Chapter 2
Robust Stochastic Stability

This chapter investigates the robust output feedback H_∞ control for a class of uncertain Markovian jump linear systems with mode-dependent time-varying time delays. With known bounds of the system uncertainties and the control gain variations, we develop the sufficient conditions to guarantee the robust stochastic stability and the γ-disturbance H_∞ attenuation for the closed-loop system. These conditions can be solved by `LMI Toolbox` efficiently. Note here that the control design is based on the measured Markovian jumping parameter r_t^o that may be inconsistent with the true jumping parameter r_t due to the measurement noises.

2.1 Introduction

Robust stability for time-delayed Markovian jump systems with uncertainties has always been a challenging problem and has been widely investigated so far. In this field H_∞ design has been one popular tool for uncertain delayed Markovian jump system due to its capability of dealing with disturbance attenuation [2, 7, 8]. For example, in [7], the results for the robust stochastic stability and γ-suboptimal H_∞ state-feedback controller design were presented. In [2], a sufficient condition for robust stochastic stability and H_∞-disturbance attenuation was derived for a class of uncertain delayed Markovian jump linear systems based on the Lyapunov functional method, where the uncertainties are of the norm-bounded type. In [8] the delay-dependent H_∞ control problem was considered by adopting a descriptor model transformation method and a new bounding inequality.

In most existing works, the jumping parameters are often assumed to be precisely known. This assumption is usually not true in practice while the system states can often be observed. Therefore, the Wonham filter can be used to estimate the jumping parameters using the given system matrices. To address this problem, the adaptive stabilization was studied in [6], where the existence condition and the adaptive certainty

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equivalence feedback control were proposed by the parameter estimation technique of nonlinear filters. On the other hand, imprecise measurements are often present in analog systems and quantization error sometimes can not be ignored in digital control systems, making precise control implementation almost impossible. To make it worse, the overall systems will have poor stability margins if these robust control strategies are not properly implemented, which applies to common techniques such as H_∞, l_1 or μ synthesis, etc.

On a parallel line, time delays often exist in practical systems such as mechanical systems, chemical processes, neural networks. Delays can deteriorate the system performance or even unstabilize the system. For the stability analysis and controller design of such delayed systems the Lyapunov-Krasovskii functionals (LKFs) approaches are widely used [14–16]. In order to reduce the conservatism caused by model transformations and inequalities, many new techniques were proposed for uncertain time delay systems [11, 12, 22, 23]. In [24], the free-weighting matrix method was proposed to bound the cross product terms and it can reduce the conservatism greatly.

In this chapter, we consider the problem of robust output-feedback H_∞ control for a class of uncertain Markovian jump linear systems with mode-dependent time-varying delays. We also consider the measurement errors of the jumping parameters, which are always inevitable due to the detection delays and false alarm of the identification algorithms [20]. The robust stochastic stability analysis and H_∞ disturbance attenuation design are given by using the measurement value of the jumping parameters directly.

2.2 Uncertain Markovian Jump Linear Systems with Time Delays

Consider the following uncertain Markovian jump linear stochastic systems with mode-dependent time-varying delays,

$$\begin{cases} \dot{x}(t) = [A_1(r_t) + \Delta_{A_1}(r_t, t)]x(t) + [A_2(r_t) + \Delta_{A_2}(r_t, t)]x(t - \tau_{r_t}(t)) \\ \qquad +[B_1(r_t) + \Delta_{B_1}(r_t, t)]u(t) + B_2(r_t)w(t), \\ z(t) = [C(r_t) + \Delta_C(r_t, t)]x(t), \\ x(s) = f(s), \quad r_s = r_0, \quad s \in [-2\mu, \ 0], \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^{m_3}$ $u(t) \in \mathbb{R}^{m_1}$ are the system states, system outputs, and control inputs, respectively. $A_1(r_t) \in R^{n\times n}$, $A_2(r_t) \in \mathbb{R}^{n\times n}$, $B_1(r_t) \in \mathbb{R}^{n\times m_1}$, $B_2(r_t) \in \mathbb{R}^{n\times m_2}$, $C(r_t) \in \mathbb{R}^{m_3\times n}$ are known real matrices denoting the nominal system parameters, and $\Delta_{A_1}(r_t, t) \in R^{n\times n}$, $\Delta_{A_2}(r_t, t) \in R^{n\times n}$, $\Delta_{B_1}(r_t, t) \in R^{n\times m_1}$, $\Delta_C(r_t, t) \in R^{m_3\times n}$ are unknown matrices representing the model uncertainties [2, 7, 9]. $w(t) \in \mathbb{R}^{m_2}$ is the exogenous disturbance input which satisfies $w(t) \in L_2[0, \infty)$. $f(t) \in \mathbb{R}^n$ is a continuous function denoting the initial states. r_t is a continuous-time

Markov chain that takes value in finite set $\mathscr{S} = \{1, 2, \ldots, N\}$ with the transition rate matrix Π defined in (1.3). $\tau_{r_t}(t)$ represents the mode-dependent time-varying delay that satisfies

$$0 < \tau_{r_t}(t) \leq \mu_{r_t} \leq \mu < \infty, \quad \dot{\tau}_{r_t}(t) \leq h_{r_t} < 1, \quad \forall\, r_t \in \mathscr{S} \tag{2.2}$$

where μ_{r_t} and h_{r_t} are upper bounds of $\tau_{r_t}(t)$ and $\dot{\tau}_{r_t}(t)$, for given $r_t \in \mathscr{S}$. μ is the common upper bounded and can be set as $\mu = \max\limits_{i \in \mathscr{S}}\{\mu_i\}$.

The following assumption is necessary to establish the main results.

Assumption 2.1 The uncertain parameters can be written as follows [27]:

$$\begin{aligned}
\Delta_{A_1}(r_t, t) &= H_1(r_t)F(r_t, t)E_1(r_t),\\
\Delta_{A_2}(r_t, t) &= H_1(r_t)F(r_t, t)E_2(r_t),\\
\Delta_{B_1}(r_t, t) &= H_1(r_t)F(r_t, t)E_3(r_t),\\
\Delta_{C}(r_t, t) &= H_2(r_t)F(r_t, t)E_4(r_t),
\end{aligned}$$

where $H_1(r_t) \in \mathbb{R}^{n \times n_f}$, $H_2(r_t) \in \mathbb{R}^{m_3 \times n_f}$, $E_1(r_t) \in \mathbb{R}^{n_f \times n}$, $E_2(r_t) \in \mathbb{R}^{n_f \times n}$, $E_3(r_t) \in \mathbb{R}^{n_f \times m_1}$ and $E_4(r_t) \in \mathbb{R}^{n_f \times n}$ are known real matrices, while $F(r_t, t) \in \mathbb{R}^{n_f \times n_f}$ are the uncertain matrix functions satisfying

$$F^T(r_t, t)F(r_t, t) \leq I, \quad \forall\, r_t \in \mathscr{S}. \tag{2.3}$$

Remark 2.1 As an extension of the **matching condition**, the structure of the uncertainties in Assumption 2.1 is widely used in the literature on robust control and robust filter, see e.g. [1–4, 7–9, 27]. How the uncertain matrix functions $F(r_t, t)$ affect the nominal parameters $A_1(r_t)$, $A_2(r_t)$, $B_1(r_t)$, $C(r_t)$ can be characterized by $H_1(r_t)$, $H_2(r_t)$, $E_1(r_t)$, $E_2(r_t)$, $E_3(r_t)$ and $E_4(r_t)$

In practical control systems, the environmental noises, external disturbance and other modelling uncertainties unavoidably cause detection delays and false alarms when we identify the activated system mode. Similar to [19, 20], we adopt two stochastic processes to describe the above phenomena. One process, denoted by r_t, is used to characterize the actual system mode in (2.1), and the other one, denoted by r_t^o, represents the mode we observed or measured in the practical systems. The difference between r_t and r_t^o are mainly caused by two kinds of measurement errors, i.e. the detection delays and false alarms. The following models are used to describe these measurement errors.

- The probability of jump from i to j conditional on r_t, denoted by r_t^o, can be written as

$$P\left\{ r_{t+\Delta}^o = j \,\middle|\, \begin{array}{l} r_s^o = i \\ r_{t_0} = j \\ r_{t_0-} = i \\ s \in [t_0, t] \end{array} \right\} = \begin{cases} \pi_{ij}^0 \Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}^0 \Delta + o(\Delta), & i = j \end{cases} \tag{2.4}$$

In fact, r_t^o can be seen as an exponentially distributed random variable with rate π_{ij}^o. The parameters π_{ij}^o can be obtained by evaluating observed sample paths, and

$$\pi_{ii}^0 = -\sum_{j \neq i} \pi_{ij}^0, \quad (\pi_{ij}^0 \geq 0,\, j \neq i). \tag{2.5}$$

- Although r_t remains at i, r_t^o can still occasionally transmit from i to j. Similarly, we also use an independent exponential distribution with mean $1/\pi_{ij}^1$ to describe this scenario

$$P\left\{ r_{t+\Delta}^o = j \,\middle|\, \begin{array}{l} r_s^o = i \\ s \in [t_0, t] \end{array} \right\} = \begin{cases} \pi_{ij}^1 \Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}^1 \Delta + o(\Delta), & i = j \end{cases} \tag{2.6}$$

where π_{ij}^1 is the false alarm rate, which can also be evaluated from observed sample paths, and satisfies

$$\pi_{ii}^1 = -\sum_{j \neq i} \pi_{ij}^1, \quad (\pi_{ij}^1 \geq 0,\, j \neq i). \tag{2.7}$$

For simplicity, we simplify $M(r_t^o, r_t, t)$ as $M_{ji}(t)$ when $r_t^o = j, r_t = i,\ j, i \in \mathscr{S}$, and let the initial time $t_0 = 0$, then the initial conditions can be written as $x(0) = x_0$, r_0 and r_0^o. Note that all these initial value are deterministic.

The following dynamic output feedback controllers are to be designed.

$$\begin{cases} \dot{\widehat{x}}(t) = A_3(r_t^o)\widehat{x}(t) + B_3(r_t^o)z(t), \\ u(t) = K(r_t^o)\widehat{x}(t), \end{cases} \tag{2.8}$$

where $\widehat{x}(t) \in \mathbb{R}^n$ is the states of the controllers, and $A_3(r_t^o)$, $B_3(r_t^o)$, $K(r_t^o)$ are the unknown matrices of the controllers with appropriate dimensions to be determined.

Practically, it is impossible to implement the above controllers precisely. So, in this chapter, the controllers with imprecise implementation are described as

$$u(t) = [I + \alpha(r_t)\phi(r_t, t)]K(r_t^o)\widehat{x}(t), \tag{2.9}$$

where $\alpha(r_t)\phi(r_t, t)$ represent the additive errors that affect the controller gains. $\alpha(r_t)$ is a positive constant and $\phi(r_t, t)$ satisfies

$$\phi^T(r_t, t)\phi(r_t, t) \le I, \quad \forall r_t \in \mathscr{S}.$$

Remark 2.2 Notice that the designed controllers are dependent on the measured jumping parameter r_t^o. To reconfigure the controllers, the switching of controller gains $K(r_t^o)$ is based on r_t^o. However, the evolution of the dynamic systems follows the actual mode r_t, and therefore, the variations of the controller gains depend on r_t and have nothing to do with r_t^o.

