

Chapter 2

Fundamentals of Bistatic SAR Imaging Algorithms

Abstract The formulation of a point target spectrum in synthetic aperture radar (SAR) is a key step in SAR focusing algorithms, which exploits the processing efficiency in the frequency domain. The general bistatic SAR range equation has a double-square-root (DSR) term that makes it difficult to derive an analytical expression of the bistatic two-dimensional spectrum. Many researchers overcome this difficulty by developing an approximation of the two-dimensional bistatic point target reference spectrum (BPTRS) and using the result to develop efficient frequency domain focusing algorithms for bistatic SAR data. In this chapter, three BPTR spectra are discussed and derived in detail. These consist of the Loffeld bistatic formula (LBF), the extended Loffeld bistatic formula (ELBF), and the method of series reversion (MSR). These three formulations are based on the principle of stationary phase (POSP).

2.1 Introduction

For bistatic SAR focusing, time-domain methods can be used to focus echo signals for any acquisition scenario with any arbitrary flight trajectories without approximation error [1]. These methods are used to produce computerized tomography in medical imaging and geophysical tomography in geophysical imaging. However, these methods incur heavy computational cost that scales with an order of $O(N^3)$, where N^2 is the number of pixels in the image [1]. By substituting the time-consuming superposition integrals in the space-time domain by fast convolution in the frequency domain, the processing efficiency can be greatly improved. For the monostatic case, an analytical solution exists for the point target spectrum [2]. Many efficient monostatic algorithms [2] are developed based on this analytical monostatic point target spectrum. These frequency-domain algorithms achieve a computational cost that scales on the order of $O(N^2 \log_2 N)$.

Deriving the point target spectrum is basically an inversion problem that must be solved to arrive at a frequency representation of the time domain signal history of the point target trajectory. For the bistatic case, there is no analytical solution for the

point target spectrum due to the double-square-root (DSR) function in the range equation. This bistatic range history has the form of a flat-top hyperbola [3]. As there is no analytical solution for the BPTRS, it can only be determined approximately or numerically. Several approximate bistatic point target reference spectra and bistatic imaging methods have been derived based on these approximate analytical spectra. In this section, three generic approaches are discussed.

The first method transforms the bistatic data to a monostatic equivalent as a preprocessing step. After this preprocessing step, the data can be focused by a monostatic focusing algorithm. In [4], a preprocessing technique known as dip move out (DMO) [5] used in geophysical imaging was applied to bistatic SAR to produce an approximate spectrum. DMO refers to the difference in the arrival times or travel times of a reflected wave, measured by geophone receivers at two different offset locations; a seismic processing technique compensates for this DMO. Rocca made use of this technique to transform an azimuth-invariant bistatic configuration into an equivalent monostatic configuration. Nonetheless, this method is limited to processing the bistatic case in which the receiver and transmitter have identical velocities and flight paths. In [6], Bamler used an equivalent velocity to model the bistatic configuration with a monostatic equivalent. This method of transforming the bistatic configuration to a monostatic configuration tends to restrict the allowable bistatic geometry. What can be inferred from these two methods is that the approximate spectra modeled are not able to account for the spectrum components brought about as the geometry becomes increasingly less pseudo-monostatic. In any case, such methods can be considered a quick and effective solution if an existing monostatic SAR processor is to be used without significant modification. In fact, the method [6] was applied to focus the spaceborne bistatic SAR missions of the satellites TanDEM-X and TerraSAR-X [7, 8], where the velocities of the two SAR satellites can be considered to be almost identical and their baselines to remain fairly constant during the imaging interval.

A second method of solving this inversion problem is to solve it numerically [6–10]. These algorithms are similar to the monostatic w - k algorithm [2], except that they make use of numerical methods when calculating the DSR phase term. In [6], Bamler et al. proposed a focusing algorithm that replaces the analytical bistatic SAR transfer function with a numerical equivalent. This algorithm is able to handle the azimuth-invariant case, including squint. The drawback of this method is that it will require more numerical terms for more extreme bistatic cases and, hence, increase the complexity and computational load of the algorithm.

The last method derives the two-dimensional spectrum directly using the principle of stationary phase (POSP). A detailed treatment of this stationary phase method can be found in Appendix C. An approximate analytical solution for the general bistatic two-dimensional frequency spectrum was proposed in [3]. Instead of just having a pseudomonostatic component as in [9, 10], this analytical formulation accounts for the bistatic spectral component by having two phase components in the spectrum—a quasimonostatic phase term and a bistatic term with a bistatic deformation term. Such a formulation suggests a step to remove the bistatic deformation term followed by a quasimonostatic focusing step. This method,

known as the Loffeld bistatic formula (LBF), is similar to the DMO method that uses a Rocca smile operator to transform the data from a bistatic configuration to a monostatic configuration. A more accurate formulation, the extended Loffeld bistatic formula (ELBF), was proposed to solve more extreme bistatic cases with squint geometry. In [11], a power series method for the general bistatic case was formulated. The spectrum is derived based on the method of series reversion (MSR), which gives a more accurate formulation of the stationary point in the form of a power series. The accuracy of this method depends on the number of terms used in the power series. Unlike LBF and ELBF, this method does not split the spectra components into monostatic or bistatic components. For the trivial case of monostatic geometry, both the LBF and ELBF can be shown to derive exact analytical formulation of the monostatic case, while the MSR will leave the formulation as a power series.

As a summary of the BPTRS, the DMO method in [4] is an efficient method that requires a simple pre-processing step, and it can be applied for a bistatic configuration, which is more pseudomonostatic, i.e., when the signal history is of the azimuth-variant type. Such methods imply that both the receiver and transmitter platforms have fairly identical velocities, so that the baseline is fixed during the imaging interval. Numerical methods are used to derive the BPTRS [6–10] by representing the DSR term numerically. Numerical methods, although fairly flexible, tend to be fairly time consuming and are less convenient when one wants to appreciate the system implications when compared to an analytical formulation of the BPTRS. The formulation in [6–9] can handle azimuth-invariant bistatic configurations, whereas the method presented in [10] can cope with some azimuth-variant cases. The LBF and ELBF account for the bistatic phase terms and are more accurate than the former method. Another method using power series expansion, MSR [11], shows good focusing performance in [12, 13].

2.2 Two-Dimensional Bistatic Point Target Reference Spectrum

A bistatic synthetic aperture radar (BiSAR) system is characterized by different locations for transmitter and receiver that offer some degrees of flexibility in designing BiSAR missions. The transmitter (illuminator) and receiver (passive) can assume different motion trajectories. This increased flexibility in designing bistatic SAR missions comes at a cost of increased complexity with respect to the processing of bistatic SAR raw data to SAR images.

Deriving the point target reference spectrum is a key step for efficient frequency-domain-based monostatic SAR processing algorithms. In the bistatic case, the range history of any point target consists of the individual range history contributions of transmitter and receiver. The classical monostatic hyperbolic range

history and the analytical correspondence between azimuth frequency and azimuth time being closely related to a certain point in the trajectory history are no longer valid and cannot be applied to the bistatic configuration. Both transmitter and receiver trajectories are, in general, quite different, and thus the bistatic spectrum is significantly different from that of a monostatic spectrum. Although an exact analytical solution for a BPTRS does not exist, an approximation can be made of the range history and an approximate formulation can be derived. In the next section, the analytical formulations LBF, ELBF, and MSR are discussed in detail.

A. Bistatic Geometry

To derive the BPTRS for an arbitrary point, the geometry of a general bistatic configuration must be defined. The geometry of a general bistatic SAR configuration is shown in Fig. 2.1, and it is composed of two moving platforms illuminating a point target P on the ground. The footprint of the antenna beam is formed from the overlapping areas of the beam footprints from both the transmitter and the receiver. This overlapping beam footprint is known as the composite antenna beam footprint. The three-dimensional coordinate system is defined by the x - y plane representing the surface of the earth and the z axis pointing away from the Earth.

Each platform moves along with different trajectories at different velocities: the transmitter travels with a velocity v_T and the receiver assumes a velocity of v_R , with the subscripts “ T ” and “ R ” denoting transmitter and receiver parameters, respectively. The azimuth time is denoted by τ , and τ_0 is the azimuth time instant when transmitter or receiver “sees” the point target at the closest distance, or the point of closest approach (PCA). The position of the point target is implicitly specified by

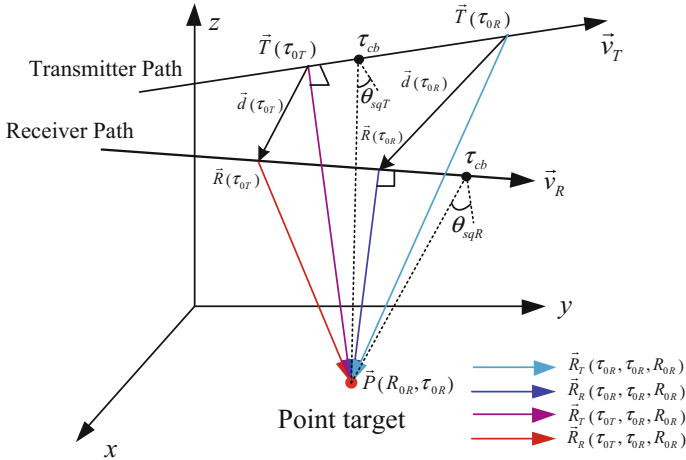


Fig. 2.1 Bistatic geometry

the time instant when it is perpendicular from the receiver track. The corresponding slant range is represented by $R_{OR} = |\vec{R}_R(\tau_{OR}, R_{OR}, \tau_{OR})|$. From Fig. 2.1, we have

$$\vec{R}_R(\tau, R_{OR}, \tau_{OR}) = \vec{R}_R(\tau_{OR}, R_{OR}, \tau_{OR}) - \vec{v}_R \cdot (\tau - \tau_{OR}) \quad (2.1)$$

where

$$\vec{R}_R(\tau_{OR}, R_{OR}, \tau_{OR}) \perp \vec{v}_R \quad (2.2)$$

The vector $\vec{R}_R(\tau_{OR}, R_{OR}, \tau_{OR})$ represents the slant range vector from receiver to point target, which is orthogonal to the receiver velocity v_R . τ_{OR} is the azimuth time when the point target is at this PCA. Likewise, for the transmitter range history,

$$\vec{R}_T(\tau, R_{OT}, \tau_{OT}) = \vec{R}_T(\tau_{OT}, R_{OT}, \tau_{OT}) - \vec{v}_T \cdot (\tau - \tau_{OT}) \quad (2.3)$$

where

$$\vec{R}_T(\tau_{OT}, R_{OT}, \tau_{OT}) \perp \vec{v}_T. \quad (2.4)$$

The vector $\vec{R}_T(\tau_{OT}, R_{OT}, \tau_{OT})$ is the slant range vector from transmitter to point target, which is orthogonal to the transmitter velocity v_T . The time τ_{OT} is the azimuth time, where the transmitter platform reaches the PCA with the point target. Without a loss of generality, the reference parameters with subscripts “OT” and “OR” of this bistatic geometry can be assumed to be constant.

