum and subcooled water leaves steam drum into the reactor core; and (3) average values of density and enthalpy of the water in the steam drum. Mass and energy balance equations of the steam drums are respectively represented by

$$e_{pv} \frac{dV_w}{dt} + e_{pp} \frac{dP}{dt} = - \sum_{i=1}^{17} (q_{d_i} - q_{r_i}) + q_f - q_s, \quad (2.7)$$

$$e_{xv} \frac{dV_w}{dt} + e_{xp} \frac{dP}{dt} = q_f h_f + x q_r h_s + (1 - x) q_r h_w - q_d h_d - q_s h_s, \quad (2.8)$$

where q_f , q_s , q_r , and q_d are average values of feed water, steam, saturated steam, and subcooled water flow rates, respectively, and V_w is the volume of water in the

Table 2.4 Constant coefficients of thermal hydraulics model

Node No.	e_{vx_i}
1	2.4406
2, 5, 6, 9	1.0909
3, 4, 7, 8	1.2160
10, 13, 14, 17	1.9861
11, 12, 15, 16	1.1677
e_{x_i}	0.5114

steam drum. Finally applying energy balance equation to water volume in steam drum yields

$$e_{p_i} \frac{dP}{dt} + e_{v_i} \frac{dV_w}{dt} + e_{x_i} \frac{dh_d}{dt} = q_f h_f + (1-x)q_r h_w - q_d h_d. \quad (2.9)$$

Complete thermal hydraulics model is given by set of equations (2.6)–(2.9). Further, drum pressure and water volume are being regulated at respective set points by plant control system. Hence, derivatives of P and V_w vanish from Eqs. (2.6) to (2.9). The above equations, therefore, reduce to

$$e_{vx_i} \frac{dx_i}{dt} = Q_i - q_{d_i}(h_w - h_d) - q_{d_i} x_i h_c, \quad (2.10)$$

$$e_{x_i} \frac{dh_d}{dt} = q_f(\hat{k}_2 h_f - \hat{k}_1) - q_d(\hat{k}_2 h_d - \hat{k}_1), \quad (2.11)$$

where $\hat{k}_2 = \frac{h_c}{h_c}$ and $\hat{k}_1 = h_w \hat{k}_2$. Values of e_{vx_i} and e_{x_i} are listed in Table 2.4. The coolant flow rate through the channels is the function of normalized nodal powers, and is given as

$$q_{d_i} = \left\{ k_1 \left[\frac{Q_i}{Q_{i_0}} \right]^3 + k_2 \left[\frac{Q_i}{Q_{i_0}} \right]^2 + k_3 \left[\frac{Q_i}{Q_{i_0}} \right] + k_4 \right\} q_{d_{i_0}} \quad (2.12)$$

where $k_1 = 0.2156$, $k_2 = -0.5989$, $k_3 = 0.48538$, and $k_4 = 0.8988$. Q_{i_0} denotes the power produced by i th node under full power operation and $q_{d_{i_0}}$ is the corresponding coolant flow rate.

2.2.3 Reactivity Feedbacks

The reactivity term ρ_i in (2.1) is expressed as

$$\rho_i = \rho_{i_u} + \rho_{i_x} + \rho_{i_\alpha}, \quad (2.13)$$

where ρ_{i_u} is the reactivity introduced by the control rods, ρ_{i_x} is the reactivity feedback due to xenon, and ρ_{i_α} is the reactivity feedback due to coolant void fraction. The reactivity contributed by the movement of the RRs is expressed as

$$\rho_{i_u} = \begin{cases} (-10.234H_i + 676.203) \times 10^{-6}, & \text{if } i = 2, 4, 6, 8. \\ 0 & \text{elsewhere.} \end{cases} \quad (2.14)$$

The xenon reactivity feedback in node i can be expressed as

$$\rho_{i_x} = \frac{\bar{\sigma}_{x_i} X_i}{\Sigma_{a_i}}. \quad (2.15)$$

The reactivity contribution by the coolant void fraction is

$$\begin{aligned} \rho_{i_\alpha} = & -5 \times 10^{-3} (9.2832x_i^5 - 27.7192x_i^4 + 31.7643x_i^3 \\ & - 17.7389x_i^2 + 5.2308x_i + 0.0792). \end{aligned} \quad (2.16)$$

The reactivity contributed by the coolant, fuel, and moderator temperature feedbacks are neglected due to their lesser significance. Equations (2.1)–(2.5), (2.10), and (2.11) constitute complete coupled neutronics–thermal hydraulics model of AHWR. Seventeen equations each of power, delayed neutron precursor, xenon, iodine concentrations, and exit quality, four equations of RR positions, and one equation of downcomer enthalpy result into 90 nonlinear first-order differential equations. Four control signals to RRs and feed flow rate are input variables with 17 nodal powers and total power as output variables. The nodal powers and coolant flow rates are constants as given in Table 2.5 under steady-state full power operation. The equilibrium positions of all RRs are 66.1% inside the core. Coolant enters the core at a temperature of 260 °C and feed water enters the steam drum at 130 °C. The operating pressure of the main heat transport system is 7 MPa. Equilibrium values of other variables like delayed neutron precursor, iodine and xenon concentrations, exit quality, and feed flow rate can easily be computed from the steady-state forms of respective equations.

2.3 Linearization and State-Space Representation

The set of nonlinear equations given by (2.1)–(2.5), (2.10), and (2.11) can be linearized around steady-state operating conditions (H_{k_0} , X_{i_0} , I_{i_0} , h_{d_0} , C_{i_0} , x_{i_0} , Q_{i_0}) and the linear equations, so obtained, can be represented in standard state-space form. For this, define the state vector as

$$\mathbf{z} = [\mathbf{z}_H^T \mathbf{z}_X^T \mathbf{z}_I^T \delta h_d \mathbf{z}_C^T \mathbf{z}_x^T \mathbf{z}_Q^T]^T, \quad (2.17)$$

Table 2.5 Nodal powers and coolant flow rates under full power operation

Node No.	Steady-state values of	
	Power (MWt)	Coolant flow rate (kg/s)
1	91.8743	187.32
2, 5, 6, 9	54.9991	130.20
3, 4, 7, 8	55.7410	125.38
10, 13, 14, 17	42.6967	97.06
11, 12, 15, 16	53.7146	125.78
Total	920.480	2101.0

where $\mathbf{z}_H = [\delta H_2 \delta H_4 \delta H_6 \delta H_8]^T$ and the rest $\mathbf{z}_\xi = [(\delta \xi_1/\xi_{1_0}) \cdots (\delta \xi_{17}/\xi_{17_0})]^T$, $\xi = X, I, C, x, Q$, in which δ denotes the deviation from respective steady-state value of the variable. Likewise, define the input vector as

$$\mathbf{u} = [\delta v_2 \delta v_4 \delta v_6 \delta v_8]^T \quad (2.18)$$

and output vector as

$$\mathbf{y} = [y_T \ y_1 \ \cdots \ y_{17}]^T, \quad (2.19)$$

where $y_T = \sum_{i=1}^{17} \frac{\delta Q_i}{\sum_{j=1}^{17} Q_{j0}}$ and $y_i = \frac{\delta Q_i}{Q_{i0}}$ correspond to normalized total reactor power and nodal powers, respectively. Then, the system given by (2.1)–(2.5), (2.10) and (2.11) can be expressed in standard linear state-space form as

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u} + \mathbf{B}_{fw}\delta q_{fw}, \quad (2.20)$$

$$\mathbf{y} = \mathbf{M}\mathbf{z}, \quad (2.21)$$

where q_{fw} is feed water flow rate. The characteristic matrix $\mathbf{A}$ is of 90th order, expressed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{XX} & \mathbf{A}_{XI} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{XQ} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{II} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{IQ} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{hh} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{hQ} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{CC} & \mathbf{0} & \mathbf{A}_{CQ} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{xh} & \mathbf{0} & \mathbf{A}_{xx} & \mathbf{A}_{xQ} \\ \mathbf{A}_{QH} & \mathbf{A}_{QX} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{QC} & \mathbf{A}_{Qx} & \mathbf{A}_{QQ} \end{bmatrix} \quad (2.22)$$

where the first row corresponds to four rows of zeros and $\mathbf{0}$ stands for null matrix of appropriate dimensions. Remaining submatrices are given as follows.