Apply the control law (2.8) to system (2.1) and denote $\xi(t) = [x^T(t), \widehat{x}^T(t)]^T$, we obtain the closed-loop system

$$\begin{cases} \dot{\xi}(t) = \overline{A}_1(r_t^o, r_t, t)\xi(t) + \overline{A}_2(r_t)I_0\xi(t - \tau_{r_t}(t)) + \overline{B}_2(r_t)w(t), \\ z(t) = [C(r_t) + \Delta_C(r_t, t)]I_0\xi(t), \\ I_0\xi(s) = f(s), \quad r_s = r_0, \quad s \in [-2\mu,\ 0], \end{cases} \tag{2.10}$$

where

$$\overline{A}_{1ji} = \begin{bmatrix} A_{1i} + \Delta_{A_{1i}}(t) & \left(B_{1i} + \Delta_{B_{1i}}(t)\right)(I + \alpha_i\phi_i(t))\,K_j \\ B_{3j}\left(C_i + \Delta_{C_i}(t)\right) & A_{3j} \end{bmatrix} \in \mathbb{R}^{2n\times 2n},$$

$$\overline{A}_{2i} = \begin{bmatrix} A_{2i} + \Delta_{A_{2i}}(t) \\ 0 \end{bmatrix} \in \mathbb{R}^{2n\times n}, \quad \overline{B}_{2i} = \begin{bmatrix} B_{2i} \\ 0 \end{bmatrix} \in \mathbb{R}^{2n\times m_2},$$

$$I_0 = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n\times 2n}, \, for\ each\ r_t^o = j, r_t = i, \quad \forall i, j \in \mathscr{S}.$$

The objectives of this chapter are as follows:

(i) *Robust stabilization*: Determine the nominal controller gains $K(r_t^o)$ in (2.9) and establish sufficient conditions for the system (2.1) such that the overall closed-loop system (2.10) is robustly exponentially stable in the mean square sense;
(ii) *H_∞ control problem*: Given a constant scalar $\gamma > 0$, determine the nominal control gain $K(r_t^o)$ in (2.9) and establish the sufficient conditions such that the resulting closed-loop system (2.10) is robustly stochastically stable with disturbance attenuation level γ under zero initial condition ($x(0) = 0$), that is

$$J = E\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right]dt\right\} < 0, \forall\, w(t) \neq 0, w(t) \in \mathscr{L}_2[0, \infty). \tag{2.11}$$

2.3 Robust Control

In this section, we study the exponential mean-square stability of the time-delayed uncertain Markovian jump linear system (2.10) with $w(t) = 0$. The following lemmas are needed in deriving the stability conditions.

Lemma 2.1 *[17] Schur complement: Consider the following matrix of appropriate dimension*

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}, \qquad Q_{22} > 0, \tag{2.12}$$

then Q is positive definite if and only if $Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T > 0$.

Lemma 2.2 *[25] Given matrices $Q = Q^T$, H, E and $R = R^T > 0$ of appropriate dimensions, then*

$$Q + HFE + E^T F^T H^T < 0$$

for all F satisfying $F^T F \leq R$, if and only if there exits some $\rho > 0$ such that

$$Q + \rho H H^T + \rho^{-1} E^T R E < 0.$$

Lemma 2.3 *[19] The infinitesimal generator $\mathfrak{L}$ of random processes can be defined as follows.*

For the following jump systems

$$\dot{x}(t) = f(x(t), u(t), r_t^o, r_t, t),$$

suppose that $f(\cdot)$ is continuous for all its variables within their domain of definition, and satisfies the usual growth and smoothness hypothesis, $g(x(t), r_t^o, r_t, t)$ is a scalar continuous function of t and $x(t)$, $\forall r_t^o, r_t \in \mathscr{S}$. Then, the infinitesimal generator $\mathfrak{L}$ of the random process $\{x(t), r_t^o, r_t, t\}$ can be described as follows:

- *For $r_t^o = r_t = i$, we have*

$$\begin{aligned}
&\mathfrak{L}g(x(t), i, i, t) \\
&= \lim_{\triangle \to 0} \frac{1}{\triangle} \left[E\left\{ g(x(t+\triangle), r_{t+\triangle}^o, r_{t+\triangle}, t+\triangle) | x(t) = x, r_t^o = i, r_t = i, t \right\} \right. \\
&\quad \left. - g(x, i, i, t) \right] \\
&= g_t(x, i, i, t) + f^T(x, u(t), i, i, t) g_x(x, i, i, t) + \sum_{j=1}^{N} \pi_{ij} g(x, i, j, t) \qquad (2.13) \\
&\quad + \sum_{j=1}^{N} \pi_{ij}^1 g(x, j, i, t).
\end{aligned}$$

- *For $r_t^o = j \neq r_t = i$, we have*

$$
\begin{aligned}
&\mathfrak{L}g(x(t), j, i, t)\\
&= \lim_{\triangle \to 0} \frac{1}{\triangle}\left[E\left\{g(x(t+\triangle), r^o_{t+\triangle}, r_{t+\triangle}, t+\triangle)|x(t)=x, r_t^o = j, r_t = i, t\right\}\right.\\
&\quad \left. -g(x, j, i, t)\right]\\
&= g_t(x, j, i, t) + f^T(x, u(t), j, i, t) g_x(x, j, i, t)\\
&\quad + \pi^0_{ji} g(x, i, i, t) - \pi^0_{ji} g(x, j, i, t).
\end{aligned}
\tag{2.14}
$$

Theorem 2.1 *Consider the uncertain delayed Markovian jump linear system with $w(t) = 0$. If there exist symmetric positive-definite matrices P_{ij}, Q, Z, positive semi-definite matrices X_{ji}, real matrices K_j, Y_{ji}, T_{ji} that are of appropriate dimensions and positive constants ρ_{1ji}, ρ_{2ji}, ρ_{3ji} such that*

$$
\overline{\mathbf{W}}_{ji} = \begin{bmatrix}
L_1 & P_{1ji}B_{1i} + \rho_{3ji}E_{1i}^T E_{3i} & 0 & P_{1ji}H_{1i} & 0 & K_j^T\\
L_2 & 0 & K_j^T & 0 & V_{ji}^T H_{2i} & 0\\
L_3 & \rho_{3ji}E_{2i}^T E_{3i} & 0 & 0 & 0 & 0\\
L_4 & \mu Z B_{1i} & 0 & \mu Z H_{1i} & 0 & 0\\
L_5 & -I + \rho_{3ji}E_{3i}^T E_{3i} & 0 & 0 & 0 & 0\\
L_6 & 0 & -I + \rho_{1ji}\alpha_i^2 I & 0 & 0 & 0\\
L_7 & 0 & 0 & -\rho_{3ji} I & 0 & 0\\
L_8 & 0 & 0 & 0 & -\rho_{2ji} I & 0\\
L_9 & 0 & 0 & 0 & 0 & -\rho_{1ji} I
\end{bmatrix} < 0 \tag{2.15}
$$

$$
\Gamma_{ji} = \begin{bmatrix}
X_{1ji} & X_{2ji} & I_0^T Y_{ji}\\
X_{2ji}^T & X_{3ji} & T_{ji}\\
Y_{ji}^T I_0 & T_{ji}^T & Z
\end{bmatrix} \geq 0, \quad \forall\, i, j \in \mathscr{S}. \tag{2.16}
$$

where

$$
\begin{bmatrix} L_1\\ L_2\\ L_3\\ L_4\\ L_5\\ L_6\\ L_7\\ L_8\\ L_9 \end{bmatrix}
= \begin{bmatrix}
\Phi_{11} + \rho_{3ji}E_{1i}^T E_{1i} + \rho_{2ji}E_{4i}^T E_{4i} & C_i^T V_{ji} + \mu X_{1ji}^2 & \Phi_{13} + \rho_{3ji}E_{1i}^T E_{2i} & \mu A_{1i}^T Z\\
V_{ji}^T C_i + \mu X_{1ji}^{2^T} & U_{ji} + U_{ji}^T + \Phi_{22} & \mu X_{2ji}^2 & 0\\
\Phi_{13}^T + \rho_{3ji}E_{2i}^T E_{1i} & \mu X_{2ji}^{2^T} & \Phi_{33} + \rho_{3ji}E_{2i}^T E_{2i} & \mu A_{2i}^T Z\\
\mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z\\
B_{1i}^T P_{1ji} + \rho_{3ji}E_{3i}^T E_{1i} & 0 & \rho_{3ji}E_{3i}^T E_{2i} & \mu B_{1i}^T Z\\
0 & K_j & 0 & 0\\
H_{1i}^T P_{1ji} & 0 & 0 & \mu H_{1i}^T Z\\
0 & H_{2i}^T V_{ji} & 0 & 0\\
K_j & 0 & 0 & 0
\end{bmatrix},
$$

$$X_{ji} = \begin{bmatrix} X_{1ji} & X_{2ji} \\ X_{2ji}^T & X_{3ji} \end{bmatrix} = \begin{bmatrix} X_{1ji}^1 & X_{1ji}^2 & X_{2ji}^1 \\ {X^2}_{1ji}^T & X_{1ji}^3 & X_{2ji}^2 \\ {X^1}_{2ji}^T & {X^2}_{2ji}^T & X_{3ji} \end{bmatrix}$$

with

$$\Phi_{11} = \begin{cases} \text{if } \ j = i \\ A_{1i}^T P_{1ii} + P_{1ii} A_{1i} + \sum_{j=1}^{N} \pi_{ij} P_{1ij} + \sum_{j=1}^{N} \pi_{ij}^1 P_{1ji} + Y_{ii} + Y_{ii}^T + (1+\eta\mu) Q + \mu X_{1ii}^1 \\ \text{if } \ j \neq i \\ A_{1i}^T P_{1ji} + P_{1ji} A_{1i} + \pi_{ji}^0 (P_{1ii} - P_{1ji}) + Y_{ji} + Y_{ji}^T + (1 + \eta\mu) Q + \mu X_{1ji}^1 \end{cases}$$

$$\Phi_{22} = \begin{cases} \sum_{j=1}^{N} \pi_{ij} P_{2ij} + \sum_{j=1}^{N} \pi_{ij}^1 P_{2ji} + \mu X_{1ii}^3, & \text{if } \ j = i \\ \pi_{ji}^0 (P_{2ii} - P_{2ji}) + \mu X_{1ji}^3, & \text{if } \ j \neq i \end{cases}$$

$$\Phi_{13} = P_{1ji} A_{2i} - Y_{ji} + T_{ji}^T + \mu X_{2ji}^1, \quad \Phi_{33} = -T_{ji} - T_{ji}^T - (1 - h_i) Q + \mu X_{3ji},$$

$$\eta = \max_{i \in \mathscr{S}} \{|\pi_{ii}|\}, \quad V_{ji} = B_{3j}^T P_{2ji}, \quad U_{ji} = A_{3j}^T P_{2ji},$$

then the systems (2.10) are exponentially stable in the mean-square sense.

Proof Consider the nominal time-delayed jump linear system Σ_0 without disturbance:

$$\Sigma_0 : \quad \begin{cases} \dot{\xi}(t) = \widehat{A}_1(r_t^o, r_t)\xi(t) + \widehat{A}_2(r_t) I_0 \xi(t - \tau_{r_t}(t)), \\ z(t) = [C(r_t) + \Delta_C(r_t, t)] I_0 \xi(t), \\ I_0 \xi(s) = f(s), \quad r_s = r_0, \quad s \in [-2\mu, \ 0] \end{cases} \tag{2.17}$$

where

$$\widehat{A}_1(r_t^o, r_t) = \begin{bmatrix} A_1(r_t) & B_1(r_t) K(r_t^o) \\ B_3(r_t^o) C(r_t) & A_3(r_t^o) \end{bmatrix} \in \mathbb{R}^{2n \times 2n},$$

$$\widehat{A}_2(r_t) = \begin{bmatrix} A_2(r_t) \\ 0 \end{bmatrix} \in \mathbb{R}^{2n \times n}.$$

It is worth pointing out that $\{(\xi(t), r_t^o, r_t), \ t \geq 0\}$ is non-Markovian due to the time delay $\tau_{r_t}(t)$. However, if we define a process $\{(\xi_t, r_t^o, r_t), t \geq 0)\}$ that taking values in $\mathscr{C}_0$, where $\xi_t = \{\xi(\theta + t) \mid -2\mu \leq \theta \leq 0\}$, $\mathscr{C}_0 = \bigcup_{i,j \in \mathscr{S}} \mathscr{C}[-2\mu, 0] \times \{i, j\}$, and $\mathscr{C}[-2\mu, 0]$ denotes the space of continuous functions on interval $[-2\mu, 0]$, then we can show that $\{(\xi_t, r_t^o, r_t), t \geq 0)\}$ is a strong Markov process with state space $\mathscr{C}_0$ [27].