B. Slant Range Histories

The scalar range history of the bistatic trajectory history is given by

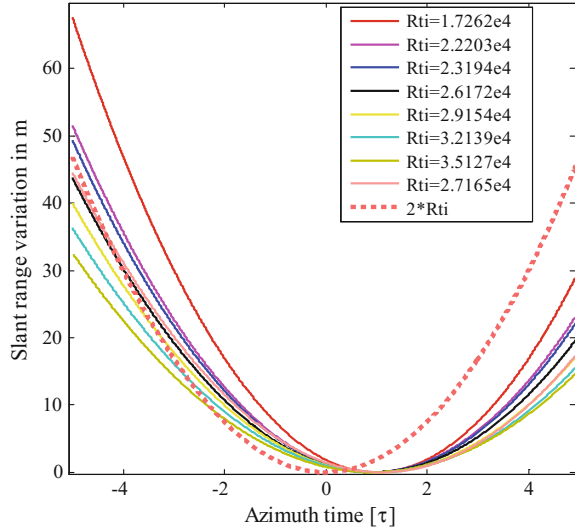
$$R_b(\tau, R_{OR}, \tau_{OR}) = R_R(\tau, R_{OR}, \tau_{OR}) + R_T(\tau, R_{OR}, \tau_{OR}). \quad (2.5)$$

The bistatic range history has a pair of square-root terms representing the sum of the hyperbolic range of the transmitter to the point target and the hyperbolic range of the receiver to the point target, which is known as the double-square-root (DSR) term and is written as

$$R_b(\tau, R_{OR}, \tau_{OR}) = \sqrt{R_{OT}^2 + v_T^2(\tau - \tau_{OT})^2} + \sqrt{R_{OR}^2 + v_R^2(\tau - \tau_{OR})^2}. \quad (2.6)$$

Figure 2.2 shows the slant range histories of different bistatic configurations. The monostatic range history is represented by the dashed-line plot, while the rest of the plots are bistatic configurations. It can be seen from the figure that the bistatic trajectory is slightly “flatter” around the stationary point, and thus it is given the name flat top hyperbola in geophysical processing.

Fig. 2.2 Bistatic slant range histories for different point target slant ranges



2.2.1 Loffeld Bistatic Formula (LBF)

In 2004, Loffeld proposed a vectorial model for expressing transmitter and receiver trajectories for an arbitrary general bistatic configuration [3]. The point target response is first modeled in the space–time domain and then transformed or inverted to the frequency domain by POSP to give the BPTRS. In the reference spectrum, two phasor functions can be identified, the first resembling some quasimonostatic contribution and the second being a bistatic deformation phasor transforming into an elliptical arc in the spatial domain. The multiplication of the two phasors in the frequency domain is transformed into a convolution-like operation in the space–time domain, indicating that the bistatic dataset can be expressed as a convolution (range and azimuth variant) mapping of a monostatic dataset. In this regard, this formulation extends the solution for the constant baseline case. The LBF removes the bistatic deformation term and is similar in operation to the geophysical pre-processing associated with “Rocca’s smile operator” [4]. In fact, the formulation for [4] is a special case of the more general LBF formulation.

By inspecting the range trajectory in Fig. 2.2, the following three observations can be made:

- (1) The overall slant range histories lose their hyperbolic form and look different than the monostatic hyperbolic slant range history. The reason for this is that the sum of two hyperbolas is no longer a hyperbola but that of a flat top hyperbola.
- (2) The azimuth points, where the overall slant range is a minimum, vary with R_{OR} ; we observe an azimuth shift of the minimum.
- (3) The shape of the overall slant range history changes in quality, and not only in scaling.

The general bistatic slant range history is essentially quite different from the monostatic slant range history; hence, bistatic SAR processing cannot be achieved by purely monostatic approaches.

A. Range Histories

From Eq. (2.5), we see that the Doppler history (being proportional to the range rate history) of a point target actually consists of the two individual contributions of transmitter and receiver:

$$\dot{R}_b(\tau, R_{0R}, \tau_{0R}) = \dot{R}_R(\tau, R_{0R}, \tau_{0R}) + \dot{R}_T(\tau, R_{0R}, \tau_{0R}). \quad (2.7)$$

For the individual scalar range rates, we then have

$$\begin{aligned} \dot{R}_R(\tau, R_{0R}, \tau_{0R}) \cdot R_R(\tau, R_{0R}, \tau_{0R}) &= \vec{\dot{R}}_R(\tau, R_{0R}, \tau_{0R}) \cdot \vec{R}_R(\tau, R_{0R}, \tau_{0R}) \\ &= -\vec{v}_R \cdot \dot{R}_R(\tau, R_{0R}, \tau_{0R}), \end{aligned} \quad (2.8)$$

$$\begin{aligned} \dot{R}_T(\tau, R_{0R}, \tau_{0R}) \cdot R_T(\tau, R_{0R}, \tau_{0R}) &= \vec{\dot{R}}_T(\tau, R_{0R}, \tau_{0R}) \cdot \vec{R}_T(\tau, R_{0R}, \tau_{0R}) \\ &= -\vec{v}_T \cdot \dot{R}_T(\tau, R_{0R}, \tau_{0R}). \end{aligned} \quad (2.9)$$

If the transmitter path was used as a reference, analogous to Eq. (2.8), we would have

$$\underbrace{\dot{R}_T(\tau, R_{0T}, \tau_{0T}) \cdot R_T(\tau, R_{0T}, \tau_{0T})}_{v_T^2 \cdot (\tau - \tau_{0T})} = -\vec{v}_T \cdot \vec{R}_T(\tau, R_{0R}, \tau_{0R}). \quad (2.10)$$

At time τ_{0R} , this gives

$$v_T^2 \cdot (\tau_{0R} - \tau_{0T}) = -\vec{v}_T \cdot \left(\vec{R}_R(\tau_{0R}, R_{0R}, \tau_{0R}) + \vec{d}(\tau_{0R}) \right), \quad (2.11)$$

from which we obtain the time difference between the individual PCAs,

$$(\tau_{0R} - \tau_{0T}) = -\frac{\vec{v}_T \cdot \left(\vec{R}_R(\tau_{0R}, R_{0R}, \tau_{0R}) + \vec{d}(\tau_{0R}) \right)}{v_T^2} = -a_0. \quad (2.12)$$

Assuming that the receiver and transmitter are perfectly synchronized, the signal echo delay from transmitter to point target and back to the receiver is given by

$$t_0(\tau, R_{0R}, \tau_{0R}) = \frac{R_T(\tau, R_{0R}, \tau_{0R}) + R_R(\tau, R_{0R}, \tau_{0R})}{c}. \quad (2.13)$$

However, note that the synchronization of transmitter and receiver over large and time-varying distances is not a trivial task. The signal received from a point target at (R_{OR}, τ_{OR}) after down-conversion is given by

$$g(t, \tau, R_{OR}, \tau_{OR}) = \sigma(R_{OR}, \tau_{OR}) \cdot w(\tau - \tau_{cb}) \times s_l(t - t_0(\tau, R_{OR}, \tau_{OR})) \cdot \exp[-j2\pi f_0 t_0(\tau, R_{OR}, \tau_{OR})], \quad (2.14)$$

where t is the range time and $w(\tau - \tau_{cb})$ is the window centered on τ_{cb} on the azimuth time axis describing the time interval in which the point target is both illuminated by the transmitter and within the antenna footprint of the receiver (composite beam footprint). $s_l(\cdot)$ is the transmitted chirp signal with modulation rate K_r . Applying a Fourier transform (FT) to the signal to invert the signal from the range time domain to the range frequency domain, we obtain

$$G(f, \tau, R_{OR}, \tau_{OR}) = \sigma(R_{OR}, \tau_{OR}) \cdot w(\tau - \tau_{cb}) \cdot S_l(f) \times \exp\left[-j2\pi \frac{(f + f_0)}{c} \cdot R_T(\tau, R_{OR}, \tau_{OR})\right] \times \exp\left[-j2\pi \frac{(f + f_0)}{c} \cdot R_R(\tau, R_{OR}, \tau_{OR})\right]. \quad (2.15)$$

where $S_l(f) = \exp\{-j\pi f^2/K_r\}$. We again see the individual contributions of transmitter and receiver in Eq. (2.15). Transforming from the azimuth time to the azimuth frequency domain, we obtain

$$G(f, f_\tau, R_{OR}, \tau_{OR}) = \sigma(R_{OR}, \tau_{OR}) \cdot S_l(f) \cdot I(f, f_\tau, R_{OR}, \tau_{OR}), \quad (2.16)$$

where

$$I(f, f_\tau, R_{OR}, \tau_{OR}) = \int_{-\infty}^{\infty} w(\tau - \tau_{cb}) \cdot \exp[-j\phi_b(\tau, f_\tau, R_{OR}, \tau_{OR})] d\tau, \quad (2.17)$$

$$\begin{aligned} \phi_b(\tau, f_\tau, R_{OR}, \tau_{OR}) &= \phi_T(\tau, f_\tau, R_{OR}, \tau_{OR}) + \phi_R(\tau, f_\tau, R_{OR}, \tau_{OR}) \\ &= 2\pi \cdot \left[\frac{(f + f_0)}{c} \cdot R_T(\tau, R_{OR}, \tau_{OR}) + \frac{f_{\tau, \tau}}{2} \right] \\ &\quad + 2\pi \cdot \left[\frac{(f + f_0)}{c} \cdot R_R(\tau, R_{OR}, \tau_{OR}) + \frac{f_{\tau, \tau}}{2} \right]. \end{aligned} \quad (2.18)$$

Substituting Eqs. (2.18) into (2.17), we obtain

$$\begin{aligned}
I(f, f_\tau, R_{0R}, \tau_{0R}) &= \int_{-\infty}^{\infty} w(\tau - \tau_{cb}) \cdot \exp[-j(\phi_T(\tau, f_\tau) + \phi_R(\tau, f_\tau))] d\tau \\
&\cong \exp[-j(\phi_T(\tilde{\tau}_T, f_\tau) + \phi_R(\tilde{\tau}_R, f_\tau))] \cdot I_2(f, f_\tau),
\end{aligned} \tag{2.19}$$

where

$$\begin{aligned}
I_2(f, f_\tau) &= \int_{-\infty}^{\infty} w(\tau - \tau_{cb}) \\
&\times \exp\left[-\frac{j}{2} \cdot \left(\ddot{\phi}_T(\tilde{\tau}_T) \cdot (\tau - \tilde{\tau}_T)^2 + \ddot{\phi}_R(\tilde{\tau}_R) \cdot (\tau - \tilde{\tau}_R)^2\right)\right] d\tau,
\end{aligned} \tag{2.20}$$

where $\tilde{\tau}_R, \tilde{\tau}_T$ are the points of the stationary phases of the receiver and transmitter phase histories given by

$$\begin{aligned}
\tilde{\tau}_T : \dot{\phi}_T(\tilde{\tau}_T, f_\tau, R_{0R}, \tau_{0R}) &= 0, \\
\tilde{\tau}_R : \dot{\phi}_R(\tilde{\tau}_R, f_\tau, R_{0R}, \tau_{0R}) &= 0.
\end{aligned} \tag{2.21}$$

The quadratic Taylor series expansion [2] around the point of stationary phase is an implicit assumption when employing the POSP to derive the monostatic point target reference spectrum. Here, we have two halves of two monostatic phase histories that are expanded in two Taylor series around the individual points of stationary phase.