$$\begin{aligned}
\mathbf{A}_{QC} &= \frac{\beta}{\ell} \mathbf{E}_{17}; \\
\mathbf{A}_{QQ}(i, j) &= \begin{cases} \frac{1}{\ell} \left(-\sum_{k=1}^{17} \alpha_{ki} \frac{Q_{k0}}{Q_{i0}} - \beta \right) & \text{if } i = j \\ \frac{1}{\ell} \alpha_{ji} \frac{Q_{j0}}{Q_{i0}} & \text{if } i \neq j \end{cases} \\
\mathbf{A}_{QX} &= \text{diag} [a_{qx1} \ a_{qx2} \ \cdots \ a_{qx17}], \text{ where } a_{qxi} = -\frac{1}{\ell} \left(\frac{\bar{\sigma}_{xi} X_{i0}}{\Sigma_{ai}} \right); \\
\mathbf{A}_{Qx} &= \frac{1}{\ell} \text{diag} [k_{\alpha 1} Q_{10} \ k_{\alpha 2} Q_{20} \ \cdots \ k_{\alpha 17} Q_{170}]; \\
\text{where } k_{\alpha i} &= -5 \times 10^{-3} \left(46.416x_{i0}^4 - 110.8787x_{i0}^3 + 95.229x_{i0}^2 - 35.4779x_{i0} + 5.2308 \right); \\
\mathbf{A}_{CQ} &= \lambda \mathbf{E}_{17}; \\
\mathbf{A}_{CC} &= -\mathbf{A}_{CQ}; \\
\mathbf{A}_{IQ} &= \lambda_I \mathbf{E}_{17}; \\
\mathbf{A}_{II} &= -\mathbf{A}_{IQ}; \\
\mathbf{A}_{XQ} &= \lambda_X \mathbf{E}_{17} - \text{diag} \left[\lambda_I \frac{I_{10}}{X_{10}} \ \cdots \ \lambda_I \frac{I_{170}}{X_{170}} \right]; \\
\mathbf{A}_{XX} &= -\text{diag} [\lambda_X + \bar{\sigma}_{X1} Q_{10} \ \lambda_X + \bar{\sigma}_{X2} Q_{20} \ \cdots \ \lambda_X + \bar{\sigma}_{X17} Q_{170}]; \\
\mathbf{A}_{XI} &= \lambda_I \text{diag} \left[\frac{I_{10}}{X_{10}} \ \cdots \ \frac{I_{170}}{X_{170}} \right]; \\
\mathbf{A}_{hh} &= -\hat{k}_1 \frac{qd_0}{e_{xh} h_w}; \\
\mathbf{A}_{hQ} &= \frac{\hat{k}_1}{e_{xh}} (3k_1 + 2k_2 + k_3) \left(\frac{1}{h_{d0}} - \frac{1}{h_w} \right) [qd_{10} \ qd_{20} \ \cdots \ qd_{170}]; \\
\mathbf{A}_{xh} &= \left[\frac{qd_{10} h_{d0}}{e_{vx1} x_{10}} \ \frac{qd_{20} h_{d0}}{e_{vx2} x_{20}} \ \cdots \ \frac{qd_{170} h_{d0}}{e_{vx17} x_{170}} \right]^T; \\
\mathbf{A}_{xx} &= -h_c \text{diag} \left[\frac{qd_{10}}{e_{vx1}} \ \frac{qd_{20}}{e_{vx2}} \ \cdots \ \frac{qd_{170}}{e_{vx17}} \right]; \\
\mathbf{A}_{xQ} &= \text{diag} [a_{xq1} \ a_{xq2} \ \cdots \ a_{xq17}]; \\
\text{where } a_{xqi} &= \frac{1}{e_{vx_i} x_{i0}} (Q_{i0} - (h_w - h_{d0} + x_{i0} h_c) q_{di0} (3k_1 + 2k_2 + k_3)); \\
\mathbf{A}_{QH}(i, j) &= \begin{cases} \frac{-10.23 \times 10^{-6} Q_{i0}}{\ell} & \text{for } i = 2, 4, 6, 8; \ j = i/2, \\ 0 & \text{elsewhere.} \end{cases}
\end{aligned}$$

Matrix $\mathbf{B}$ of dimension (90×4) is as follows

$$\mathbf{B} = [\mathbf{B}_H^T \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}]^T \quad (2.23)$$

where $\mathbf{B}_H$ is a diagonal matrix of dimensional (4×4) , with κ as diagonal entries and all other submatrices are zero. Matrix $\mathbf{B}_{fw}$ is of dimension (90×1) with $b_2 = \hat{k}_1 q_{f_0} \left(\frac{h_{f_0} - 1}{e_{x_i} h_{d0}} \right)$ on the 39th row and remaining all entries zero.

The normalized total reactor power and the nodal powers are measured and they constitute the output vector $\mathbf{y}$. Hence, in (2.21), matrix $\mathbf{M}$ is of dimension (18×90) , given by

$$\mathbf{M} = [\mathbf{M}_1 \mathbf{M}_2] \quad (2.24)$$

where $\mathbf{M}_1$ is a null matrix of dimension (18×73) and

$$\mathbf{M}_2 = \begin{bmatrix} \frac{Q_{10}}{\sum_{j=1}^{17} Q_{j0}} & \frac{Q_{20}}{\sum_{j=1}^{17} Q_{j0}} & \cdots & \frac{Q_{170}}{\sum_{j=1}^{17} Q_{j0}} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}. \quad (2.25)$$

2.4 Linear System Properties

2.4.1 Stability

The linear system (2.20) is said to be asymptotically stable if

$$\Re\{\varphi_i(\mathbf{A})\} < 0, \quad \forall i,$$

where $\varphi_i(\mathbf{A})$ is an eigenvalue of $\mathbf{A}$. Unstable modes of the system are those φ_i 's such that $\Re\{\varphi_i(\mathbf{A})\} \geq 0$ [3]. Hence, stability can be assessed by checking the eigenvalues of the corresponding open-loop linear system matrix $\mathbf{A}$ defined by (2.22). The eigenvalues of system matrix $\mathbf{A}$ are given in Table 2.6. It can be observed that the system has six eigenvalues with positive real parts (eigenvalues 1–6) besides the four eigenvalues at the origin (eigenvalues 7–10), which indicates instability. The unstable eigenvalues are represented by shaded region in Table 2.6. Hence, it is necessary to devise a closed-loop control that effectively maintains the total power of the reactor while the xenon-induced oscillations are being controlled.

2.4.2 Controllability

The linear system (2.20) with order n is said to be controllable, if and only if

$$\text{rank} \{[\mathbf{A} - \varphi_i \mathbf{E}_n \mathbf{B}]\} = n, \quad \forall i,$$

where φ_i is i th eigenvalue of $\mathbf{A}$ and $\mathbf{E}_n$ is an identity matrix of dimension n . If this is satisfied, then it is commonly mentioned that $(\mathbf{A}, \mathbf{B})$ pair is controllable. Uncontrollable modes of the system are those φ_i 's for which $\text{rank} \{[\mathbf{A} - \varphi_i \mathbf{E}_n \mathbf{B}]\} \neq n$.

Table 2.6 Open-loop eigenvalues with thermal hydraulics feedback

Sr. No.	Eigenvalues	Sr. No.	Eigenvalues	Sr. No.	Eigenvalues
1	7.4551×10^{-3}	39	-1.2514×10^{-2}	66	-1.6316×10^{-1}
2–3	$(8.8268 \pm j1.8656) \times 10^{-5}$	40	-1.6108×10^{-2}	67	-1.6325×10^{-1}
4–5	$(8.0470 \pm j2.4129) \times 10^{-5}$	41	-5.0954×10^{-2}	68	-1.6405×10^{-1}
6	3.9654×10^{-6}	42	-5.1159×10^{-2}	69	-1.6576×10^{-1}
7–10	0	43	-5.7730×10^{-2}	70	-1.8037×10^{-1}
11–12	$(-3.5182 \pm j7.7577) \times 10^{-5}$	44	-5.7893×10^{-2}	71	-1.8049×10^{-1}
13	-3.7781×10^{-5}	45	-5.9707×10^{-2}	72	-1.8122×10^{-1}
14–15	$(-3.7785 \pm j7.6475) \times 10^{-5}$	46	-5.9723×10^{-2}	73	-1.8395×10^{-1}
16	-3.7993×10^{-5}	47	-6.0344×10^{-2}	74	-7.2516
17	-4.0124×10^{-5}	48	-6.0642×10^{-2}	75	-3.2844×10^1
18	-4.1520×10^{-5}	49	-6.1848×10^{-2}	76	-3.3372×10^1
19	-4.2245×10^{-5}	50	-6.1942×10^{-2}	77	-6.6599×10^1
20	-4.4204×10^{-5}	51	-6.2200×10^{-2}	78	-6.8323×10^1
21	-4.7476×10^{-5}	52	-6.2380×10^{-2}	79	-9.3653×10^1
22	-4.8866×10^{-5}	53	-6.2458×10^{-2}	80	-9.4612×10^1
23–24	$(-6.4855 \pm j5.3109) \times 10^{-5}$	54	-6.2608×10^{-2}	81	-1.0868×10^2
25–26	$(-6.5890 \pm j5.4696) \times 10^{-5}$	55	-6.2865×10^{-2}	82	-1.1705×10^2
27–28	$(-7.3359 \pm j3.9319) \times 10^{-5}$	56	-6.2893×10^{-2}	83	-1.6967×10^2
29–30	$(-7.7407 \pm j2.9929) \times 10^{-5}$	57	-1.1714×10^{-1}	84	-1.7568×10^2
31	-1.4107×10^{-4}	58	-1.4712×10^{-1}	85	-1.9497×10^2
32	-1.4624×10^{-4}	59	-1.4713×10^{-1}	86	-2.1110×10^2
33	-1.5717×10^{-4}	60	-1.4809×10^{-1}	87	-2.1904×10^2
34	-1.6524×10^{-4}	61	-1.4849×10^{-1}	88	-2.3591×10^2
35	-1.6720×10^{-4}	62	-1.5580×10^{-1}	89	-2.7163×10^2
36	-1.7308×10^{-4}	63	-1.5585×10^{-1}	90	-2.7626×10^2
37	-1.8807×10^{-4}	64	-1.5662×10^{-1}		
38	-1.8870×10^{-4}	65	-1.5759×10^{-1}		

The condition of controllability governs the existence of a complete solution to the control system design problem [6]. Designing a controller to stabilize an unstable system and to achieve any specified transient response characteristics may not be possible if the system is uncontrollable. In case of AHWR, with matrices **A** and **B** given by equations (2.22) and (2.23) respectively, test has been performed, which results that the pair (**A**, **B**) is controllable, i.e., $\text{rank} \{[\mathbf{A} - \varphi_i \mathbf{E}_n \quad \mathbf{B}]\} = n, \forall i$, where $n = 90$.