Consider the following LKFs candidate:

$$V(\xi_t, r_t^o, r_t, t) = V_1 + V_2 + V_3 + V_4, \tag{2.18}$$

where

$$\begin{aligned}
V_1 &= \xi^T(t)P(r_t^o, r_t)\xi(t) = x^T(t)P_1(r_t^o, r_t)x(t) + \widehat{x}^T(t)P_2(r_t^o, r_t)\widehat{x}(t),\\
V_2 &= \int_{t-\tau_{r_t}(t)}^{t} x^T(s)Qx(s)ds\\
V_3 &= \eta\int_{-\mu}^{0}\int_{t+\theta}^{t} x^T(s)Qx(s)dsd\theta\\
V_4 &= \int_{-\mu}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\theta.
\end{aligned}$$

For both cases of $r_t^o = r_t = i$ and $r_t^o = j, r_t = i, j \neq i$, we obtain their respective results according to the definition of the infinitesimal generator $\mathfrak{L}$ in Lemma 2.3.

Case I. $r_t^o = r_t = i$

We can find that

$$\begin{aligned}
\mathfrak{L}V_1 &= \xi^T(t)\left[\widehat{A}_{1ii}^T P_{ii} + P_{ii}\widehat{A}_{1ii} + \sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^1 P_{ji}\right]\xi(t)\\
&\quad + \xi^T(t)P_{ii}\widehat{A}_{2i}x(t-\tau_i(t)) + x^T(t-\tau_i(t))\widehat{A}_{2i}^T P_{ii}\xi(t),\\
\mathfrak{L}V_2 &= \xi^T(t)I_0^T Q I_0\xi(t) - (1-\dot{\tau}_i(t))x^T(t-\tau_i(t))Qx(t-\tau_i(t))\\
&\quad + \sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t} x^T(s)Qx(s)ds\\
&\le \xi^T(t)I_0^T Q I_0\xi(t) - (1-h_i)x^T(t-\tau_i(t))Qx(t-\tau_i(t))\\
&\quad + \sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t} x^T(s)Qx(s)ds,\\
\mathfrak{L}V_3 &= \eta\mu\xi^T(t)I_0^T Q I_0\xi(t) - \eta\int_{t-\mu}^{t} x^T(s)Qx(s)ds,\\
\mathfrak{L}V_4 &= \mu\dot{\xi}^T(t)I_0^T Z I_0\dot{\xi}(t) - \int_{t-\mu}^{t}\dot{x}^T(s)Z\dot{x}(s)ds\\
&\le \mu\dot{\xi}^T(t)I_0^T Z I_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds.
\end{aligned}$$

Combining (1.4) and (2.2), we obtain

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}x^T(s)Qx(s)ds \le \sum_{j=1,j\neq i}^{N}\pi_{ij}\int_{t-\mu}^{t}x^T(s)Qx(s)ds$$
$$= -\pi_{ii}\int_{t-\mu}^{t}x^T(s)Qx(s)ds \le \eta\int_{t-\mu}^{t}x^T(s)Qx(s)ds. \tag{2.19}$$

To overcome the conservativeness in selecting the optimal weighting matrices between the terms in the Newton-Leibniz formula, the following condition is presented [24]:

$$2\left[x^T(t)Y + x^T(t-d(t))T\right]\left[x(t) - \int_{t-d(t)}^{t}\dot{x}(s)ds - x(t-d(t))\right] = 0,$$

where the free weighting matrices Y and T indicate the relationship between the terms in the above formula, and they can easily be selected by means of linear matrix inequalities.

The following conditions are also employed to complete the proof.

$$\mu\zeta^T(t)X(r_t^o, r_t)\zeta(t) - \int_{t-\tau_{p_{mj,t}}(t)}^{t}\zeta^T(t)X(r_t^o, r_t)\zeta(t)ds \ge 0, \tag{2.20}$$

$$2\left[\xi^T(t)I_0^TY(r_t^o, r_t) + x^T(t-\tau_{r_t}(t))T(r_t^o, r_t)\right]\times$$
$$\left[I_0\xi(t) - \int_{t-\tau_{r_t}(t)}^{t}\dot{x}(s)ds - x(t-\tau_{r_t}(t))\right] = 0, \tag{2.21}$$

where $\zeta^T(t) = [\xi^T(t) \;\; x^T(t-\tau_{r_t}(t))]$, and $X(r_t^o, r_t)$ are defined in Theorem 2.1.

We have

$$\mathfrak{L}V(\xi_t, i, i, t) \le \xi^T(t)\left[\widehat{A}_{1ii}^T P_{ii} + P_{ii}\widehat{A}_{1ii} + \sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^1 P_{ji}\right]\xi(t)$$
$$+ \xi^T(t)P_{ii}\widehat{A}_{2i}x(t-\tau_i(t)) + x^T(t-\tau_i(t))\widehat{A}_{2i}^T P_{ii}\xi(t)$$
$$+ (1+\eta\mu)\xi^T(t)I_0^TQI_0\xi(t) - (1-h_i)x^T(t-\tau_i(t))Qx(t-\tau_i(t))$$
$$+ \mu\dot{\xi}^T(t)I_0^TZI_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds$$
$$+ 2\left[\xi^T(t)I_0^TY(r_t^o, r_t) + x^T(t-\tau_{r_t}(t))T(r_t^o, r_t)\right]\Big[I_0\xi(t)$$

$$
\begin{aligned}
&- \int_{t-\tau_{r_t}(t)}^{t} \dot{x}(s)ds - x(t-\tau_{r_t}(t))\Big] + \mu\zeta^T(t)X(r_t^o, r_t)\zeta(t) \\
&- \int_{t-\tau_{r_t}(t)}^{t} \zeta^T(t)X(r_t^o, r_t)\zeta(t)ds \\
&= \zeta^T(t)\Xi_{ii}\zeta(t) - \int_{t-\tau_i(t)}^{t} \chi^T(t,s)\Gamma_{ii}\chi(t,s)ds, \qquad (2.22)
\end{aligned}
$$

where

$$
\begin{aligned}
&\chi^T(t,s) = [\xi^T(t) \;\; x^T(t-\tau_i(t)) \;\; \dot{x}^T(s)], \\
&\Xi_{ii} = \begin{bmatrix} \widehat{\Phi}_{11} + \mu\widehat{A}_{1ii}^T I_0^T Z I_0 \widehat{A}_{1ii} & \widehat{\Phi}_{12} + \mu\widehat{A}_{1ii}^T I_0^T Z I_0 \widehat{A}_{2i} \\ \widehat{\Phi}_{12}^T + \mu\widehat{A}_{2i}^T I_0^T Z I_0 \widehat{A}_{1ii} & \widehat{\Phi}_{22} + \widehat{A}_{2i}^T I_0^T Z I_0 \widehat{A}_{2i} \end{bmatrix}, \\
&\widehat{\Phi}_{11} = \begin{cases} \widehat{A}_{1ii}^T P_{ii} + P_{ii}\widehat{A}_{1ii} + \sum_{j=1}^{N} \pi_{ij}P_{ij} + \sum_{j=1}^{N} \pi_{ij}^1 P_{ji} + I_0^T Y_{ii} I_0 \\ \qquad + I_0 Y_{ii}^T I_0^T + (1+\eta\mu)I_0^T Q I_0 + \mu X_{1ii}, & (if \;\; j = i) \\ \widehat{A}_{1ji}^T P_{ji} + P_{ji}\widehat{A}_{1ji} + q_{ji}^0(P_{ii} - P_{ji}) + I_0^T Y_{ji} I_0 + I_0 Y_{ji}^T I_0^T \\ \qquad + (1+\eta\mu)I_0^T Q I_0 + \mu X_{1ji}, & (if \;\; j \neq i) \end{cases} \\
&\widehat{\Phi}_{12} = P_{ji}\widehat{A}_{2i} - I_0^T Y_{ji} + I_0^T T_{ji}^T + \mu X_{2ji}, \qquad (2.23) \\
&\widehat{\Phi}_{22} = -T_{ji} - T_{ji}^T - (1-h_i)Q + \mu X_{3ji}, \\
&\eta = \max_{i\in\mathscr{S}}\{|\pi_{ii}|\}.
\end{aligned}
$$

If $\Xi_{ii} < 0$, $\Gamma_{ii} \geq 0$, then for each $i \in S$ and any scalar $\beta > 0$, we obtain

$$
\mathfrak{L}[e^{\beta t}V(\xi_t, i, i, t)] \leq -\alpha_1 e^{\beta t}\|\xi(t)\|^2 + \beta e^{\beta t}V(\xi_t, i, i, t), \;\; \forall\, i \in \mathscr{S}, \beta > 0, \qquad (2.24)
$$

where $\alpha_1 = \min_{i\in\mathscr{S}}\{\lambda_{min}(-\Xi_{ii})\}$.

Similar to [27], we can verify that

$$
\begin{aligned}
V(\xi_t, i, i, t) \leq\; & \lambda_{max}(P_{ii})\|\xi(t)\|^2 + \lambda_{max}(Q)\int_{t-\tau_i(t)}^{t} \|x(s)\|^2 ds \\
&+ \eta\lambda_{max}(Q)\int_{-\mu}^{0}\int_{t+\theta}^{t} \|x(s)\|^2 ds d\theta + \lambda_{max}(Z)\int_{-\mu}^{0}\int_{t+\theta}^{t} \|\dot{x}(s)\|^2 ds d\theta \\
\leq\; & \lambda_{max}(P_{ii})\|\xi(t)\|^2 + (\eta\mu + 1)\lambda_{max}(Q)\int_{t-\mu}^{t} \|x(s)\|^2 ds \\
&+ \mu\lambda_{max}(Z)\int_{t-\mu}^{t} \|\dot{x}(s)\|^2 ds.
\end{aligned}
$$

Noticing that in nominal system Σ_0:

$$\dot{\xi}(t) = \widehat{A}_{1ii}\xi(t) + \widehat{A}_{2i} I_0 \xi(t - \tau_{r_t}(t))$$

and letting $\alpha_2 = \max_{i \in \mathscr{S}} \left\{ 2\|\widehat{A}_{1ii}\|^2 \right\}$, $\alpha_3 = \max_{i \in \mathscr{S}} \left\{ 2\|\widehat{A}_{2i}\|^2 \right\}$, it yields

$$\|\dot{\xi}(t)\|^2 \leq \alpha_2 \|\xi(t)\|^2 + \alpha_3 \|x(t - \tau_{r_t}(t))\|^2.$$

This, together with (2.24), gives

$$\begin{aligned} \mathfrak{L}[e^{\beta t} V(\xi_t, i, i, t)] \leq & (-\alpha_1 + \alpha_4 \beta) e^{\beta t} \|\xi(t)\|^2 \\ & + \alpha_3 \mu \lambda_{max}(Z) \beta e^{\beta t} \int_{t-\mu}^{t} \|x(s - \tau_{r_s}(s))\|^2 ds \\ & + \beta e^{\beta t} \big[(\mu \eta + 1) \lambda_{max}(Q) + \alpha_2 \mu \lambda_{max}(Z) \big] \int_{t-\mu}^{t} \|x(s)\|^2 ds, \end{aligned} \tag{2.25}$$

where $\alpha_4 = \max_{i \in \mathscr{S}} \{\lambda_{max}(P_{ii})\}$.