Now, reasoning that the sum of two quadratic functions is again a (shifted and scaled) quadratic function, we may write, for the phase in Eq. (2.20) as in Eq. (2.22),

$$\begin{aligned}
\ddot{\phi}_T(\tilde{\tau}_T) \cdot (\tau - \tilde{\tau}_T)^2 + \ddot{\phi}_R(\tilde{\tau}_R) \cdot (\tau - \tilde{\tau}_R)^2 &= \frac{\ddot{\phi}_T(\tilde{\tau}_T) \cdot \ddot{\phi}_R(\tilde{\tau}_R)}{\ddot{\phi}_T(\tilde{\tau}_T) + \ddot{\phi}_R(\tilde{\tau}_R)} \cdot (\tilde{\tau}_T - \tilde{\tau}_R)^2 \\
&+ \left(\ddot{\phi}_T(\tilde{\tau}_T) + \ddot{\phi}_R(\tilde{\tau}_R)\right) \cdot (\tau - \tilde{\tau}),
\end{aligned} \tag{2.22}$$

where the common (bistatic) point of stationary phase is given by

$$\tilde{\tau} = \frac{\ddot{\phi}_T(\tilde{\tau}_T) \cdot \tilde{\tau}_T + \ddot{\phi}_R(\tilde{\tau}_R) \cdot \tilde{\tau}_R}{\ddot{\phi}_T(\tilde{\tau}_T) + \ddot{\phi}_R(\tilde{\tau}_R)}. \tag{2.23}$$

Solving Eq. (2.20) by POSP and substituting Eqs. (2.22) and (2.23), we obtain, from Eq. (2.17),

$$\begin{aligned}
I(f, f_\tau, R_{0R}, \tau_{0R}) &\cong w(\tau - \tau_{cb}) \\
&\times \exp \left[-j \underbrace{(\phi_T(\tilde{\tau}_T, f_\tau) + \phi_R(\tilde{\tau}_R, f_\tau))}_{\Psi_1(f, f_\tau)} \right] \\
&\times \exp \left[-j \underbrace{\frac{\ddot{\phi}_T(\tilde{\tau}_T) \cdot \ddot{\phi}_R(\tilde{\tau}_R)}{2 \ddot{\phi}_T(\tilde{\tau}_T) + \ddot{\phi}_R(\tilde{\tau}_R)} \cdot (\tilde{\tau}_T - \tilde{\tau}_R)^2}_{\Psi_2(f, f_\tau)} \right] \\
&\times \frac{\sqrt{2\pi}}{\sqrt{\ddot{\phi}_T(\tilde{\tau}_T) + \ddot{\phi}_R(\tilde{\tau}_R)}} \cdot \exp \left(-j \frac{\pi}{4} \right). \tag{2.24}
\end{aligned}$$

Equation (2.24) conceptually shows how we can combine the two monostatic-phase and Doppler histories of the transmitter and receiver to the BPTRS.

We must formulate the individual points of stationary phase, the corresponding phase arguments in those points, and the second phase derivatives. For the receiver, the derivation is straightforward and has been documented in the open literature [2].

When considering the transmitter, the coupling is essential, yet the calculation and combination of the individual terms is lengthy and must be omitted. The stationary points clearly depend on the azimuth frequency; hence, the first term in Eq. (2.24), $\omega(\tau - \tau_{cb})$, defines azimuth bandwidth and bistatic Doppler centroid frequency as the center frequency of the azimuth reference spectrum. The analysis of this window is lengthy, and exact equations determining Doppler bandwidth and Doppler centroid are difficult to derive. However, they can be approximated just in the case of the stationary phase formulation:

$$\begin{aligned}
\Psi_1(f, f_\tau) &= \phi_T(\tilde{\tau}_T, f_\tau) + \phi_R(\tilde{\tau}_R, f_\tau) \\
&= \pi f_\tau \cdot (2\tau_{0R} + a_0) \\
2\pi \frac{R_{0R}}{c} &\times \left[\sqrt{\underbrace{(f + f_0)^2 - f_\tau^2 \frac{c^2}{4v_R^2}}_{F_R(f, f_\tau)}} + \sqrt{\underbrace{(f + f_0)^2 - f_\tau^2 \frac{c^2}{4v_T^2}}_{F_T(f, f_\tau)}} \cdot a_2 \right], \tag{2.25}
\end{aligned}$$

where the coefficients a_0 , a_2 determining the “bistatic grade” are given by

$$a_0 = \frac{(\vec{R}_R(\tau_{0R}) + \vec{d}(\tau_{0R})) \cdot \vec{v}_T}{v_T^2} = \tau_{0T} - \tau_{0R}, \tag{2.26}$$

$$a_2 = \sqrt{\left(\vec{e}_R(\tau_{0R}) + \frac{\vec{d}(\tau_{0R})}{R_{0R}}\right)^2 - v_T^2 \frac{a_0^2}{R_{0R}^2}} = \frac{R_{0T}}{R_{0R}}. \quad (2.27)$$

After some lengthy algebra, the second exponential phase term is found,

$$\begin{aligned} \Psi_2(f, f_\tau) &= \frac{\phi_T''(\tilde{\tau}_T) \cdot \phi_R''(\tilde{\tau}_R)}{\phi_T''(\tilde{\tau}_T) + \phi_R''(\tilde{\tau}_R)} \cdot (\tilde{\tau}_T - \tilde{\tau}_R)^2 \\ &= \frac{2\pi}{R_{0R}c} \cdot \frac{v_T^2 v_R^2}{(f + f_0)^2} \\ &\quad \times \frac{F_T^{3/2}(f, f_\tau) \cdot F_R^{3/2}(f, f_\tau)}{v_T^2 F_T^{3/2}(f, f_\tau) + a_2 v_R^2 F_R^{3/2}(f, f_\tau)} \cdot (\tilde{\tau}_T - \tilde{\tau}_R)^2, \end{aligned} \quad (2.28)$$

where the difference between the stationary points is

$$\begin{aligned} (\tilde{\tau}_T - \tilde{\tau}_R)^2 &= \left[a_0 - f_\tau \cdot \frac{c}{2v_R^2 v_T^2} \cdot \frac{R_{0R}}{\sqrt{F_T(f, f_\tau) \cdot F_R(f, f_\tau)}} \right. \\ &\quad \left. \times \left[v_R^2 \cdot F_R^{1/2}(f, f_\tau) a_2 - v_T^2 \cdot F_T^{1/2}(f, f_\tau) \right] \right]. \end{aligned} \quad (2.29)$$

In the monostatic case, the right-hand side of Eq. (2.29) vanishes, the stationary points of transmitter and receiver coincide, and the second phase term vanishes. Thus, the bistatic reference spectrum given in Eq. (2.24) “collapses” to the trivial monostatic reference spectrum if the bistatic baseline goes to zero and if the velocities are equivalent.

2.2.2 Extended Loffeld Bistatic Formula

The LBF consists of two components: a quasimonostatic phase term and a bistatic deformation phase term. Owing to the fact that a second-order approximation is used in the course of calculating the bistatic spectrum, and that equally weighted contributions of the transmitter and receiver to the azimuth modulation are assumed, LBFs tend to be inaccurate in extreme bistatic configurations (e.g., space-borne/airborne configurations, such as a satellite used as a transmitter/receiver and aircraft used as receiver/transmitter).

In the space-borne/airborne configuration, azimuth signals from the space-borne and airborne platforms only cover a part of the total synthetic aperture time, particularly for the airborne case. Therefore, the airborne case has a very small time-bandwidth product (TBP), which might result in the failure of the POSP. The different TBPs of slant range histories result in unequal contributions of the transmitter and receiver to the overall phase modulation of the azimuth signal.

For the ELBF, the individual TBP of the transmitter and receiver is used as a factor to weigh contributions of the transmitter and receiver to the common bistatic point of stationary phase. This weighted phase is then used to substitute for the real point of stationary phase of BiSAR. From this formulation of phase and by applying the POSP, the BPTRS in the general configuration can be derived.

The received signal from a point target located at (τ_{0R}, R_{0R}) (see Fig. 2.1) after demodulation is given by

$$g(\tau, t, \tau_{0R}, R_{0R}) = \sigma(\tau_{0R}, R_{0R}) w(\tau - \tau_{cb}) s_l \left(t - \frac{R_R(\tau) + R_T(\tau)}{c} \right) \exp \left[-j2\pi \frac{R_R(\tau) + R_T(\tau)}{\lambda} \right]. \quad (2.30)$$

Performing a two-dimensional FT to Eq. (2.30) gives

$$G(f_\tau, f, \tau_{0R}, r_{0R}) = \sigma(\tau_{0R}, r_{0R}) S_1(f) \int w(\tau - \tau_{cb}) \exp[-j\phi_b(\tau, f)] d\tau, \quad (2.31)$$

where $S_1(f)$ is the spectrum of the transmitted signal, and the bistatic phase history $\phi_b(\tau, f)$ is given by

$$\phi_b(\tau, f) = 2\pi(f + f_0) \frac{R_R(\tau) + R_T(\tau)}{c} + 2\pi f_\tau \tau \quad (2.32)$$

and $R_R(\tau)$, $R_T(\tau)$ is given in (2.6). From Eq. (2.31), it can be seen that a double square-root (DSR) term is included in the integral of Eq. (2.31), which makes it difficult to apply the POSP to obtain the BPTRS. To circumvent the limitation of the DSR in Eq. (2.31), we split the bistatic phase history into two components, i.e., the phase history of the receiver, $\phi_R(\tau, f)$, and the phase history of the transmitter, $\phi_T(\tau, f)$:

$$\phi_b(\tau, f) = \phi_R(\tau, f) + \phi_T(\tau, f). \quad (2.33)$$

In [3], $\phi_R(\tau, f)$ and $\phi_T(\tau, f)$ are defined as

$$\begin{aligned} \phi_R(\tau, f) &= 2\pi \left[\frac{f + f_0}{c} R_R(\tau) + \frac{f_\tau \tau}{2} \right], \\ \phi_T(\tau, f) &= 2\pi \left[\frac{f + f_0}{c} R_T(\tau) + \frac{f_\tau \tau}{2} \right]. \end{aligned} \quad (2.34)$$

From Eq. (2.34), it can be seen that both range equations are restricted to the equal contributions to the instantaneous Doppler frequency f_τ . In [14], we formulated $\phi_R(\tau, f)$ and $\phi_T(\tau, f)$ as

$$\begin{aligned}\phi_R(\tau, f) &= 2\pi \left[\frac{f+f_0}{c} R_R(\tau) + k_R f_{\tau} \tau \right], \\ \phi_T(\tau, f) &= 2\pi \left[\frac{f+f_0}{c} R_T(\tau) + k_T f_{\tau} \tau \right],\end{aligned}\quad (2.35)$$

where the weighting factors k_R and k_T are defined by using the ratio of the TBP of the platform to the total TBP in [14, 15]. Note that these factors will always satisfy the equation $k_R + k_T = 1$. However, the ELBF would show a limitation in the moderate- or high-squint configuration since the effect of squint angles on the instantaneous Doppler frequency is neglected.