2.4.3 Observability

The linear system (2.20)–(2.21) of order n is said to be observable, if and only if

$$\text{rank} \left\{ \begin{bmatrix} \mathbf{A} - \varphi_i \mathbf{E}_n \\ \mathbf{M} \end{bmatrix} \right\} = n, \quad \forall i.$$

If this is satisfied, then it is commonly mentioned that $(\mathbf{A}, \mathbf{M})$ pair is observable. Unobservable modes of the system are those φ_i 's for which $\text{rank} \left\{ \begin{bmatrix} \mathbf{A} - \varphi_i \mathbf{E}_n \\ \mathbf{M} \end{bmatrix} \right\} \neq n$. Observability denotes whether every state can be determined from the observation of the outputs over a finite interval of time [6]. The concept of observability thus helps in solving the problem of reconstructing unmeasured state variables from the measured variables. This plays a significant role in control system design, since the information of all the state variables is many a times essential for designing a suitable controller. The observability test of AHWR model has been carried out using pair $(\mathbf{A}, \mathbf{M})$ given by (2.22) and (2.24) and it is found that $\text{rank} \left\{ \begin{bmatrix} \mathbf{A} - \varphi_i \mathbf{E}_n \\ \mathbf{M} \end{bmatrix} \right\} = n, \forall i$. This concludes that system given by (2.20)–(2.21) is observable.

2.5 Vectorization of AHWR Model

For brevity, the equations governing the coupled neutronics–thermal hydraulics model of AHWR are revisited here:

$$\frac{dQ_i}{dt} = (\rho_i - \alpha_{ii} - \beta) \frac{Q_i}{\ell} + \sum_{j=1}^{17} \alpha_{ji} \frac{Q_j}{\ell} + \lambda C_i, \quad (2.26)$$

$$\frac{dC_i}{dt} = \frac{\beta}{\ell} Q_i - \lambda C_i, \quad (2.27)$$

$$\frac{dI_i}{dt} = \gamma_I \Sigma_{f_i} Q_i - \lambda_I I_i, \quad (2.28)$$

$$\frac{dX_i}{dt} = \gamma_X \Sigma_{f_i} Q_i + \lambda_I I_i - (\lambda_X + \bar{\sigma}_{X_i} Q_i) X_i, \quad (2.29)$$

$$\frac{dH_k}{dt} = \kappa v_k, \quad (2.30)$$

$$e_{v_{x_i}} \frac{dx_i}{dt} = Q_i - q_{d_i}(h_w - h_d) - q_{d_i} x_i h_c, \quad (2.31)$$

$$e_{x_i} \frac{dh_d}{dt} = q_f(\hat{k}_2 h_f - \hat{k}_1) - q_d(\hat{k}_2 h_d - \hat{k}_1), \quad (2.32)$$

where $i = 1, 2, \dots, 17$; $k = 2, 4, 6, 8$; and

$$q_{d_i} = \left\{ k_1 \left[\frac{Q_i}{Q_{i_0}} \right]^3 + k_2 \left[\frac{Q_i}{Q_{i_0}} \right]^2 + k_3 \left[\frac{Q_i}{Q_{i_0}} \right] + k_4 \right\} q_{d_{i_0}}, \quad (2.33)$$

$$\rho_i = \rho_{i_u} + \rho_{i_x} + \rho_{i_a}, \quad (2.34)$$

$$\rho_{i_u} = \begin{cases} (-10.234H_i + 676.203) \times 10^{-6}, & \text{if } i = 2, 4, 6, 8 \\ 0 & \text{elsewhere,} \end{cases} \quad (2.35)$$

$$\rho_{i_x} = \frac{\bar{\sigma}_{X_i} X_i}{\Sigma_{a_i}}, \quad (2.36)$$

$$\rho_{i_a} = -5 \times 10^{-3} (9.2832x_i^5 - 27.7192x_i^4 + 31.7643x_i^3 - 17.7389x_i^2 + 5.2308x_i + 0.0792). \quad (2.37)$$

The dynamic equations (2.26)–(2.37) can be written in vector/matrix form to implement in MATLAB/Simulink environment [8]. For that rearrange Eq. (2.26) of nodal powers as

$$\frac{dQ_i}{dt} = \frac{1}{\ell} \left[\rho_i Q_i - \alpha_{ii} Q_i - \beta Q_i + \sum_{j=1}^{17} \alpha_{ji} Q_j + \lambda \ell C_i \right]. \quad (2.38)$$

In the above equation ℓ , β are constants, ρ_i , α_{ii} , C_i , and Q_i are column vectors and α_{ji} is a matrix. However, the terms $\rho_i Q_i$, $\alpha_{ii} Q_i$, βQ_i , $\sum_{j=1}^{17} \alpha_{ji} Q_j$ and $\lambda \ell C_i$ are all column vectors of the same dimensions. If scalar multiplication is denoted by ‘ $\cdot$ ’, element-wise multiplication is denoted by ‘ $\odot$ ’, and array multiplication is denoted by ‘ $*$ ’ then (2.38) can be rewritten as

$$\frac{dQ_i}{dt} = \frac{1}{\ell} \cdot \left[\rho_i \odot Q_i - \alpha_{ii} \odot Q_i - \beta \cdot Q_i + \sum_{j=1}^{17} \alpha_{ji} Q_j + (\lambda \ell) \cdot C_i \right]. \quad (2.39)$$

Above equation is implemented using only one integrator, instead of 17 different integrators. Simulink of MATLAB automatically expands the equation to appropriate size as shown in Fig. 2.2. Initial values of nodal powers can be inserted in the integrator in vector form by double clicking ‘integrator’ block.

Similarly, the delayed neutron precursor, iodine, xenon concentrations, and rod position dynamics can be structured in vector/matrix form as

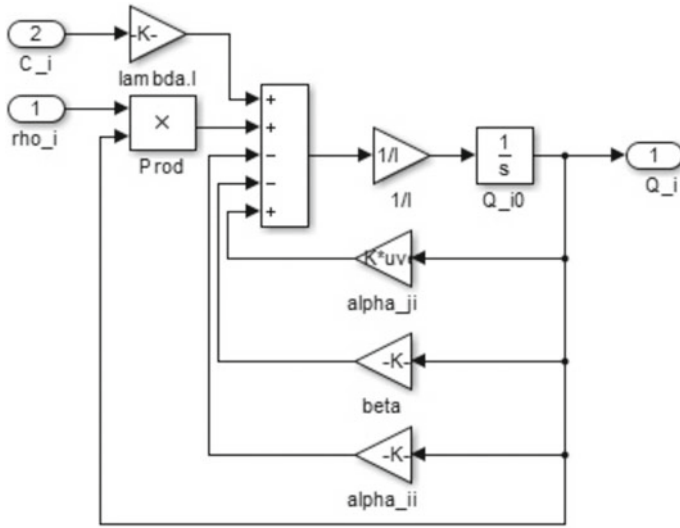


Fig. 2.2 Implementation of nodal powers

$$\frac{dC_i}{dt} = \frac{\beta}{\ell} \cdot Q_i - \lambda \cdot C_i, \quad (2.40)$$

$$\frac{dI_i}{dt} = (\gamma_I \cdot \Sigma_{fi}) \odot Q_i - \lambda_I \cdot I_i, \quad (2.41)$$

$$\frac{dX_i}{dt} = (\gamma_X \cdot \Sigma_{fi}) \odot Q_i + \lambda_I \cdot I_i - (\lambda_X + \bar{\sigma}_{Xi} \odot Q_i) \odot X_i, \quad (2.42)$$

$$\frac{dH_k}{dt} = \kappa \cdot v_k, \quad k = 2, 4, 6, 8; \quad i = 1, 2, \dots, 17. \quad (2.43)$$

After defining vector gains, above Eqs.(2.40)–(2.43) can be easily realized in Simulink as a core neutronics model subsystem block, as depicted in Fig. 2.3. In the similar manner, the thermal hydraulics model involving dynamics of exit quality and downcomer enthalpy, given by (2.31) and (2.32), is represented in vectorized form as

$$\frac{dx_i}{dt} = \frac{1}{e_{vx_i}} \odot [Q_i - q_{d_i} \cdot (h_w - h_d) - (q_{d_i} \odot x_i) \cdot h_c], \quad (2.44)$$

$$\frac{dh_d}{dt} = \frac{1}{e_{x_i}} \odot [q_f \cdot (\hat{k}_2 h_f - \hat{k}_1) - q_d \cdot (\hat{k}_2 h_d - \hat{k}_1)] \quad (2.45)$$

and employed in Simulink as given in Fig. 2.4. The instantaneous coolant flow rate through the channels can be evaluated as per (2.33) and is represented in Fig. 2.4 as ‘coolant flowrate’ subsystem block. Reactivity on account of RR movements is the function of its position, as defined by (2.30) and (2.35). These two equations can be collectively implemented as represented in Fig. 2.5. All these equations can be