Using Dynkin's formula [18], for any $T > 0, \beta > 0$, and each $r_t^o = r_t = i,\ i \in \mathscr{S}$, it follows that

$$\begin{aligned} & E\big\{ e^{\beta T} V(\xi_T, r_t^o, r_t, T) \,\big|\xi_0, r_0^o, r_0, 0 \,\big\} \\ & = V(\xi_0, r_0^o, r_0, 0) + E\Big\{ \int_0^T \mathfrak{L}[e^{\beta s} V(\xi_s, i, i, s)] ds \,\big|\xi_0, r_0^o, r_0, 0 \,\Big\}. \end{aligned}$$

Since the initial time values $x(0) = x_0, r_0$ and r_0^o are deterministic, ξ_0 is also deterministic. Substituting (2.25) into above gives

$$\begin{aligned} & E\big\{ e^{\beta T} V(\xi_T, r_t^o, r_t, T) \big\} \\ & \leq V(\xi_0, r_0^o, p_0, 0) + E\Bigg\{ (-\alpha_1 + \alpha_4 \beta) \int_0^T e^{\beta t} \|\xi(t)\|^2 dt \\ & + \beta \big[(\mu \eta + 1) \lambda_{max}(Q) + \alpha_2 \mu \lambda_{max}(Z) \big] \int_0^T e^{\beta t} \int_{t-\mu}^{t} \|x(s)\|^2 ds dt \\ & + \alpha_3 \mu \lambda_{max}(Z) \beta \int_0^T e^{\beta t} \int_{t-\mu}^{t} \|x(s - \tau_{r_s}(s))\|^2 ds dt \Bigg\}. \end{aligned} \tag{2.26}$$

Let $\overline{\theta} = t - \tau_i(t)$. The following inequalities

$$\begin{cases} \dot{\tau}_i(t) = \dfrac{d\tau_i(t)}{dt} \leq h_i < 1, \\ dt \leq \dfrac{1}{1-h_i} d\overline{\theta}, \end{cases} \tag{2.27}$$

yields

$$\begin{aligned} &\int_0^T e^{\beta t} \int_{t-\mu}^t \|x(s)\|^2 ds dt \\ &\leq \int_{-\mu}^0 \mu e^{\beta(s+\mu)} \|x(s)\|^2 ds + \int_0^{T-\mu} \mu e^{\beta(s+\mu)} \|x(s)\|^2 ds + \int_{T-\mu}^T \mu e^{\beta(s+\mu)} \|x(s)\|^2 ds \\ &= \mu \int_{-\mu}^T e^{\beta(t+\mu)} \|x(t)\|^2 dt \leq \mu \int_{-\mu}^T e^{\beta(t+\mu)} \|\xi(t)\|^2 dt, \end{aligned} \tag{2.28}$$

$$\begin{aligned} &\int_0^T e^{\beta t} \int_{t-\mu}^t \|x(s - \tau_{r_s}(s))\|^2 ds dt \\ &\leq \int_{-\mu}^0 \mu e^{\beta(s+\mu)} \|x(s - \tau_{r_s}(s))\|^2 ds + \int_0^{T-\mu} \mu e^{\beta(s+\mu)} \|x(s - \tau_{r_s}(s))\|^2 ds \\ &\quad + \int_{T-\mu}^T \mu e^{\beta(s+\mu)} \|x(s - \tau_{r_s}(s))\|^2 ds \\ &= \mu \int_{-\mu}^T e^{\beta(t+\mu)} \|x(t - \tau_{r_t}(t))\|^2 dt \leq \frac{1}{1-h_i} \mu \int_{-2\mu}^T e^{\beta(\overline{\theta}+2\mu)} \|x(\overline{\theta})\|^2 d\overline{\theta} \\ &= \frac{1}{1-h_i} \mu \int_{-2\mu}^T e^{\beta(t+2\mu)} \|x(t)\|^2 dt \leq \frac{1}{1-h_i} \mu \int_{-2\mu}^T e^{\beta(t+2\mu)} \|\xi(t)\|^2 dt. \end{aligned} \tag{2.29}$$

Substituting (2.28) and (2.29) into (2.26) leads to

$$\begin{aligned} &E\big\{e^{\beta T} V(\xi_T, r_t^o, r_t, T)\big\} \\ &\leq V(\xi_0, r_0^o, p_0, 0) + E\bigg\{(-\alpha_1 + \alpha_4\beta) \int_0^T e^{\beta t} \|\xi(t)\|^2 dt + \beta\big[(\mu\eta + 1)\lambda_{max}(Q) \\ &\quad + \alpha_2\mu\lambda_{max}(Z)\big]\mu \int_{-\mu}^T e^{\beta(t+\mu)} \|\xi(t)\|^2 dt + \frac{\alpha_3\mu^2\lambda_{max}(Z)\beta}{1-h_i} \int_{-2\mu}^T e^{\beta(t+2\mu)} \|\xi(t)\|^2 dt\bigg\} \\ &\leq V(\xi_0, r_0^o, r_0, 0) + E\bigg\{\alpha_5\beta e^{\beta\mu} \int_{-\mu}^0 \|\xi(t)\|^2 dt + \alpha_6\beta e^{2\beta\mu} \int_{-2\mu}^0 \|\xi(t)\|^2 dt \\ &\quad + [-\alpha_1 + \alpha_4\beta + \alpha_5\beta e^{\beta\mu} + \alpha_6\beta e^{2\beta\mu}] \int_0^T e^{\beta t} \|\xi(t)\|^2 dt\bigg\}, \end{aligned}$$

where $\alpha_5 = \big[(\mu\eta + 1)\lambda_{max}(Q) + \alpha_2\mu\lambda_{max}(Z)\big]\mu$, and $\alpha_6 = \frac{\alpha_3\mu^2\lambda_{max}(Z)}{1-h_i}$.

Choose $\beta > 0$ such that

$$-\alpha_1 + \alpha_4\beta + \alpha_5\beta e^{\beta\mu} + \alpha_6\beta e^{2\beta\mu} \le 0.$$

Then, we have

$$E\left\{e^{\beta T} V(\xi_T, r_t^o, r_t)\right\} \le c, \tag{2.30}$$

where $c = V(\xi_0, r_0^o, p_0, 0) + E\left\{\alpha_5\beta e^{\beta\mu}\int_{-\mu}^{0}\|\xi(t)\|^2 dt + \alpha_6\beta e^{2\beta\mu}\int_{-2\mu}^{0}\|\xi(t)\|^2 dt\right\}$.

Hence, the LMIs $\Xi_{ii} < 0$, $\Gamma_{ii} \ge 0$ guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square, for $r_t^o = r_t = i, \forall i \in \mathscr{S}$.

Case II. $r_t^o = j, r_t = i$, and $j \ne i$

Following similar lines as in the proof of Case I, we obtain

$$\begin{aligned}
&\mathfrak{L}V(x_t, j, i, t)\\
&\le \xi^T(t)\left[\widehat{A}_{1ji}^T P_{ji} + P_{ji}\widehat{A}_{1ji} + q_{ji}^0(P_{ii} - P_{ji})\right]\xi(t)\\
&+ \xi^T(t)P_{ji}\widehat{A}_{2i}x(t-\tau_i(t)) + x^T(t-\tau_i(t))\widehat{A}_{2i}^T P_{ji}\xi(t)\\
&+ (1+\eta\mu)\xi^T(t)I_0^T Q I_0\xi(t) - (1-h_i)x^T(t-\tau_i(t))Qx(t-\tau_i(t))\\
&+ \mu\dot{\xi}^T(t)I_0^T Z I_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds\\
&\le \xi^T(t)\Xi_{ji}\xi(t) - \int_{t-\tau_i(t)}^{t}\chi^T(t,s)\Gamma_{ji}\chi(t,s)ds,
\end{aligned} \tag{2.31}$$

where

$$\Xi_{ji} = \begin{bmatrix} \widehat{\Phi}_{11} + \mu\widehat{A}_{1ji}^T I_0^T Z I_0\widehat{A}_{1ji} & \widehat{\Phi}_{12} + \mu\widehat{A}_{1ji}^T I_0^T Z I_0\widehat{A}_{2i} \\ \widehat{\Phi}_{12}^T + \mu\widehat{A}_{2i}^T I_0^T Z I_0\widehat{A}_{1ji} & \widehat{\Phi}_{22} + \widehat{A}_{2i}^T I_0^T Z I_0\widehat{A}_{2i} \end{bmatrix},$$

and the LMIs $\Xi_{ji} < 0$, $\Gamma_{ji} \ge 0$ guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square, for $r_t^o = j, r_t = i$, and $j \ne i$, $\forall j, i \in S$.

Applying the Schur complement, one sees that for any $i, j \in S$, $\Xi_{ji} < 0$ implies

$$\begin{bmatrix} \widehat{\Phi}_{11} & \widehat{\Phi}_{12} & \mu\widehat{A}_{1ji}^T I_0^T Z \\ \widehat{\Phi}_{12}^T & \widehat{\Phi}_{22} & \mu\widehat{A}_{2i}^T I_0^T Z \\ \mu Z I_0\widehat{A}_{1ji} & \mu Z I_0\widehat{A}_{2i} & -\mu Z \end{bmatrix} < 0, \tag{2.32}$$

which is equivalent to the following condition:

$$\begin{bmatrix} \Phi_{11} & C_i^T B_{3j}^T P_{2ji} + \mu X_{1ii}^2 & \Phi_{13} & \mu A_{1i}^T Z \\ P_{2ji} B_{3j} C_i + \mu X_{1ii}^{2^T} & A_{3j}^T P_{2ji} + P_{2ji} A_{3j} + \Phi_{22} & \mu X_{2ii}^2 & 0 \\ \Phi_{13}^T & \mu X_{2ii}^{2^T} & \Phi_{33} & \mu A_{2i}^T Z \\ \mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z \end{bmatrix}$$
$$+ \begin{bmatrix} P_{1ji} B_{1i} \\ 0 \\ 0 \\ \mu Z B_{1i} \end{bmatrix} I \begin{bmatrix} 0 & K_j & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} B_{1i}^T P_{1ji} & 0 & 0 & \mu B_{1i}^T Z \end{bmatrix} < 0. \quad (2.33)$$

By Lemma 2.2, a sufficient condition guaranteeing (2.33) is that there exists a positive number $\rho_{ji} > 0$ such that

$$\rho_{ji} \begin{bmatrix} \Phi_{11} & C_i^T B_{3j}^T P_{2ji} + \mu X_{1ii}^2 & \Phi_{13} & \mu A_{1i}^T Z \\ P_{2ji} B_{3j} C_i + \mu X_{1ii}^{2^T} & A_{3j}^T P_{2ji} + P_{2ji} A_{3j} + \Phi_{22} & \mu X_{2ii}^2 & 0 \\ \Phi_{13}^T & \mu X_{2ii}^{2^T} & \Phi_{33} & \mu A_{2i}^T Z \\ \mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z \end{bmatrix}$$
$$+\rho_{ji}^2 \begin{bmatrix} P_{1ji} B_{1i} \\ 0 \\ 0 \\ \mu Z B_{1i} \end{bmatrix} I \begin{bmatrix} B_{1i}^T P_{1ji} & 0 & 0 & \mu B_{1i}^T Z \end{bmatrix} + \begin{bmatrix} 0 \\ K_i^T \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} 0 & K_i & 0 & 0 \end{bmatrix} < 0. \quad (2.34)$$

Replacing $\rho_{ji} P_{1ji}$, $\rho_{ji} P_{2ji}$, $\rho_{ji} Q$, $\rho_{ji} Z$, $\rho_{ji} X_{ji}$, $\rho_{ji} Y_{ji}$ and $\rho_{ji} T_{ji}$ with P_{1ji}, P_{2ji}, Q, Z, X_{ji}, Y_{ji} and T_{ji}, respectively, and applying the Schur complement shows that (2.34) is equivalent to

$$\mathbf{W}_{ji} = \begin{bmatrix} \Phi_{11} & C_i^T V_{ji} + \mu X_{1ii}^2 & \Phi_{13} & \mu A_{1i}^T Z & P_{1ji} B_{1i} & 0 \\ V_{ji}^T C_i + \mu X_{1ii}^{2^T} & U_{ji} + U_{ji}^T + \Phi_{22} & \mu X_{2ii}^2 & 0 & 0 & K_j^T \\ \Phi_{13}^T & \mu X_{2ii}^{2^T} & \Phi_{33} & \mu A_{2i}^T Z & 0 & 0 \\ \mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z & \mu Z B_{1i} & 0 \\ B_{1i}^T P_{1ji} & 0 & 0 & \mu B_{1i}^T Z & -I & 0 \\ 0 & K_j & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (2.35)$$

with $j, i \in \mathscr{S}$. Hence, the LMIs (2.16) (2.35) guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square for $r_t^o = j, r_t = i, \ \forall j, i \in S$.