To further improve the result of [3], we redefine the individual slant range histories as

$$\begin{aligned}\phi_R(\tau, f) &= 2\pi \left[\frac{f+f_0}{c} R_R(\tau) + f_{\tau R} \tau \right], \\ \phi_T(\tau, f) &= 2\pi \left[\frac{f+f_0}{c} R_T(\tau) + f_{\tau T} \tau \right],\end{aligned}\quad (2.36)$$

where $f_{\tau R}$ and $f_{\tau T}$ represent the individual instantaneous Doppler frequencies contributed by receiver and transmitter, respectively, and are assumed to be the unknown variables. It is clear that $f_{\tau R} + f_{\tau T} = f_{\tau}$ is the identical equation. Subsequently, we expand $\phi_R(\tau, f)$ and $\phi_T(\tau, f)$ in a Taylor series around $\tilde{\tau}_R$ and $\tilde{\tau}_T$ (note that the first-order terms are zero):

$$\begin{aligned}\phi_R(\tau, f) &\approx \phi_R(\tilde{\tau}_R, f) + \frac{1}{2} \ddot{\phi}_R(\tilde{\tau}_R, f) (\tau - \tilde{\tau}_R)^2 + \frac{1}{6} \dddot{\phi}_R(\tilde{\tau}_R, f) (\tau - \tilde{\tau}_R)^3, \\ \phi_T(\tau, f) &\approx \phi_T(\tilde{\tau}_T, f) + \frac{1}{2} \ddot{\phi}_T(\tilde{\tau}_T, f) (\tau - \tilde{\tau}_T)^2 + \frac{1}{6} \dddot{\phi}_T(\tilde{\tau}_T, f) (\tau - \tilde{\tau}_T)^3,\end{aligned}\quad (2.37)$$

where stationary points $\tilde{\tau}_R$ and $\tilde{\tau}_T$ are the solution to $\dot{\phi}_R(\tau, f) = 0$ and $\dot{\phi}_T(\tau, f) = 0$, respectively. Solving for the stationary points, we obtain

$$\tilde{\tau}_R = \tau_{0R} - \frac{cR_{0R}f_{\tau R}}{v_R^2 F_R}, \quad \tilde{\tau}_T = \tau_{0T} - \frac{cR_{0T}f_{\tau T}}{v_T^2 F_T}, \quad (2.38)$$

where $\tilde{\tau}_R$ and $\tilde{\tau}_T$ are the stationary points of $\phi_R(\tau, f)$ and $\phi_T(\tau, f)$. Equation (2.38) represent the individual time-Doppler correspondences between the azimuth time variables $\tilde{\tau}_R$, $\tilde{\tau}_T$ and the Doppler frequency variables $f_{\tau R}$, $f_{\tau T}$, respectively. F_R and F_T are defined as

$$F_R = \sqrt{(f+f_0)^2 - \left(\frac{cf_{\tau R}}{v_R} \right)^2}, \quad F_T = \sqrt{(f+f_0)^2 - \left(\frac{cf_{\tau T}}{v_T} \right)^2}. \quad (2.39)$$

The following derivations show how a more accurate time-Doppler correspondence can be formulated. To use the finite-order polynomial model in Eq. (2.37) (e.g., the zeroth order or second order) and obtain the accurate $\tilde{\tau}_R$ and $\tilde{\tau}_T$ to hold $(\tau - \tilde{\tau}_R) \approx (\tau - \tilde{\tau}_T) \approx 0$, we define the following Euclidean norm on $\mathbf{R}^2$:

$$\|E\|^2 = (\tau - \tilde{\tau}_R)^2 + (\tau - \tilde{\tau}_T)^2. \quad (2.40)$$

This norm can be used to formulate the new approximation expressions of the individual slant range histories. To attain the goal of neglecting the second- and higher-order terms in Eq. (2.37), we should keep $\|E\|^2$ as small as possible. Thus, we would like

$$\|E\|^2 = 0. \quad (2.41)$$

This equation does not have an analytical solution, and an approximate solution can be found by using the least-square methods. Therefore, the minimum of the sum of two square terms in Eq. (2.41) can be obtained by setting the gradient of $\|E\|^2$ to zero. Since $\|E\|^2$ contains two parameters, there are two gradient equations, i.e.,

$$\begin{aligned} \frac{\partial \|E\|^2}{\partial f_{\tau R}} &= -2 \frac{\partial \tilde{\tau}_R}{\partial f_{\tau R}} (\tau - \tilde{\tau}_R) - 2 \frac{\partial \tilde{\tau}_T}{\partial f_{\tau R}} (\tau - \tilde{\tau}_T) = 0, \\ \frac{\partial \|E\|^2}{\partial f_{\tau T}} &= -2 \frac{\partial \tilde{\tau}_R}{\partial f_{\tau T}} (\tau - \tilde{\tau}_R) - 2 \frac{\partial \tilde{\tau}_T}{\partial f_{\tau T}} (\tau - \tilde{\tau}_T) = 0. \end{aligned} \quad (2.42)$$

In order to achieve the approximated formulation of $f_{\tau R}$ and $f_{\tau T}$, we must formulate the partial derivatives of $\tilde{\tau}_R$ and $\tilde{\tau}_T$ with respect to $f_{\tau R}$ and $f_{\tau T}$. The partial derivatives can be expressed as

$$\frac{\partial \tilde{\tau}_R}{\partial f_{\tau R}} = -\frac{cR_{0R}}{v_R^2} \left[\frac{(f+f_0)^2}{F_R^{3/2}} \right], \quad \frac{\partial \tilde{\tau}_R}{\partial f_{\tau T}} = \frac{cR_{0R}}{v_R^2} \left[\frac{(f+f_0)^2}{F_R^{3/2}} \right], \quad (2.43)$$

$$\frac{\partial \tilde{\tau}_T}{\partial f_{\tau T}} = -\frac{cR_{0T}}{v_T^2} \left[\frac{(f+f_0)^2}{F_T^{3/2}} \right], \quad \frac{\partial \tilde{\tau}_T}{\partial f_{\tau R}} = \frac{cR_{0T}}{v_T^2} \left[\frac{(f+f_0)^2}{F_T^{3/2}} \right]. \quad (2.44)$$

Substituting Eqs. (2.43) and (2.44) into (2.42) yields

$$\begin{aligned} \frac{cR_{0R}}{v_R^2} \left[\frac{(f+f_0)^2}{F_R^{3/2}} \right] (\tau - \tilde{\tau}_R) - \frac{cR_{0R}}{v_R^2} \left[\frac{(f+f_0)^2}{F_R^{3/2}} \right] (\tau - \tilde{\tau}_T) &= 0, \\ \frac{cR_{0T}}{v_T^2} \left[\frac{(f+f_0)^2}{F_T^{3/2}} \right] (\tau - \tilde{\tau}_R) - \frac{cR_{0T}}{v_T^2} \left[\frac{(f+f_0)^2}{F_T^{3/2}} \right] (\tau - \tilde{\tau}_T) &= 0. \end{aligned} \quad (2.45)$$

Simplifying Eq. (2.45) by removing the equivalent terms in Eq. (2.45) gives

$$(\tau - \tilde{\tau}_R) = (\tau - \tilde{\tau}_T) \Leftrightarrow \tilde{\tau}_R = \tilde{\tau}_T. \quad (2.46)$$

Before substituting Eqs. (2.38) into (2.46), $\tilde{\tau}_R$ and $\tilde{\tau}_T$ can be approximately expressed as

$$\begin{aligned} \tilde{\tau}_R &\approx \tau_{0R} - \frac{R_{0R}}{v_R} \tan \theta_{SR} - \frac{\lambda R_{0R}}{v_R^2 \cos^3 \theta_{SR}} (f_{\tau R} - f_{DcR}), \\ \tilde{\tau}_T &\approx \tau_{0T} - \frac{R_{0T}}{v_T} \tan \theta_{ST} - \frac{\lambda R_{0T}}{v_T^2 \cos^3 \theta_{ST}} (f_{\tau T} - f_{DcT}), \end{aligned} \quad (2.47)$$

where f_{DcR} and f_{DcT} represent the Doppler centroid of receiver and transmitter at the composite beam center crossing time, respectively. They can be accurately formulated by Eq. (2.52).

By using (2.42) and substituting Eqs. (2.47) into (2.46), the resulting relationship between $f_{\tau R}$ and $f_{\tau T}$ is

$$\frac{(f_{\tau T} - f_{DcT})}{(f_{\tau R} - f_{DcR})} = \frac{\frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}}}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}}} = \frac{\frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2}. \quad (2.48)$$

Combining Eq. (2.48) and the identical equation $f_{\tau R} + f_{\tau T} = f_\tau$, we can express the individual instantaneous Doppler frequencies as

$$\begin{aligned} f_{\tau R} &= \frac{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2 + \frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2} (f_\tau - f_{DcR} - f_{DcT}) + f_{DcR}, \\ f_{\tau T} &= \frac{\frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2 + \frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2} (f_\tau - f_{DcR} - f_{DcT}) + f_{DcT}. \end{aligned} \quad (2.49)$$

By introducing the factors k_R and k_T defined in Eq. (2.51), we can obtain Eq. (2.50). In Eq. (2.48), we introduce the square term of the composite synthetic aperture time in the denominator and numerator. It seems that the contributions of transmitter and receiver to the instantaneous Doppler frequency in the base band are proportional to their TBPs. This is because the accuracy of the time-Doppler relationship derived by the POSP is determined by the TBP [16, 17]. We have

$$\begin{aligned} f_{\tau R} &= k_R (f_\tau - f_{DcR} - f_{DcT}) + f_{DcR}, \\ f_{\tau T} &= k_T (f_\tau - f_{DcR} - f_{DcT}) + f_{DcT}, \end{aligned} \quad (2.50)$$

where

$$k_R = \frac{TBP_R}{TBP_R + TBP_T} = \frac{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2 + \frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2}, \quad (2.51)$$

$$k_T = \frac{TBP_T}{TBP_R + TBP_T} = \frac{\frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2}{\frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} T_a^2 + \frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}} T_a^2},$$

$$\begin{aligned} f_{DcR} &= \frac{(f + f_0)}{c} v_R \sin \theta_{SR}, \\ f_{DcT} &= \frac{(f + f_0)}{c} v_T \sin \theta_{ST}. \end{aligned} \quad (2.52)$$

In Eq. (2.50), $f_{\tau R}$ and $f_{\tau T}$ are of the minimum error. Therefore, substituting Eqs. (2.50) into (2.38) yields the accurate individual time-Doppler correspondences, i.e., $\tilde{\tau}_R$ and $\tilde{\tau}_T$. These two accurate correspondences can lead to $\|E\|^2 \approx 0$, which means that $\tilde{\tau}_R \approx \tilde{\tau}_T \approx \tau$.¹ Using $\|E\|^2 \approx 0$, $\phi_R(\tau, f)$ and $\phi_T(\tau, f)$ can be as

$$\begin{aligned} \phi_R(\tau, f) &\approx \phi_R(\tilde{\tau}_R, f), \\ \phi_T(\tau, f) &\approx \phi_T(\tilde{\tau}_T, f). \end{aligned} \quad (2.53)$$

The preceding zeroth-order approximation is validated by the simulation experiments in the high-squint geometry in [3]. The resulting phase error functions introduced by the zeroth-order Taylor approximation can be approximately represented by neglecting the quadratic terms:

$$\begin{aligned} E_{RWS}(\tau, f) &= |\phi_R(\tau, f) - \phi_R(\tilde{\tau}_R, f)| \approx \left| \frac{1}{2} \ddot{\phi}_R(\tilde{\tau}_R, f) (\tau - \tilde{\tau}_R)^2 \right|, \\ E_{TWS}(\tau, f) &= |\phi_T(\tau, f) - \phi_T(\tilde{\tau}_T, f)| \approx \left| \frac{1}{2} \ddot{\phi}_T(\tilde{\tau}_T, f) (\tau - \tilde{\tau}_T)^2 \right|. \end{aligned} \quad (2.54)$$

Substituting Eqs. (2.53) into (2.33) yields the bistatic slant range history as

$$\phi_b(\tau, f) \approx \phi_R(\tilde{\tau}_R, f) + \phi_T(\tilde{\tau}_T, f). \quad (2.55)$$

From Eq. (2.55), it can be seen that the present bistatic slant range history is represented by the zeroth-order polynomials and is independent of the azimuth time variable τ . Therefore, the integral in Eq. (2.31) can be readily solved to derive the desired BPTRS:

¹In addition, the bistatic stationary point can also be related by $\tilde{\tau}_R \approx \tilde{\tau}_T \approx \tau \approx \tilde{\tau}_p$ as $\tilde{\tau}_p$ represents the bistatic time-Doppler correspondence.

$$G(f_\tau, f, \tau_{0R}, r_{0R}) = \sigma(\tau_{0R}, r_{0R}) w(\tilde{\tau}_p - \tau_{cb}) S_1(f) \exp[-j\Psi_B(f_\tau, f)]. \quad (2.56)$$

The bistatic phase $\Psi_B(f_\tau, f)$ is given as

$$\begin{aligned} \Psi_B(f_\tau, f) &= \phi_R(\tilde{\tau}_R, f) + \phi_T(\tilde{\tau}_T, f) \\ &= 2\pi(f_{\tau T}\tau_{0T} + f_{\tau R}\tau_{0R}) \\ &\quad + \frac{2\pi}{c} \left(R_{0R} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau R}}{v_R}\right)^2} \right. \\ &\quad \left. + R_{0T} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau T}}{v_T}\right)^2} \right). \end{aligned} \quad (2.57)$$

Comparing Eq. (2.57) and the monostatic spectrum in [2], it can be shown that $\Psi_B(f_\tau, f)$ would become the monostatic spectrum by setting $\tau_{0R} = \tau_{0T}$, $v_R = v_T$, and $R_{0R} = R_{0T}$ where the bistatic configuration reduces to a monostatic case. In comparison with the bistatic spectrum in [3], Eq. (2.57) does not have the bistatic deformation term, and thus appears simpler. For the applications of the proposed zeroth-order model, the algorithm presented in [15] can be directly applied based on the formulation in Eq. (2.57) without a “blocking” operation. In particular, it is linearly dependent on the slant range variables, which will facilitate its application for bistatic SAR processing [2].

