

Chapter 2

Geological Fractures and Geophysical Assumptions

Abstract Studying geological fractures from seismic data not only requires performing forward modeling and backward inversion, but also required an understanding of the rock mechanisms and some geophysical hypotheses related to fractures. In this chapter, many indispensable fundamental concepts are reviewed that are associated with fracture formations, fracture types, fracture parameters, fracture detections, fracture equivalent hypotheses (effective fractures), fractured media composition, and decomposition and fracture interface boundary conditions especially, rock deformations, Anderson's principal stresses classification (Trans Edinb Geol Soc 8: 387–402, 1905), petrophysics technologies, linear slip theory (Schoenberg in J Acoust Soc Am 68: 1516–1521, 1980), Schoenberg and Muir calculus theory (Geophysics 54: 581–589, 1989), welded contact boundary conditions, and nonwelded contact boundary conditions are explored and integrated to form a substantial body of material supporting the geological fractured media forward modeling in Chap. 3 and seismic inversion in Chap. 4 of this book.

Keywords Stress–strain tensors • Rock deformation • Stiffness-compliances • Fracture formation • Fracture parameters • Fracture detection • Well logs • Backus average theory • Linear slip theory • Schoenberg-Muir theory • Boundary conditions • Host medium • Fractured medium • Isotropy • Intrinsic anisotropy • Induced anisotropy

2.1 Rock Deformations

2.1.1 Stress Tensor

Stress and strain tensors are keys for deeply understanding the fracture mechanisms. In earth science, permanently fractured rock usually experiences three successive stages when it is continually subjected to forces.

Traction is the force per area acting within a deformable material.

$$\mathbf{T} = \frac{\mathbf{F}}{S}, \quad (2.1a)$$

$$T_i = \sum_{j=1}^3 \sigma_{ij} n_j, \quad (2.1b)$$

where $\mathbf{T}$ is traction as the force per unit surface area, $\mathbf{F}$ is a force, and S is an area subjected to the force, $\boldsymbol{\sigma}$ is stress tensor, n is surface. $i, j = 1, 2, 3$. In general, an arbitrary stress $\boldsymbol{\sigma}$ is a second rank tensor consisting of nine components, and can be expressed in matrix form as

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}. \quad (2.2)$$

σ_{ij} are the components of the tensor in the Cartesian coordinate system. Index “ i ” describes the direction of the force component, and index “ j ” denotes the direction that is perpendicular to the surface on which the force acts (Krebes 2006). For example, the stress component σ_{12} describes a force component parallel to (1)-axis and acting on the surface (the (1) & (3)-plane) that is normal to (2)-axis. In general, σ_{11} , σ_{22} and σ_{33} are so-called normal stresses because these components are normally acting to the surfaces, while σ_{12} , σ_{13} , σ_{21} , σ_{23} , σ_{31} and σ_{32} are so-called shear stresses that are tangentially acting on the surfaces. The components of the stress are shown in Fig. 2.1 on which bold arrows indicate the normal stress components and light arrows indicate the tangential ones. The stress tensor $\boldsymbol{\sigma}$ is commonly expressed in terms of Voigt notation ($11 \rightarrow 1, 22 \rightarrow 2, 33 \rightarrow 3, 23 \rightarrow 4, 31 \rightarrow 5, 12 \rightarrow 6$) which also makes use of the symmetry of the tensor ($13 \rightarrow 31, 23 \rightarrow 32, 12 \rightarrow 21$) in the Cartesian coordinate system.

$$\begin{aligned} \boldsymbol{\sigma} &= [\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{23} \quad \sigma_{31} \quad \sigma_{12}]^T \\ &= [\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4 \quad \sigma_5 \quad \sigma_6]^T \end{aligned} \quad (2.3)$$

2.1.2 Strain Tensor

The components of the strain tensor, $\boldsymbol{\varepsilon}$, are defined in terms of the relative changes in the displacement components of the deformed material subjected to the stresses. Similar to the stress tensor, the strain tensor is symmetric, and has nine components: the normal strains ε_{11} , ε_{22} , ε_{33} and the shear strains ε_{12} , ε_{13} , ε_{21} , ε_{23} , ε_{31} , ε_{32} . The normal strains measure relative changes in displacement along a specific direction. For instance, ε_{22} can be given as

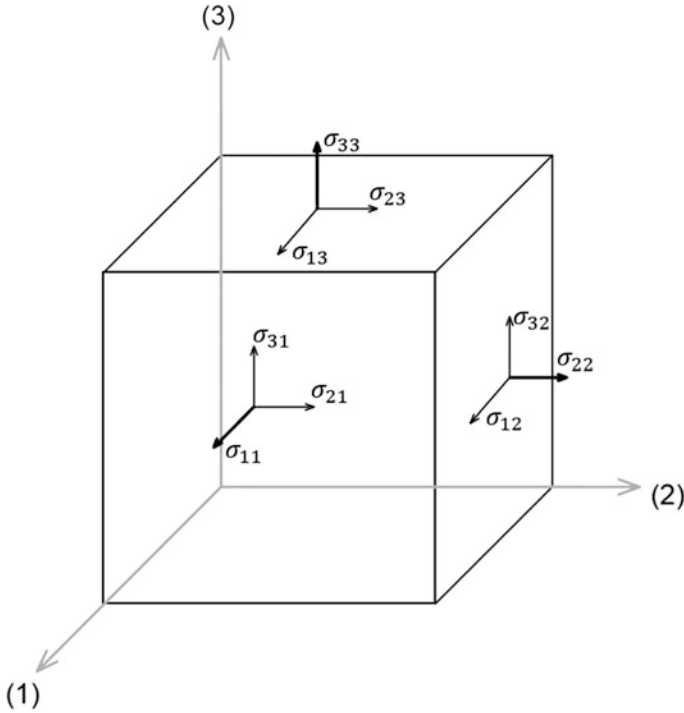


Fig. 2.1 A sketch of the nine components of the stress tensor at a point in a Cartesian coordinate system. The *bold arrows* indicate the normal components of the stress. The *light arrows* indicate the tangential components of the stress

$$\varepsilon_{22} = \frac{\Delta u_2}{\Delta l_2}, \quad (2.4)$$

where, Δl_2 is the original length and Δu_2 is the change of displacement along (2)-axis direction (Fig. 2.2a).

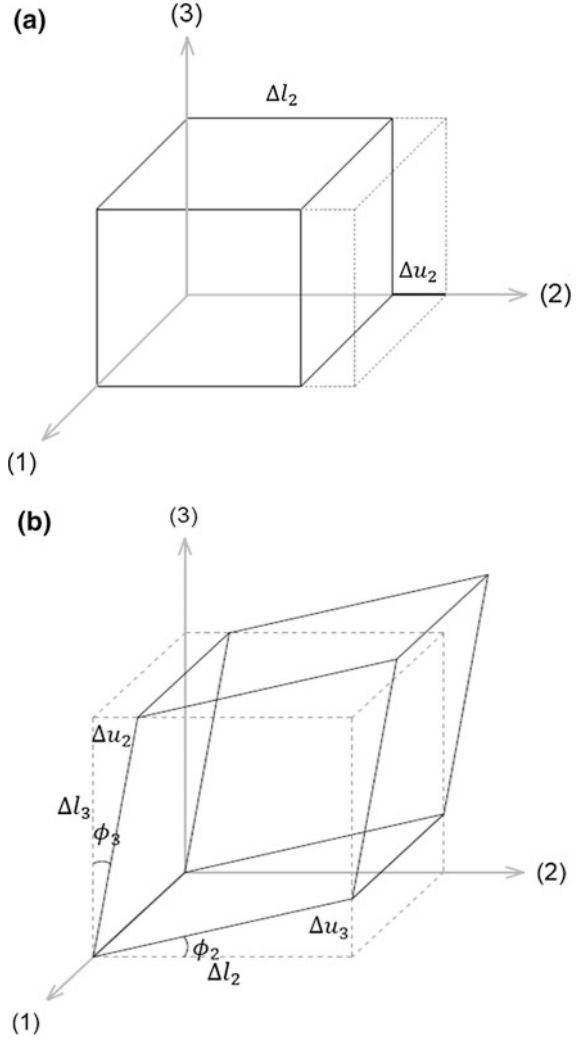
The shear strains simply measure the material distortion of the shape in (small) angles with respect to a specific direction. Figure 2.2b shows simultaneous shear strains along both the (2)-axis and the (3)-axis. The change in shape of the material is due to the applied force that can be described by

$$\varepsilon_{32} = \frac{1}{2}(\phi_2 + \phi_3) \approx \frac{1}{2} \left(\frac{\Delta u_3}{\Delta l_2} + \frac{\Delta u_2}{\Delta l_3} \right), \quad (2.5)$$

where the definitions of Δl_2 and Δu_2 are the same as in Eq. (2.4). Δl_3 is the original length and Δu_3 is the change in displacement along (3)-axis direction. ϕ_2 and ϕ_3 are material shape distortions in angles related to (2)-axis and (3)-axis, respectively (Fig. 2.2b).

Fig. 2.2 a A sketch of the normal strain showing a size change in the Cartesian coordinate system. Δl_2 is the original length and Δu_2 is the change in displacement along the 2-axis direction.

b Diagram of the tangential (shear) strains in shape change in the Cartesian coordinate system. Δl_2 , Δl_3 are original lengths and Δu_2 , Δu_3 are changes in displacement along (2)-axis, (3)-axis direction respectively. ϕ_2 and ϕ_3 are material shape distortions in angles respect to (2)-axis and (3)-axis, respectively

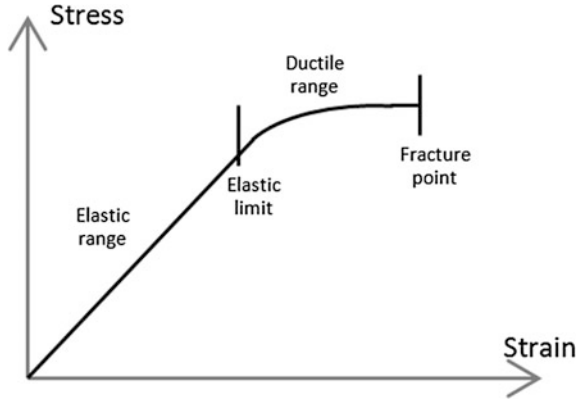


The strain tensor ε , can also be expressed in Voigt notation rule which makes use of the symmetry in the Cartesian coordinate system as well, i.e.,

$$\begin{aligned} \varepsilon &= [\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad \varepsilon_{23} \quad \varepsilon_{31} \quad \varepsilon_{12}]^T \\ &= [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_4 \quad \varepsilon_5 \quad \varepsilon_6]^T. \end{aligned} \quad (2.6)$$

In the infinitesimal limit, the strain is given by (Krebes 2006)

Fig. 2.3 Diagram of stress–strain curve and rock deformation stages. The stages involve reversible elastic deformation and irreversible ductile deformation and fracture deformation



$$\varepsilon_{ij} = \frac{1}{2}(\phi_j + \phi_i) \approx \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (2.7)$$

The rock strains represent either reversible deformations where an object can return to its original size and shape once the exerting stress is removed or irreversible deformations where an object is permanently distorted even if the exerting stress disappears. The deformation stages actually depend on the ratio of the stress and strain. Figure 2.3 shows a stress and strain function and the deformations (Nelson 2003).