Then, for the uncertain time-delayed jump linear system (2.10) without disturbance, replacing A_{1i}, A_{2i}, B_{1i} and K_j in (2.35) with $A_{1i} + H_{1i} F_i(t) E_{1i}$, $A_{2i} + H_{1i} F_i(t) E_{2i}$, $B_{1i} + H_{1i} F_i(t) E_{3i}$ and $K_j + \alpha_i \phi_i(t) K_j$, we can obtain that (2.35) for system (2.10) is equivalent to the following condition:

$$
\mathbf{W}_{ji} + \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} P_{ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix}^T
$$

$$
+ \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
$$

$$
+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix} \phi_i(t) \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \phi_i^T(t) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix}^T < 0. \tag{2.36}
$$

By Lemma 2.2, a sufficient condition guaranteeing (2.36) is that there exist positive numbers $\rho_{1ji} > 0$, $\rho_{2ji} > 0$, $\rho_{3ji} > 0$ such that

$$
\mathbf{W}_{ji} + \rho_{3ji}^{-1} \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix}^T + \rho_{3ji} \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix}^T
$$

$$
+ \rho_{2ji}^{-1} \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \rho_{2ji} \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
$$

$$
+ \rho_{1ji} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix}^T + \rho_{1ji}^{-1} \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{2.37}
$$

With the Schur complement one can show that (2.15) is equivalent to (2.37) for all $r_t^o = j, r_t = i, \quad \forall j, i \in \mathscr{S}$. This completes the proof.

Remark 2.3 It can be seen that the condition in (2.32) is nonlinear in the design parameters A_{3j}, B_{3j}, K_j and P_{ji}. In non-delayed systems, these types of nonlinearities have been eliminated by some appropriate change of control variables with the general form of P_{ji} as follows [13, 21]:

$$P_{ji} = \begin{bmatrix} P_{1ji} & P_{2ji} \\ P_{2ji}^T & P_{3ji} \end{bmatrix}, \qquad \forall j, i \in \mathscr{S}. \tag{2.38}$$

To deal with the output feedback control problem for time-delay systems, there are always some parameters coupled with their inverse which is required to be fixed a priori, see, e.g., [10, 26]. In this chapter, if we partition P_{ji} as (2.38) and use the linearizing change of variable approach as in [26] for condition (2.32), the design parameters Y_{ji}, X_{1ji}^1, Y_{ji}^{-1}, $X_{1ji}^{1^{-1}}$ will occur in the same inequality.

Then, if we were to transfer the control design problem into the framework of LMI, we have to fix these parameters a priori, which makes the obtaining of the optimal relationships between the terms in the Newton-Leibniz formula (2.20) and (2.21) almost impossible.

To obtain an easier design technique, we choose P_{ji} to be diagonal block matrices

$$P_{ji} = \begin{bmatrix} P_{1ji} & 0 \\ 0 & P_{2ji} \end{bmatrix}, \quad \forall j, i \in \mathscr{S}.$$

It is reasonable to choose Lyapunov parameters P_{1ji} for plant states $x(t)$ and P_{2ji} for control systems states $\widehat{x}(t)$, respectively. We can obtain the optimal free weighting matrices by solving the corresponding linear matrix inequalities without the need to fix any design parameters, leading to less conservative results.

2.4 Robust H_∞ Disturbance Attenuation

In this section, we consider robust H_∞ disturbance attenuation for the time-delayed uncertain jump linear systems (2.10).

Theorem 2.2 *The time-delayed uncertain jump linear systems (2.10) is stochastically stable with γ-disturbance H_∞ attenuation (2.11), and the output feedback control law (2.8) is robust if there exist symmetric positive-definite matrices P_{1ji}, P_{2ji}, Q, Z, symmetric positive semi-definite matrices $\overline{X}_{ji} \geq 0$, constants $\rho_{1ji} > 0$, $\rho_{2ji} > 0$, $\rho_{3ji} > 0$ and appropriately dimensioned matrices K_j, Y_{ji}, T_{ji}, N_{ji} such that*

$$\begin{bmatrix} \overline{L}_1 & \mu A_{1i}^T \widehat{Z} & \widehat{P}_{1ji} B_{1i} + \rho_{3ji} E_{1i}^T E_{3i} & 0 & C_i^T & \widehat{P}_{1ji} H_{1i} & 0 & 0 \\ \overline{L}_2 & 0 & 0 & K_j^T & 0 & 0 & V_{ji}^T H_{2i} & K_j^T \\ \overline{L}_3 & \mu A_{2i}^T \widehat{Z} & \rho_{3ji} E_{2i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\ \overline{L}_4 & \mu B_{2i}^T \widehat{Z} & 0 & 0 & 0 & 0 & 0 & 0 \\ \overline{L}_5 & -\mu \widehat{Z} & \mu \widehat{Z} B_{1i} & 0 & 0 & \mu \widehat{Z} H_{1i} & 0 & 0 \\ \overline{L}_6 & \mu B_{1i}^T \widehat{Z} & -I + \rho_{3ji} E_{3i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\ \overline{L}_7 & 0 & 0 & -I + \rho_{1ji} \alpha_i^2 I & 0 & 0 & 0 & 0 \\ \overline{L}_8 & 0 & 0 & 0 & -\rho_{4ji} I & 0 & H_{2i} & 0 \\ \overline{L}_9 & \mu H_{1i}^T \widehat{Z} & 0 & 0 & 0 & -\rho_{3ji} I & 0 & 0 \\ \overline{L}_{10} & 0 & 0 & 0 & H_{2i}^T & 0 & -\rho_{2ji} I & 0 \\ \overline{L}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{1ji} I \end{bmatrix} < 0, \tag{2.39}$$

$$\overline{\Gamma}_{ji} = \begin{bmatrix} \widehat{X}_{11ji} & \widehat{X}_{12ji} & \widehat{X}_{13ji} & I_0^T \widehat{Y}_{ji} \\ \widehat{X}_{12ji}^T & \widehat{X}_{22ji} & \widehat{X}_{23ji} & \widehat{T}_{ji} \\ \widehat{X}_{13ji}^T & \widehat{X}_{23ji}^T & \widehat{X}_{33ji} & \widehat{N}_{ji} \\ \widehat{Y}_{ji}^T I_0 & \widehat{T}_{ji}^T & \widehat{N}_{ji}^T & \widehat{Z} \end{bmatrix} \geq 0, \quad \forall i, j \in \mathscr{S}, \tag{2.40}$$

where

$$\begin{bmatrix} \overline{L}_1 \\ \overline{L}_2 \\ \overline{L}_3 \\ \overline{L}_4 \\ \overline{L}_5 \\ \overline{L}_6 \\ \overline{L}_7 \\ \overline{L}_8 \\ \overline{L}_9 \\ \overline{L}_{10} \\ \overline{L}_{11} \end{bmatrix} = \begin{bmatrix} \Psi_{11} + \rho_{3ji} E_{1i}^T E_{1i} + \rho_{2ji} E_{4i}^T E_{4i} & C_i^T V_{ji} + \mu \widehat{X}_{11ji}^2 & \Psi_{13} + \rho_{3ji} E_{1i}^T E_{2i} & \Psi_{14} \\ \mu \widehat{X}_{11ji}^{2^T} + V_{ji}^T C_i & U_i + U_i^T + \Psi_{22} & \mu \widehat{X}_{12ji}^2 & \mu \widehat{X}_{13ji}^2 \\ \Psi_{13}^T + \rho_{3ji} E_{2i}^T E_{1i} & \mu \widehat{X}_{12ji}^{2^T} & \Psi_{33} + \rho_{3ji} E_{2i}^T E_{2i} & \Psi_{34} \\ \Psi_{14}^T & \mu \widehat{X}_{13ji}^{2^T} & \Psi_{34}^T & \Psi_{44} - \gamma^2 I \\ \mu \widehat{Z} A_{1i} & 0 & \mu \widehat{Z} A_{2i} & \mu \widehat{Z} B_{2i} \\ B_{1i}^T \widehat{P}_{1ji} + \rho_{3ji} E_{3i}^T E_{1i} & 0 & \rho_{3ji} E_{3i}^T E_{2i} & 0 \\ 0 & K_j & 0 & 0 \\ C_i & 0 & 0 & 0 \\ H_{1i}^T \widehat{P}_{1ji} & 0 & 0 & 0 \\ 0 & H_{2i}^T V_{ji} & 0 & 0 \\ 0 & K_j & 0 & 0 \end{bmatrix},$$

$$\overline{X}_{ji} = \begin{bmatrix} \overline{X}_{11ji} & \overline{X}_{12ji} & \overline{X}_{13ji} \\ \overline{X}_{12ji}^T & \overline{X}_{22ji} & \overline{X}_{23ji} \\ \overline{X}_{13ji}^T & \overline{X}_{23ji}^T & \overline{X}_{33ji} \end{bmatrix} = \begin{bmatrix} \overline{X}_{11ji}^1 & \overline{X}_{11ji}^2 & \overline{X}_{12ji}^1 & \overline{X}_{13ji}^1 \\ \overline{X}_{11ji}^{2^T} & \overline{X}_{11ji}^3 & \overline{X}_{12ji}^2 & \overline{X}_{13ji}^2 \\ \overline{X}_{12ji}^{1^T} & \overline{X}_{12ji}^{2^T} & \overline{X}_{22ji} & \overline{X}_{23ji} \\ \overline{X}_{13ji}^{1^T} & \overline{X}_{13ji}^{2^T} & \overline{X}_{23ji}^T & \overline{X}_{33ji} \end{bmatrix},$$

with

$$\Psi_{11} = \begin{cases} (\text{if} \quad j = i) \\ A_{1i}^T \widehat{P}_{1ii} + \widehat{P}_{1ii} A_{1i} + \widehat{Y}_{ii} + \widehat{Y}_{ii}^T + (1+\eta\mu)\widehat{Q} + \mu \widehat{X}_{11ii}^1 + \sum_{j=1}^N \pi_{ij} \widehat{P}_{1ij} + \sum_{j=1}^N \pi_{ij}^1 \widehat{P}_{1ji}, \\ (\text{if} \quad j \neq i) \\ A_{1i}^T \widehat{P}_{1ji} + \widehat{P}_{1ji} A_{1i} + \widehat{Y}_{ji} + \widehat{Y}_{ji}^T + (1+\eta\mu)\widehat{Q} + \mu \widehat{X}_{11ji}^1 + \pi_{ji}^0 (\widehat{P}_{1ii} - \widehat{P}_{1ji}), \end{cases}$$

$$\Psi_{22} = \begin{cases} \sum_{j=1}^N \pi_{ij} \widehat{P}_{2ij} + \sum_{j=1}^N \pi_{ij}^1 \widehat{P}_{2ji} + \mu \widehat{X}_{11ii}^3, & \text{if} \ \ j = i \\ \pi_{ji}^0 (\widehat{P}_{2ii} - \widehat{P}_{2ji}) + \mu \widehat{X}_{11ji}^3, & \text{if} \ \ j \neq i \end{cases}$$