2.2.3 Method of Series Reversion

In 2007, Neo proposed a two-dimensional point target spectrum for an arbitrary bistatic synthetic aperture radar configuration. His method makes use of series reversion to expand the range history into a power series. The stationary phase is then used to invert the power series, and finally a FT pair is used to derive the point target spectrum. The accuracy of the spectrum is controlled by keeping enough terms in the power series expansion.

To simplify the derivation of the two-dimensional (2D) bistatic spectrum, let the aperture center τ_{cb} be at the origin, and the baseband signal can be expressed by

$$g(\tau, t, \tau_{0R}, R_{0R}) = \sigma(\tau_{0R}, R_{0R}) w(\tau) s_l \left(t - \frac{R_R(\tau) + R_T(\tau)}{c} \right) \exp \left[-j2\pi \frac{R_R(\tau) + R_T(\tau)}{\lambda} \right]. \quad (2.58)$$

The first step is to remove the linear phase and the linear range cell migration (LRCM). This reason for this step will become apparent when we apply the series

reversion at a later step. After removal of these terms, the point target signal in the time domain becomes

$$g_A(t, \tau) = s_l \left(t - \frac{R_1(\tau)}{c} \right) w(\tau) \exp \left(-j2\pi \frac{R_1(\tau)}{\lambda} \right), \quad (2.59)$$

where $\sigma(\tau_{0R}, R_{0R})$ is absorbed into the above equation. In addition,

$$R_1(\tau) = R_{cen} + k_2 \tau^2 + k_3 \tau^3 + k_4 \tau^4 + \dots \quad (2.60)$$

and

$$\begin{aligned} R_{cen} &= R_{Tcen} + R_{Rcen} \\ &= \sqrt{R_{0T}^2 + v_T^2 \tau_{0T}^2} + \sqrt{R_{0R}^2 + v_R^2 \tau_{0R}^2}. \end{aligned} \quad (2.61)$$

Using the property of Taylor series, we have

$$k_2 = \frac{1}{2!} \left(\frac{dR_T^2(\tau)}{d\tau^2} + \frac{dR_R^2(\tau)}{d\tau^2} \right) \Big|_{\tau=0}, \quad (2.62)$$

$$k_3 = \frac{1}{3!} \left(\frac{dR_T^3(\tau)}{d\tau^3} + \frac{dR_R^3(\tau)}{d\tau^3} \right) \Big|_{\tau=0}, \quad (2.63)$$

$$k_4 = \frac{1}{4!} \left(\frac{dR_T^4(\tau)}{d\tau^4} + \frac{dR_R^4(\tau)}{d\tau^4} \right) \Big|_{\tau=0}. \quad (2.64)$$

The coefficients k_1 , k_2 , and k_3 are evaluated at the aperture center. The derivatives of the transmitter range are given by

$$\frac{dR_T^2(\tau)}{d\tau^2} \Big|_{\tau=0} = \frac{R_{0T}^2 V_T^2}{R_{Tcen}^3} = \frac{V_T^2 \cos^2 \theta_{sqT}}{R_{Tcen}}, \quad (2.65)$$

$$\frac{dR_T^3(\tau)}{d\tau^3} \Big|_{\tau=0} = \frac{3R_{0T}^2 \tau_{0T} V_T^4}{R_{Tcen}^5} = \frac{3V_T^3 \cos^2 \theta_{sqT} \sin \theta_{sqT}}{R_{Tcen}^2}, \quad (2.66)$$

$$\begin{aligned} \frac{dR_T^4(\tau)}{d\tau^4} \Big|_{\tau=0} &= \frac{3R_{0T}^2 V_T^4 (4\tau_{0T}^2 V_T^2 - R_{0T}^2)}{R_{Tcen}^7} \\ &= \frac{3V_T^4 \cos^2 \theta_{sqT} (4 \sin^2 \theta_{sqT} - \cos^2 \theta_{sqT})}{R_{Tcen}^3}. \end{aligned} \quad (2.67)$$

Similar equations can be written for the derivatives of the receiver range $R_r(\tau)$.

As an example, if the number of terms in the trajectory was kept up to the fourth order in Eq. (2.60) and the range spectrum was expanded up to the fourth azimuth frequency term, the 2D point target spectrum is given by

$$G'_A(f, \tau) = S_1(f)w(\tau) \exp \left\{ -j2\pi \frac{(f+f_0)R_1(\tau)}{c} \right\}. \quad (2.68)$$

Next, we perform an azimuth FT to invert the function to the frequency domain. Using POSP (see Appendix A), azimuth frequency is related to azimuth time by

$$-\frac{c}{f+f_0}f_\tau = 2k_2\tau + 3k_3\tau^2 + 4k_4\tau^3 + \dots \quad (2.69)$$

We can derive an expression of τ in terms of f_τ by using the series reversion (see Appendix B). In the backward function given by Eq. (2.117), we replace x by τ , y by $\left(-\frac{c}{f+f_0}f_\tau\right)$, and substitute the coefficients of x by the coefficients of τ . Inverting this power series, we arrive at

$$\begin{aligned} \tau(f_\tau) = & A_1 \cdot \left(-\frac{c}{f+f_0}f_\tau\right) + A_2 \cdot \left(-\frac{c}{f+f_0}f_\tau\right)^2 \\ & + A_3 \cdot \left(-\frac{c}{f+f_0}f_\tau\right)^3 + \dots \end{aligned} \quad (2.70)$$

The rationale for removal of the linear phase term and LRCM becomes clear at this step. In order to apply the series reversion directly in Eq. (2.69), we should remove the constant term in the forward function since the constant term is absent in the backward function Eq. (2.117). Since the linear phase term and the LRCM term are removed, there is no constant term left after applying the azimuth FT to Eq. (2.68).

Using Eq. (2.70) with Eq. (2.68), we obtain the 2D spectrum of $g_A(t, \tau)$:

$$\begin{aligned} G_A(f, f_\tau) = & S_1(f)W_{az}(f_\tau) \exp \{-j2\pi f_\tau \tau(f_\tau)\} \\ & \exp \left\{ -j2\pi \frac{(f_0+f)}{c} R_1(\tau(f_\tau)) \right\}, \end{aligned} \quad (2.71)$$

where $W_{az}(\cdot)$ represents the shape of the Doppler spectrum and is approximately a scaled version of the azimuth time envelope $w(\cdot)$. To obtain the 2D point target spectrum for $g(t, \tau)$, we reintroduce the LRCM and linear phase into $g_A(t, \tau)$ in Eq. (2.59), and obtain

$$\begin{aligned}
g(t, \tau) &= g_A \left(t - \frac{k_1 \tau}{c}, \tau \right) \exp \left(-j2\pi \frac{f_0 k_1}{c} \tau \right) \\
&= s_l \left(t - \frac{R_l(\tau) + k_1 \tau}{c} \right) w(\tau) \exp \left(-j2\pi \left(\frac{f_0 R_l(\tau)}{c} + \frac{f_0 k_1 \tau}{c} \right) \right),
\end{aligned} \tag{2.72}$$

where

$$k_1 = \left. \frac{dR_T(\tau)}{d\tau} \right|_{\tau=0} + \left. \frac{dR_R(\tau)}{d\tau} \right|_{\tau=0}. \tag{2.73}$$

The derivatives Eq. (2.73) at the aperture center are given by

$$\left. \frac{dR_T(\tau)}{d\tau} \right|_{\tau=0} = -V_T \sin \theta_{sqT}, \tag{2.74}$$

$$\left. \frac{dR_R(\tau)}{d\tau} \right|_{\tau=0} = -V_R \sin \theta_{sqR}. \tag{2.75}$$

To derive the 2D point target spectrum for $g(t, \tau)$, we use the FT skew and shift properties [2]:

$$\begin{aligned}
g(t, \tau) &\leftrightarrow G(f, f_\tau), \\
g(t, \tau) \exp \{-j2\pi f_k \tau\} &\leftrightarrow G(f, f_\tau + f_k), \\
g(t - k\tau, \tau) &\leftrightarrow G(f, f_\tau + kf),
\end{aligned} \tag{2.76}$$

where g is a 2D time function, G its corresponding frequency function, and k and f_k are constants. Applying these FT pairs to Eqs. (2.71) and (2.72), we arrive at the desired 2D point target spectrum,

$$G(f, f_\tau) = G_A \left(f, f_\tau + (f + f_0) \frac{k_1}{c} \right). \tag{2.77}$$

The accuracy of the spectrum is limited by the number of terms used in the expansion of Eq. (2.77). In general, we would like to limit the uncompensated phase error to be within $\pm \frac{\pi}{4}$, in order to avoid significant deterioration of the image quality.

2.2.4 Two-Dimensional Principle of Stationary Phase

As is analyzed in the above section, the LBF is the first approximated BPTRS. However, for the LBF the same contributions of the transmitter and receiver to the

total azimuth modulation are assumed, which is not always valid for the general bistatic configurations. To circumvent this limitation, the azimuth TBPs are introduced into the ELBF mode and better accuracy in the spectrum is obtained for the general geometry where azimuth modulations are unequal for the transmitter and receiver. A 2D POSP is used in the 2D frequency domain to achieve an approximate one-to-one correspondence between time and Doppler frequency. By derivation, it can be seen that the spectrum contains two hyperbolic range-azimuth coupling terms and thus is very similar to the monostatic spectrum. It shows the characteristic of the conventional monostatic SAR in addition to an additional azimuth scaling term. Therefore, it makes the common Doppler-based monostatic processing algorithms readily suitable to handle the BiSAR data in the moderate-squint, azimuth-variant configurations with two moving platforms.