2.1.3 Stages of the Rock Deformation

For a small stress exerted on the rock, Hooke's law entirely describes the linear elastic behavior on the first elastic stage of the reversible rock deformation (Fig. 2.3). The general mathematical equation for Hooke's law is

$$\sigma_{ij} = \sum_{k=1}^3 \sum_{l=1}^3 C_{ijkl} \varepsilon_{kl}, \quad (2.8)$$

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl}), \quad (2.9)$$

where the quantity C_{ijkl} is called the material stiffness tensor or material modulus tensor that is a measurement of the resistance offered by an elastic body to deformation, and where $i, j, k, l = 1, 2, 3$. λ and μ are known as the Lamé physical parameters, and δ_{ij} is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}. \quad (2.10)$$

With Voigt notation, Hooke's law in Eq. (2.8) can be rewritten as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}, \quad (2.11)$$

where

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix}. \quad (2.11a)$$

An elastic homogeneous isotropic medium's physical properties are invariant in all directions. For such a medium, the stiffness tensor $\mathbf{C}$ simplifies to

$$\mathbf{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}. \quad (2.12)$$

Note that there are only two independent elastic moduli in isotropic media. Hooke's law describes the situation of an elastic isotropic medium when it is subjected to small stress, and gives

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} = \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (2.13)$$

where a sum is performed over k . In a 2D x - z Cartesian coordinate system,

$$\sigma_{xz} = \mu \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right). \quad (2.14)$$

$$\sigma_{zz} = \lambda \frac{\partial u_x}{\partial x} + (\lambda + 2\mu) \left(\frac{\partial u_z}{\partial z} \right). \quad (2.15)$$

If stress is persistently exerted, the elastic deformation proceeds to the ductile deformation stage wherein the size and shape changes are irreversible (Nelson 2003). With further exertion of stress, the deformation status reaches the fracture stage where the rock has been broken (Fig. 2.3). At the fracture point, the rock strain is over the strain threshold since real materials are not infinitely rigid.

2.1.4 Stresses and Fractures

Thein-situ stresses can be divided into three perpendicular principal compressive stresses including one vertical stress and two horizontal stresses. Anderson (1951) recognized that the principal stresses orientation ($\sigma_1 > \sigma_2 > \sigma_3$) could vary with geological movement in the upper crust of the earth. Once the three principal stresses deviate from a level of equilibrium, the fractures are possibly created in which the direction of the fracture is parallel to the direction of the maximum stress and perpendicular to the direction of minimum stress. An isotropic medium will be reformed into an anisotropic medium with a new balance of three principal stresses. In fact, in the deeper area of the upper crust, the maximum principal stress is deemed to be produced by compression from the overburden deposition, while the minimum stress will likely be one of the horizontal stresses. Thus most of the fractures in the reservoir are oriented in the vertical or nearly vertical direction because the maximum stress decides the direction of the fracture. Figure 2.4 shows

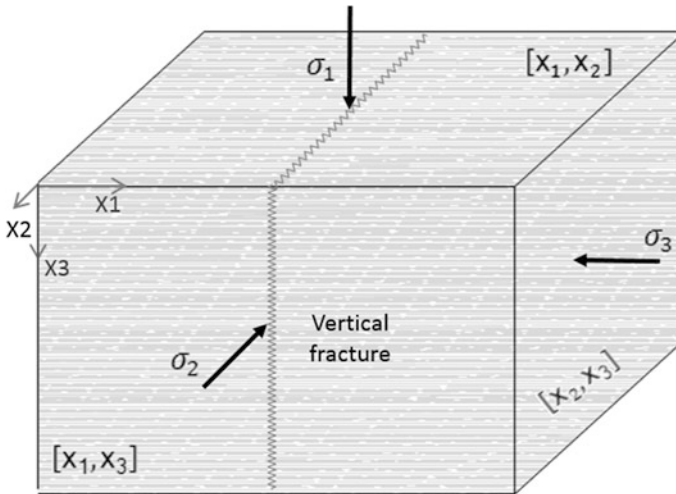


Fig. 2.4 Schematic of vertical fracture with three principal compressive stresses ($\sigma_1 > \sigma_2 > \sigma_3$)

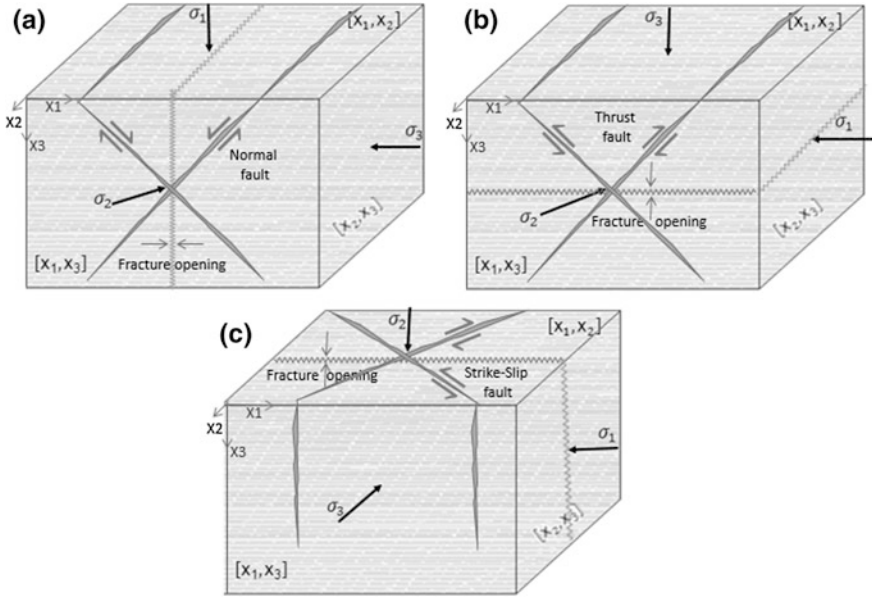


Fig. 2.5 Schematic of the direction relationship between the fractures and the corresponding faults (Anderson 1951) when they are under the same relationship of three principal compressive stresses ordered as $\sigma_1 > \sigma_2 > \sigma_3$

a schematic for three compressive principal stresses and a vertical fracture. If $\sigma_1 > \sigma_2 > \sigma_3$, the vertical fracture plane is parallel to the $[x_2, x_3]$ plane and is perpendicular to the x_1 -axis. Figure 2.5 illustrates the direction relationship between the fractures and the corresponding faults (Anderson 1951) when they are under the same relationship of three principal compressive stresses ordered as $\sigma_1 > \sigma_2 > \sigma_3$. Figure 2.5a shows a vertical fracture (normal to x_1 -axis) and a normal fault. Figure 2.5b shows a horizontal fracture (normal to x_3 -axis) and a thrust fault. Figure 2.5c shows a vertical fracture (normal to x_2 -axis) and a strike-slip fault.

2.2 Geological Fractures in the Reservoir

Subsurface geological formations are confined under in-situ stresses. Therefore, the media in the reservoir can be deformed and fractured.

2.2.1 Fracture Parameters

As we know, the geological fracture is a plane along which rocks show partially lost cohesion when stresses act on it. The fracture planes may exhibit a little displacement or may not exhibit any movement. A slight movement on the fracture plane will produce a fracture opening, which may remain unfilled and result in an increased permeability or may get subsequently filled by secondary minerals or some fluids. Fracture openings, or apertures, and may vary from very thin (0.001–0.5 mm) to relatively wide (1 inch) where fractures are propped open by some minerals in the reservoir (Aguilera 1998).

The subsurface stress is a key to controlling the well-drilling performance and this can be indicated by the fracture orientation. Thus, the most important parameters of a fracture set are the orientation and density which are considered more critical than the fracture opening to describe the fracture sets in a geophysical fracture study. Conventionally, geoscientists describe vertical or nearly vertical fracture orientation that is perpendicular to the direction of minimum compressive stress and the fracture plane as an isotropic plane. The different fractures have different lengths or heights that possibly can span hundreds of meters, and this may result in a lower value of the fracture density in the survey, while it is a fact that there are quite a lot fractures in this area. Fracture density is defined as the number of fractures per meter in a certain direction. For a very sparse fracture set the fracture density could be less than 0.75 m^{-1} , and for a tight fracture set may exceed 10 m^{-1} . Typical values of fracture density for carbonate reservoirs are $1\text{--}20 \text{ m}^{-1}$ (Bakulin et al. 2000a, b, c). The higher the fracture density is, the higher the permeability is—if the fractures are conductive in the reservoir (Singhal and Gupta 1999).

Figure 2.6 shows two vertical fractures with orientations of φ_1 and φ_2 respect to x_2 -axis. It also illustrates the fracture length, fracture height, and fracture opening parameters.

It is clear that fracture aperture is much shorter than seismic wavelengths that are on the order of tens and hundreds of meters. Therefore, in geoscience, a fracture model can effectively be a transversely isotropic model that neglects finite fracture openings, fracture shape, and fracture microstructure. The parameters of such a fractured model will depend on the orientation and intensity of the fracture set(s), as well as on the elastic properties of the host media.

2.2.2 Fracture Detection by Integrated Methods

Fracture detection is an integrated method that incorporates geology, petrophysics, and geophysics technologies. Geological outcrops clearly indicate the near surface fractures from which the directions of local maximum stress can be inferred. Petrophysical well logs directly acquired from the reservoir can delineate the

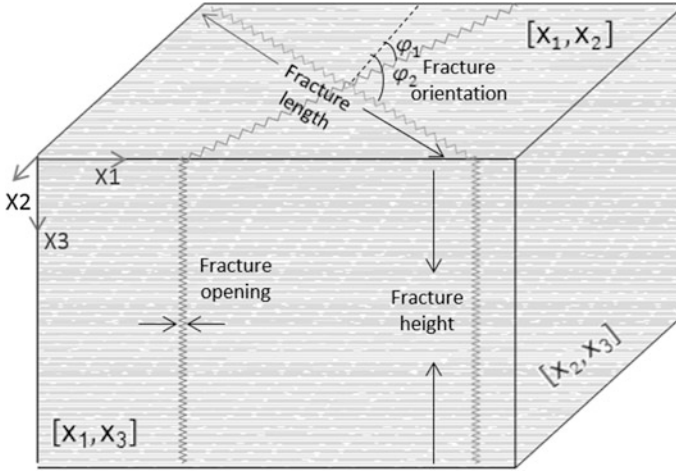


Fig. 2.6 Schematic of two vertical fractures model with parameters of fracture orientation, fracture width and fracture height parameters

fractures through integrated interpretations of logs with a high resolution. Geophysical seismic data provide densely sampled reflection signals from the reservoir that can be used to seek the anisotropic zones indicating possible fractures. In PP-data, COCA gathers (common offset common azimuth), as well as amplitude and velocity variation with offset and azimuth (AVO/VVAZ/AVAZ) data can be used to locate potential fractures. In PS-data, investigation of time lags between fast and slow shear waves is a method to identify the fractures for the reservoirs.

2.2.3 Fracture Delineation by Petrophysical Data

Since the well logs directly measure data from the reservoir, well logs are the most reliable and can be “hard data” for constraining the interpretation of surface seismic data.

2.2.3.1 Core Analysis

The core analysis technique involves the examination of core samples from the zone of interest and performing laboratory examinations. Thus it is the best suited technique for studying fractures on a fine scale and detailed local fracture properties: fracture length and width, the mineralization infilling and un-filling of the fracture, the orientation and density of the fractures, and the contact relationship of the fracture to the host media.

2.2.3.2 Temperature Log

The Earth's temperature gradually increases with depth. A continuous temperature log curve that sharply changes to a cooler temperature can be used to identify the fracture zone. Typically, where the cooler drilling fluid enters into formation, there will be a cool temperature anomaly. A pre-fracturing gradient contrasts with the post-fracturing gradient showing a cool anomaly thought to be the fracture zone (Rider 2002).

2.2.3.3 Caliper Log

Caliper tools measure accurately the hole size, shape, and orientation. A borehole shape named a "breakout" is considered to be an identification for the fracture because breakouts show the orientation of horizontal stress $\sigma_{\min}$ [or a $SH_{\min}$ stress (Rider 2002)]. So the detected fractures should be oriented in the direction of maximum horizontal stress $\sigma_{\max}$ [or a $SH_{\max}$ stress (Rider 2002)].

2.2.3.4 Density Log

A method of fracture detection is to compare the porosity from the density log with the sonic log. The density tool records the bulk density that includes intergranular and fracture porosities. However, the sonic log just measures intergranular porosity because the sound wave takes the shortest path from emitter to receiver which avoids the fracture. So if the density porosity changes much more than sonic log porosity, it means that fractures are present (Rider 2002).

2.2.3.5 Dipmeter Log

The dipmeter log can show an open fracture from a dipmeter micro-resistivity curve. The indicator is a conductive anomaly due to the invasion of drilling mud. The fracture may be given an orientation because the dipmeter pad bearings are known (Rider 2002).

