

Contents

Part I Conservative Systems

1	Two Coupled Oscillators.	3
1.1	Limiting Phase Trajectories of Two Weakly Coupled Identical Nonlinear Oscillators	3
1.1.1	The Model and Main Asymptotic Equations of Motion.	3
1.1.2	Analytical Solution for LPT	9
1.1.3	Beating Close to LPTs	16
1.2	Effect of the Frequency Detuning Between the Oscillators	16
1.2.1	Equations of Motion and Explicit Approximate Solutions	17
1.2.2	Stationary States and LPTs	19
1.2.3	Critical Parameters.	23
	References.	25
2	Two-Particle Systems Under Conditions of Sonic Vacuum	27
2.1	Weakly Coupled Oscillators Under Conditions of Local Sonic Vacuum	28
2.1.1	Evidence of Energy Localization and Exchange in Coupled Oscillators in the State of Sonic Vacuum.	28
2.1.2	Energy Localization.	29
2.1.3	Complete Energy Exchanges (Strong Beating Response)	30
2.1.4	Asymptotic Analysis of Resonance Motion.	31
2.1.5	Fixed Points and NNMs in the Neighborhood of Resonance	32
2.1.6	Limiting Phase Trajectories	34
2.1.7	Numerical Analysis of the Fundamental Model.	37

2.2	Non-local Sonic Vacuum	43
2.2.1	The Model.	43
2.2.2	Two-Particle System ($n = 2$): Slow Transverse Oscillations	47
2.2.3	Slow Flow Reduction of the Dynamics.	48
2.2.4	Stationary and Non-stationary Dynamics.	52
2.2.5	Analytical Approximations of the LPTs on the Two-Torus	56
2.2.6	Mixed Slow/Fast Axial Oscillations for $n = 2$	60
2.2.7	Global Dynamics.	62
	References.	66
3	Emergence and Bifurcations of LPTs in the Chain of Three Coupled Oscillators.	67
3.1	“Hard” Nonlinearity	67
3.1.1	Bifurcations of Limiting Phase Trajectories and Routes to Chaos in the Anharmonic Chain of the Three Coupled Particles	67
3.1.2	Nonlinear Normal Modes (NNMs)	69
3.1.3	Emergence and Bifurcations of Limiting Phase Trajectories (LPTs) in the System of Three Coupled Oscillators	71
3.1.4	Numerical Results	74
3.1.5	Spatially Localized Pulsating Regimes	77
3.2	“Soft” Nonlinearity.	80
4	Quasi-One-Dimensional Nonlinear Lattices.	85
4.1	Finite Fermi–Pasta–Ulam Oscillatory Chain	86
4.1.1	The Model.	86
4.1.2	Basic Asymptotic	88
4.1.3	From “Waves” to “Particles”.	92
4.1.4	Analytical Solution for the LPTs.	100
4.2	Klein–Gordon Lattice	102
4.2.1	The Model.	102
4.2.2	Asymptotic Analysis	105
4.3	Intense Energy Exchange and Localization in Periodic FPU Dimer Chains	111
4.3.1	The Model.	113
4.3.2	Intensive Energy Exchanges: Linear Case ($N = 1, \alpha = 0$).	114
4.3.3	Complete Energy Exchanges and Localization: Nonlinear Case ($N = 1, \alpha > 0$)	119

4.3.4	Extension to the Higher Number of Light Particles ($N > 1, \alpha > 0$)	124
	Appendix	136
	References.	137
5	Localized Nonlinear Excitations and Inter-chain Energy Exchange	141
5.1	Linear Chains with Weak Coupling	142
5.2	Nonlinear Chains	144
5.2.1	Chains with Nonlinearity, Compatible with Coupling	147
	References.	152
Part II Extensions to Non-conservative Systems		
6	Duffing Oscillators	155
6.1	Duffing Oscillator with Harmonic Forcing Near 1:1 Resonance.	156
6.1.1	Main Equations and Definitions.	156
6.1.2	Stationary States, LPTs, and Critical Parameters	158
6.1.3	Non-smooth Approximations of Strongly Nonlinear Oscillatory Modes	162
6.1.4	Analysis with Taking into Account the Energy Dissipation.	165
6.2	Duffing Oscillator Subjected to Biharmonic Forcing Near the Primary Resonance.	166
6.2.1	Equations of Fast and Slow Motion	167
6.2.2	LPTs of Slow Motion in a Non-dissipative System.	168
6.2.3	Super-Slow Dynamics	169
6.2.4	Relaxation Oscillations in a Lightly Damped System	171
6.3	Super-Harmonic Resonance	177
6.3.1	Equations of Motion	177
6.3.2	Super-Harmonic Resonance in the Non-dissipative System.	180
	References.	185
7	Non-conventional Synchronization of Weakly Coupled Active Oscillators	187
7.1	Main Equations.	188
7.1.1	Coupled Active Oscillators	189
7.2	NNMs and LPTs Symmetries	190
7.3	Analysis of the Phase Plane and Analytical Solutions	190
	References.	194