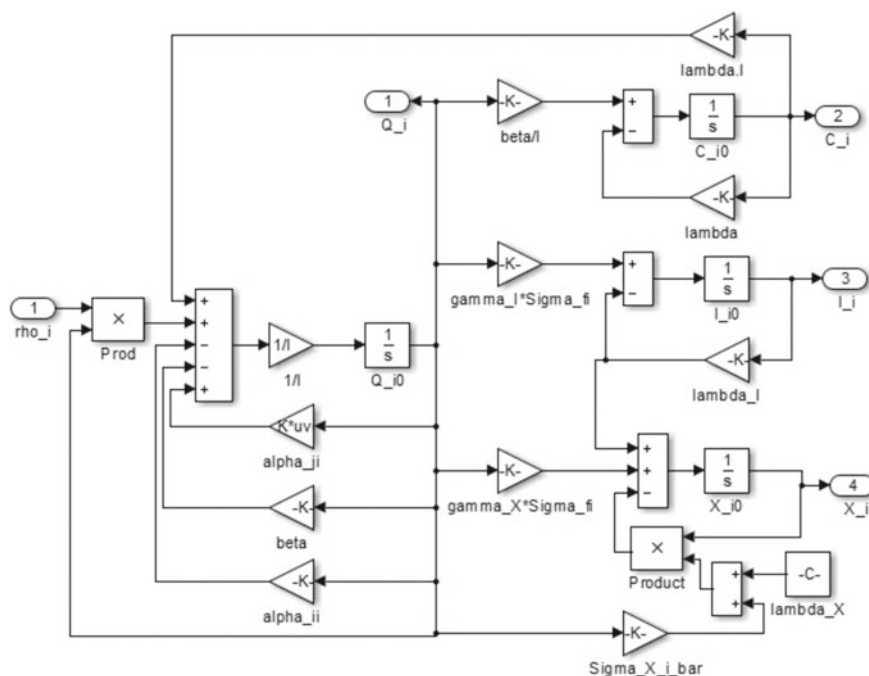


Fig. 2.3 Block of core neutronics model

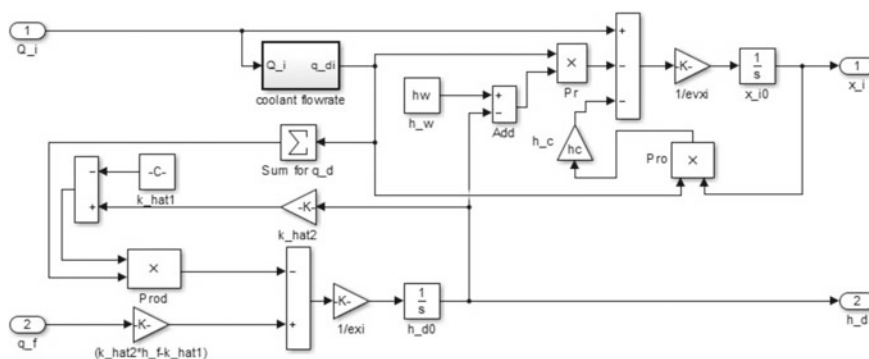


Fig. 2.4 Block of thermal hydraulics model

combined together, in the form of different subsystems, considering proper relationship between different variables and reactivity feedbacks, to form complete AHWR model. This complete model, shown in Fig. 2.6, eventually leads to an automatic program that solves nonlinear equations (2.26)–(2.32). Advantages of this type of model constructed in Simulink are:

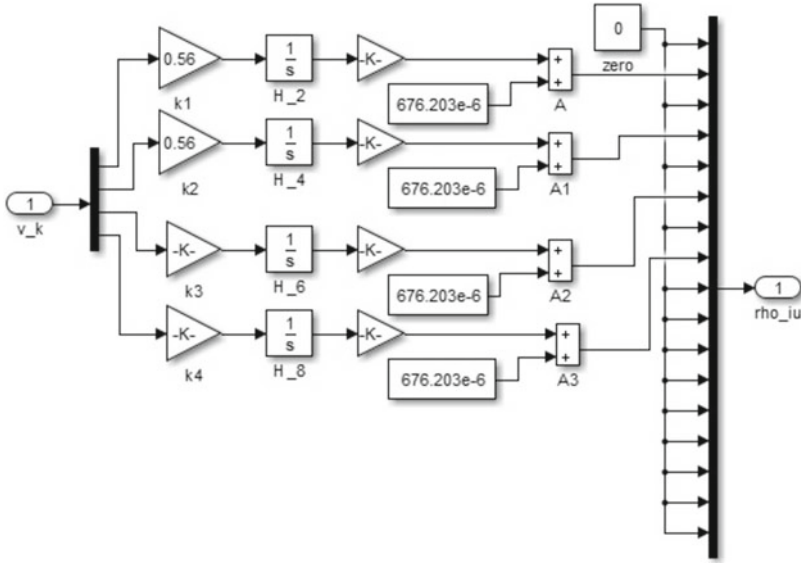


Fig. 2.5 Block of reactivity due to control rod positions

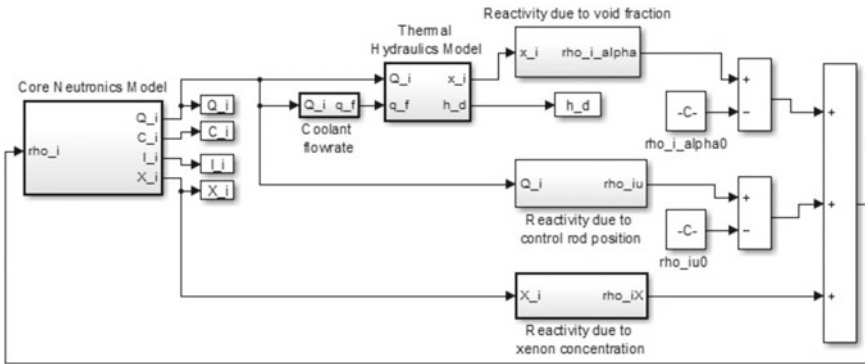


Fig. 2.6 Complete AHWR model

1. It can be used for different types of reactors with different number of nodes, provided that the coupling coefficients matrix and reactivity feedbacks are modeled properly,
2. Variations of any variable with respect to time or any other variable can be studied by applying 'scope' block across that variable,
3. Visualization of calculations and their recording for further applications is possible,

4. Different methods of solving nonlinear differential equations with different time steps can be studied, and
5. The computations are performed in much less time compared to the transient duration.

2.6 Static Output Feedback Control for AHWR

As mentioned in Sect. 2.4.1, the existence of multiple eigenvalues at the origin and eigenvalues with positive real part depicts instabilities in the AHWR. Hence, it is necessary to devise a closed-loop control that efficiently maintains the total power of the reactor core while the xenon-induced oscillations are being controlled.

Generally feedback of total power is sufficient to control small and medium size nuclear reactors, however, large reactors, like AHWR, require feedback of spatial power distribution along with the total power feedback for effective spatial control. Based on these considerations, let us consider the input $\mathbf{u}$ in (2.20) of the form

$$\mathbf{u} = -\mathbf{K}\mathbf{y} = -\mathbf{K} * \mathbf{y}, \quad (2.46)$$

where $\mathbf{K}$ is a (4×18) matrix. With the above control and (2.21), the system (2.20) becomes

$$\dot{\mathbf{z}} = (\mathbf{A} - \mathbf{BKM})\mathbf{z} + \mathbf{B}_{fw}\delta q_{fw} = \hat{\mathbf{A}}\mathbf{z} + \mathbf{B}_{fw}\delta q_{fw}, \quad (2.47)$$

where $\hat{\mathbf{A}} = (\mathbf{A} - \mathbf{BKM})$.