2.2.3.6 Image Log

An image log is a computer-created image based on acoustic reflectivity or electrical conductivity. To analyze the fracture, it is common to use an acoustic image in which raw acoustic travel times and amplitudes are processed to a color image presentation (Rider 2002).

2.2.4 Fracture-Induced Anisotropy and Intrinsic Anisotropy

Anisotropy means that the physical properties of a medium are directionally dependent. Isotropy, as opposed to anisotropy, means that the physical properties of the medium are identical in all directions. In seismology, the anisotropy properties commonly can be classified into intrinsic anisotropy and induced anisotropy.

The intrinsic anisotropy may be caused in the original formation in sedimentary zones when the layers are substantially thinner than the seismic wavelength. It assumes that the layers have a welded contact and the waves should meet the perfectly welded boundary conditions. Intrinsic anisotropy is usually described by Thomsen's anisotropic parameters (1986).

Induced anisotropy appears because a medium is subject to regional stresses that cause the medium to crack and attain a new system of equilibrium stresses in the subsurface. The cracking action causes a welded continuous medium to change to a nonwelded discontinuous medium satisfying the nonwelded contact boundary conditions. From theoretical work and field observations, the fractured medium exhibits induced anisotropy (Crampin and Bamford 1977; Lefeuvre 1993; Lynn et al. 1996; Ramos and Davis 1997; Rueger 1996). The induced anisotropy shows azimuthal dependence that is visible as a sinusoidal variation in the seismic data in which the travel time is a function of the azimuth (due to velocity variation) in the common offset and common azimuth cube (COCA). The shear wave splitting phenomenon further indicates the azimuthal dependence of the induced anisotropy in that the shear wave split into a fast shear wave that the polarization is parallel to the direction of the fracture, and into a slower shear wave that the polarization is perpendicular the direction of the fracture. Both polarizations are orthogonal to the wave propagation direction (Crampin 1985).

The fracture causes the induced anisotropy that can be described by the fracture weakness parameters that appear in the five independent moduli of the fractured medium in next Sect. (2.3).

2.2.4.1 Anisotropic Parameters and Stiffness

In 1986, Thomsen introduced three dimensionless weak-anisotropic parameters: ε_{Th} , δ_{Th} , and γ_{Th} that are related to the five independent moduli for TI media as

$$\gamma_{Th} \equiv \frac{C_{66} - C_{44}}{2C_{44}}, \quad (2.16a)$$

$$\varepsilon_{Th} \equiv \frac{C_{11} - C_{33}}{2C_{33}}, \quad (2.16b)$$

$$\delta_{Th} \equiv \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}, \quad (2.16c)$$

also,

$$\alpha_{p0} \equiv \sqrt{C_{33}/\rho}, \quad (2.16d)$$

$$\beta_{p0} \equiv \sqrt{C_{44}/\rho}, \quad (2.16e)$$

where γ_{Th} describes the fractional difference between the horizontal and vertical SH-wave velocities, ε_{Th} describes the fractional difference between the horizontal and vertical P-wave velocities, and δ_{Th} denotes the variation of P-wave velocity with phase angle for nearly vertical propagation (Thomsen 1986). α_{p0} and β_{p0} are vertical P-wave and S-wave velocities, respectively.

Based on the definitions of the weak-anisotropy coefficients in Eqs. (2.16a–2.16e), the notations for the VTI (transversely isotropic medium with vertical symmetric axis) and HTI (transversely isotropic medium with horizontal symmetric axis) anisotropy coefficients are often identified as $\gamma, \varepsilon, \delta$, and $\gamma^{(v)}, \varepsilon^{(v)}, \delta^{(v)}$ respectively, and they have been widely used in the geophysical literature to describe wave propagation in anisotropic media.

Tsvankin (1997a, b) introduced the anisotropic coefficients for an orthorhombic medium with the superscript notations (1), (2) and (3) for three-dimensional symmetric planes using Thomson's-type weak-anisotropic parameters (Fig. 2.7). The superscript indicates the direction of the axis that is normal to the symmetry

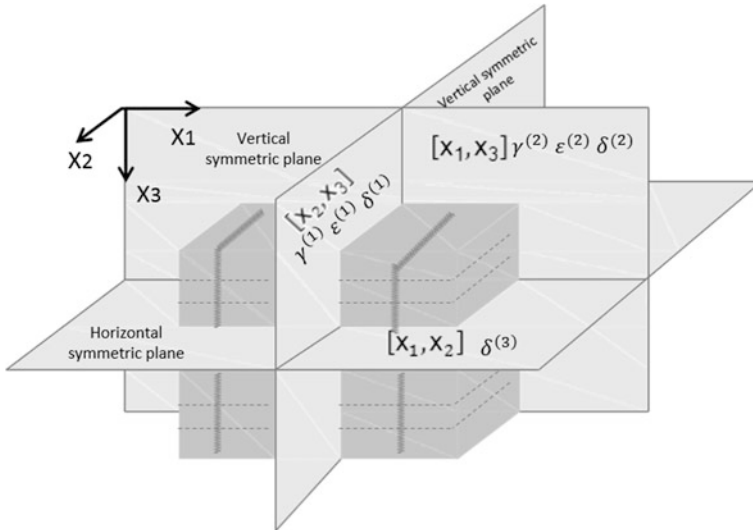


Fig. 2.7 Sketch of an orthorhombic media model composed of two vertical fractures embedded in a layered host medium (VTI). Two vertical symmetric planes and one horizontal symmetric plane are determined by the vertical fracture orientation and the horizontal layered medium

plane. For example, the notation (1) means that the x_1 -axis is normal to the symmetry plane.

The weak-anisotropy coefficients in the fracture plane (a vertical symmetric plane) of the orthorhombic medium that is parallel to $[x_2, x_3]$ and normal to the x_1 -axis are

$$\gamma^{(1)} \equiv \frac{C_{66_orth} - C_{55_orth}}{2C_{55_orth}}, \quad (2.17a)$$

$$\varepsilon^{(1)} \equiv \frac{C_{22_orth} - C_{33_orth}}{2C_{33_orth}}, \quad (2.17b)$$

$$\delta^{(1)} \equiv \frac{(C_{23_orth} + C_{44_orth})^2 - (C_{33_orth} - C_{44_orth})^2}{2C_{33_orth}(C_{33_orth} - C_{44_orth})}. \quad (2.17c)$$

The weak-anisotropy coefficients in the normal fracture plane (another vertical symmetric plane, see Table 2.2) that is parallel to $[x_1, x_3]$ and normal to the x_2 -axis are

$$\gamma^{(2)} \equiv \frac{C_{66_orth} - C_{44_orth}}{2C_{44_orth}}, \quad (2.18a)$$

$$\varepsilon^{(2)} \equiv \frac{C_{11_orth} - C_{33_orth}}{2C_{33_orth}}, \quad (2.18b)$$

$$\delta^{(2)} \equiv \frac{(C_{13_orth} + C_{55_orth})^2 - (C_{33_orth} - C_{55_orth})^2}{2C_{33_orth}(C_{33_orth} - C_{55_orth})}. \quad (2.18c)$$

The anisotropic coefficient in another normal fracture plane (a horizontal symmetric plane) that is parallel to $[x_1, x_2]$ and normal to the x_3 axis is

$$\delta^{(3)} \equiv \frac{(C_{12_orth} + C_{66_orth})^2 - (C_{11_orth} - C_{66_orth})^2}{2C_{11_orth}(C_{11_orth} - C_{66_orth})}. \quad (2.19)$$

Note that anisotropic coefficients $\gamma^{(2)}$, $\varepsilon^{(2)}$, and $\delta^{(2)}$ have the same formulae as Eqs. (2.16a–2.16c) for TI medium anisotropic coefficients. It means that the anisotropic coefficient $\gamma^{(2)}$, $\varepsilon^{(2)}$ and $\delta^{(2)}$ would coincide with the coefficients $\gamma^{(v)}$, $\varepsilon^{(v)}$ and $\delta^{(v)}$ for the simplest HTI model if the symmetry axis is orientated in the x_1 -direction. In the isotropy plane, therefore, the fracture does not affect wave velocities and the propagation direction, and the anisotropic coefficients will have the same constant values as the host VTI anisotropic medium (Tsvankin 1997a).

For fractured media, when the fractures have a certain direction (one or several), and the wavelengths are much greater than the fracture opening, it is feasible that the induced anisotropic problem described by the fracture weakness parameters can

be transformed into the intrinsic anisotropic problem described by Thomson's anisotropy parameters with Tsvankin's notations (see Sect. 2.3).

2.3 Fracture Related Geophysical Assumptions

Relative to the seismic wavelength, a fracture is regarded as a weakness plane. Thus, a seismological fractured medium model should neglect finite fracture openings, fracture microstructure, and it can be equivalent to an effective anisotropic medium.

2.3.1 Backus Average Theory

Backus (1962) presented a long wavelength equivalent theory that describes a finely stratified homogeneous medium which is effectively equivalent to an anisotropic medium, and is named an "effective anisotropic medium". The effective anisotropic medium might be a transversely isotropic medium with a vertical symmetry axis (VTI) or transversely isotropic medium with a horizontal symmetry axis (HTI). In Fig. 2.8, there is a picture of Backus's theory (1962): the thin layers are parallel to the horizontal x_1 -axis and the media properties vary with vertical x_3 -axis. The medium thickness H must be long enough so that the elastic properties of the medium vary appreciably over H . Also it must be smaller than the smallest wavelength in order to replace the layered medium by an anisotropic medium where the density is the average density over H and the elastic parameters are an algebraic

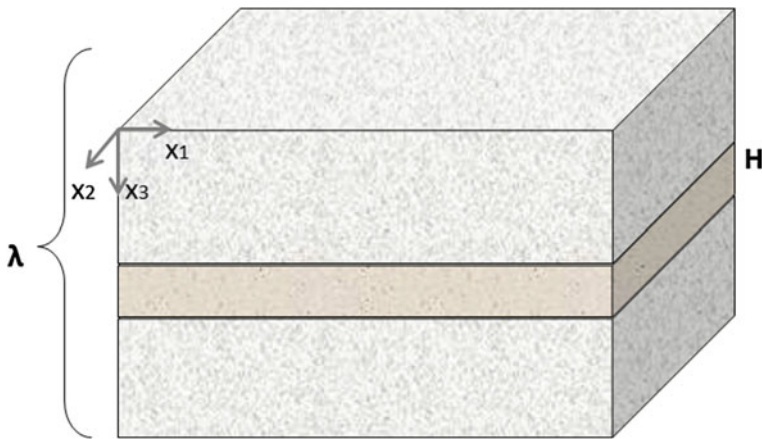


Fig. 2.8 Sketch of the long wavelength equivalent medium. H is the medium width. λ is seismic wavelength. $\lambda \gg H$

combination of the parameters from the original layered medium. The theory concludes that the elastic moduli of the equivalent medium can be expressed, in the long wavelength assumption, as thickness-weighted averages of the moduli of the thin layers of the stratified medium. The Backus average theory has been verified numerically by Carcione et al (1991).