$$\Psi_{13} = \widehat{P}_{1ji} A_{2i} - \widehat{Y}_{ji} + \widehat{T}_{ji}^T + \mu \widehat{X}_{12ji}^1, \qquad \Psi_{14} = \widehat{P}_{1ji} B_{2i} + \widehat{N}_{ji}^T + \mu \widehat{X}_{13ji}^1,$$

$$\Psi_{33} = -\widehat{T}_{ji} - \widehat{T}_{ji}^T - (1 - h_i)\widehat{Q} + \mu \widehat{X}_{22ji}, \quad \Psi_{34} = -\widehat{N}_{ji}^T + \mu \widehat{X}_{23ji},$$

$$\Psi_{44} = \mu \widehat{X}_{33ji}, \quad \eta = \max_{i \in \mathscr{S}} \{|\pi_{ii}|\}, \quad V_{ji} = B_{3j}^T \widehat{P}_{2ji}, \quad U_{ji} = A_{3j}^T \widehat{P}_{2ji},$$

$$\left[\widehat{P}_{1ji} \ \ \widehat{P}_{2ji} \ \ \widehat{Q} \ \ \widehat{Z} \ \ \widehat{Y}_{ji} \ \ \widehat{T}_{ji} \ \ \widehat{N}_{ji}\right] = \rho_{4ji}^{-1} \left[P_{1ji} \ \ P_{2ji} \ \ Q \ \ Z \ \ Y_{ji} \ \ T_{ji} \ \ N_{ji}\right],$$

$$\begin{bmatrix} \widehat{X}_{11ji}^1 & \widehat{X}_{11ji}^2 & \widehat{X}_{12ji}^1 & \widehat{X}_{13ji}^1 \\ \widehat{X}_{11ji}^{2^T} & \widehat{X}_{11ji}^3 & \widehat{X}_{12ji}^2 & \widehat{X}_{13ji}^2 \\ \widehat{X}_{12ji}^{1^T} & \widehat{X}_{12ji}^{2^T} & \widehat{X}_{22ji} & \widehat{X}_{23ji} \\ \widehat{X}_{13ji}^{1^T} & \widehat{X}_{13ji}^{2^T} & \widehat{X}_{23ji}^T & \widehat{X}_{33ji} \end{bmatrix} = \rho_{4ji}^{-1} \begin{bmatrix} \overline{X}_{11ji}^1 & \overline{X}_{11ji}^2 & \overline{X}_{12ji}^1 & \overline{X}_{13ji}^1 \\ \overline{X}^{2^T}_{11ji} & \overline{X}_{11ji}^3 & \overline{X}_{12ji}^2 & \overline{X}_{13ji}^2 \\ \overline{X}^{1^T}_{12ji} & \overline{X}^{2^T}_{12ji} & \overline{X}_{22ji} & \overline{X}_{23ji} \\ \overline{X}^{1^T}_{13ji} & \overline{X}^{2^T}_{13ji} & \overline{X}_{23ji}^T & \overline{X}_{33ji} \end{bmatrix}.$$

Proof For the nominal time-delayed jump linear system Σ_1 with disturbance:

$$\Sigma_1 : \quad \begin{cases} \dot{\xi}(t) = \widehat{A}_1(r_t^o, r_t)\xi(t) + \widehat{A}_2(r_t) I_0 \xi(t - \tau_{r_t}(t)) + \widehat{B}_2(r_t) w(t), \\ z(t) = [C(r_t) + \Delta_C(r_t, t)] I_0 \xi(t), \\ I_0 \xi(s) = f(s), \quad r_s = r_0, \quad s \in [-\mu, \ 0], \end{cases} \tag{2.41}$$

where

$$\widehat{B}_2(r_t) = \begin{bmatrix} B_2(r_t) \\ 0 \end{bmatrix}.$$

Let $\overline{\zeta}^T(t) = [\xi^T(t) \ \ x^T(t - \tau_i(t)) \ \ w^T(t)]$. Take the Lyapunov function candidate as (2.18), and employ the following conditions

$$\mu \overline{\zeta}^T(t) \overline{X}(r_t^o, r_t) \overline{\zeta}(t) - \int_{t-\tau_{r_t}(t)}^t \overline{\zeta}^T(t) \overline{X}(r_t^o, r_t) \overline{\zeta}(t) ds \geq 0,$$

$$2\left[\xi^T(t) I_0^T Y(r_t^o, r_t) + x^T(t - \tau_{r_t}(t)) T(r_t^o, r_t) + w^T(t) N(r_t^o, r_t)\right] \times \left[I_0 \xi(t) - \int_{t-\tau_{r_t}(t)}^t \dot{x}(s) ds - x(t - \tau_{r_t}(t))\right] = 0, \tag{2.42}$$

we can then obtain

$$\mathfrak{L}V(x_t, j, i, t) \leq \overline{\zeta}^T(t)\overline{\Xi}_{ji}\overline{\zeta}(t) - \int_{t-\tau_i(t)}^{t} \overline{\chi}^T(t,s)\overline{\Gamma}_{ji}\overline{\chi}(t,s)ds, \tag{2.43}$$

where

$$\overline{\Xi}_{ji} = \begin{bmatrix} \overline{\Psi}_{11} + \mu\widehat{A}_{1ji}^T I_0^T Z I_0 \widehat{A}_{1ji} & \overline{\Psi}_{12} + \mu\widehat{A}_{1ji}^T I_0^T Z I_0 \widehat{A}_{2i} & \overline{\Psi}_{13} + \widehat{A}_{1ji}^T I_0^T Z I_0 \widehat{B}_{2i} \\ \overline{\Psi}_{12}^T + \mu\widehat{A}_{2i}^T I_0^T Z I_0 \widehat{A}_{1ji} & \overline{\Psi}_{22} + \mu\widehat{A}_{2i}^T I_0^T Z I_0 \widehat{A}_{2i} & \overline{\Psi}_{23} + \mu\widehat{A}_{2i}^T I_0^T Z I_0 \widehat{B}_{2i} \\ \overline{\Psi}_{13}^T + \mu\widehat{B}_{2i}^T I_0^T Z I_0 \widehat{A}_{1ji} & \overline{\Psi}_{23}^T + \mu\widehat{B}_{2i}^T I_0^T Z I_0 \widehat{A}_{2i} & \overline{\Psi}_{33} + \mu\widehat{B}_{2i}^T I_0^T Z I_0 \widehat{B}_{2i} \end{bmatrix}$$

$$\overline{\Psi}_{11} = \begin{cases} \widehat{A}_{1ii}^T P_{ii} + P_{ii}\widehat{A}_{1ii} + I_0^T Y_{ii} I_0 + I_0 Y_{ii}^T I_0^T + (1+\eta\mu) I_0^T Q I_0 & \\ \quad +\mu\overline{X}_{11ii} + \sum_{j=1}^{N} \pi_{ij} P_{ij} + \sum_{j=1}^{N} \pi_{ij}^1 P_{ji}, & \text{if } j = i \\ \widehat{A}_{1ji}^T P_{ji} + P_{ji}\widehat{A}_{1ji} + I_0^T Y_{ji} I_0 + I_0 Y_{ji}^T I_0^T + (1+\eta\mu) I_0^T Q I_0 & \\ \quad +\mu\overline{X}_{11ji} + q_{ji}^0 (P_{ii} - P_{ji}), & \text{if } j \neq i \end{cases}$$

$$\overline{\Psi}_{12} = P_{ji}\widehat{A}_{2i} - I_0^T Y_{ji} + I_0^T T_{ji}^T + \mu\overline{X}_{12ji},$$

$$\overline{\Psi}_{22} = -T_{ji} - T_{ji}^T - (1-h_i)Q + \mu\overline{X}_{22ji},$$

$$\overline{\Psi}_{13} = P_{ji}\widehat{B}_{2i} + I_0^T N_{ji}^T + \mu\overline{X}_{13ji}, \ \overline{\Psi}_{23} = -N_{ji}^T + \mu\overline{X}_{23ji}, \ \overline{\Psi}_{33} = \overline{X}_{33ji},$$

$$\eta = \max_{i\in\mathscr{S}}\{|\pi_{ii}|\}, \overline{\chi}^T(t,s) = [\xi^T(t) \ \ x^T(t-\tau_i(t)) \ \ w^T(t) \ \ \dot{x}^T(s)].$$

Using Dynkin's formula again [18], we obtain

$$E\left\{\int_0^T \mathfrak{L}V(x_s, r_s^o, r_s, s)ds\right\} = E\{V(x_T, r_t^o, r_t, T)\} - E\{V(x_0, r_0^o, r_0, 0)\}.$$

Under the zero initial condition ($x(0) = 0$), we have

$$E\{V(x_0, r_0^o, r_0, 0)\} = 0.$$

Thus, for any $w(t) \in L_2[0 \ \ \infty)$, one sees that

$$\begin{aligned} J &= E\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathfrak{L}V(x_t, r_t^o, r_t, t)\right]dt\right\} - E\{V(x_T, r_t^o, r_t, T)\} \\ &\leq E\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathfrak{L}V(x_t, r_t^o, r_t, t)\right]dt\right\}. \end{aligned} \tag{2.44}$$

Substituting (2.43) into the above inequality gives

$$J \le E\Big\{\int_0^T \Big[\overline{\zeta}^T(t)\Big(\overline{\Xi}_{ji} + \begin{bmatrix} I_0^T C_i^T C_i I_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix}\Big)\overline{\zeta}(t) - \int_{t-\tau_i(t)}^{t} \overline{\chi}^T(t,s)\overline{\Gamma}_{ji}\overline{\chi}(t,s)ds\Big]dt\Big\}.$$

By Lemma 2.2 and the Schur complement we obtain (2.40), and

$$\begin{bmatrix} \Psi_{11}+\rho_{4ji}^{-1}C_i^T C_i & C_i^T B_{3j}^T \widehat{P}_{2ji} + \mu\widehat{X}_{11ji}^2 & \Psi_{13} & \Psi_{14} & \mu A_{1i}^T\widehat{Z} & \widehat{P}_{1ji}B_{1i} & 0 \\ \widehat{P}_{2ji}B_{3j}C_i + \mu\widehat{X}_{11ji}^{2^T} & A_{3j}^T\widehat{P}_{2ji} + \widehat{P}_{2ji}A_{3j} + \Psi_{22} & \mu\widehat{X}_{12ji}^2 & \mu\widehat{X}_{13ji}^2 & 0 & 0 & K_j^T \\ \Psi_{13}^T & \mu\widehat{X}_{12ji}^{2^T} & \Psi_{33} & \Psi_{34} & \mu A_{2i}^T\widehat{Z} & 0 & 0 \\ \Psi_{14}^T & \mu\widehat{X}_{13ji}^{2^T} & \Psi_{34}^T & \Psi_{44}-\gamma^2 I & \mu B_{2i}^T\widehat{Z} & 0 & 0 \\ \mu\widehat{Z}A_{1i} & 0 & \mu\widehat{Z}A_{2i} & \mu\widehat{Z}B_{2i} & -\mu\widehat{Z} & \widehat{Z}B_{1i} & 0 \\ B_{1i}^T\widehat{P}_{1ji} & 0 & 0 & 0 & \mu B_{1i}^T\widehat{Z} & -I & 0 \\ 0 & K_j & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (2.45)$$

guarantee $J < 0$ for any $w(t) \neq 0$ (and $w(t) \in L_2[0,\infty)$), which also guarantee γ-disturbance H_∞ attenuation (2.11) of the closed-loop system Σ_1 from $w(t)$ to $z(t)$.