2.6.1 Total Power Feedback

First consider that

$$\mathbf{K} = [\bar{\mathbf{K}}_T \ \mathbf{0} \ \cdots \ \mathbf{0}] \quad (2.48)$$

in which $\mathbf{0}$ represents vectors of (4×1) dimension and $\bar{\mathbf{K}}_T = [K_T \ K_T \ K_T \ K_T]^T$ such that the feedback gain corresponding to total power is K_T for all RRs and is zero corresponding to nodal powers. The total power feedback control scheme is depicted in Fig. 2.7. The stability characteristic of the system (2.47) is investigated by varying the value of K_T and for $K_T = 12.5$, the gross behavior of the system seems stable though the system can show spatial instability. To reveal this, a transient involving a spatial power disturbance was simulated using vectorized nonlinear model of the reactor obtained from Eqs. (2.26)–(2.37) developed in MATLAB/Simulink. It was assumed that the reactor was operating initially at full power, with control signal

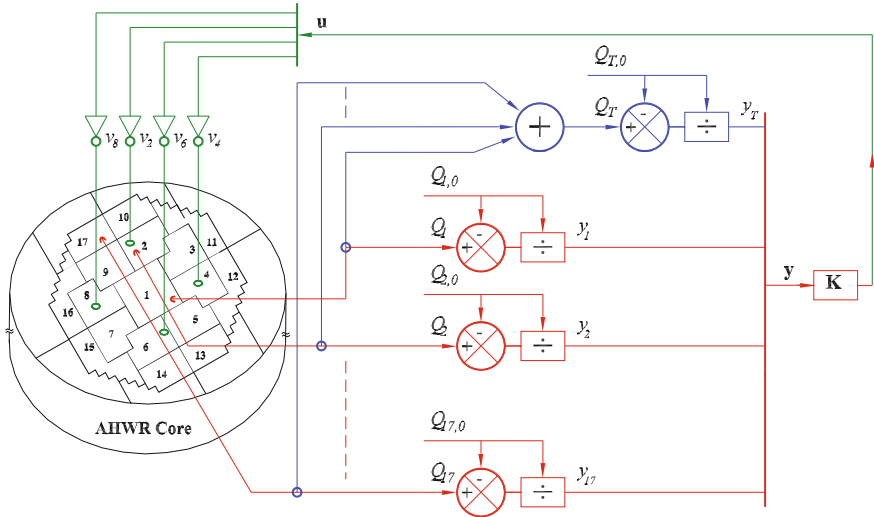


Fig. 2.7 Output feedback control scheme for AHWR ($Q_{T,0}$ and Q_T are respectively steady-state and instantaneous values of total reactor power)

generated by (2.46). The RR2 which was initially at its equilibrium position was driven out by about 1% by giving proper control signal. Immediately after that RR2 was driven back to its original position and thereafter left under the influence of controller. The response of the model to this disturbance was investigated in terms of variations in total reactor power and tilts in the first and second azimuthal modes defined as

$$\text{First azimuthal tilt} = \frac{Q_L - Q_R}{\sum_{i=1}^{17} \frac{Q_i}{2}} \times 100\% \quad (2.49)$$

$$\text{where } Q_L = \frac{1}{2} Q_1 + \sum_{i=6}^9 Q_i + \sum_{i=14}^{17} Q_i,$$

$$Q_R = \frac{1}{2} Q_1 + \sum_{i=2}^5 Q_i + \sum_{i=10}^{13} Q_i;$$

$$\text{Second azimuthal tilt} = \frac{Q_{p1} - Q_{p2}}{\sum_{i=1}^{17} \frac{Q_i}{2}} \times 100\% \quad (2.50)$$

$$\text{where } Q_{p1} = \frac{1}{2} Q_1 + Q_2 + Q_3 + Q_6 + Q_7 + Q_{10} + Q_{11} + Q_{14} + Q_{15},$$

$$Q_{p2} = \frac{1}{2} Q_1 + Q_4 + Q_5 + Q_8 + Q_9 + Q_{12} + Q_{13} + Q_{16} + Q_{17}.$$

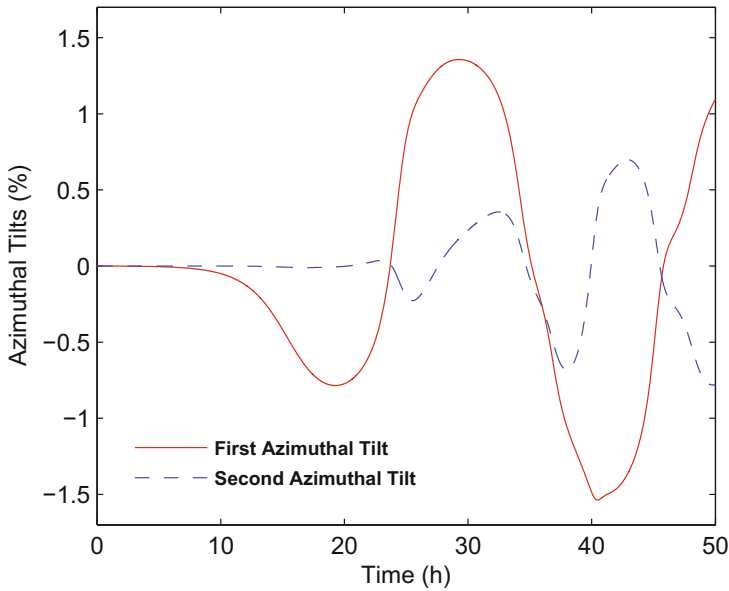


Fig. 2.8 Unstable modes of spatial instability

From simulation results, it was noticed that although the total power is being regulated at steady state, the power distribution in the core experiences oscillations. In 38 h, the first and second azimuthal tilts raise to the amplitudes of the order of 1.4 and 0.75% respectively as shown in Fig. 2.8. The period of the oscillations for first and second azimuthal modes are observed to be 20 and 12 h, respectively. If coolant void fraction reactivity feedback is not modeled, the amplitudes of first and second azimuthal tilts are observed to be markedly higher than as given in Fig. 2.8. Whereas, if xenon reactivity feedback is removed, then no oscillations were observed in power distribution, signifying that these are certainly xenon-induced spatial oscillations. These spatial oscillations and following local overpowers pose a potential threat to the fuel integrity of any nuclear reactor and hence necessitate control. Therefore, it is needed to devise an appropriate spatial power controller for AHWR.

2.6.2 Spatial Power Feedback

As observed in the Sect. 2.6.1, the AHWR system is showing spatial instability even with total power feedback. This is because the system (2.47) has still four eigenvalues with positive real parts and three eigenvalues at the origin. Hence, in addition to total power feedback, feedback of spatial power is required. Here, spatial stabilization of AHWR system is achieved with the feedback of nodal powers, in which RRs are placed along with total power feedback. Thus, in (2.48), $\mathbf{K}$ that was restricted

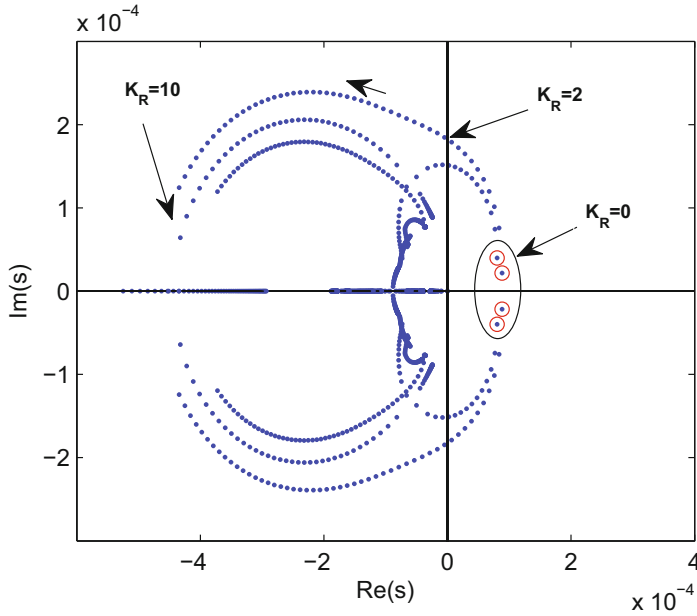


Fig. 2.9 Effect of feedback of nodal powers wherein RRs are located on location of unstable eigenvalues

to contain nonzero values only in the first column, will now be allowed to have nonzero values in other locations. This can be realized in such way that feedback gain corresponding to total power is K_T and feedback gain to power in nodes 2, 4, 6, and 8 is K_R , that is

$$\mathbf{K} = \begin{bmatrix} K_T & 0 & K_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_T & 0 & 0 & 0 & K_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_T & 0 & 0 & 0 & 0 & 0 & K_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2.51)$$

Figure 2.9 shows the locus of both stable and unstable eigenvalues near origin when K_R was increased progressively from zero. It is observed that all the unstable eigenvalues are stabilized for $K_R \geq 2$, which proves that the feedback of nodal powers in which RRs are placed can be effectively used to stabilize the system. Most of the eigenvalues near to the origin are found to have settled at their respective new locations for $K_R \approx 10$. With this consideration value of K_R is selected as 10. Closed-loop eigenvalues with $K_R = 10$ and $K_T = 12.5$ are found to be in the left half of s-plane, as listed in Table 2.7. This shows that the control law (2.46) stabilizes the system (2.47) with $\mathbf{K}$ given by (2.51). Spatial power feedback control scheme can be implemented with the same Fig. 2.7, where $\mathbf{K}$ is given by (2.51) [9].