2.3.2 Stress, Strain in the Stratified Layers

For a stratified medium, in the long wavelength limit, all components of stress acting on the layering plane, i.e., σ_{3h}, σ_{4h} and σ_{5h} ($h = 1 \dots n$, with h denoting different layers) and all components of strain lying in the layering plane, i.e., $\varepsilon_{1h}, \varepsilon_{2h}$ and ε_{6h} , are the same in all the layers across medium H. The notations of stress and strain are in Voigt form. The other components of the stress and strain, i.e., $\sigma_{1h}, \sigma_{2h}, \sigma_{6h}$ and $\varepsilon_{3h}, \varepsilon_{4h}, \varepsilon_{5h}$, are different from layer to layer. In other words, some components of stress and strain are layer-independent, i.e.,

$$\begin{bmatrix} \sigma_{3h} \\ \sigma_{4h} \\ \sigma_{5h} \end{bmatrix} = \begin{bmatrix} \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{bmatrix} = \sigma, \quad (2.20a)$$

$$\begin{bmatrix} \varepsilon_{1h} \\ \varepsilon_{2h} \\ \varepsilon_{6h} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \varepsilon, \quad (2.20b)$$

whereas other components of stress and strain are layer-dependent, i.e.,

$$\begin{bmatrix} \sigma_{1h} \\ \sigma_{2h} \\ \sigma_{6h} \end{bmatrix} = \sigma_h, \quad (2.21a)$$

$$\begin{bmatrix} \varepsilon_{3h} \\ \varepsilon_{4h} \\ \varepsilon_{5h} \end{bmatrix} = \varepsilon_h. \quad (2.21b)$$

The thickness-weighted average value for the layer-dependent components will be the total value over full thickness H (Schoenberg and Douma 1988). Thus, the layer-dependent stress components are

$$\bar{\sigma}_1 = \frac{1}{H} \sum_{h=1}^n H_h \sigma_{1h}, \quad (2.22a)$$

$$\bar{\sigma}_2 = \frac{1}{H} \sum_{h=1}^n H_h \sigma_{2h}, \quad (2.22b)$$

$$\bar{\sigma}_6 = \frac{1}{H} \sum_{h=1}^n H_h \sigma_{6h}, \quad (2.22c)$$

the layer-dependent strain components are

$$\bar{\epsilon}_3 = \frac{1}{H} \sum_{h=1}^n H_h \epsilon_{3h}, \quad (2.23a)$$

$$\bar{\epsilon}_4 = \frac{1}{H} \sum_{h=1}^n H_h \epsilon_{4h}, \quad (2.23b)$$

$$\bar{\epsilon}_5 = \frac{1}{H} \sum_{h=1}^n H_h \epsilon_{5h}, \quad (2.23c)$$

where H_h is the thickness and l_n is the relative thickness of the composed thin layer, and $l_1 + l_2 + \dots + l_n = 1$. In Eqs. (2.22a) and (2.23a), the overhead bar denotes that the layer-dependent components of stress and strain have been done using the thickness-weighted average. The linear relationship of the stress and strain in Eq. (2.11) can be separated into two parts that are individually for the layer-dependent and layer-independent stress components with corresponding strain components and the thickness-weighted average for the stiffness

$$\bar{\sigma} = C_{\mathbf{TT}} \bar{\epsilon} + C_{\mathbf{TN}} \bar{\epsilon}, \quad (2.24a)$$

$$\sigma = C_{\mathbf{NT}} \bar{\epsilon} + C_{\mathbf{NN}} \bar{\epsilon}, \quad (2.24b)$$

where $C_{\mathbf{TN}}$ is the transpose of the corresponding $C_{\mathbf{NT}}$, and

$$C_{\mathbf{TT}} = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{21} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix}, \quad (2.25a)$$

$$C_{\mathbf{NN}} = \begin{bmatrix} C_{33} & C_{34} & C_{35} \\ C_{34} & C_{44} & C_{45} \\ C_{35} & C_{45} & C_{55} \end{bmatrix}, \quad (2.25b)$$

$$C_{\mathbf{TN}} = \begin{bmatrix} C_{13} & C_{14} & C_{15} \\ C_{23} & C_{24} & C_{25} \\ C_{36} & C_{46} & C_{56} \end{bmatrix}. \quad (2.25c)$$

2.3.3 Stiffness and Compliances of the Fracture

In an effective medium, when a thin layer composing the medium is permanently deformed or fractured, this layer will be soft, and the layer-dependent strain components $\bar{\epsilon}$ will be enlarged. The layer-independent strain components ϵ are the same as the corresponding components in the host medium with a constant value. To represent the fracture, Eq. (2.24b) can be approximated as

$$\sigma_f \approx C_{NNf} \bar{\epsilon}. \quad (2.26)$$

Equation (2.26) describes a pure fracture feature. It turns out that the fracture feature is mainly affected by the stiffness C_{NNf} rather than by the stiffness C_{TT} and C_{TN} . C_{NNf} is approximately linearly proportional to the layer-independent stresses and layer-dependent strains. Now it is assumed that the fracture plane is isotropic (no roughness) and the fracture behavior is invariant with rotation with respect to an axis normal to the fracture (fracture system), and the fracture compliance is S (Schoenberg 1988, 1995). The fracture stiffness C_{NNf} and fracture compliance S can be transformed into each other as follows:

$$\begin{aligned} C_{NNf}^{-1} &= \begin{bmatrix} C_{33} & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & C_{55} \end{bmatrix}^{-1} \\ &= S = \begin{bmatrix} S_N & 0 & 0 \\ 0 & S_H & 0 \\ 0 & 0 & S_V \end{bmatrix}, \end{aligned} \quad (2.27)$$

where S_N is the normal fracture compliance that relates to the normal displacement and the normal stress, S_V and S_H are the tangential fracture compliances along the vertical (x_3 -axis) and horizontal (x_1 -axis) axes, respectively. Equation (2.27) is the so-called rotation-invariant fracture system. Because the two tangential fracture compliances are equal in the rotation-invariant fracture system, S_T may be used ($S_T = S_V = S_H$) as the tangential compliance that relates to the shear displacement and the shear stress. The fracture compliances S_N and S_T are nonnegative and have the dimension length/stress within a discontinuous medium.

In 1988, Schoenberg and Douma (1988) presented the dimensionless fracture compliances that link the dimensioned fracture compliances to the unfractured host medium compliances:

$$E_T \equiv C_{44H} S_T, \quad (2.28a)$$

$$E_N \equiv C_{33H} S_N, \quad (2.28b)$$

where E_N and E_T are the normal and tangential dimensionless fracture compliances. The subscript “ H ” denotes the host medium. To simplify the fracture compliances, Hsu and Schoenberg (1993) introduce the dimensionless quantities

$$\Delta_N = \frac{E_N}{1 + E_N}, \quad (2.29a)$$

$$\Delta_T = \frac{E_T}{1 + E_T}, \quad (2.29b)$$

$$0 \leq \Delta_N, \quad \Delta_T \leq 1. \quad (2.29c)$$

The quantities Δ_N, Δ_T relate the fracture compliance to the total compliance of a fractured medium that is algebraically a summation of a host medium and a fracture. Δ_N, Δ_T are also the so-called normal and tangential weakness of the fracture when the fracture is treated as an infinite slip interface (Bakulin et al. 2000a, b, c).

2.3.4 Linear Slip Interface and Fracture

A linear slip interface can be used to model a fracture that is based on the Backus average (1962). In 1980, Michael Schoenberg depicted a physical mechanism of the linear slip interface: an isotropic thin layer inserted into a homogeneous isotropic host medium to construct a perfectly welded layered medium (effective anisotropic VTI media). This isotropic thin layer thickness l' should be very small compared to a minimum wavelength λ , and its impedance Z' should be much lower with respect to the host medium impedance Z . The reflection and transmission coefficients of this isotropic thin layer are R' and T' . Once the thickness and impedance of the inserted isotropic thin layer approach to zero, the constructed layered medium (Fig. 2.9) is transformed into a medium that is effectively equal to a linear slip interface embedded in a homogeneous isotropic host medium in which the displacement discontinuity and stress continuity across the linear slip interface satisfy the nonwelded contact boundary conditions (Schoenberg 1980). At this point, the transformed medium is named the effective fractured medium, and the slip interface equals to a fracture. The reflection coefficient R and transmission coefficient T of the effective fractured medium will be $R = R'$ and $T = T'$.

The linear slip theory appreciably provides a convenient effective fractured medium model in geophysics that allows us to study the fracture from seismic signatures since the fracture is possibly simulated as an infinite weakness surface. Pyrak-Nolte et al. (1990a, b) and Hsu and Schoenberg (1993) have confirmed the validity of this model by some laboratory experimental verifications. In addition, Peterson et al. (1993) have shown that results obtained from small-scale cross-well experiments appear to agree with this model. In fact, the fracture outcrops show that a fracture as a nonwelded interface or weakness plane separates a medium into two

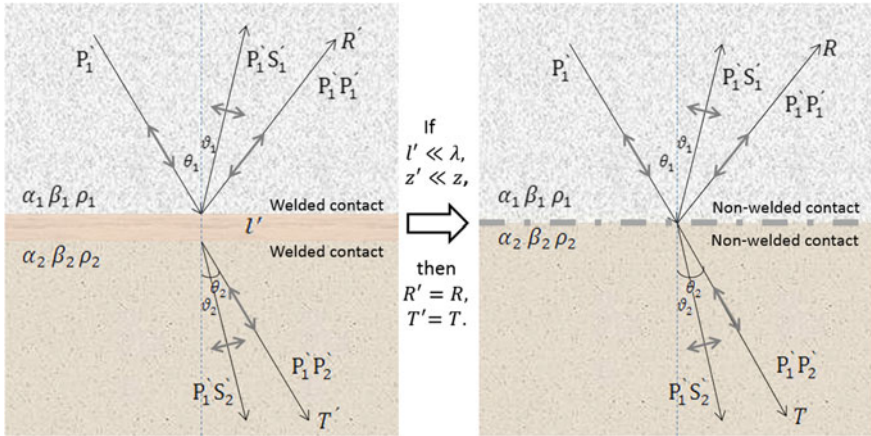


Fig. 2.9 Diagram of physical mechanism of the linear slip interface. Once $l' < \lambda$, $z' < z$, then $R' = R$, $T' = T$

half spaces that exactly coincide with the linear slip interface. The linear slip interface theory satisfies the nonwelded contact boundary conditions. Therefore, a linear slip interface obviously can be adapted to simulate a fracture which unites a geometrical phenomenon of the geology and a mechanical property of the medium. In the effective fractured medium model, the fracture is just an infinitely extended weakness plane or a highly compliant layer, regardless of its shape and microstructure and the porosity information.

Sayers (2009) used the ratio between the normal and tangential compliances as an indicator for the fluid content, e.g., $\frac{S_N}{S_T} \approx 1$ for dry fractures. Schoenberg (1980) suggested that a fluid filled fracture can be approximated by letting $S_N = 0$ and $S_T \neq 0$, which is equivalent to requiring normal displacement continuity. The case of $S_N = 0$ and $S_T \propto \infty$, meaning that there is no shear stress across the interface, and is equivalent to the solid–fluid boundary condition: both the normal stress and normal displacement are continuous.

2.3.5 Schoenberg-Muir Calculus Theory

Schoenberg and Muir (1989) extended the Backus average approach to develop a matrix formalism that enables a simple calculation of the stiffness of the rock, and then achieves a description for the composition and decomposition of the effective anisotropic medium. First, the rock stiffness of a layer can be mapped to an element of an Abelian group that includes two scalars and three 3×3 matrices. Then summation or subtraction is used to calculate the corresponding elements from the media that need to be calculated. Finally, the summed or the subtracted elements are

inverted into the rock stiffness to describe a reconstructed medium. Under the Backus theory (1962), a numerical simulation was performed to show that the Schoenberg-Muir theory is valid from the kinematic (travel times) and dynamic (amplitudes) viewpoints for a small crack aspect ratio or fracture opening, very long flat parallel fractures and thin layered media (Carcione et al. 2012). It concludes that a fracture as an infinitely extended weakness plane is an element for assembling a fractured medium from a fracture and a host medium. A fractured medium can be separated into a fracture and a host medium

$$\begin{aligned} \text{Fracture} + \text{Host medium} &\rightarrow \text{Fractured medium} \\ \text{Fractured medium} &\rightarrow \text{Fracture} + \text{Host medium.} \end{aligned}$$

In this view, geophysically a horizontally fractured medium (the VTI case) can be composed as a horizontal slip interface (a horizontal fracture) within an isotropic homogeneous host medium. A vertically fractured medium consists of a vertical linear slip interface (a vertical fracture) and an isotropic homogeneous host medium. A more realistic orthorhombic medium can be formed by either embedding a vertical slip interface into a transversely isotropic host medium with a symmetric vertical axis (VTI) or embedding a horizontal slip interface into a transversely isotropic host medium with a symmetric horizontal axis (HTI).

Schoenberg and Muir's calculus approach has been further developed into an algorithm that calculates the rock compliances instead of the rock stiffnesses by Hood (1991) and Nichols et al (1989).