Then, replacing A_{1i}, A_{2i}, B_{1i}, C_i and K_j in (2.45) with $A_{1i} + H_{1i}F_i(t)E_{1i}$, $A_{2i} + H_{1i}F_i(t)E_{2i}$, $B_{1i} + H_{1i}F_i(t)E_{3i}$, $C_i + H_{2i}F_i(t)E_{4i}$ and $K_j + \alpha_i\phi_i(t)K_j$ and using the similar proof of Theorem 2.1, we can easily verify that the control $u(t) = K(r_t^o)x(t)$ guarantees γ-disturbance H_∞ attenuation (2.11) of the closed-loop system (2.10) from $w(t)$ to $z(t)$, if the coupled linear matrix inequalities (2.39) and (2.40) are satisfied. This completes the proof.

In the case that the jumping parameter process can be directly and precisely measured; that is, $r_t = r_t^o$, $\forall t \in [0,\ \infty)$, the closed-loop system (2.10) is specialized as

$$\begin{cases} \dot{\xi}(t) = \widetilde{A}_1(r_t,t)\xi(t) + \widetilde{A}_2(r_t)I_0\xi(t-\tau_{r_t}(t)) + \widetilde{B}_2(r_t)w(t), \\ z(t) = [C(r_t) + \Delta_C(r_t,t)]I_0\xi(t), \\ I_0\xi(s) = f(s), \quad r_s = r_0, \quad s \in [-2\mu,\ 0], \end{cases} \quad (2.46)$$

where

$$\widetilde{A}_{1i} = \begin{bmatrix} A_{1i} + \Delta_{A_{1i}}(t) & \left(B_{1i} + \Delta_{B_{1i}}(t)\right)\left(I + \alpha_i \phi_i(t)\right) K_i \\ B_{3i}\left(C_i + \Delta_{C_i}(t)\right) & A_{3i} \end{bmatrix} \in \mathbb{R}^{2n \times 2n},$$

$$\widetilde{A}_{2i} = \begin{bmatrix} A_{2i} + \Delta_{A_{2i}}(t) \\ 0 \end{bmatrix} \in \mathbb{R}^{2n \times n}, \ \ \widetilde{B}_{2i} = \begin{bmatrix} B_{2i} \\ 0 \end{bmatrix} \in \mathbb{R}^{2n \times m_2}, \ \ I_0 = \begin{bmatrix} I\, 0 \end{bmatrix} \in \mathbb{R}^{n \times 2n}$$

for each $p_t = i, \ \ \forall i \in \mathscr{S}$.

Then by Theorem 2.2, we have the following corollary.

Corollary 2.1 *The time-delayed uncertain jump linear systems (2.10) is stochastically stable with γ-disturbance H_∞ attenuation (2.11), and the output feedback control law (2.8) is robust if the jumping parameter process can be directly and precisely measured, and there exist symmetric positive-definite matrices P_{1i}, P_{2i}, Q, Z, symmetric positive semi-definite matrices $\widetilde{X}_i \geq 0$, constants $\rho_{1i} > 0$, $\rho_{2i} > 0$, $\rho_{3i} > 0$ and appropriately dimensioned matrices K_i, Y_i, T_i, N_i such that*

$$\begin{bmatrix} \widetilde{L}_1 & \mu A_{1i}^T \widehat{Z} & \widehat{P}_{1i} B_{1i} + \rho_{3i} E_{1i}^T E_{3i} & 0 & C_i^T & \widehat{P}_{1i} H_{1i} & 0 & 0 \\ \widetilde{L}_2 & 0 & 0 & K_i^T & 0 & 0 & V_i^T H_{2i} & K_i^T \\ \widetilde{L}_3 & \mu A_{2i}^T \widehat{Z} & \rho_{3i} E_{2i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\ \widetilde{L}_4 & \mu B_{2i}^T \widehat{Z} & 0 & 0 & 0 & 0 & 0 & 0 \\ \widetilde{L}_5 & -\mu \widehat{Z} & \mu \widehat{Z} B_{1i} & 0 & 0 & \mu \widehat{Z} H_{1i} & 0 & 0 \\ \widetilde{L}_6 & \mu B_{1i}^T \widehat{Z} & -I + \rho_{3i} E_{3i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\ \widetilde{L}_7 & 0 & 0 & -I + \rho_{1i} \alpha_i^2 I & 0 & 0 & 0 & 0 \\ \widetilde{L}_8 & 0 & 0 & 0 & -\rho_{4i} I & 0 & H_{2i} & 0 \\ \widetilde{L}_9 & \mu H_{1i}^T \widehat{Z} & 0 & 0 & 0 & -\rho_{3i} I & 0 & 0 \\ \widetilde{L}_{10} & 0 & 0 & 0 & H_{2i}^T & 0 & -\rho_{2i} I & 0 \\ \widetilde{L}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{1i} I \end{bmatrix} < 0 \qquad (2.47)$$

$$\widetilde{\Gamma}_i = \begin{bmatrix} \widehat{X}_{11i} & \widehat{X}_{12i} & \widehat{X}_{13i} & I_0^T \widehat{Y}_i \\ \widehat{X}_{12i}^T & \widehat{X}_{22i} & \widehat{X}_{23i} & \widehat{T}_i \\ \widehat{X}_{13i}^T & \widehat{X}_{23i}^T & \widehat{X}_{33i} & \widehat{N}_i \\ \widehat{Y}_i^T I_0 & \widehat{T}_i^T & \widehat{N}_i^T & \widehat{Z} \end{bmatrix} \geq 0, \ \ \forall i \in \mathscr{S}, \qquad (2.48)$$

where

$$\begin{bmatrix} \widetilde{L}_1 \\ \widetilde{L}_2 \\ \widetilde{L}_3 \\ \widetilde{L}_4 \\ \widetilde{L}_5 \\ \widetilde{L}_6 \\ \widetilde{L}_7 \\ \widetilde{L}_8 \\ \widetilde{L}_9 \\ \widetilde{L}_{10} \\ \widetilde{L}_{11} \end{bmatrix} = \begin{bmatrix} \widetilde{\Psi}_{11} + \rho_{3i} E_{1i}^T E_{1i} + \rho_{2i} E_{4i}^T E_{4i} & C_i^T V_i + \mu \widehat{X}_{11i}^2 & \widetilde{\Psi}_{13} + \rho_{3i} E_{1i}^T E_{2i} & \widetilde{\Psi}_{14} \\ \mu \widehat{X}_{11i}^{2^T} + V_i^T C_i & U_i + U_i^T + \widetilde{\Psi}_{22} & \mu \widehat{X}_{12i}^2 & \mu \widehat{X}_{13i}^2 \\ \widetilde{\Psi}_{13}^T + \rho_{3i} E_{2i}^T E_{1i} & \mu \widehat{X}_{12i}^{2^T} & \widetilde{\Psi}_{33} + \rho_{3i} E_{2i}^T E_{2i} & \widetilde{\Psi}_{34} \\ \widetilde{\Psi}_{14}^T & \mu \widehat{X}_{13i}^{2^T} & \widetilde{\Psi}_{34}^T & \widetilde{\Psi}_{44} - \gamma^2 I \\ \mu \widehat{Z} A_{1i} & 0 & \mu \widehat{Z} A_{2i} & \mu \widehat{Z} B_{2i} \\ B_{1i}^T \widehat{P}_{1i} + \rho_{3i} E_{3i}^T E_{1i} & 0 & \rho_{3i} E_{3i}^T E_{2i} & 0 \\ 0 & K_i & 0 & 0 \\ C_i & 0 & 0 & 0 \\ H_{1i}^T \widehat{P}_{1i} & 0 & 0 & 0 \\ 0 & H_{2i}^T V_i & 0 & 0 \\ 0 & K_i & 0 & 0 \end{bmatrix},$$

$$\widetilde{X}_i = \begin{bmatrix} \widetilde{X}_{11i} & \widetilde{X}_{12i} & \widetilde{X}_{13i} \\ \widetilde{X}_{12i}^T & \widetilde{X}_{22i} & \widetilde{X}_{23i} \\ \widetilde{X}_{13i}^T & \widetilde{X}_{23i}^T & \widetilde{X}_{33i} \end{bmatrix} = \begin{bmatrix} \widetilde{X}_{11i}^1 & \widetilde{X}_{11i}^2 & \widetilde{X}_{12i}^1 & \widetilde{X}_{13i}^1 \\ \widetilde{X}_{11i}^{2^T} & \widetilde{X}_{11i}^3 & \widetilde{X}_{12i}^2 & \widetilde{X}_{13i}^2 \\ \widetilde{X}_{12i}^{1^T} & \widetilde{X}_{12i}^{2^T} & \widetilde{X}_{22i} & \widetilde{X}_{23i} \\ \widetilde{X}_{13i}^{1^T} & \widetilde{X}_{13i}^{2^T} & \widetilde{X}_{23i}^T & \widetilde{X}_{33i} \end{bmatrix},$$

$$\widetilde{\Psi}_{11} = A_{1i}^T \widehat{P}_{1i} + \widehat{P}_{1i} A_{1i} + \widehat{Y}_i + \widehat{Y}_i^T + (1+\eta\mu)\widehat{Q} + \mu \widehat{X}_{11i}^1 + \sum_{j=1}^N \pi_{ij} \widehat{P}_{1j},$$

$$\widetilde{\Psi}_{22} = \sum_{j=1}^N \pi_{ij} \widehat{P}_{2j} + \mu \widehat{X}_{11i}^3, \quad \widetilde{\Psi}_{13} = \widehat{P}_{1i} A_{2i} - \widehat{Y}_i + \widehat{T}_i^T + \mu \widehat{X}_{12i}^1, \quad \widetilde{\Psi}_{44} = \mu \widehat{X}_{33i},$$

$$\widetilde{\Psi}_{14} = \widehat{P}_{1i} B_{2i} + \widehat{N}_i^T + \mu \widehat{X}_{13i}^1, \quad \widetilde{\Psi}_{33} = -\widehat{T}_i - \widehat{T}_i^T - (1-h_i)\widehat{Q} + \mu \widehat{X}_{22i},$$

$$\widetilde{\Psi}_{34} = -\widehat{N}_i^T + \mu \widehat{X}_{23i}, \quad \eta = \max_{i \in \mathscr{S}} \{|\pi_{ii}|\}, \quad V_i = B_{3i}^T \widehat{P}_{2i}, \quad U_i = A_{3i}^T \widehat{P}_{2i},$$

$$\left[\widehat{P}_{1i} \;\; \widehat{P}_{2i} \;\; \widehat{Q} \;\; \widehat{Z} \;\; \widehat{Y}_i \;\; \widehat{T}_i \;\; \widehat{N}_i\right] = \rho_{4i}^{-1} \left[P_{1i} \;\; P_{2i} \;\; Q \;\; Z \;\; Y_i \;\; T_i \;\; N_i\right],$$

$$\begin{bmatrix} \widehat{X}_{11i}^1 & \widehat{X}_{11i}^2 & \widehat{X}_{12i}^1 & \widehat{X}_{13i}^1 \\ \widehat{X}_{11i}^{2^T} & \widehat{X}_{11i}^3 & \widehat{X}_{12i}^2 & \widehat{X}_{13i}^2 \\ \widehat{X}_{12i}^{1^T} & \widehat{X}_{12i}^{2^T} & \widehat{X}_{22i} & \widehat{X}_{23i} \\ \widehat{X}_{13i}^{1^T} & \widehat{X}_{13i}^{2^T} & \widehat{X}_{23i}^T & \widehat{X}_{33i} \end{bmatrix} = \rho_{4i}^{-1} \begin{bmatrix} \widetilde{X}_{11i}^1 & \widetilde{X}_{11i}^2 & \widetilde{X}_{12i}^1 & \widetilde{X}_{13i}^1 \\ \widetilde{X}_{11i}^{2^T} & \widetilde{X}_{11i}^3 & \widetilde{X}_{12i}^2 & \widetilde{X}_{13i}^2 \\ \widetilde{X}_{12i}^{1^T} & \widetilde{X}_{12i}^{2^T} & \widetilde{X}_{22i} & \widetilde{X}_{23i} \\ \widetilde{X}_{13i}^{1^T} & \widetilde{X}_{13i}^{2^T} & \widetilde{X}_{23i}^T & \widetilde{X}_{33i} \end{bmatrix}.$$