Performing the 2D FT to (2.14) gives

$$G(f_\tau, f, \tau_{0R}, R_{0R}) = \sigma(\tau_{0R}, R_{0R}) S_I(f) \int w(\tau - \tau_{cb}) \exp \left[-2\pi(f + f_0) \frac{R_R(\tau) + R_T(\tau)}{c} \right] \exp(-j2\pi f_\tau \tau) d\tau, \quad (2.78)$$

where $S_I(f)$ is the spectrum of the transmitted signal. From Eq. (2.78), it can be seen that a DSR term is included in the integral of Eq. (2.78), which makes it difficult to apply the POSP to solve the integral [2, 11–13, 18]. To circumvent the limitation in Eq. (2.78), we replace two hyperbolic square-root terms with the following Fourier decompositions [14] and [19].

$$\begin{aligned} W_R \left[\frac{\tau - \tau_{cb}}{T_{sc}} \right] \exp \left[-2\pi \frac{(f + f_0)}{c} R_R(\tau) \right] \\ = \int W_T \left[\frac{\tilde{\tau}_T(f_{\tau T}) - \tau_{cb}}{T_{sc}} \right] \exp[-j\Phi_T(f_{\tau T}, f)] \exp[j2\pi f_{\tau T} \tau] d\tilde{\tau}_T, \end{aligned} \quad (2.79)$$

where $f_{\tau R}$ and $f_{\tau T}$ denote two azimuth frequency variables and represent contributions of the range equations of receiver and transmitter to the instantaneous Doppler frequency f_τ . Thus, we always have $f_{\tau R} + f_{\tau T} = f_\tau$. Φ_R and Φ_T are defined as

$$\begin{aligned} \Phi_R(f_{\tau R}, f) &= 2\pi f_{\tau R} \tau_{0R} + 2\pi \frac{R_{0R}}{c} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau R}}{v_R} \right)^2}, \\ \Phi_T(f_{\tau T}, f) &= 2\pi f_{\tau T} \tau_{0T} + 2\pi \frac{R_{0T}}{c} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau T}}{v_T} \right)^2}. \end{aligned} \quad (2.80)$$

$\tilde{\tau}_R(f_{\tau R})$ and $\tilde{\tau}_T(f_{\tau T})$ are the individual time-Doppler correspondences of receiver and transmitter, respectively, and are given by

$$\begin{aligned}
\tilde{\tau}_R(f_{\tau R}) &= \tau_{0R} - \frac{cR_{0R}}{v_R^2} \frac{f_{\tau R}}{\sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau R}}{v_R}\right)^2}}, \\
\tilde{\tau}_T(f_{\tau T}) &= \tau_{0T} - \frac{cR_{0T}}{v_T^2} \frac{f_{\tau T}}{\sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau T}}{v_T}\right)^2}}.
\end{aligned} \tag{2.81}$$

Substituting Eqs. (2.79)–(2.81) into (2.78) yields

$$\begin{aligned}
G(f_\tau, f, \tau_{0R}, R_{0R}) &= \sigma(\tau_{0R}, R_{0R}) S_1(f) \int \exp(-j2\pi f_\tau \tau) \\
&\times \left[\iint W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau T} - f_{DcT}}{K_{aT} T_{sc}} \right] \right. \\
&\times \exp(j2\pi f_\tau \tau) \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\} df_{\tau R} df_{\tau T} d\tau.
\end{aligned} \tag{2.82}$$

From Eq. (2.82), we find that $f_{\tau R}$ and $f_{\tau T}$ are centered on the Doppler centroid frequencies f_{DcR} and f_{DcT} , and have a width of $K_{aR} T_{sc}$ and $K_{aT} T_{sc}$, respectively. They are given by

$$\begin{aligned}
f_{DcR} &= \frac{v_R \sin \theta_{SR}}{c/(f + f_0)} & f_{DcT} &= \frac{v_T \sin \theta_{ST}}{c/(f + f_0)}, \\
K_{aR} &= \frac{v_R^2 \cos^3 \theta_{SR}}{\lambda R_{0R}} & K_{aT} &= \frac{v_T^2 \cos^3 \theta_{ST}}{\lambda R_{0T}}.
\end{aligned} \tag{2.83}$$

The 2D POSP can be applied to solve the double integral in Eq. (2.82). Letting the first partial derivatives of the phase of Eq. (2.82) with respect to $f_{\tau R}$ and $f_{\tau T}$ be zero, we obtain

$$\begin{cases} \frac{\partial [\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f) - 2\pi(f_{\tau R} + f_{\tau T})\tau]}{\partial f_{\tau R}} = 0 \\ \frac{\partial [\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f) - 2\pi(f_{\tau R} + f_{\tau T})\tau]}{\partial f_{\tau T}} = 0 \end{cases}. \tag{2.84}$$

Using the identical equation $f_\tau = f_{\tau R} + f_{\tau T}$ in Eq. (2.84), we can approximately determine the values of $f_{\tau R}$ and $f_{\tau T}$ as

$$\begin{aligned}
f_{\tau R} &= k_R(f_\tau - f_{DcR} - f_{DcT}) + f_{DcR}, \\
f_{\tau T} &= k_T(f_\tau - f_{DcR} - f_{DcT}) + f_{DcT},
\end{aligned} \tag{2.85}$$

where k_R and k_T are given by

$$\begin{cases} k_R = \frac{K_{aR}}{K_{aR} + K_{aT}} \\ k_T = \frac{K_{aT}}{K_{aR} + K_{aT}} \end{cases}. \quad (2.86)$$

Substituting Eq. (2.85) in the double-integral term of Eq. (2.82) and disregarding the complex factor give

$$\begin{aligned} & \iint W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] \\ & \quad \exp(j2\pi f_{\tau} \tau) \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\} df_{\tau R} df_{\tau T} \\ &= W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] \\ & \quad \exp[j2\pi(f_{\tau R} + f_{\tau T})\tau] \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\}. \end{aligned} \quad (2.87)$$

We further substitute Eqs. (2.87) into (2.82), and obtain

$$\begin{aligned} G(f_{\tau}, f, \tau_{0R}, R_{0R}) &= \sigma(\tau_{0R}, R_{0R}) S_1(f) \times \int W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau T} - f_{DcT}}{K_{aT} T_{sc}} \right] \\ & \quad \exp(-j2\pi f_{\tau} \tau) \exp[j2\pi(f_{\tau R} + f_{\tau T})\tau] \\ & \quad \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\} d\tau \\ &= \sigma(\tau_{0R}, R_{0R}) S_1(f) \int W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau T} - f_{DcT}}{K_{aT} T_{sc}} \right] \\ & \quad \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\} d\tau. \end{aligned} \quad (2.88)$$

Since the phase term in the integrand of Eq. (2.88) is independent of the variable τ , Eq. (2.88) can be further expressed as

$$\begin{aligned} G(f_{\tau}, f, \tau_{0R}, R_{0R}) &= \sigma(\tau_{0R}, R_{0R}) S_1(f) W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] \\ & \quad W_T \left[\frac{f_{\tau T} - f_{DcT}}{K_{aT} T_{sc}} \right] \exp\{-j[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]\}. \end{aligned} \quad (2.89)$$

For simplicity, we introduce a composite window function in the Doppler domain, given by $W_a \left[\frac{f_{\tau} - f_{DcR} - f_{DcT}}{K_{aR} T_{sc} + K_{aT} T_{sc}} \right] = W_R \left[\frac{f_{\tau R} - f_{DcR}}{K_{aR} T_{sc}} \right] W_T \left[\frac{f_{\tau T} - f_{DcT}}{K_{aT} T_{sc}} \right]$. If we assume that the transmitted signal is a chirp signal with a positive modulation rate K_r , Eq. (2.89) can be rewritten as

$$G(f_\tau, f, \tau_{0R}, R_{0R}) = \sigma(\tau_{0R}, R_{0R}) W_r(f) W_a \left[\frac{f_\tau - f_{DcR} - f_{DcT}}{K_{aR} T_{sc} + K_{aT} T_{sc}} \right] \exp\{-j\Psi_B(f_\tau, f, R_{0R})\}, \quad (2.90)$$

where $W_r(f)$ is the range frequency envelope. The BPTRS is defined as

$$\begin{aligned} \Psi_B(f_\tau, f, R_{0R}) = & \pi \frac{f^2}{K_r} + 2\pi(f_{\tau R} \tau_{0R} + f_{\tau T} \tau_{0T}) \\ & + 2\pi \left[\frac{R_{0R}}{c} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau R}}{v_R}\right)^2} + \frac{R_{0T}}{c} \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau T}}{v_T}\right)^2} \right]. \end{aligned} \quad (2.91)$$

From Eq. (2.91), we find that the bistatic spectrum similarly contains two hyperbolic range-azimuth coupling terms. Comparing Eq. (2.91) and the monostatic spectrum in [2], it is seen that Eq. (2.91) will degenerate to the monostatic spectrum when the bistatic configuration reduces to a monostatic configuration. The availability of this expression allows one to accurately formulate the range migration trajectory and azimuth-matched filter in the Doppler domain.

However, the foregoing derivation is restricted on a point target. In order to focus the entire scene, we must further accommodate the space variation and range-azimuth coupling of the derived spectrum. To highlight the space variation and coupling, some transformations will be introduced in the following.

For clarity, we use the geometric image transformation to formulate τ_{0T} as a function of τ_{0R} and R_{0R} :

$$\begin{aligned} \tau_{0T} &= p_{10} + p_{11} R_{0R} + p_{12} \tau_{0R}, \\ p_{01} &= \tau_{0T}(0, R_{0R}), \\ p_{11} &= \left. \frac{\partial \tau_{0T}(\tau_{0R}, R_{0R})}{\partial R_{0R}} \right|_{\tau_{0R}=0}, \quad R_{0R} = R_{RR}, \\ p_{12} &= \left. \frac{\partial \tau_{0T}(\tau_{0R}, R_{0R})}{\partial \tau_{0R}} \right|_{\tau_{0R}=0}, \quad R_{0R} = R_{RR}. \end{aligned} \quad (2.92)$$

In Eq. (2.92), R_{RR} and R_{RT} are defined as the reference slant ranges of receiver and transmitter. Generally, they are set to be the midrange values.

Substituting Eqs. (2.92) into (2.91) yields

$$\begin{aligned} \Psi_B(f_\tau, f, R_{0R}) = & \pi \frac{f^2}{K_r} + 2\pi \beta_A \tau_{0R} f_\tau + 2\pi(p_{10} + p_{11} R_{0R}) k_T f_\tau \\ & + 2\pi \left[\frac{R_{0R}}{c} F_R + \frac{R_{0T}}{c} F_T \right] + \overline{\Phi}_{RCM}(f) + \Phi_{Res}(\tau_{0R}, R_{0R}), \end{aligned} \quad (2.93)$$

where F_R and F_T are defined as

$$F_R = \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau R}}{v_R}\right)^2}, \quad F_T = \sqrt{(f + f_0)^2 - \left(\frac{cf_{\tau T}}{v_T}\right)^2}. \quad (2.94)$$

β_A refers to the scaling factor in the azimuth-time domain that is introduced by the variant relative position in azimuth between transmitter and receiver. It is formulated as

$$\beta_A = k_R + p_{12}k_T. \quad (2.95)$$

$\overline{\Phi}_{RCM}(f)$ denotes a range cell migration (RCM) term and is expressed as

$$\begin{aligned} \overline{\Phi}_{RCM}(f) &= -2\pi \frac{(k_T v_R \sin \theta_{SR} - k_R v_T \sin \theta_{ST})}{c} \tau_{0T} f \\ &\quad + 2\pi \frac{(k_T v_R \sin \theta_{SR} - k_R v_T \sin \theta_{ST})}{c} \tau_{0R} f \\ &= 2\pi \frac{R_{0T}}{c} \frac{\mu_{T1} \mu_{T2}}{D_T} f + 2\pi \frac{R_{0R}}{c} \frac{\mu_{R1} \mu_{R2}}{D_R} f, \end{aligned} \quad (2.96)$$

where

$$D_R = \sqrt{1 - \mu_{R1}^2}, \quad D_T = \sqrt{1 - \mu_{T1}^2}, \quad (2.97)$$

$$\begin{aligned} \mu_c &= \frac{(k_T v_R \sin \theta_{SR} - k_R v_T \sin \theta_{ST})}{\lambda}, \\ \mu_{R1} &= \frac{\lambda}{v_R} [k_R f_{\tau} + \mu_c], \quad \mu_{R2} = \frac{\lambda}{v_R} \mu_c, \\ \mu_{T1} &= \frac{\lambda}{v_T} [k_T f_{\tau} - \mu_c], \quad \mu_{T2} = -\frac{\lambda}{v_T} \mu_c. \end{aligned} \quad (2.98)$$

$\Phi_{\text{Res}}(\tau_{0R}, R_{0R})$ is the residual phase term, and given as

$$\begin{aligned} \Phi_{\text{Res}}(\tau_{0R}, R_{0R}) &= -2\pi \frac{(k_T v_R \sin \theta_{SR} - k_R v_T \sin \theta_{ST})}{\lambda} \\ &\quad (p_{10} + p_{11}R_{0R} + p_{12}\tau_{0R}) + 2\pi \frac{(k_T v_R \sin \theta_{SR} - k_R v_T \sin \theta_{ST})}{\lambda} \tau_{0R}. \end{aligned} \quad (2.99)$$

This term is independent of the range and azimuth frequency variables, and it can be ignored if a magnitude image is the final product. If the phase information is

required (e.g., a bistatic interferometer), this residual term needs to be compensated in the image domain by using a phase multiplication.