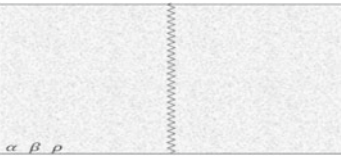
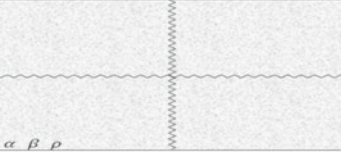
Table 2.1 gives the types of media contact that will be involved in this book. Throughout the book, the expression "a fractured medium" means that the host medium is a uniform homogeneous isotropic medium, while the expression "fractured medium with impedance contrast" implies that homogeneous isotropic media with impedance contrasts are the host media.

2.3.6 Horizontally Fractured Medium Moduli (VTI)

The stiffness matrix of an isotropic medium (Eq. 2.12) can be rewritten in a special order of the elements by moving the column six into column three and then moving the row six into row three as

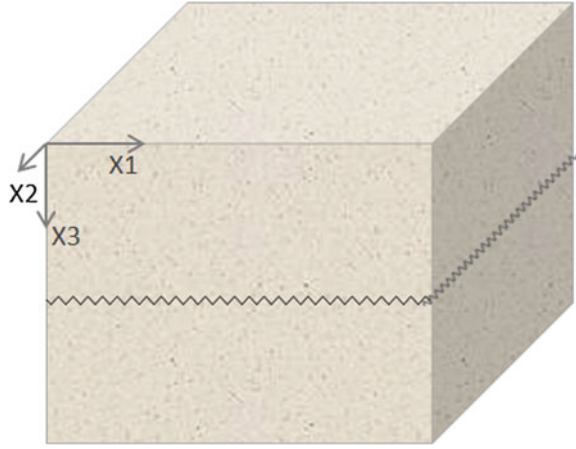
$$C_{H_iso} = \begin{bmatrix} C_{11} & C_{12} & 0 & C_{13} & 0 & 0 \\ C_{21} & C_{22} & 0 & C_{23} & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 & 0 \\ C_{31} & C_{32} & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}. \quad (2.30)$$

Table 2.1 Types of media contact

 Fracture interface	 Impedance contrast interface
 Horizontally fractured medium	 Horizontally fractured medium with impedance contrast
 Vertically fractured medium	 Vertically fractured medium with impedance contrast
 Orthogonally fractured medium	 Orthogonally fractured medium with impedance contrast

Note that the nine elements of the top left of the matrix in Eq. (2.30) form the stiffness matrix C_{TT} , while the nine elements (3×3) of the bottom right of the matrix in Eq. (2.30) form the stiffness matrix C_{NN} (Appendix A). Following the matrix formalism of algebra and Abelian (commutative) group theory (Schoenberg and Muir 1989), the effective elastic moduli of the fractured VTI media (Fig. 2.10) can be computed as

Fig. 2.10 Horizontally fractured medium model. It is formed by a horizontal fracture interface and a uniform homogeneous isotropic host medium



$$\mathbf{G}_{\text{VTI}} = \mathbf{G}_{H_iso} + \mathbf{G}_{h_f}, \quad (2.31)$$

where $\mathbf{G}_{H_iso}$ the group stiffness of the homogeneous isotropic host medium, and $\mathbf{G}_{h_f}$ is the group stiffness of the horizontal fracture that was discussed and given in Eq. (2.27). The effective elastic stiffness $\mathbf{C}_{\text{VTI}}$ of the horizontally fractured media with the parameters of tangential and normal weaknesses of the fracture (Eqs. 2.29a) is (see Appendix A)

$$\mathbf{C}_{\text{VTI}} = \begin{bmatrix} (\lambda + 2\mu) \left[1 - \frac{\lambda^2}{(\lambda + 2\mu)^2} \Delta_N \right] & \lambda \left(1 - \frac{\lambda}{\lambda + 2\mu} \Delta_N \right) & \lambda(1 - \Delta_N) & 0 & 0 & 0 \\ \lambda \left(1 - \frac{\lambda}{\lambda + 2\mu} \Delta_N \right) & (\lambda + 2\mu) \left[1 - \frac{\lambda^2}{(\lambda + 2\mu)^2} \Delta_N \right] & \lambda(1 - \Delta_N) & 0 & 0 & 0 \\ \lambda(1 - \Delta_N) & \lambda(1 - \Delta_N) & (\lambda + 2\mu)(1 - \Delta_N) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \mu(1 - \Delta_H) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu(1 - \Delta_V) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \end{bmatrix}. \quad (2.32)$$

This fractured elastic stiffness shows that the fractured medium is a TI medium with five independent moduli ($C_{\text{VTI}12} = C_{\text{VTI}11} - C_{\text{VTI}66}$) and the fracture properties are independent of the properties of the host medium (Hsu 1993).

2.3.6.1 Horizontal Fracture Anisotropy

A conclusion from Sect. 2.4.2 is that the effective composite media are TI media since the stiffness has five independent moduli. Hereby, the composed horizontally fractured medium is equivalent to an effective anisotropic VTI medium in which the fracture is part of a rotationally invariant fracture system and causes anisotropy. The stiffness of this effective horizontally fractured medium has been presented in Eq. (2.32). Substituting some elements in Eq. (2.32) into the anisotropic coefficients defined in Eq. (2.16a), the anisotropic coefficients caused by a fracture in terms of the fracture dimensionless weakness parameters are (Schoenberg and Douma 1988)

$$\gamma = \frac{\Delta_T}{2(1 - \delta_T)} \approx \frac{\Delta_T}{2}, \quad (2.33a)$$

$$\varepsilon = \frac{2g(1 - g)\Delta_N}{1 - \delta_N} \approx 2g(1 - g)\Delta_N, \quad (2.33b)$$

$$\delta = 2(1 - g) \frac{g(\Delta_N - \Delta_T)}{(1 - \Delta_N) - g(1 - \Delta_T)} \approx 2(1 - g)g(\Delta_N - \Delta_T), \quad (2.33c)$$

where g is

$$g = \frac{\mu_H}{\lambda_H + 2\mu_H}. \quad (2.33d)$$

2.3.7 Vertically Fractured Medium Moduli (HTI)

Cui and Lines (2011) showed that the elastic moduli of the HTI can be computed from the elastic moduli of a transversely isotropic medium with a vertical symmetry axis (a VTI medium) by using a Bond transformation (Winterstein 1990). Hence, rotating the horizontal fracture system in Eq. (2.27) by 90° with respect to the Y axis (Fig. 2.11), the elastic moduli of the vertical fracture system should be

$$\begin{aligned} \mathbf{S}_{v_f} &= \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{55} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} S_N & 0 & 0 \\ 0 & S_V & 0 \\ 0 & 0 & S_H \end{bmatrix}. \end{aligned} \quad (2.34)$$

Fig. 2.11 Coordinates Rotation. A horizontal interface is rotated into a vertical interface by a rotation of 90° with respect to the Y axis

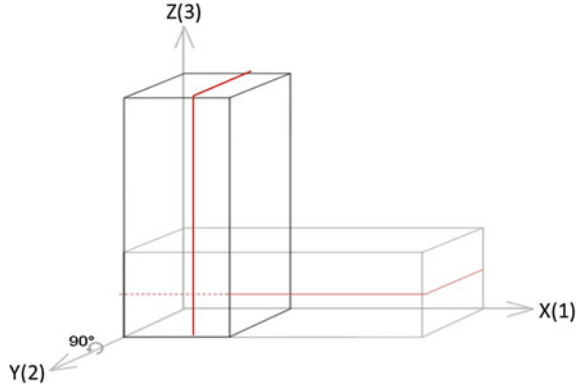


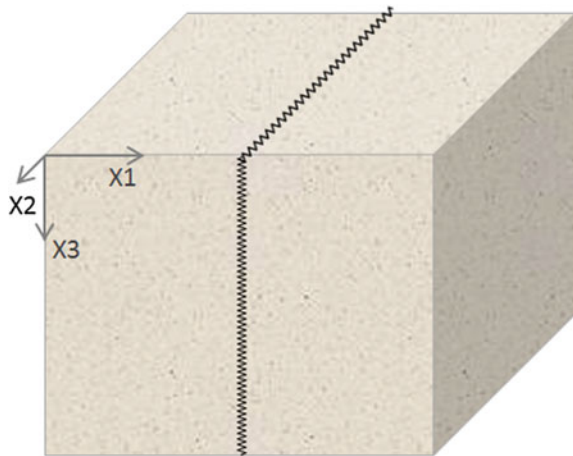
Figure 2.11 shows a vertical fracture that is normal to the $X(1)$ -axis or parallel to $[Y(2), Z(3)]$ -plane with a horizontal symmetry axis. It is rotated from a horizontal fracture.

Following the Schoenberg and Muir (1989) procedure to simplify the calculation of the effective elastic moduli of the vertically fractured media (HTI) (Fig. 2.12), we have:

$$G_{HTI} = G_{H_iso} + G_{v_f}. \quad (2.35)$$

G_{H_iso} is the group stiffness of the homogeneous isotropic host medium, G_{v_f} is the group stiffness of the vertical fracture. Consequently, the elastic stiffness for a vertically fractured medium (HTI) is (Appendix A)

Fig. 2.12 Vertically fractured medium model. It is formed by a vertical fracture interface and a uniform homogeneous isotropic host medium



$$\mathbf{C}_{\text{HTI}} = \begin{bmatrix}
(\lambda + 2\mu)(1 - \Delta_N) & \lambda(1 - \Delta_N) & \lambda(1 - \Delta_N) \\
\lambda(1 - \Delta_N) & (\lambda + 2\mu) \left[1 - \frac{\lambda^2}{(\lambda + 2\mu)^2} \Delta_N \right] & \lambda \left(1 - \frac{\lambda}{\lambda + 2\mu} \Delta_N \right) \\
\lambda(1 - \Delta_N) & \lambda \left(1 - \frac{\lambda}{\lambda + 2\mu} \Delta_N \right) & (\lambda + 2\mu) \left[1 - \frac{\lambda^2}{(\lambda + 2\mu)^2} \Delta_N \right] \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\mu & 0 & 0 \\
0 & \mu(1 - \Delta_V) & 0 \\
0 & 0 & \mu(1 - \Delta_H)
\end{bmatrix}. \quad (2.36)$$

The elements of Eq. (2.36) have $C_{\text{HTI}22} = C_{\text{HTI}33}$, $C_{\text{HTI}12} = C_{\text{HTI}13}$, $C_{\text{HTI}55} = C_{\text{HTI}66}$ and $C_{\text{HTI}23} = C_{\text{HTI}33} - 2C_{\text{HTI}44}$. Thus effective vertically fractured media are TI media with five independent moduli. Note that $C_{44_HTI} = \mu$ is the same as the homogeneous isotropic host medium stiffness. It means that only the elements corresponding to the fracture system describe the fracture in the stiffness for the effective vertically fractured medium. This effective vertically fractured medium (HTI) is the simplest azimuthally anisotropic medium that provides valuable insights into the study of the fracture behavior of the seismic signature variation with the azimuthal fractures.

2.3.7.1 Vertical Fracture Anisotropy

This vertically fractured medium may cause anisotropy that would be seen and observed on seismic signature as travel time and amplitude variations. The stiffness tensor of this vertically fractured medium is expressed by the fracture weakness in Eq. (2.36). Substituting some elements in Eq. (2.36) into Eq. (2.16a), the fracture-caused anisotropic coefficients can be conveniently described by the anisotropic parameters as

$$\gamma^{(v)} = -\frac{\Delta_T}{2}, \quad (2.37a)$$

$$\varepsilon^{(v)} = -\frac{2g(1-g)\Delta_N}{1 - (1-2g)^2\Delta_N} \approx -2g(1-g)\Delta_N, \quad (2.37b)$$

$$\begin{aligned}\delta^{(v)} &= -\frac{2g(1 - (1 - 2g)\Delta_N)((1 - 2g)\Delta_N + \Delta_T)}{\left(1 - (1 - g)^2\right)\left(1 + \frac{1}{1+g}(g\Delta_T - (1 - 2g)\Delta_N)\right)} \\ &\approx -2g((1 - 2g)\Delta_N + \Delta_T).\end{aligned}\quad (2.37c)$$

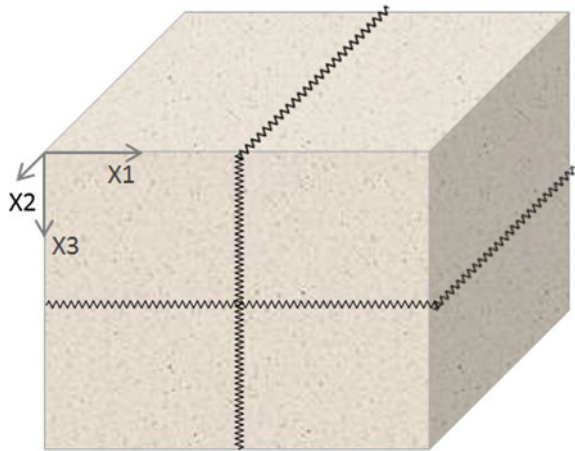
In Eq. (2.37a), anisotropic parameters $\gamma^{(v)}$, $\varepsilon^{(v)}$ and $\delta^{(v)}$ are always negative in a HTI model, thus the fracture weakness parameters always have nonnegative values. The fracture characterization can be determined from the surface seismic signatures through the anisotropic representations in the seismic data.