Table 2.7 Closed-loop eigenvalues of AHWR model

Sr. No.	Eigenvalues	Sr. No.	Eigenvalues	Sr. No.	Eigenvalues
1	-2.8773×10^{-5}	37	-5.8834×10^{-2}	69	-1.751×10^{-1}
2	-2.8773×10^{-5}	38	-5.9364×10^{-2}	70	-1.8053×10^{-1}
3	-2.8773×10^{-5}	39	-5.966×10^{-2}	71	-1.8078×10^{-1}
4	-2.8773×10^{-5}	40	-5.9845×10^{-2}	72	-1.8221×10^{-1}
5	-4.0085×10^{-5}	41	-6.1191×10^{-2}	73	-2.6345×10^{-1}
6	-4.0237×10^{-5}	42	-6.1316×10^{-2}	74	-6.9796
7	-4.3283×10^{-5}	43–44	$(-5.9085 \pm j1.6901) \times 10^{-2}$	75	-3.2787×10^1
8	-4.3593×10^{-5}	45	-6.1803×10^{-2}	76	-3.3327×10^1
9	-6.755×10^{-5}	46	-6.1872×10^{-2}	77	-6.6584×10^1
10	-7.1457×10^{-5}	47	-6.235×10^{-2}	78	-6.8302×10^1
11–12	$(-7.6810 \pm j3.0862) \times 10^{-5}$	48	-6.2387×10^{-2}	79	-9.3652×10^1
13–14	$(-7.6926 \pm j3.0637) \times 10^{-5}$	49	-6.2702×10^{-2}	80	-9.4604×10^1
15–16	$(-2.0967 \pm j8.2771) \times 10^{-5}$	50	-6.2723×10^{-2}	81	-1.0867×10^2
17–18	$(-3.7556 \pm j7.6706) \times 10^{-5}$	51–52	$(-6.5120 \pm j2.2626) \times 10^{-2}$	82	-1.1704×10^2
19–20	$(-6.4610 \pm j5.6114) \times 10^{-5}$	53–54	$(-6.8026 \pm j2.3457) \times 10^{-2}$	83	-1.6966×10^2
21–22	$(-3.5386 \pm j7.7973) \times 10^{-5}$	55	-8.4982×10^{-2}	84	-1.7567×10^2
23–24	$(-5.6889 \pm j0.8659) \times 10^{-6}$	56	-1.1257×10^{-1}	85	-1.9496×10^2
25	-9.4731×10^{-5}	57	-1.3375×10^{-1}	86	-2.1109×10^2
26	-1.0049×10^{-4}	58	-1.4715×10^{-1}	87	-2.1903×10^2
27	-1.5741×10^{-4}	59	-1.4718×10^{-1}	88	-2.359×10^2
28	-1.5881×10^{-4}	60	-1.4844×10^{-1}	89	-2.7162×10^2
29	-1.7621×10^{-4}	61	-1.5045×10^{-1}	90	-2.7625×10^2
30	-1.7705×10^{-4}	62	-1.5592×10^{-1}		
31	-2.3573×10^{-4}	63	-1.5602×10^{-1}		
32	-2.3586×10^{-4}	64	-1.5749×10^{-1}		
33	-2.4991×10^{-4}	65	-1.6038×10^{-1}		
34	-2.5026×10^{-4}	66	-1.6324×10^{-1}		
35	-1.5734×10^{-4}	67	-1.6339×10^{-1}		
36	-5.7761×10^{-2}	68	-1.6499×10^{-1}		

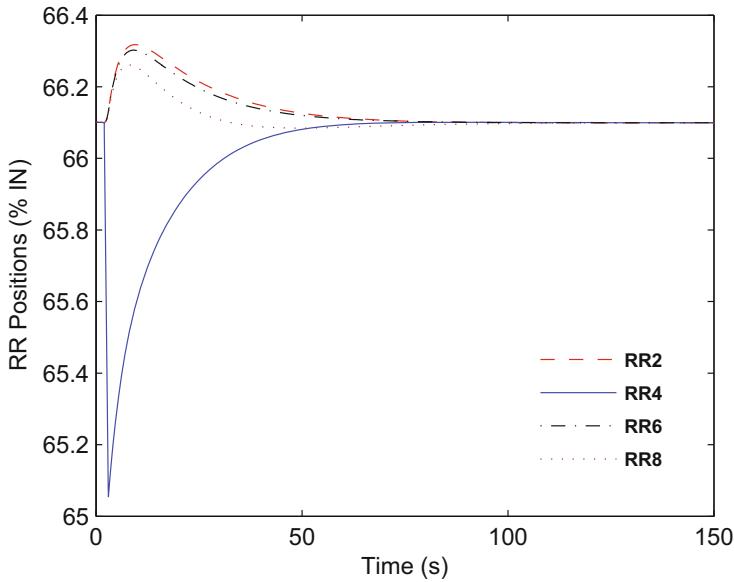


Fig. 2.10 Effect of RR disturbance on regulating rod positions

2.6.3 Transient Simulations

The controller performance was examined by simulating the nonlinear system of AHWR. Assuming that the reactor is operating at full power steady-state condition, with control signal generated by (2.46) and $\mathbf{K}$ given by (2.51), shortly, RR4, initially under auto control was driven out by almost 1% manually by giving proper control signal after 2 s and left under the influence of automatic control thereafter. As shown in the Fig. 2.7, deviations in nodal powers from their respective equilibrium values are given as input to the feedback gain $\mathbf{K}$. This gain is designed such that it utilizes only the deviations of nodal powers in which RRs are placed, along with total power deviation. From the transient simulation, it was noticed that the RRs are driven back to their equilibrium position, as shown in Fig. 2.10. As a result of the controller action, disturbance in total power and spatial powers are suppressed in around 80 s. The variation in total power is depicted in Fig. 2.11 and variations in nodal powers, measured in terms of azimuthal tilts, are shown in Fig. 2.12. Since the transient is of very short duration, amplitudes of first and second azimuthal tilts are, respectively, 0.0128 and 0.0048%.

In one more transient, the reactor is assumed to be operating at steady-state full power condition, i.e., at 920.48 MW¹ with nodal powers as given in Table 2.5. Concentrations of iodine, xenon, and delayed neutron precursor are also in equilibrium with the respective nodal powers. Now, the demand power is decreased uniformly

¹MW is used to represent Mega Watt Thermal.

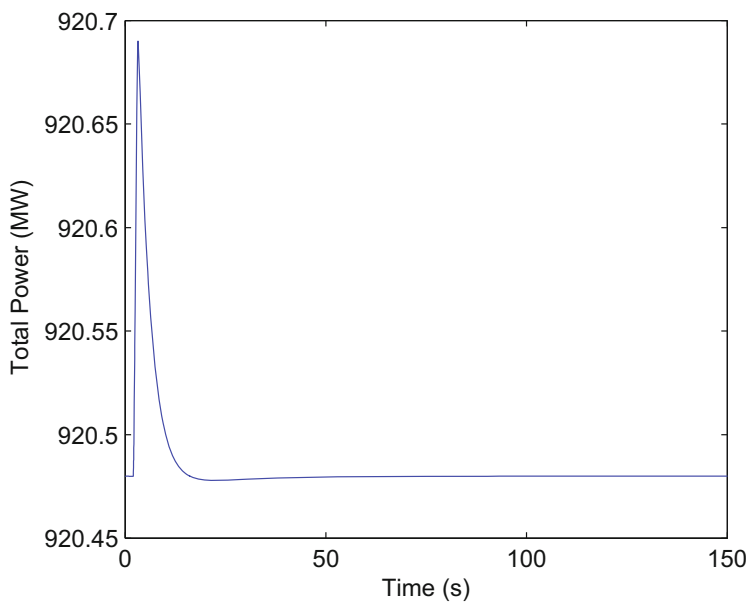


Fig. 2.11 Effect of RR disturbance on total reactor power

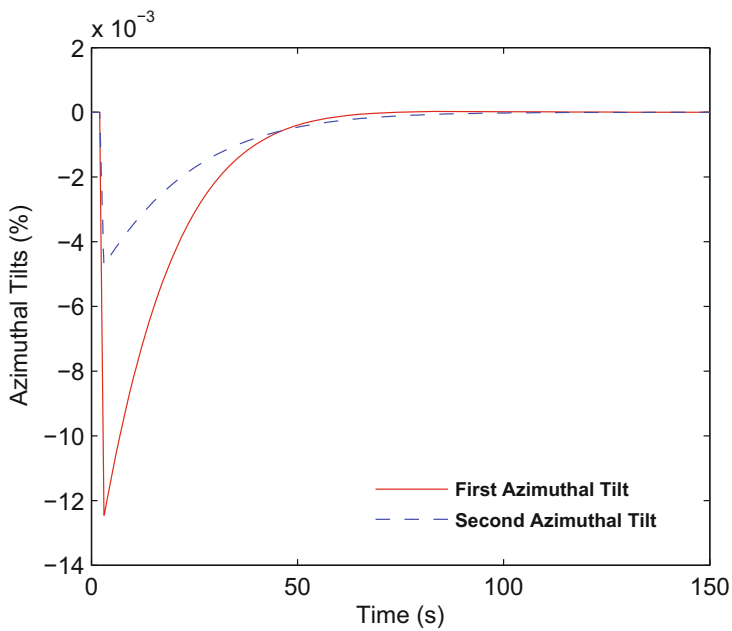


Fig. 2.12 Effect of RR disturbance on azimuthal tilts

at the rate of 1.5 MW/s to 828.43 MW, in around 61 s and thereafter held steady. Figures 2.13, 2.14, 2.15, 2.16, and 2.17 represent respective variations in total power, nodal powers, nodal xenon, and delayed neutron precursor concentrations during the transient period. From Fig. 2.13 it is noticed that the total power is maintained close to the demand power. The total power variation during initial 0.06 h (216 s) is shown in Fig. 2.14. The total power is near about 822.42 MW at 0.02 h (72 s) and it settles within $\pm 0.12\%$ of new demand power approximately in next 90 s. The nodal powers achieve new steady-state values within about 90 s as represented in Fig. 2.15 and do not show any variation during the remaining prolonged simulation. The xenon concentrations become stable at their respective new steady-state values in about 50 h. The delayed neutron precursor concentrations take only 90 s to achieve new steady state (Fig. 2.17). Though the change in stabilization time for delayed neutron precursor and xenon concentrations is several hours, this does not create any intricacy in simulation, which is achieved in Simulink proficiently using suggested vectorization of modeling equation.