To formulate the space variation and range-azimuth coupling, and further handle the variation and coupling, we expand F_R and F_T in the second-order Taylor series in terms of f/f_0 , giving the following results:

$$\begin{aligned} F_R &\approx D_R f_0 + \frac{(1 - \mu_{R1}\mu_{R2})}{D_R} f - \frac{(\mu_{R1} - \mu_{R2})^2}{2f_0 D_R^3} f^2, \\ F_T &\approx D_T f_0 + \frac{(1 - \mu_{T1}\mu_{T2})}{D_T} f - \frac{(\mu_{T1} - \mu_{T2})^2}{2f_0 D_T^3} f^2. \end{aligned} \quad (2.100)$$

Substituting Eqs. (2.100) into (2.93), we can decompose Ψ_B into the range compression, range cell migration, azimuth compression, and azimuth scaling terms, denoted by the subscripts R_c , RCM , AC , and AS , respectively:

$$\Psi_B(f_\tau, f, R_{0R}) = \Phi_{RC}(f_\tau, f) + \Phi_{RCM}(f_\tau, f, R_{0R}) + \Phi_{AC}(f_\tau, R_{0R}) + \Phi_{AS}(f_\tau), \quad (2.101)$$

$$\Phi_{RC}(f_\tau, f) \approx \pi \frac{f^2}{K_r} - \pi \frac{f^2}{K_{SRC}}, \quad (2.102)$$

$$\begin{aligned} \Phi_{RCM}(f_\tau, f) &= \frac{2\pi}{c} \left[\frac{R_{0R}(1 - \mu_{R1}\mu_{R2})}{D_R} + \frac{R_{0T}(1 - \mu_{T1}\mu_{T2})}{D_T} \right] f + \bar{\Phi}_{RCM}(f) \\ &= \frac{2\pi}{c} \left[\frac{R_{0R}}{D_R} + \frac{R_{0T}}{D_T} \right] f, \end{aligned} \quad (2.103)$$

$$\Phi_{AC}(f_\tau, R_{0R}) = 2\pi(p_{10} + p_{11}R_{0R})k_T f_\tau + \frac{2\pi}{\lambda}(R_{0R}D_R + R_{0T}D_T), \quad (2.104)$$

$$\Phi_{AS}(f_\tau) = 2\pi\beta_A\tau_{0R}f_\tau, \quad (2.105)$$

$$\frac{1}{K_{SRC}} = \left[R_{RR} \frac{(\mu_{R1} - \mu_{R2})^2}{cf_0 D_R^3} + R_{RT} \frac{(\mu_{T1} - \mu_{T2})^2}{cf_0 D_T^3} \right]. \quad (2.106)$$

Some short remarks concerning Eqs. (2.101)–(2.106) will be helpful to understand the characteristics of the space-variant terms and coupling terms:

- The first term of Φ_{RC} is quadratic in the range frequency variable, and thus responsible for the range modulation. The second term of Φ_{RC} is not only quadratic in the range frequency variable, but also dependent on the azimuth frequency variable, and thus denotes the range-azimuth coupling. Since the range-azimuth coupling is weakly dependent on the variant component of the slant range [2], we neglect the dependency of Φ_{RC} on the range-variant component. The resulting range-invariant quadratic range-azimuth coupling term is

well known as the secondary range compression (SRC) term [2]. It can be corrected together with the range compression in the two-dimensional frequency domain with a fixed slant range variable of the reference values.

- Φ_{RCM} is linearly dependent on the range frequency variable and identifies the range cell migration trajectory of target. Thus, it represents the range-variant feature of the derived spectrum within the scene. From Eq. (2.103), it can be seen that the echoed signal of the target follows a curve that is the sum of two curvature-variant curves in range. To accurately focus the echoed signal from the entire scene, this range-variant curve must be straightened along the swath, i.e., $\left[\frac{R_{0R}}{D_R} + \frac{R_{0T}}{D_T}\right] \rightarrow R_{0R} + R_{0T}$. For the range Doppler algorithm (RDA) [2], it will be corrected by the range-variant interpolation in the range-Doppler domain (generally, an eight-point *sinc* interpolation kernel appears to be sufficient [2]). For the chirp scaling algorithm (CSA) [2], the chirp transformation is applied to correct the differential component of the total RCM, and the bulk component is corrected by using the phase multiplication.
- Φ_{AC} represents the range-variant azimuth modulation. It must be removed with the range-variant matched filtering after range cell migration correction (RCMC) in the range-Doppler domain. The accommodation of this range-variant azimuth modulation can result in accurately focusing the bistatic SAR data in the azimuth direction.
- Φ_{AS} denotes the additional azimuth scaling term compared to the conventional monostatic SAR. Although this scaling factor does affect the azimuth resolution, it results in the fact that the focused targets are scaled in the azimuth-time domain. To rescale the SAR image in time domain, one may use the inverse short Fourier transform (ISFT) [20] to correct the factor. Otherwise, the inverse Fourier transform (IFT) can be used to directly transform the data into the image domain.

The resulting phase error in Eq. (2.101) introduced by the quadratic approximation, referencing Eq. (2.100), can be expressed as

$$\begin{aligned} \Phi_E = 2\pi \left\{ \frac{R_{0R}}{c} \left[F_R - D_R f_0 - \frac{(1 - \mu_{R1}\mu_{R2})}{D_R} f \right] + R_{RR} \frac{(\mu_{R1} - \mu_{R2})^2}{2f_0 D_R^3} f^2 \right\} \\ + 2\pi \left\{ \frac{R_{0T}}{c} \left[F_T - D_T f_0 - \frac{(1 - \mu_{T1}\mu_{T2})}{D_T} f \right] + R_{RT} \frac{(\mu_{T1} - \mu_{T2})^2}{2f_0 D_T^3} f^2 \right\}. \end{aligned} \quad (2.107)$$

Because the truncated error terms can be approximately represented by the quadratic and cubic terms, Eq. (2.107) can be simplified as

$$\begin{aligned} \Phi_E = & \frac{\pi}{c} \left\{ (R_{RR} - R_{0R}) \frac{(\mu_{R1} - \mu_{R2})^2}{4f_0 D_R^3} B_r^2 + R_{0R} \frac{(\mu_{R1} - \mu_{R2})^2 (1 - \mu_{R1} \mu_{R2})}{8f_0^2 D_R^5} B_r^3 \right\} \\ & + \frac{\pi}{c} \left\{ (R_{RT} - R_{0T}) \frac{(\mu_{T1} - \mu_{T2})^2}{4f_0 D_T^3} B_r^2 + R_{0T} \frac{(\mu_{T1} - \mu_{T2})^2 (1 - \mu_{T1} \mu_{T2})}{8f_0^2 D_T^5} B_r^3 \right\}, \end{aligned} \quad (2.108)$$

where B_r denotes the bandwidth of the transmitted chirp signal. Finally, a phase error of $\pi/4$ is usually used as an upper limit to obtain good focusing quality [2],

$$|\Phi_E| \leq \frac{\pi}{4}. \quad (2.109)$$

Appendix A: The Principle of Stationary Phase

In this Appendix, the approximate analytical form of the Fourier transform of a linear frequency-modulated (FM) signal will be derived using the principle of stationary phase (POSP).

Let $g(\tau)$ be a FM signal whose modulation is either linear or approximately linear:

$$p(\tau) = w_r(\tau) \exp(j\phi_r(\tau)), \quad (2.110)$$

where $w_r(\tau)$ represents the real-valued envelope and $\phi_r(\tau)$ is the demodulated phase of the signal. To simplify the derivation, it is assumed that the envelope varies very slowly with time, compared with the variation of the phase.

The FT of the signal, $P(f_\tau)$, is written as

$$\begin{aligned} P(f_\tau) &= \int_{-\infty}^{\infty} p(\tau) \exp\{-j2\pi f_\tau \tau\} d\tau \\ &= \int_{-\infty}^{\infty} p(\tau) \exp\{j\theta(\tau)\} d\tau, \end{aligned} \quad (2.111)$$

where the phase due to the FT, $-2\pi f_\tau \tau$, has been absorbed into a single-phase term:

$$\theta(\tau) = \phi(\tau) - 2\pi f_\tau \tau. \quad (2.112)$$

The phase in the integrand contains of the quadratic- and higher-order terms. The analytical form of the integral is difficult to derive by conventional means. However, the approximate FT may be obtained by using the POSP. Based on the assumption that $w_r(\tau)$ varies slowly, where the phase $\theta(\tau)$ is rapidly varying, the envelope $w_r(\tau)$ is almost constant over one complete phase cycle. Then, the main

component of the integral comes from approximately the stationary phase point. The rest of the components vary so rapidly that their net contributions are negligible. The stationary point can be determined by finding the stationary point of the phase,

$$\frac{d\theta(\tau)}{d\tau} = \frac{d(\phi(t) - 2\pi f_\tau \tau)}{d\tau} = 0. \quad (2.113)$$

From the preceding equation, the relation between frequency f_τ and time τ can be determined. This equation must be inverted to obtain an analytical function for τ expressed in terms of f_τ , denoted $\tau(f_\tau)$. Stating the result of the derivation, which is detailed in [14] and [21], the spectrum of the signal is given by

$$P(f_\tau) = C_1 W_r(f_\tau) \exp \{j\Theta(f_\tau) \pm \pi/4\}, \quad (2.114)$$

- where C_1 is a constant and can usually be ignored;
- $W_r(f_\tau)$ is the frequency domain envelope, which is a scaled version of the time domain envelope $w_r(\tau)$,

$$W_r(f_\tau) = w_r[\tau(f_\tau)], \quad (2.115)$$

- and $\Theta(f_\tau)$ is the frequency domain phase, which is also a scaled version of the time domain phase, $\theta(\tau)$:

$$\Theta(f_\tau) = \theta[\tau(f_\tau)]. \quad (2.116)$$

Appendix B: Series Reversion

Series reversion is the computation of the coefficients of the inverse function given those of the forward power series Eq. (2.26). For a function expressed in a series with no constant term, which means that $a_0 = 0$,

$$y = a_1x + a_2x^2 + a_3x^3 + \dots. \quad (2.117)$$

The series expansion of the inverse function is given by

$$x = A_1y + A_2y^2 + A_3y^3 + \dots. \quad (2.118)$$

Substituting (2.27) into (2.26), the following equation is obtained:

$$y = a_1 A_1 y + (a_2 A_1^2 + a_1 A_2) y^2 + (a_3 A_1^3 + 2a_1 A_1 A_2 + a_1 A_3) y^3 + \dots \quad (2.119)$$

By equating terms, the coefficients of the inverse function are

$$\begin{aligned} A_1 &= a_1^{-1}, \\ A_2 &= -a_1^{-1} a_2, \\ A_3 &= a_1^{-5} (2a_2^2 - a_1 a_3), \\ &\dots \end{aligned} \quad (2.120)$$

The formula for the n th coefficient is given in [22], as summarized in the handbook [23].

Appendix C: Two-Dimensional Principle of Stationary Phase

This appendix gives an approximation for the analytical solution of the double integral in Eq. (2.82) and derives Eqs. (2.85)–(2.86).