2.3.8 Orthogonally Fractured Medium Moduli (VTI + HTI)

A reconstructed effective orthorhombic medium (the symmetry of a brick) is usually a combination of a VTI medium and a HTI medium that has been considered as a more realistic model to characterize the reservoir (Fig. 2.13). The reason is that geological sediments deposited into a horizontally layered medium show transversely isotropic elastic behavior with a vertical symmetry axis (VTI), while the maximum principal stress is a vertical one of compression from the overburden deposition that generated the vertical fractures with a horizontal symmetry axis. Therefore, an orthorhombic medium model is a more reasonable model to describe the subsurface structures. Based on the Schoenberg and Muir calculus, an orthogonally fractured medium can be treated as a vertical slip interface plus a VTI host medium

$$G_{\text{Orth}} = G_{H_VTI} + G_{v_f}. \quad (2.38)$$

Fig. 2.13 Orthogonally fractured medium model. It is formed by a vertical fracture interface and horizontally fractured host medium



G_{H_VTI} is the group stiffness of the VTI host medium that could include the stiffness of a horizontally fractured medium in Eq. (2.36) as well. G_{v_f} is the group element of the vertical fracture and has the same expression as in Eq. (2.34) (Appendix A).

$$\mathbf{C}_{Orth} = \begin{bmatrix} C_{11H_VTI}(1 - \Delta_N) & C_{12H_VTI}(1 - \Delta_N) & C_{13H_VTI}(1 - \Delta_N) & \\ C_{12H_VTI}(1 - \Delta_N) & C_{11H_VTI} \left(1 - \frac{C_{12H_VTI}^2}{C_{11H_VTI}^2} \Delta_N \right) & C_{13H_VTI} \left(1 - \frac{C_{12H_VTI}}{C_{11H_VTI}} C_{12H_VTI} \Delta_N \right) & \\ C_{31H_VTI}(1 - \Delta_N) & C_{13H_VTI} \left(1 - \frac{C_{112H_VTI}}{C_{11H_VTI}} \Delta_N \right) & C_{33H_VTI} \left(1 - \frac{C_{13H_VTI}^2}{C_{11H_VTI} C_{33H_VTI}} \Delta_N \right) & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ C_{44H_VTI} & 0 & 0 & \\ 0 & C_{44H_VTI}(1 - \Delta_V) & 0 & \\ 0 & 0 & C_{66H_VTI}(1 - \Delta_H) & \end{bmatrix}. \quad (2.39)$$

Equation (2.39) gives the stiffness of an orthogonally fractured medium. An effective orthogonally fractured medium has nine independent moduli that depend on the five independent moduli of the TI host medium and the three fracture compliances of the fracture. A more complex method for solving for the stiffness of the orthorhombic medium is given in Appendix A by considering a composite of a VTI medium and a HTI anisotropic media.

2.3.8.1 Orthorhombic Fractures Anisotropy

The geometry of Fig. 2.13 shows that the orthorhombic fracture model has two orthogonal vertically symmetric planes $[x_2, x_3]$, $[x_1, x_3]$ and one horizontally symmetric plane $[x_1, x_2]$ which are normal to the x_1 -axis, x_2 -axis and x_3 -axis, respectively. Tsvankin (1997b) proved that the anisotropy in the symmetric planes of orthorhombic media can be completely described by known intrinsic anisotropy parameters, and all conclusions about anisotropy in VTI and HTI media remain valid for symmetric planes in orthorhombic media. If the properties of all vertical planes are identical, an orthorhombic medium can be reduced to a VTI medium. The stiffness tensor of this orthorhombic fractured medium is expressed through the fracture weakness in Eq. (2.39). Substituting necessary elements in Eq. (2.39) into

Eq. (2.17a) yields the anisotropic coefficients for the vertical symmetry $[x_2, x_3]$ -plane

$$\gamma^{(1)} = \gamma_H + \frac{\Delta_V - \Delta_H}{2}, \quad (2.40a)$$

$$\varepsilon^{(1)} = \varepsilon_H, \quad (2.40b)$$

$$\delta^{(1)} = \delta_H. \quad (2.40c)$$

The subscript H of anisotropic parameters denotes the host medium, a horizontally fractured medium. Therefore, the anisotropic coefficients in the $[x_2, x_3]$ -plane coincide with those of the horizontally fractured VTI host medium. It is seen that the $[x_2, x_3]$ -plane represents an isotropic plane with a horizontal symmetric x_1 -axis in this orthorhombic medium in which all of the velocities are not affected by the fracture and remain constant for all propagation directions (Tsvankin 1997b). If the fracture system is rotationally invariant, we have $\Delta_V = \Delta_H$, and then $\gamma^{(1)} = \gamma_H$.

Substituting some elements in Eq. (2.39) into Eq. (2.18a) gives the anisotropic coefficients for another vertical symmetry $[x_1, x_3]$ -plane that is normal to the x_2 -axis

$$\gamma^{(2)} = \gamma_H - \frac{\Delta_H}{2}, \quad (2.41a)$$

$$\varepsilon^{(2)} = \varepsilon_H - 2g(1 - g)\Delta_N, \quad (2.41b)$$

$$\delta^{(2)} = \delta_H - 2g[(1 - 2g)\Delta_N + \Delta_V]. \quad (2.41c)$$

Note that the $[x_1, x_3]$ -plane involves two distinguishing anisotropic coefficients. One is the corresponding anisotropic coefficient of the horizontally fractured VTI host medium with subscript “ H ”, while the other anisotropic coefficient that results from the vertical fracture that approximately equals the coefficients $\gamma^{(v)}, \varepsilon^{(v)}, \delta^{(v)}$ describing a vertically fractured HTI model. This composite result is what we expected.

The anisotropic coefficients of the horizontal symmetry $[x_1, x_2]$ -plane are obtained by substituting necessary elements in Eq. (2.39) into Eq. (2.19)

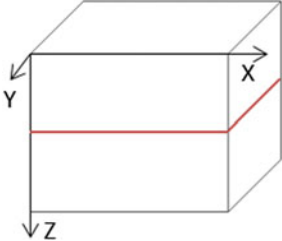
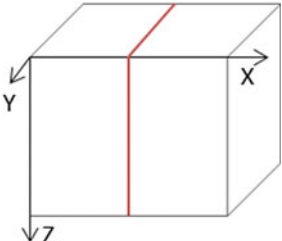
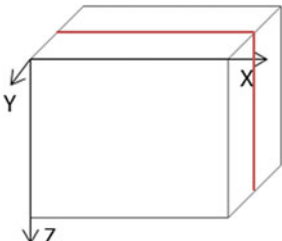
$$\delta^{(3)} = 2g[\Delta_N - \Delta_V]. \quad (2.42)$$

Note that $\delta^{(3)}$ does not relate to the anisotropic coefficients of host medium because the horizontal symmetry $[x_1, x_2]$ -plane is an isotropic plane in the horizontally fractured VTI host media.

2.4 Boundary Conditions

Boundary condition stands for a constraint function that must be satisfied along a boundary (Sheriff 2002). At the boundary, all waves are constrained such that kinematic displacements and dynamic stresses on one side of the boundary are related to displacements and stresses on the other side by certain boundary conditions. The direction of the boundary and contact properties of the two sides of the boundary decide the wave expressions in terms of the displacement and stress. Table 2.2 provides boundary types in 3D and the corresponding expressions of the boundary conditions in terms of displacement $\mathbf{u}$ and the stresses $\boldsymbol{\sigma}$, in which $\mathbf{u}^{\pm} = u_i^{\pm}$, $\boldsymbol{\sigma} = \sigma_{ij}^{\pm}$, $i = j = x, y, z$.

Table 2.2 Boundary conditions in 3D

Type of the boundaries	Welded boundary conditions	Nonwelded boundary conditions
z-normal boundary 	$\begin{cases} u_x^+ - u_x^- = 0 \\ \sigma_{xz}^+ - \sigma_{xz}^- = 0 \end{cases}$ $\begin{cases} u_z^+ - u_z^- = 0 \\ \sigma_{zz}^+ - \sigma_{zz}^- = 0 \end{cases}$ $\begin{cases} u_y^+ - u_y^- = 0 \\ \sigma_{yz}^+ - \sigma_{yz}^- = 0 \end{cases}$	$\begin{cases} u_x^+ - u_x^- = S_T \sigma_{xz}^+ \\ \sigma_{xz}^+ = \sigma_{xz}^- \end{cases}$ $\begin{cases} u_z^+ - u_z^- = S_N \sigma_{zz}^+ \\ \sigma_{zz}^+ = \sigma_{zz}^- \end{cases}$ $\begin{cases} u_y^+ - u_y^- = S_T \sigma_{yz}^+ \\ \sigma_{yz}^+ = \sigma_{yz}^- \end{cases}$
x-normal boundary 	$\begin{cases} u_x^+ - u_x^- = 0 \\ \sigma_{xx}^+ - \sigma_{xx}^- = 0 \end{cases}$ $\begin{cases} u_z^+ - u_z^- = 0 \\ \sigma_{zx}^+ - \sigma_{zx}^- = 0 \end{cases}$ $\begin{cases} u_y^+ - u_y^- = 0 \\ \sigma_{yx}^+ - \sigma_{yx}^- = 0 \end{cases}$	$\begin{cases} u_x^+ - u_x^- = S_N \sigma_{xx}^+ \\ \sigma_{xx}^+ = \sigma_{xx}^- \end{cases}$ $\begin{cases} u_z^+ - u_z^- = S_T \sigma_{zx}^+ \\ \sigma_{zx}^+ = \sigma_{zx}^- \end{cases}$ $\begin{cases} u_y^+ - u_y^- = S_T \sigma_{yx}^+ \\ \sigma_{yx}^+ = \sigma_{yx}^- \end{cases}$
y-normal boundary 	$\begin{cases} u_x^+ - u_x^- = 0 \\ \sigma_{xy}^+ - \sigma_{xy}^- = 0 \end{cases}$ $\begin{cases} u_z^+ - u_z^- = 0 \\ \sigma_{zy}^+ - \sigma_{zy}^- = 0 \end{cases}$ $\begin{cases} u_y^+ - u_y^- = 0 \\ \sigma_{yy}^+ - \sigma_{yy}^- = 0 \end{cases}$	$\begin{cases} u_x^+ - u_x^- = S_T \sigma_{xy}^+ \\ \sigma_{xy}^+ = \sigma_{xy}^- \end{cases}$ $\begin{cases} u_z^+ - u_z^- = S_T \sigma_{zy}^+ \\ \sigma_{zy}^+ = \sigma_{zy}^- \end{cases}$ $\begin{cases} u_y^+ - u_y^- = S_N \sigma_{yy}^+ \\ \sigma_{yy}^+ = \sigma_{yy}^- \end{cases}$

2.4.1 Perfectly Welded Contact Interface

Consider a compressional wave traveling in the x - z plane. It impinges upon a perfectly welded contact interface at which the kinematic displacements and the dynamic stresses of the wave quantities on the two sides of the interface satisfy

$$\mathbf{u}^+ = \mathbf{u}^-, \quad (2.43)$$

$$\boldsymbol{\sigma}^+ = \boldsymbol{\sigma}^-, \quad (2.44)$$

where $\mathbf{u}$ is kinematic displacement and $\boldsymbol{\sigma}$ is dynamic stress. $\mathbf{u}^+ = \begin{Bmatrix} u_x^+ \\ u_z^+ \end{Bmatrix}$, $\mathbf{u}^- = \begin{Bmatrix} u_x^- \\ u_z^- \end{Bmatrix}$, $\boldsymbol{\sigma}^+ = \begin{Bmatrix} \sigma_{xz}^+ \\ \sigma_{zz}^+ \end{Bmatrix}$, $\boldsymbol{\sigma}^- = \begin{Bmatrix} \sigma_{xz}^- \\ \sigma_{zz}^- \end{Bmatrix}$. The superscripts “-” and “+” denote the two sides of the interface (Fig. 2.14). The subscripts “ x and z ” or “ xz and zz ” denote the $\mathbf{u}$ or $\boldsymbol{\sigma}$ in tangential and normal components. For example, u_x^+ or u_z^+ can be interpreted as a displacement $\mathbf{u}$ in tangential x or normal z component at the “+” side of the interface. Equations (2.43) and (2.44) must be satisfied by the incident, reflected, and transmitted waves at the interface, and are known as the perfect welded contact boundary conditions. They state that both kinematic displacements and dynamic stresses of the wave are continuous across the perfect welded interface when the wave propagates at the interface. Figure 2.14 shows the perfectly welded interface boundary conditions.