2.5 Numerical Simulation

Example 2.1 Consider a time-delayed uncertain jump linear system (2.10) in $\mathbb{R}^2$ with two regimes $r_t \in \mathscr{S} = \{1, 2\}$. For Mode 1, the dynamics of the system are described by

$$A_{11} = \begin{bmatrix} -9 & -2 \\ 1 & -6 \end{bmatrix}, \; A_{21} = \begin{bmatrix} 2.5 & -2 \\ 2 & -1.6 \end{bmatrix}, E_{11} = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}^T, E_{21} = \begin{bmatrix} 0.4 \\ 2 \end{bmatrix}^T,$$

$$E_{41} = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}^T, B_{11} = \begin{bmatrix} 0.3 \\ 2 \end{bmatrix}, B_{21} = \begin{bmatrix} 1.5 \\ 2 \end{bmatrix}, C_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}^T, H_{11} = \begin{bmatrix} -1 \\ 2 \end{bmatrix},$$

$$E_{31} = -1, \;\; H_{21} = 1, \;\; \mu_1 = 0.1, \;\; h_1 = 1, \;\; \alpha_1 = 2.$$

For Mode 2, the dynamics of the system are described by

$$A_{12} = \begin{bmatrix} 0 & -2 \\ -3 & 1 \end{bmatrix}, A_{22} = \begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}, E_{12} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}^T, E_{22} = \begin{bmatrix} -0.1 \\ 1 \end{bmatrix}^T,$$

$$E_{42} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}^T, B_{12} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, B_{22} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.6 \\ -1 \end{bmatrix}^T, H_{12} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$E_{32} = 0.3, \;\; H_{22} = 1, \;\; \mu_2 = 0.1, \;\; h_2 = 0.4, \;\; \alpha_2 = 3.$$

Let the noise attenuation level $\gamma = 1.2$, and

$$[\pi_{ij}]_{2\times 2} = \begin{bmatrix} -12 & 12 \\ 18 & -18 \end{bmatrix}, \quad [\pi_{ij}^0]_{2\times 2} = \begin{bmatrix} -2 & 2 \\ 5 & -5 \end{bmatrix}, \quad [\pi_{ij}^1]_{2\times 2} = \begin{bmatrix} -4 & 4 \\ 6 & -6 \end{bmatrix}.$$

Solving the LMIs in (2.39) and (2.40), we obtain

$$\widehat{P}_{111} = \begin{bmatrix} 0.645 & 0.230 \\ 0.230 & 0.553 \end{bmatrix}, \widehat{P}_{112} = \begin{bmatrix} 0.302 & -0.010 \\ -0.010 & 0.119 \end{bmatrix},$$
$$\widehat{P}_{121} = \begin{bmatrix} 0.701 & 0.266 \\ 0.266 & 0.808 \end{bmatrix}, \widehat{P}_{122} = \begin{bmatrix} 3.5454 & 1.127 \\ 1.127 & 1.604 \end{bmatrix},$$
$$\widehat{P}_{211} = \begin{bmatrix} 2.513 & 0.104 \\ 0.104 & 4.688 \end{bmatrix}, \quad \widehat{P}_{212} = \begin{bmatrix} 1.697 & -0.124 \\ -0.124 & 2.916 \end{bmatrix},$$
$$\widehat{P}_{221} = \begin{bmatrix} 1.910 & -1.141 \\ -1.141 & 5.488 \end{bmatrix}, \quad \widehat{P}_{222} = \begin{bmatrix} 3.291 & -2.969 \\ -2.969 & 12.955 \end{bmatrix},$$
$$\widehat{T}_{11} = \begin{bmatrix} 13.651 & 2.448 \\ 2.410 & 3.080 \end{bmatrix}, \widehat{T}_{12} = \begin{bmatrix} 13.228 & 1.842 \\ 1.188 & 2.868 \end{bmatrix},$$
$$\widehat{T}_{21} = \begin{bmatrix} 12.375 & 2.034 \\ 2.012 & 2.940 \end{bmatrix}, \widehat{T}_{22} = \begin{bmatrix} 12.829 & 1.838 \\ 1.741 & 2.872 \end{bmatrix},$$
$$\widehat{Y}_{11} = \begin{bmatrix} -8.435 & 6.694 \\ 5.939 & -9.778 \end{bmatrix}, \widehat{Y}_{12} = \begin{bmatrix} -13.236 & -1.843 \\ -1.168 & -2.868 \end{bmatrix},$$
$$\widehat{Y}_{21} = \begin{bmatrix} -13.028 & -2.350 \\ -3.585 & -3.703 \end{bmatrix}, \widehat{Y}_{22} = \begin{bmatrix} -12.808 & -1.838 \\ -0.044 & -2.816 \end{bmatrix},$$
$$U_{11} = \begin{bmatrix} -79.698 & 1.953 \\ 1.959 & -78.229 \end{bmatrix}, U_{12} = \begin{bmatrix} -160.118 & -5.683 \\ -5.807 & -171.548 \end{bmatrix},$$
$$U_{21} = \begin{bmatrix} -44.515 & -1.019 \\ 4.386 & -43.598 \end{bmatrix}, U_{22} = \begin{bmatrix} -120.141 & -1.413 \\ -1.525 & -101.532 \end{bmatrix},$$
$$\widehat{Q} = \begin{bmatrix} 0.0006 & 0.0002 \\ 0.0002 & 0.0034 \end{bmatrix}, \widehat{Z} = \begin{bmatrix} 1.197 & 0.176 \\ 0.176 & 0.276 \end{bmatrix}, V_{11} = \begin{bmatrix} 1.459 \\ -2.172 \end{bmatrix}^T,$$
$$V_{12} = \begin{bmatrix} -0.291 \\ -1.407 \end{bmatrix}^T, \quad V_{21} = \begin{bmatrix} 0.537 \\ -1.846 \end{bmatrix}^T, \quad V_{22} = \begin{bmatrix} 3.279 \\ -11.917 \end{bmatrix}^T,$$
$$\widehat{N}_{11} = \begin{bmatrix} 2.323 \\ 1.086 \end{bmatrix}^T, \quad \widehat{N}_{12} = \begin{bmatrix} -3.721 \\ -0.122 \end{bmatrix}^T, \quad \widehat{N}_{21} = \begin{bmatrix} 3.367 \\ 1.634 \end{bmatrix}^T,$$
$$\widehat{N}_{22} = \begin{bmatrix} -2.922 \\ -0.099 \end{bmatrix}^T, \quad K_1 = \begin{bmatrix} 4.384 \\ 1.868 \end{bmatrix}^T, \quad K_2 = \begin{bmatrix} -5.203 \\ 0.494 \end{bmatrix}^T,$$

$$\begin{aligned}
&\rho_{111} = 0.234, && \rho_{112} = 0.108, && \rho_{121} = 0.145, && \rho_{122} = 0.094,\\
&\rho_{211} = 0.567, && \rho_{212} = 0.185, && \rho_{221} = 0.259, && \rho_{222} = 5.643,\\
&\rho_{311} = 0.332, && \rho_{312} = 0.086, && \rho_{321} = 0.344, && \rho_{322} = 0.976,\\
&\rho_{411} = 9.452, && \rho_{412} = 8.758, && \rho_{421} = 6.488, && \rho_{422} = 4.182,
\end{aligned}$$

Therefore, by Theorem 2.2, the corresponding parameters of a suitable robust output feedback control law (2.8) can be chosen as

$$\begin{aligned}
A_{311} &= \begin{bmatrix} -31.745 & 1.476 \\ 1.126 & -16.718 \end{bmatrix}, A_{312} = \begin{bmatrix} -94.725 & -7.757 \\ -5.994 & -59.161 \end{bmatrix},\\
A_{321} &= \begin{bmatrix} -26.736 & -2.797 \\ -5.745 & -8.525 \end{bmatrix}, A_{322} = \begin{bmatrix} -46.141 & 9.496 \\ 10.683 & -10.013 \end{bmatrix},\\
B_{311} &= \begin{bmatrix} -0.601 \\ -0.478 \end{bmatrix}, B_{312} = \begin{bmatrix} 0.208 \\ -0.491 \end{bmatrix}, B_{321} = \begin{bmatrix} -0.091 \\ -0.318 \end{bmatrix},\\
B_{322} &= \begin{bmatrix} -0.209 \\ -0.872 \end{bmatrix}, K_1 = \begin{bmatrix} -4.384 \\ 1.868 \end{bmatrix}^T, K_2 = \begin{bmatrix} 5.203 \\ 0.494 \end{bmatrix}^T.
\end{aligned}$$

Example 2.2 Consider the robust stability of the uncertain system (1) with the following parameters:

$$\begin{aligned}
A_{11} &= \begin{bmatrix} a_{11} & 4 \\ 0 & -13 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -2.8 & -1.1 \\ 0.4 & -2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -0.7 & 0.2 \\ 0 & -0.1 \end{bmatrix},\\
A_{22} &= \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & -0.1 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1.2 & 0 \\ 0 & -2.1 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 & 1.1 \\ -4.2 & 0 \end{bmatrix},\\
H_{11} &= \begin{bmatrix} 1.4 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad H_{12} = \begin{bmatrix} 0 & 0.1 \\ -0.8 & -1.1 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix},\\
E_{12} &= \begin{bmatrix} 1 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.6 & 0.3 \\ 0 & -1 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} -0.2 & 0 \\ 1 & 0 \end{bmatrix},\\
E_{31} &= \begin{bmatrix} 0.3 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\\
C_2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad H_{21} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad H_{22} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\\
H_{31} &= \begin{bmatrix} 0.1 & 0.4 \\ 0 & 1.4 \end{bmatrix}, \quad H_{32} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad [\pi_{ij}]_{2\times 2} = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}.
\end{aligned}$$

Table 2.1 The maximum allowed value of time delay (μ)

h		0	0.2	0.5	1.0
$a_{11} = -2$	E.K.Boukas(2002)	0.2453	0.1522	-	-
	Theorem 3.1	0.6225	0.5795	0.4930	0.3281
$a_{11} = -8$	E.K.Boukas(2002)	1.0061	0.9421	0.5834	-
	Theorem 3.1	1.2954	1.0594	0.7242	0.3427

To compare with Theorem 9.18 in [5], Theorem 2.1 should be reduced to the conditions that the jumping parameter process can be directly and precisely measured and controller can be accurately implemented. Furthermore, we also assume that $h_1 = h_2 = h$, and

$$\Delta_{A_2}(r_t, t) = H_3(r_t)F(r_t, t)E_2(r_t),$$

$$\Delta_C(r_t, t) = H_2(r_t)F(r_t, t)E_1(r_t).$$

The corresponding results are similar to Corollary 2.1, and are omitted here. The maximum allowed value of time delay for different h obtained from Theorem 2.1 are shown in Table 2.1. For comparison, The table also lists the results obtained from Theorem 9.18 in [5]. From the example, we can find that our results show much less conservatism than those in [5], especially for the increasing of the value of h.

2.6 Summary

The problem of robust output feedback H_∞ control for time-delayed uncertain jump linear systems has been studied. We have presented sufficient conditions on the existence of output feedback control by the imperfect information r_t^o, which guarantees not only the robust exponential mean-square stability but also the γ-disturbance H_∞ attenuation for the closed loop system for all admissible parameter uncertainties and time delays. However, all of these results are established under conditions of the prior knowledge of the upper bounds of the system uncertainties. A possible direction for future work is to obtain adaptive H_∞ control laws with less knowledge of those bounds.

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