Finally, to test the performance of controller under disturbance condition, nonlinear model was once again simulated. Apart from four voltage signals to RRs, feed flow is the fifth input of the AHWR system. Change in the feed flow rate is nothing but the disturbance for the system. When reactor was operating at equilibrium condition, a 5% positive step change was introduced in feed flow as shown in Fig. 2.18 after 100 s. Due to this, the downcomer enthalpy is decreased by 0.64% as shown in Fig. 2.19 and total power experienced variations as depicted in Fig. 2.20. The total power rises from 920.48 to 920.70 MW and becomes stable at its original value in about next 100 s. It is evident from the Fig. 2.21, that in order to maintain total power at equilibrium level, all the RRs are moved inside almost by 1%. Since the variation in total power is observed to be very small, i.e., 0.02% (Fig. 2.20), variations in delayed neutron precursor and xenon concentrations are also found to be very small, as indicated in Figs. 2.22 and 2.23. Again in spite of huge difference in time constants for delayed neutron precursor and xenon concentrations, the simulations are carried out without any difficulty. This shows the efficacy of the controller.

2.7 Conclusion

Analysis of nuclear reactor system and design of suitable control require a mathematical model. In this chapter, a simplified coupled core neutronics–thermal hydraulics model of AHWR system is presented. The final model equations are represented into standard state-space form for control system studies and stability, controllability, and observability properties of the AHWR system are examined. Thereafter, nonlinear model of AHWR is developed in MATLAB/Simulink environment by vectorization and steady state is achieved by giving feedback of total power. When this model is simulated for transient condition, it is observed that total power remains constant

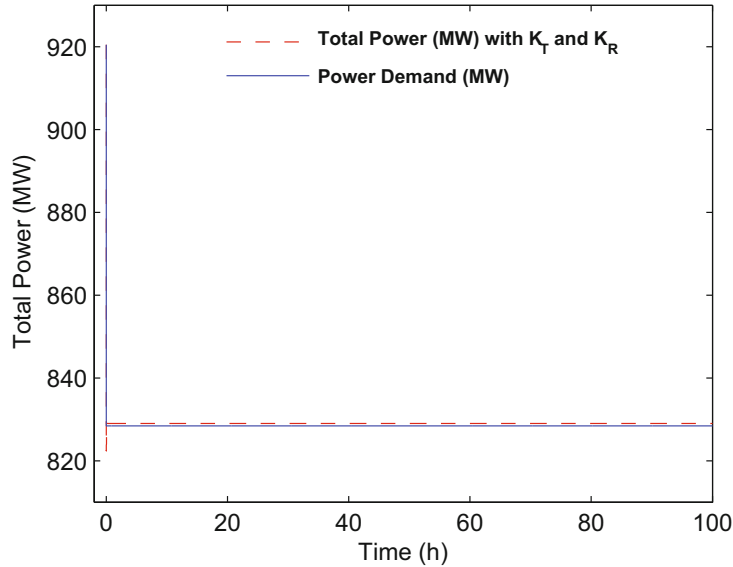


Fig. 2.13 Effect of power maneuvering on total power

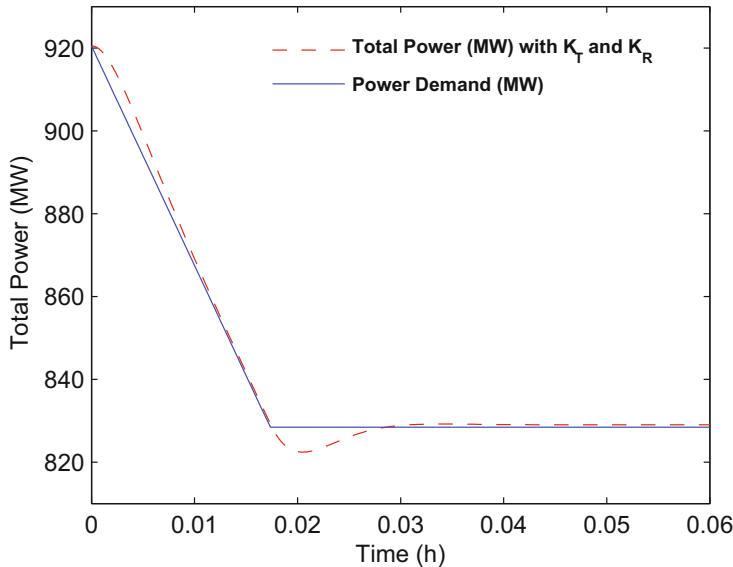


Fig. 2.14 Effect of power maneuvering on total power for initial 0.06h (216s)

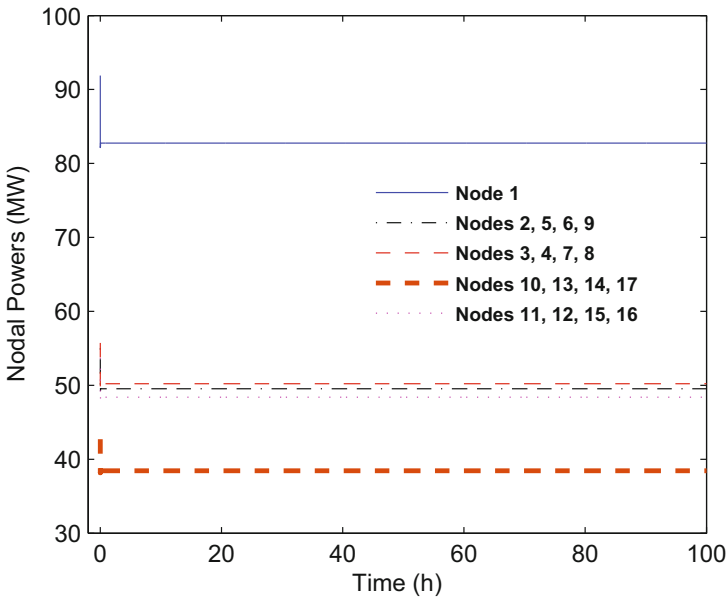


Fig. 2.15 Effect of power maneuvering on nodal powers

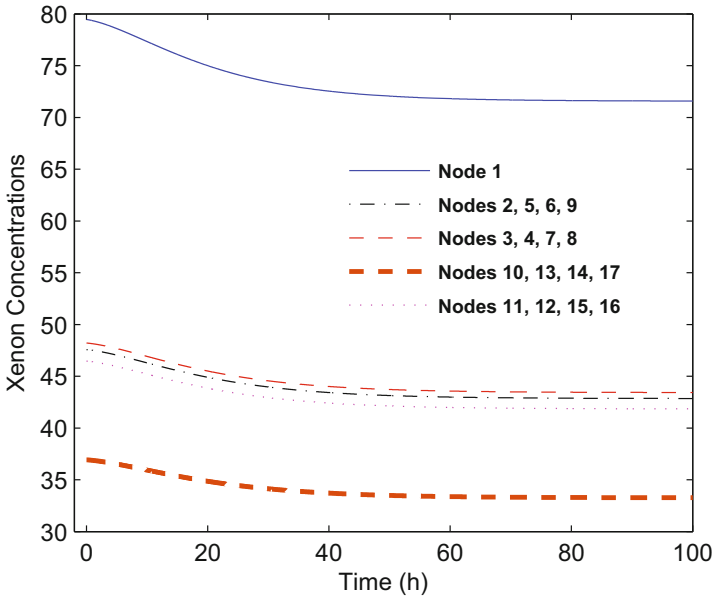


Fig. 2.16 Effect of power maneuvering on xenon concentrations

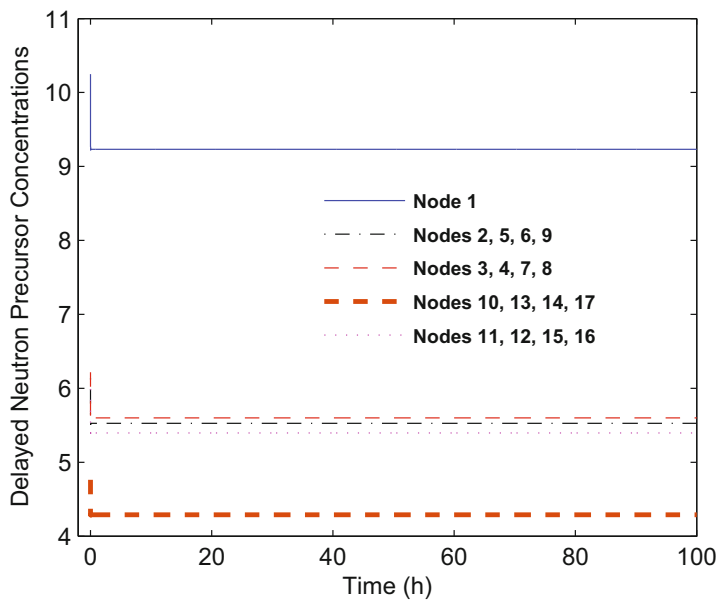


Fig. 2.17 Effect of power maneuvering on delayed neutron precursor concentrations