2.4.1.1 Reflections and Transmissions

In seismology, an incident compressional plane wave produces reflected and transmitted waves at the interface. Figure 2.15 shows all elastic waves at the

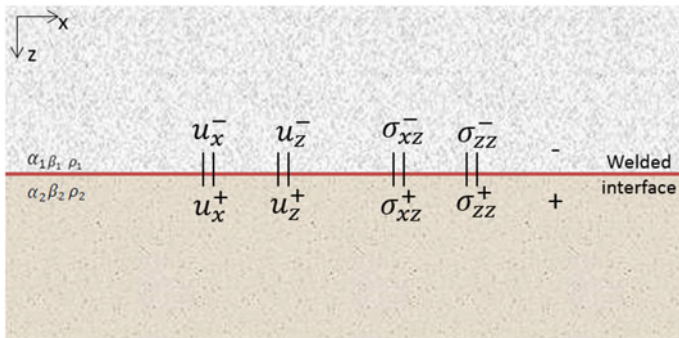


Fig. 2.14 Perfectly welded interface boundary conditions. $\mathbf{u}^+ = \mathbf{u}^-$, $\boldsymbol{\sigma}^+ = \boldsymbol{\sigma}^-$. The kinematic displacements and dynamic stresses are continuous across the welded interface

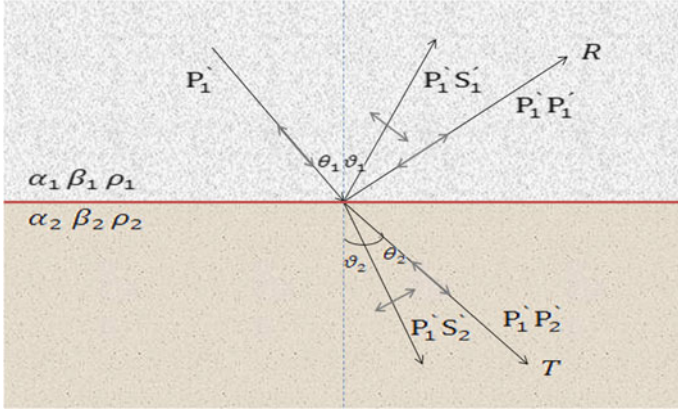


Fig. 2.15 Incident P-wave, reflected and transmitted PP and PS waves at a perfectly welded contact interface. $\theta_1, \theta_2, \vartheta_1$ and ϑ_2 are PP and PS-waves reflection and transmission angles respectively. The *single arrows* point in the direction of wave propagation. The *double arrows* indicate the direction of the wave polarization. α_n, β_n and ρ_n , $n = 1, 2$ are the media parameters

reflector in a homogeneous isotropic medium. The upper prime “’” indicates up going reflection waves, and the down prime “” means down going waves that include the incident wave in the upper medium and the transmitted waves in the lower medium. Consider a plane wave,

$$\mathbf{u} = A e^{-i\omega(t-\mathbf{s} \cdot \mathbf{x})} \mathbf{d}, \quad (2.45)$$

where A is the amplitude and is assumed to be unity, and $\mathbf{s} \cdot \mathbf{x} = s_x x + s_y y + s_z z$. Equation (2.45) represents a harmonic plane wave traveling in the $\mathbf{s}$ direction, where $\mathbf{s}$ is the slowness. $\mathbf{d}$ denotes the wave polarization, with $|\mathbf{d}| = 1$.

Substituting Eq. (2.45) into Eq. (2.43) and using Fig. 2.15, gives

$$P_1' \sin \theta_1 = -P_1' \sin \theta_1 - S_1' \cos \vartheta_1 + P_2' \sin \theta_2 + S_2' \cos \vartheta_2. \quad (2.46a)$$

$$P_1' \cos \theta_1 = P_1' \cos \theta_1 - S_1' \sin \vartheta_1 + P_2' \cos \theta_2 - S_2' \sin \vartheta_2. \quad (2.46b)$$

where P_1', P_1', P_2', S_1' and S_2' are the amplitudes of the P-incident, PP-reflected and transmitted, PS-reflected and transmitted waves, respectively, while $\theta_1, \theta_2, \vartheta_1$ and ϑ_2 are angle of PP and PS-waves reflection and transmission, respectively.

Similarly, substituting Eq. (2.45) into Eq. (2.44) and using Fig. 2.15 gives

$$P_1' x_1 \cos \theta_1 = P_1' x_1 \cos \theta_1 + S_1' \beta_1 r_1 + P_2' x_2 \cos \theta_2 + S_2' \beta_2 r_2. \quad (2.47a)$$

$$P_1' \alpha_1 r_1 = -P_1' \alpha_1 r_1 + S_1' x_1 \cos \vartheta_1 + P_2' \alpha_2 r_2 - S_2' x_2 \cos \vartheta_2. \quad (2.47b)$$

where α_n, β_n and ρ_n are the P-wave velocity, S-wave velocity and density, respectively. $x_n = 2\rho_n\beta_n^2P, r_n = \rho_n(1 - 2\beta_n^2P^2), n = 1, 2$ (Aki and Richards 1980). P is the ray parameter.

$$\frac{\sin\theta_n}{\alpha_n} = \frac{\sin\vartheta_n}{\beta_n} = P. \quad (2.48)$$

2.4.1.2 Zoeppritz Equations

Equations (2.46a) and (2.47a) are four linear equations in four unknowns, and they can be used to solve for the four unknown PP and PS reflection transmission coefficients in a homogeneous isotropic media. This system of equations can be written in matrix form as

$$\begin{bmatrix} \alpha_1 P \\ \cos \theta_1 \\ x_1 \cos \theta_1 \\ \alpha_1 r_1 \end{bmatrix} = \begin{bmatrix} -\alpha_1 P & -\cos \vartheta_1 & \alpha_2 P & \cos \vartheta_2 \\ \cos \theta_1 & -\beta_1 P & \cos \theta_2 & -\beta_2 P \\ x_1 \cos \theta_1 & \beta_1 r_1 & x_2 \cos \theta_2 & \beta_2 r_2 \\ -\alpha_1 r_1 & x_1 \cos \vartheta_1 & \alpha_2 r_2 & -x_2 \cos \vartheta_2 \end{bmatrix} \begin{bmatrix} P'P'_1 \\ P'S'_1 \\ P'P'_2 \\ P'S'_2 \end{bmatrix}. \quad (2.49)$$

These equations are known as the Zoeppritz equations (1919). The equations reveal that the amplitudes of reflected and transmitted waves are functions of the angles at the interface. For a normal incidence case, the PP-wave reflection and transmission coefficients are

$$R_w = \frac{P'_1}{P_1} = P'P'_1 = \frac{\alpha_2 \rho_2 - \alpha_1 \rho_1}{\alpha_2 \rho_2 + \alpha_1 \rho_1}, \quad (2.50a)$$

$$T_w = \frac{P'_2}{P_1} = P'P'_2 = \frac{2\alpha_1 \rho_1}{\alpha_2 \rho_2 + \alpha_1 \rho_1}. \quad (2.50b)$$

Note that there should be only transmitted waves passing through the interface in a uniform homogeneous isotropic medium at normal incidence since in that case $\alpha_2 \rho_2 - \alpha_1 \rho_1 = 0$.

2.4.2 Imperfectly Welded (Nonwelded) Contact Interface

Consider a compressional wave hitting upon a deformed (fractured) linear slip interface that is different from the perfect welded interface. At this slip interface, the incident, reflected, and transmitted waves are constrained by the imperfectly welded (nonwelded) contact boundary conditions: the dynamic stresses are continuous across the interface, but the kinematic displacements are discontinuous and the

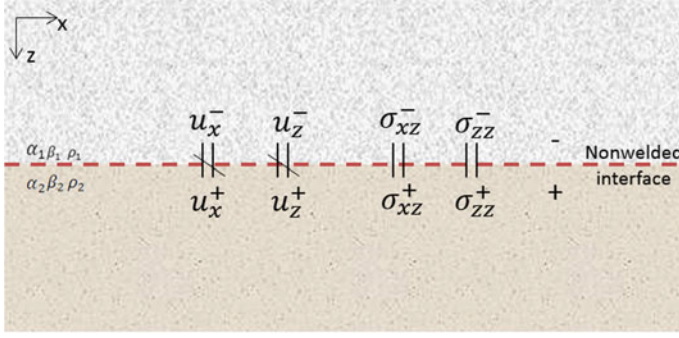


Fig. 2.16 Imperfectly welded (nonwelded) contact interface boundary conditions. $\mathbf{u}^+ \neq \mathbf{u}^-$, $\boldsymbol{\sigma}^+ = \boldsymbol{\sigma}^-$. The kinematic displacements are discontinuous, but the dynamic stresses are continuous across the nonwelded contact interface

differential displacements are linearly proportional to the corresponding stresses (Fig. 2.16):

$$\mathbf{u}^+ - \mathbf{u}^- = \mathbf{S}\boldsymbol{\sigma}^+ \quad (2.51)$$

$$\boldsymbol{\sigma}^+ = \boldsymbol{\sigma}^- \quad (2.52)$$

where $\mathbf{u}$ and $\boldsymbol{\sigma}$ are the same as in Eqs. (2.43, 2.44). The parameters $\mathbf{S} = \begin{Bmatrix} S_T \\ S_N \end{Bmatrix}$, S_T and S_N have been described in Eq. (2.27) as the tangential and normal fracture compliances of the fractured medium (Schoenberg 1980), which are the reciprocals of the rock stiffnesses. S_T is for a normally incident shear wave, and S_N is for a normally incident compressional wave (Schoenberg 1980). S_T and S_N imply that fracture deformation is a combination of tangential deformation and normal deformation.

As Eq. (2.52) shows, the stress is continuous across the interface, even though it is a nonwelded contact interface. If it was not, the equation of motion,

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (2.52a)$$

indicates that forces and accelerations would be infinite for a discontinuity in stress, which is unphysical.

2.4.2.1 Reflections and Transmissions

Similarly, a plane compressional incident wave produces reflected and transmitted waves at the linear slip interface in a homogeneous isotropic medium (Fig. 2.16).