8	Limiting Phase Trajectories and the Emergence of Autoresonance in Anharmonic Oscillators.	195
8.1	Autoresonance in a SDOF Nonlinear Oscillator	197
8.1.1	Critical Parameters.	198
8.1.2	Numerical Evidence of Capture into Resonance	202
8.2	Autoresonance Versus Localization in Weakly Coupled Oscillators.	203
8.2.1	Energy Transfer in a System with Constant Excitation Frequency	204
8.2.2	Energy Localization and Transport in a System with a Slowly Varying Forcing Frequency	207
8.2.3	Energy Transfer in a System with Slow Changes of the Natural and Excitation Frequencies	209
8.3	Autoresonance in Nonlinear Chains	211
8.3.1	The Model.	211
8.3.2	Quasi-steady States	214
8.3.3	Parametric Thresholds	215
8.3.4	Numerical Results	218
	References.	222

Part III Applications

9	Targeted Energy Transfer	227
9.1	The Model	227
9.2	Analytical Study	228
9.3	Selection of Resonance Terms and Principal Asymptotic Approximation	230
9.4	3 DOF Oscillators with the NES	235
9.5	Transient Dynamics of the Dissipative System	238
9.6	Reduction to a Model of the Single Oscillator	240
	References.	243
10	Nonlinear Energy Channeling in the 2D, Locally Resonant, Systems.	245
10.1	Unit Cell Model: High Energy Pulsations.	245
10.1.1	The Model.	246
10.1.2	Analytical Study	249
10.1.3	Numerical Verifications	263
10.1.4	Concluding Remarks	269
10.2	Unit Cell Model: Low Energy Excitation Regimes.	270
10.2.1	Numerical Evidence of the Unidirectional Energy Channeling	270
10.2.2	Theoretical Study	271

10.2.3	Numerical Verifications	286
10.2.4	Concluding Remarks	288
	Appendix	291
	References	292
11	Nonlinear Targeted Energy Transfer and Macroscopic Analogue of the Quantum Landau-Zener Effect in Coupled Granular Chains	293
11.1	Introduction	293
11.2	System Description	295
11.3	Recurrent Energy Exchange Phenomena in the System of Coupled Granular Chains	296
11.4	Nonlinear Targeted Energy Transfer and Energy Exchange: Analysis	301
11.4.1	Localization of Energy by Complete Decoupling	302
11.5	Targeted Energy Transfer Through the Landau-Zener Tunneling Effect in Space	303
11.5.1	Nonlinear Targeted Energy Transfer and Irreversible Energy Exchange: Simulation	314
11.6	Conclusions	319
	Appendix	319
	References	323
12	Forced Pendulum	327
12.1	The Model	328
12.2	Nonstationary Dynamics and Dynamical Transitions	330
12.3	Poincaré Sections and Onset of Chaotic Motion	332
	References	335
13	Classical Analog of Linear and Quasi-Linear Quantum Tunneling	337
13.1	Two Weakly Coupled Linear Oscillators	338
13.2	Approximate Analysis of Energy Transfer in the Linear System	340
13.3	Classical Analog of Quasi-Linear Quantum Tunneling	343
13.4	Moderately and Strongly Nonlinear Adiabatic Tunneling	346
13.4.1	Moderately Nonlinear Regimes	346
13.5	Strongly Nonlinear Regimes	352
	References	353
14	Strongly Nonlinear Lattices	355
14.1	The Large-Amplitude Oscillations in the Discrete Finite Frenkel–Kontorova Model	355
14.2	Large-Amplitude Nonlinear Normal Modes of the Discrete Sine-Lattices	362

14.3	Is Energy Localization Possible in the Conditions of Acoustic Vacuum?	374
14.3.1	The Model and Equations of Motion.	375
14.3.2	Two-Mode Approximation	376
14.3.3	Cluster Variables	379
14.3.4	Equations in Angular Variables in Cluster Variant	380
14.3.5	Phase Plane	381
14.3.6	Analytical solution for LPT.	382
14.3.7	Poincare Sections	384
	Appendix 1: Timescale Separation	385
	Appendix 2: Projection onto Two Modes—Formulas	387
	References.	389
15	Nonlinear Vibrations of the Carbon Nanotubes	391
15.1	Nonlinear Optical Vibrations of Single-Walled Carbon Nanotubes.	392
15.1.1	The Model.	393
15.1.2	Radial Breathing Mode	394
15.1.3	Circumferential Flexure Mode.	403
15.2	Coupling Shell- and Beam-Type Oscillations of Single-Walled Carbon Nanotubes	414
15.2.1	The Model.	414
15.2.2	Stationary Solutions.	417
15.2.3	Multi-scale Expansion	419
15.2.4	Analysis of the Steady States Solutions and Non-stationary Dynamics	422
	References.	431
	Conclusions	435

Nonstationary Resonant Dynamics of Oscillatory Chains
and Nanostructures

Manevitch, L.I.; Kovaleva, A.; Smirnov, V.; Starosvetsky,
Y.

2018, XXII, 436 p. 194 illus., 116 illus. in color.,

Hardcover

ISBN: 978-981-10-4665-0