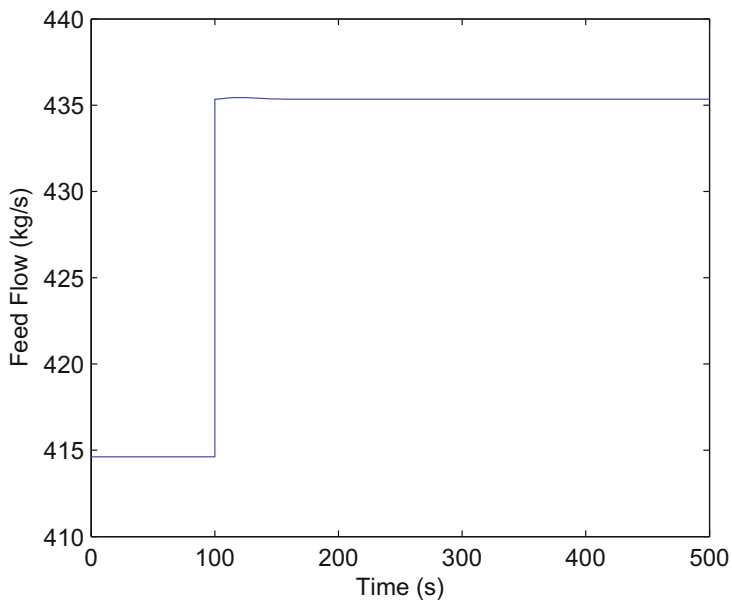


Fig. 2.18 Step change of 5% in the feed flow

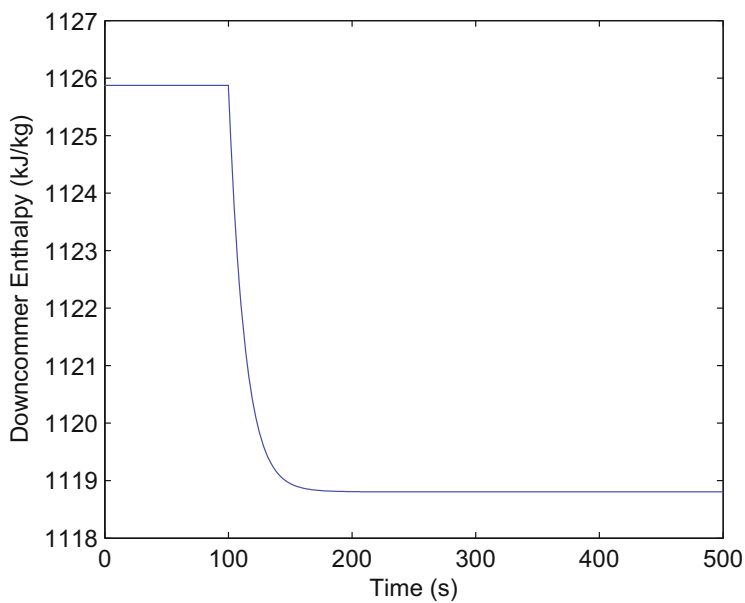


Fig. 2.19 Effect of step change in feed flow on downcomer enthalpy

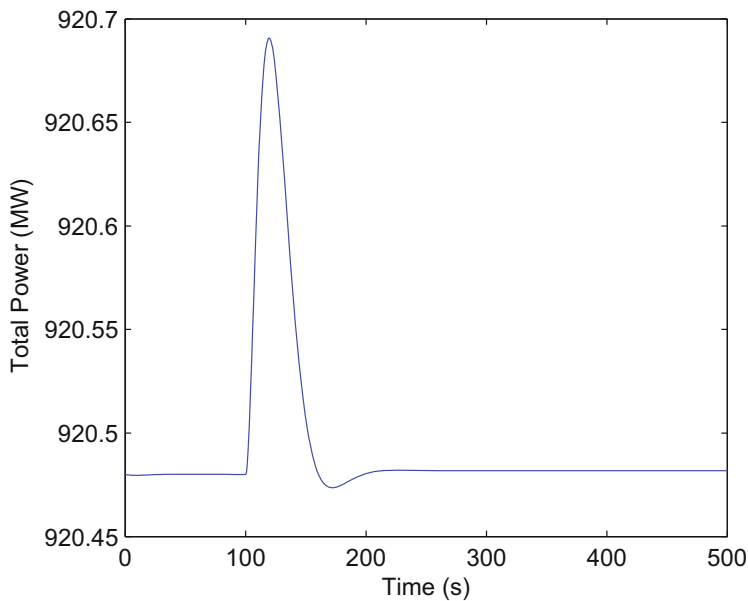


Fig. 2.20 Effect of step change in feed flow on total power

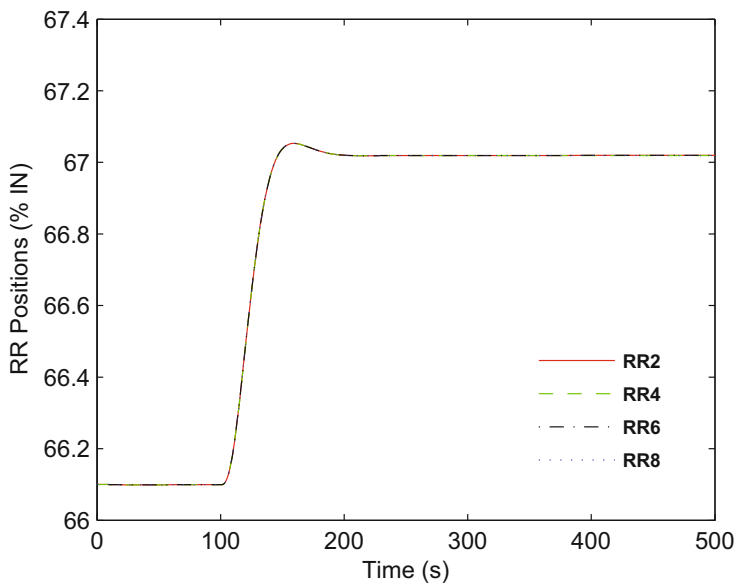


Fig. 2.21 Effect of step change in feed flow on RR positions

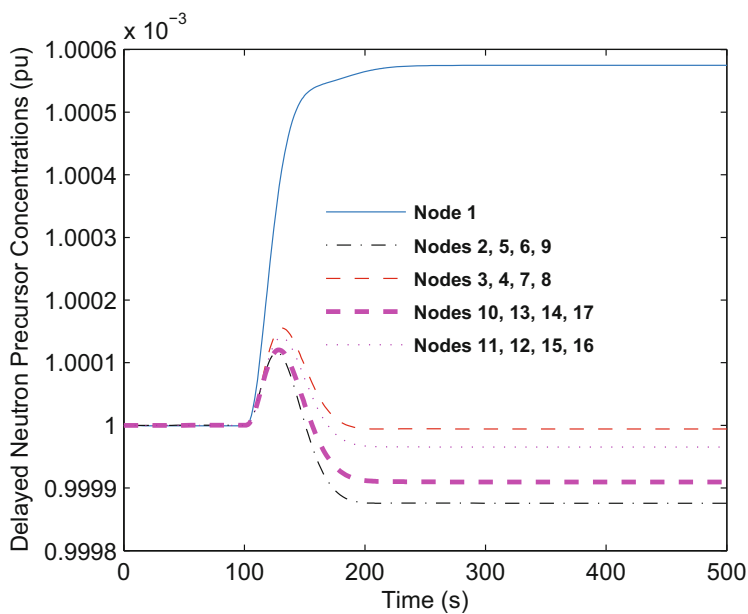


Fig. 2.22 Effect of step change in feed flow on nodal delayed neutron precursor concentrations

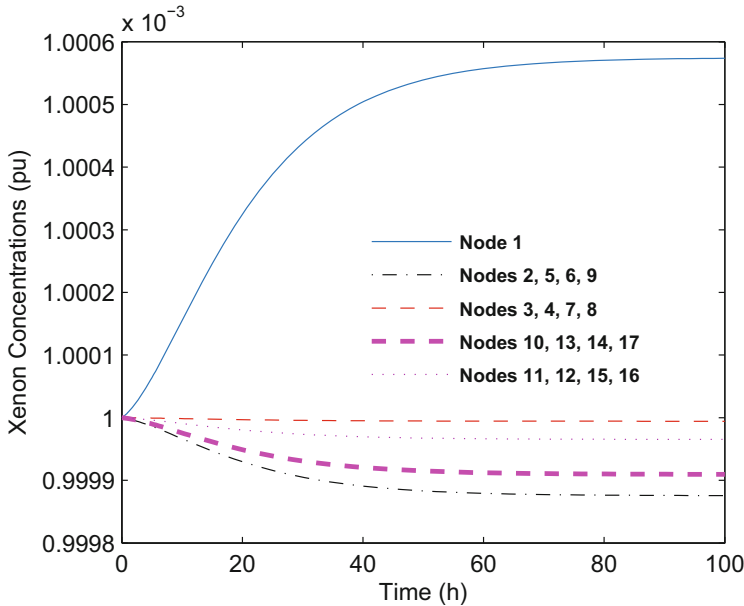


Fig. 2.23 Effect of step change in feed flow on nodal xenon concentrations

but the power in opposite regions of the core undergoes oscillations. Therefore, the feedback of nodal powers where RRs are placed is given in addition to total power feedback. Effectiveness of this static output feedback control is justified by testing the performance under different transient conditions.

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