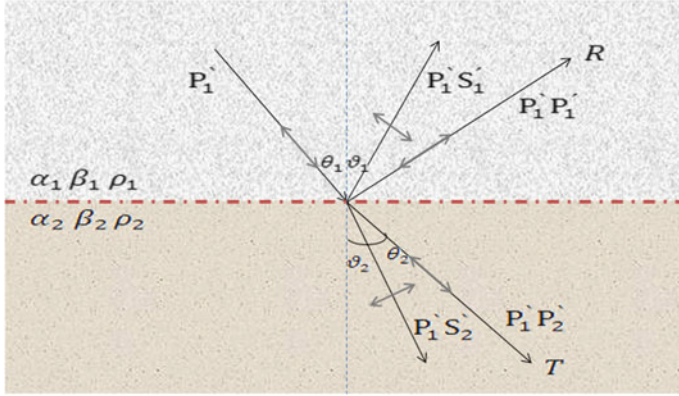


Fig. 2.17 Incident P-wave, reflected and transmitted PP and PS waves at the imperfectly welded (nonwelded) contact interface. $\theta_1, \theta_2, \varphi_1$ and φ_2 are PP and PS-waves reflection and transmission angles respectively. The *single arrows* point in the direction of wave propagation. The *double arrows* indicate the direction of the wave polarization. α_n, β_n and ρ_n , $n = 1, 2$ are the media parameters. S_T and S_N are tangential and normal fracture compliances, respectively

For a given frequency ω , substituting Eq. (2.45) into Eq. (2.51) and using Fig. 2.17 gives

$$P_1^i \sin \theta_1 = -P_1' \sin \theta_1 - S_1' \cos \varphi_1 + P_2^i (\sin \theta_2 - i\omega S_T x_2 \cos \theta_2) + S_2^i (\cos \varphi_2 - i\omega S_T \beta_2 r_2), \quad (2.53a)$$

$$P_1^i \cos \theta_1 = P_1' \cos \theta_1 - S_1' \sin \varphi_1 + P_2^i (\cos \theta_2 - i\omega S_N \alpha_2 r_2) + S_2^i (-\sin \varphi_2 + i\omega S_N x_2 \cos \varphi_2). \quad (2.53b)$$

Similarly, substituting the plane wave in Eq. (2.45) into Eq. (2.52) for the stress nonwelded boundary conditions, gives

$$P_1^i x \cos \theta_1 = P_1' x_1 \cos \theta_1 + S_1' \beta_1 r_1 + P_2^i x_2 \cos \theta_2 + S_2^i \beta_2 r_2, \quad (2.54a)$$

$$P_1^i \alpha_1 r_1 = -P_1' \alpha_1 r_1 + S_1' x_1 \cos \varphi_1 + P_2^i \alpha_2 r_2 - S_2^i x_2 \cos \varphi_2, \quad (2.54b)$$

where, the symbols $P_1^i, P_1', P_2^i, S_1', S_2^i$ and $\alpha_n, \beta_n, \rho_n$ and $x_n = 2\rho_n \beta_n^2 P, r_n = \rho_n (1 - 2\beta_n^2 P^2)$, $n = 1, 2$ in Eqs. (2.46a) and (2.47a) are the same as in Eqs. (2.53a) and (2.54a). Note that Eq. (2.54a) are identical to Eq. (2.47a). An explanation for this is that the dynamic stresses always obey the law of conservation of energy, regardless of whether contact of the interface is perfectly welded or not.

2.4.2.2 Zoeppritz Equations

Combining Eqs. (2.53a) and (2.54a) yields a system of four linear equations in four unknowns. They can be used to solve for the four unknown reflection and transmission coefficients of the PP and PS-waves at the imperfect welded interface:

$$\begin{bmatrix} \alpha_1 P \\ \cos \theta_1 \\ x_1 \cos \theta_1 \\ \alpha_1 r_1 \end{bmatrix} = \begin{bmatrix} -\alpha_1 P & -\cos \vartheta_1 & \alpha_2 P - i\omega S_T x_2 \cos \theta_2 & \cos \vartheta_2 - i\omega S_T \beta_2 r_2 \\ \cos \theta_1 & -\beta_1 P & \cos \theta_2 - i\omega S_N \alpha_2 r_2 & -\beta_2 P + i\omega S_N x_2 \cos \vartheta_2 \\ x_1 \cos \theta_1 & \beta_1 r_1 & x_2 \cos \theta_2 & \beta_2 r_2 \\ -\alpha_1 r_1 & x_1 \cos \vartheta_1 & \alpha_2 r_2 & -x_2 \cos \vartheta_2 \end{bmatrix} \begin{bmatrix} P_1^i P_1^i \\ P_1^i S_1^i \\ P_1^i P_2^i \\ P_1^i S_2^i \end{bmatrix} \quad (2.55)$$

where $i = \sqrt{-1}$. Equations (2.55), are the so-called Zoeppritz equations for the linear slip interface, and these equations are important for fracture analysis because they reveal the relationship between the wave incident angles and the reflection/transmission coefficients at the linear slip interface. Note that they have the same pattern as the original Zoeppritz equations (1919). Equation (2.55) also shows that the reflection/transmission coefficients are frequency dependent with the fracture parameters S_T and S_N (Schoenberg 1980; Chaisri and Krebs 2000; Chaisri 2002). Pyrak-Nolte et al. (1990a, b) and Chaisri and Krebs (2000) have shown that the reflected P-wave has a lower amplitude at the lower frequencies that has an apparent attenuation in the nonwelded interface.

Chaisri and Krebs (2000) and Chaisri (2002) give the conditions that the fracture parameters and the frequency should satisfy the following conditions.

$$\omega S_N \rho_2 \alpha_2 < 1, \quad (2.56a)$$

and

$$\omega S_T \rho_2 \beta_2 < 1. \quad (2.56b)$$

From Eq. (2.55), the normal incidence P-wave reflection and transmission coefficients at the slip interface in a homogeneous isotropic medium are

$$R_{\text{non-w}} = P_1^i P_1^i = \frac{Z_2 - Z_1 + i\omega S_N Z_2 Z_1}{Z_2 + Z_1 - i\omega S_N Z_2 Z_1}, \quad (2.57a)$$

$$T_{\text{non-w}} = P_1^i P_2^i = \frac{2Z_1}{Z_2 + Z_1 - i\omega S_N Z_2 Z_1}, \quad (2.57b)$$

where Z_n is the media impedance: $Z_n = \rho_n \alpha_n$. Note that reflection coefficients at a linear slip interface should be nonzero at normal incidence even if the interface is in the uniform homogeneous isotropic medium where there is no impedance contrast, i.e., $Z_2 - Z_1 = 0$. This is a substantial difference between a linear slip interface and

a welded interface. In other words, reflections from the slip interface show that a fracture itself generates a reflection which is due to the displacement discontinuity, rather than an impedance contrast at the boundary.

2.5 Conclusions

This chapter theoretically and systematically provides a link between the inherent elastic properties of the medium and the gained fracture compliance parameters of the fractured medium. Also it presents the knowledge of fracture formation, fracture parameters, and fracture detection. As well, it supplies a rock stiffness composition and decomposition framework within which a complex fractured medium stiffness can be simplified into a fracture stiffness and a host medium stiffness, and vice versa. This infers that a medium forming a fracture feature combined with a host medium is equivalent to a fractured medium, and vice versa. Additionally, it discusses the boundary conditions that are constraints for all waves at an interface, thus it is a key for next chapter's study, which is on the seismic signature response of the fracture.

Elastic tensors are the basis of understanding what will happen to a rock when it is subjected to force. In general, an elastic rock deformation first obeys Hooke's law as a reversible deformation when it is subjected to a small force, then it changes to an irreversible ductile deformation if the force keeps exerting. Once the rock strain exceeds a strain threshold, the rock deformation finally changes to a physically permanent fracture deformation. A discussion of stress and strain is presented in Sect. 2.1 for illustrating the different deformation stages. As a subsurface medium, the fracture formation similarly suffers the same deformation stages as a rock because the medium still undergoes the geological movement and overburden compressions. Commonly, most geological fractures are vertical or nearly vertical fractures because the overburden compression is the maximum stress rather than the other two horizontal stresses.

From a geophysical viewpoint, the fracture orientation and density are significantly more important parameters than the fracture openings in reservoir characterizations. The orientation is determined by the directions of maximum and minimum stress. The direction of the fracture is parallel to the direction of the maximum compressive stress and perpendicular to the direction of the minimum compressive stress. The fracture density is a measurement of the number of the fractures per meter along a certain direction, and it influences permeability evaluations: the higher the fracture density, the higher the permeability if the fractures are conductive.

The fracture detection usually is an integrated method involving geology, geophysics, and petrophysics technologies. For this integrated method, petrophysics data are regarded as the most reliable, i.e., as "hard data" to be broadly applied since the data are directly observed with higher resolution from the reservoir, even though the coverage is sparser than seismic data coverage. Petrophysics data such as core

analysis, temperature log, caliper log, density log, dipmeter log, and image log data have been individually introduced in Sect. 2.2.3 in order to have a deep knowledge of well logs that predict the fractures in the reservoir.

Backus (1962) average theory is the basis of forming effective media. A perfectly welded layered medium can be approximated as an effective TI medium (a transversely isotropic medium with a symmetric axis). It indicates that an algebraic calculation is probably applicable for the rock stiffnesses of a medium. In Sect. 2.3, three effective fractured anisotropic media stiffnesses (VTI, HTI, and orthorhombic) have been calculated and presented. The calculations show that a fractured rock stiffness can be algebraically decomposed into the fracture stiffness and the host medium stiffness, and vice versa. Analogously, a fractured medium is equivalent to a fracture feature combined with a homogeneous host medium, whereas a fracture feature plus a homogeneous host medium is equivalent to a fractured medium. In other words, a fracture can be obtained by taking a fractured medium and subtracting the host medium, or, a host medium can be obtained by getting rid of the fracture from the fractured medium. In some sense, the fractured medium with five independent moduli exhibits properties of a transversely isotropic medium with a symmetry axis.

The linear slip interface theory (Schoenberg 1980) has been studied and it is regarded as one of perfect fracture model. Pyrak-Nolte et al. (1990a, b) and Hsu and Schoenberg (1993) conducted laboratory experiments to validate the model. As a result, a horizontally fractured medium is effectively formed by a linear slip interface embedded in a homogeneous isotropic host medium. A vertically fractured medium is effectively constructed by a vertical linear slip interface inserted in a homogeneous isotropic host medium. Furthermore, the orthogonally fractured media are effectively made from a vertical linear slip interface and a transversely isotropic medium with a vertical symmetric axis (VTI) and a homogeneous host medium.

The anisotropy problem has been discussed. Based on effective medium theory, the Schoenberg-Muir calculus theory and the mechanism of the fracture formation, it has been confirmed that fractured media have the characteristics of a transversely isotropic medium and that the anisotropy is an induced anisotropy caused by tectonic movement, rather than an intrinsic anisotropy formed by natural deposition.

The anisotropy issue has been discussed and it can be described by the parameters γ , ε and δ with five independent moduli (Thomsen 1986). The fractured medium has five independent moduli that can be expressed in terms of fracture weaknesses Δ_N and Δ_T that express the fracture in terms of induced anisotropic problem. There is a way to transform the intrinsic anisotropic parameters γ , ε , δ and fracture anisotropic description Δ_N and Δ_T into each other.

The boundary conditions control the seismic reflection and transmission coefficient values at the interface. Two sets of PP and PS reflection and transmission coefficients formulae have been presented in the last section. One is for a perfectly welded contact interface boundary and the other is for an imperfectly welded (nonwelded) contact interface boundary. The imperfectly welded (nonwelded) boundary conditions require that the kinematic displacements are discontinuous

across the interface, whereas the dynamic stresses are continuous across the interface. These boundary conditions completely constrain all waves at the fracture interface. It is also demonstrated that the reflection coefficients of the nonwelded boundary are frequency dependent. In addition, for a wave normally incident propagating in a uniform homogeneous isotropic medium, there is no reflection at a perfectly welded boundary that has no impedance contrast. However, there are reflections at an imperfectly welded (nonwelded) contact boundary, since a fracture manifests itself a reflector, and the reflected waves are caused by a displacement discontinuity across the fracture, instead of by the impedance contrast at the fracture interface.

This chapter mainly concludes that a linear slip interface can model a fracture regardless of its fracture shape or microstructure, and it satisfies the nonwelded contact boundary conditions when a wave impinges on it. The fractured medium is composed by the fracture and homogeneous isotropic host medium, and vice versa. The composed horizontally and vertically fractured medium with five independent moduli and the orthogonally fractured medium with nine independent moduli shows that the fractured medium present the medium properties of a transversely isotropic medium with a symmetric axis (TI).

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Medium Inversion

Cui, X.; Lines, L.; Krebs, E.S.; Peng, S.

2018, XXVII, 153 p. 56 illus., 32 illus. in color., Hardcover

ISBN: 978-981-10-3583-8