Preface

Since the second half of the last century there has been significant development in the methods used to solve applied problems of mechanics. There are a number of reasons for this. First, as a result of the increasing needs of technology the problems to be researched have become ever more complicated in themselves. Second, there is unprecedented quantitative growth, as well as improvement of the quality of computer equipment and the expansion of its capabilities. Finally, there is the development of computational methods associated with the achievements of mathematical physics and functional analysis.

Techniques based on finite-dimensional approximation of differential and integral equations, such as the methods of finite differences, finite elements, as well as of influence matrices, have become widespread. Their advantage lies in universality and coverage of a wide class of problems. The disadvantages include the need to deal with large numerical arrays. Moreover, by splitting the calculated object into smaller and smaller elements to improve the accuracy of computation, singular matrices can be encountered [10].

The alternative approach, to which this book is devoted, does not differ in universality and requires the development of algorithms to solve each problem or a certain class of problems separately. This includes the so-called variational, projection and iterative methods which, when applied to nonlinear problems, use the procedure of linearization of original nonlinear equations. The composition of efficient algorithms to solve specific individual complicated problems demands a certain skill in the use of mathematical physics and functional analysis. However, significant results can be achieved in this way. According to the variational approach, the equations of the problem correspond to the functionality, extremality or stationarity conditions that are equivalent to the equations themselves. To construct an approximate solution to the problem, functional variation needs to be removed. To do this an approximate solution is represented as a finite series expansion in so-called coordinate functions, and then so-called projection conditions follow from functional stationarity conditions. Projection conditions, approximately replacing the original equations, essentially represent the removal of scalar products from the residual of equations and from the sequence of coordinate functions. The core question consists in choosing a system of coordinate functions. These functions should be linearly independent and possess some properties of the desired solution; for example, to satisfy all or some of the conditions of differentiability and boundary conditions. For this purpose, one can use the spectral properties of the close objective; for example, when considering an elastic body with variable inertial and elastic properties it is convenient to take the forms of free oscillations (natural forms of vibrations) of a homogeneous body as the coordinate functions. For ordinary linear differential equations the procedure based on the use of variational conditions leads to a finite system of algebraic equations. If the selected coordinate functions are not only linearly independent, but also orthonormal in a certain sense, as when using the spectral properties of the nearby problem, then the resulting system of algebraic equations will be well conditioned.

Approximate solution of an extremal problem by reducing it to a system of algebraic equations was first proposed by W. Ritz [9]. B.G. Galerkin [2], [3] pointed to the possibility of composing projection conditions, bypassing their variational justification, which greatly expanded the class of problems that can be solved using projection methods. L.V. Kantorovich extended the application of projection methods to partial differential equations [6]. In nonlinear problems, Newton proposed the iterative method of solution, based on linearization of nonlinear operations by differentiating them by retaining only the first derivative [4]. Cauchy extended the iterative method of Newton to multidimensional extremal problems, using the concept of functional gradient with determination of the step length just from the linearized extremum condition of the functional itself [1]. Thus, Cauchy laid the foundations of iterative gradient methods to solve linear and nonlinear problems. L.V. Kantorovich in his fundamental paper [4] gave the generalized interpretation of approximate methods to solve linear and nonlinear operator equations, based on the notions of functional analysis, thereby significantly extending the class of problems that can be solved using these methods. The most complete statement of approximate methods for solving operator equations is given in Russian in the book [7]. The monograph [8] is devoted to detailed description and justification of variational methods.

This book is written on the basis of the course of lectures delivered over many years at the Physics and Mechanics Faculty of Leningrad Polytechnic Institute. The part of the book dedicated to applied problems was primarily developed at the Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences.

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St. Petersburg, Los Angeles, USA Vladimir Fridman
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