Using the identical equation $f_\tau = f_{\tau R} + f_{\tau T}$ in Eq. (2.84) gives

$$\begin{cases} \frac{\partial[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]}{\partial f_{\tau R}} = 0 \\ \frac{\partial[\Phi_R(f_{\tau R}, f) + \Phi_T(f_{\tau T}, f)]}{\partial f_{\tau T}} = 0 \end{cases} \quad (2.121)$$

As $\partial\Phi_R(f_{\tau R}, f)/\partial f_{\tau R} = -\partial\Phi_R(f_{\tau R}, f)/\partial f_{\tau T}$ and $\partial\Phi_T(f_{\tau T}, f)/\partial f_{\tau T} = -\partial\Phi_T(f_{\tau T}, f)/\partial f_{\tau R}$ can be derived using Eq. (2.80), then Eq. (2.121) can be simplified as

$$\frac{\partial\Phi_R(f_{\tau R}, f)}{\partial f_{\tau R}} = -\frac{\partial\Phi_T(f_{\tau T}, f)}{\partial f_{\tau R}}. \quad (2.122)$$

To overcome the limitation of the DSR term in Eq. (2.122), we expand $\frac{\partial\Phi_R(f_{\tau R}, f)}{\partial f_{\tau R}}$ and $\frac{\partial\Phi_T(f_{\tau T}, f)}{\partial f_{\tau R}}$ as the linear functions of $(f_{\tau R} - f_{DcR})$ and $(f_{\tau R} - f_{DcT})$, respectively, given as

$$\begin{aligned} \frac{\partial\Phi_R(f_{\tau R}, f)}{\partial f_{\tau R}} &\approx 2\pi\tau_{0R} - 2\pi \left[\frac{R_{0R}}{v_R} \tan \theta_{SR} - \frac{(f_{\tau R} - f_{DcR})}{K_{aR}} \right], \\ \frac{\partial\Phi_T(f_{\tau T}, f)}{\partial f_{\tau R}} &\approx -2\pi\tau_{0T} + 2\pi \left[\frac{R_{0T}}{v_T} \tan \theta_{ST} + \frac{(f_{\tau T} - f_{DcT})}{K_{aT}} \right]. \end{aligned} \quad (2.123)$$

Substituting Eqs. (2.123) into (2.122) yields

$$\frac{(f_{\tau T} - f_{DcT})}{(f_{\tau R} - f_{DcR})} = \frac{K_{aT}}{K_{aR}}. \quad (2.124)$$

Combining Eq. (2.124) and the identical equation $f_{\tau R} + f_{\tau T} = f_{\tau}$, we can express the individual instantaneous Doppler frequencies as

$$\begin{aligned} f_{\tau R} &= K_R(f_{\tau} - f_{DcR} - f_{DcT}) + f_{DcR}, \\ f_{\tau T} &= K_T(f_{\tau} - f_{DcR} - f_{DcT}) + f_{DcT}, \end{aligned} \quad (2.125)$$

$$\begin{aligned} K_R &= \frac{v_R^2 \cos^3 \theta_{SR} / \lambda R_{0R}}{v_R^2 \cos^3 \theta_{SR} / \lambda R_{0R} + v_T^2 \cos^3 \theta_{ST} / \lambda R_{0T}}, \\ K_T &= \frac{v_T^2 \cos^3 \theta_{ST} / \lambda R_{0T}}{v_R^2 \cos^3 \theta_{SR} / \lambda R_{0R} + v_T^2 \cos^3 \theta_{ST} / \lambda R_{0T}}. \end{aligned} \quad (2.126)$$

Appendix D: Overview of Weighting Concept

This Appendix shows how the weighting model was developed for the LBF. In the derivation of the LBF, the same azimuth modulations of both platforms are assumed. When both platforms contribute unequally to the total azimuth modulation, this assumption would result in the inaccurate individual stationary points $\tilde{\tau}_R$ and $\tilde{\tau}_T$, with reference to Eq. (2.47). Thus, these inaccurate stationary points cannot represent the individual time-Doppler correspondences [24]. Therefore, the time differences $(\tau - \tilde{\tau}_R)$ and $(\tau - \tilde{\tau}_T)$ would become larger, which means that the neglected third- or higher-order phase terms in (2.40) as the functions of $(\tau - \tilde{\tau}_R)$ and $(\tau - \tilde{\tau}_T)$ would introduce a significant phase error and the second-order model in the bistatic spectrum [2] may not be accurate.

The purpose of introducing the weighting is to improve the accuracy of the quadratic model around $\tilde{\tau}_R$ and $\tilde{\tau}_T$ by obtaining the more accurate individual time-Doppler correspondences. It can be implemented by making azimuth modulations of both range equations to agree with the individual instantaneous Doppler frequencies in the respective slant range histories.

The instantaneous Doppler frequency of the bistatic range history in the space-borne/airborne configuration (i.e., a small-squint case) can be formulated as [6]

$$\begin{aligned} f_{\tau}(\tau) &= -\frac{f+f_0}{c} \frac{d}{d\tau} [R_R(\tau) + R_T(\tau)] \\ &\approx \underbrace{-\frac{f+f_0}{c} \frac{v_R^2}{R_{0R}} (\tau - \tau_{cb})}_{\text{Receiver}} - \underbrace{\frac{f+f_0}{c} \frac{v_T^2}{R_{0T}} (\tau - \tau_{cb})}_{\text{Transmitter}}. \end{aligned} \quad (2.127)$$

From Eq. (2.127), we see that the contributions of the Doppler modulations from the individual platform to the total instantaneous Doppler frequency are approximately proportional to the slopes of the respective range equations v_R^2/R_{0R} and v_T^2/R_{0T} . The LBF works well when the ratio $(v_R^2/R_{0R})/(v_T^2/R_{0T})$ is near unity. However, the LBF would be inaccurate in cases where the ratio deviates further from unity, e.g., in a space-borne/airborne configuration.

Starting from Eq. (2.127), the weighted individual phase histories are formulated as in Eq. (2.36).

To show the validity of the weighting operation, we define the phase error functions of the quadratic slant range histories of receiver and transmitter in [25] and [2] as

$$\begin{aligned} E_R(\tau, f) &\approx \phi_R(\tau, f) - \left[\phi_R(\tilde{\tau}_R, f) + \frac{1}{2} \ddot{\phi}_R(\tilde{\tau}_R, f)(\tau - \tilde{\tau}_R)^2 \right], \\ E_T(\tau, f) &\approx \phi_T(\tau, f) - \left[\phi_T(\tilde{\tau}_T, f) + \frac{1}{2} \ddot{\phi}_T(\tilde{\tau}_T, f)(\tau - \tilde{\tau}_T)^2 \right], \end{aligned} \quad (2.128)$$

$$\begin{aligned} E_{RW}(\tau, f) &= \phi_{RW}(\tau, f) - \left[\phi_{RW}(\tilde{\tau}_{RW}, f) + \frac{1}{2} \ddot{\phi}_{RW}(\tilde{\tau}_{RW}, f)(\tau - \tilde{\tau}_{RW})^2 \right], \\ E_{TW}(\tau, f) &= \phi_{TW}(\tau, f) - \left[\phi_{TW}(\tilde{\tau}_{TW}, f) + \frac{1}{2} \ddot{\phi}_{TW}(\tilde{\tau}_{TW}, f)(\tau - \tilde{\tau}_{TW})^2 \right], \end{aligned} \quad (2.129)$$

where $\tilde{\tau}_R$ and $\tilde{\tau}_T$ are defined as

$$\tilde{\tau}_R = \tau_{0R} - \frac{cR_{0R}}{2v_R^2} \frac{f_\tau}{F_R}, \quad \tilde{\tau}_T = \tau_{0T} - \frac{cR_{0T}}{2v_T^2} \frac{f_\tau}{F_T}, \quad (2.130)$$

$$F_R = \sqrt{(f + f_0)^2 - \left(\frac{cf_\tau}{2v_R} \right)^2}, \quad F_T = \sqrt{(f + f_0)^2 - \left(\frac{cf_\tau}{2v_T} \right)^2}. \quad (2.131)$$

Table 2.1 SAR system parameters

	Receiver	Transmitter
Carrier frequency (GHz)	10.13	
Range bandwidth (MHz)	100	
PRF (Hz)	650	
Composite exposure time (s)	0.5	
Doppler bandwidth (Hz)	63.4	
Velocity (m/s)	100	100
Altitude (km)	4.5	6
Depression angle (deg)	35	35
TBP	178	14
Weighting factors (k_B, k_T)	0.925	0.00745

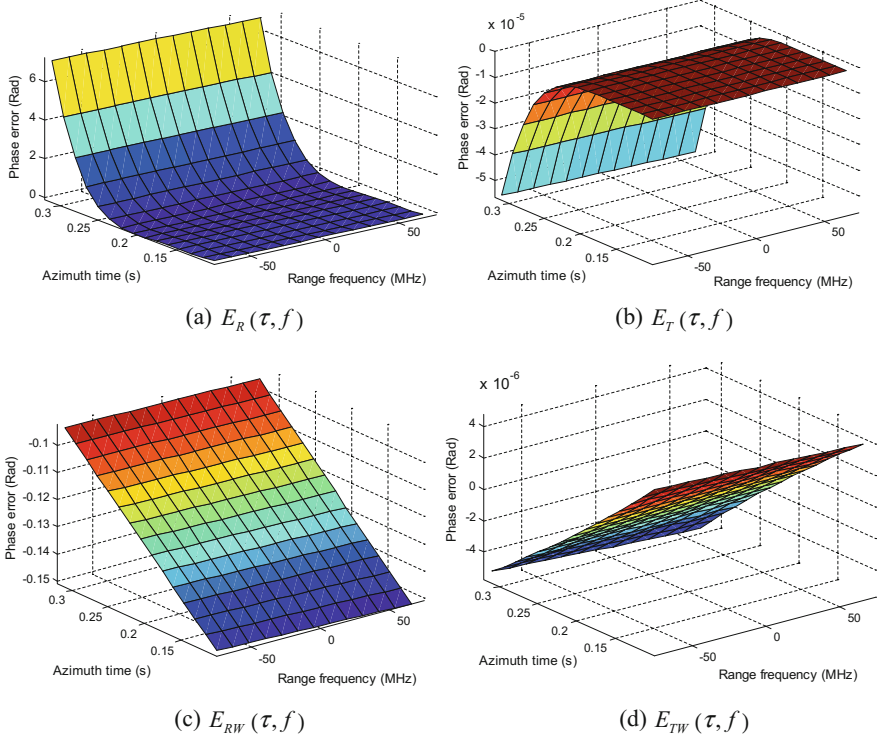


Fig. 2.3 Phase errors in the LBF and ELBF

The phase errors in the LBF (E_R and E_T) are shown in Fig. 2.3a, b; the phase errors for the ELBF (E_{RW} and E_{TW}) are shown in Fig. 2.3c, d. The system is shown in Table 2.1.

Figure 2.3a shows that the phase error appears to be nonlinear in azimuth and approximately linear in range. This is the reason the focused point target in a space-borne/airborne case computed using the LBF deteriorates visibly in azimuth [5].

Comparing Fig. 2.3a, b with Fig. 2.3 c, d, it can be seen that the weighting operation can reduce this phase error significantly and results in a marked improvement of the accuracy of the spectrum for this space-borne/airborne configuration.

In addition, the plots also show that the larger the time bandwidth product (TBP) of the slant range history, the more accurate the second-order model will be. When the TBP is larger than 100, the slant range histories can be accurately represented by its second-order approximation [6, 21].

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Bistatic SAR System and Signal Processing Technology

Wang, R.; Deng, Y.

2018, XIV, 275 p. 123 illus., Hardcover

ISBN: 978-981-10-3077-2