Chapter 2
Robust Stabilization of Single Nonlinear Time-Delay System

Abstract The problem of robust stabilization for a class of uncertain dynamic systems with multiple delayed state perturbations is considered. It is assumed that perturbations of the time-delay sections are not bounded by first-order linear functions, but bounded by high-order functions with unknown gains. And the time delay considered is time varying. Two classes of controllers are proposed. When the time derivative of each time-varying time delay is less than one, a class of adaptive state feedback controllers are proposed based on Lyapunov–Krasovskii method, which can render the closed-loop systems uniformly ultimately bounded stable. Novel nonlinear feedback controllers are developed by employing Razumikhin lemma, and the controller also can render the closed-loop systems stable in the sense of uniform ultimate boundedness. Finally, several examples are given to show the potential of the proposed techniques.

2.1 Introduction

In the control literature, for dynamic systems with delayed state perturbations, the delayed state perturbations are generally supposed to be bounded by first-order linear functions. The existing results can be divided into two classes: The bounds are known and the bounds are unknown. With the bounds known, one can employ the bounds to construct some types of stabilizing state feedback controllers or to develop some stability conditions [21, 31, 54, 55, 95, 110, 132, 141, 175, 205], and the results often come to solving LMI. If the bounds are partially known or not known, some papers appeared. In [23], the authors developed a saturation-type robust adaptive controller for a class of uncertain dynamic systems, where the uncertainty bounds were partially known. In [137], a robust adaptive feedback controller was developed for a class of uncertain linear dynamic delay systems where the bounds of the uncertainty were unknown, but the results needed to examine whether the perturbation of the delayed state can satisfy a bound. In [72], an adaptive feedback controller and a novel nonlinear feedback controller were proposed to solve the control problem of time-delay systems with the bounds of uncertainties completely unknown. However, in practical control problems, the delay perturbations may not be bounded by

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first-order function of time delay, but they are bounded by high-order polynomial and the time delay is often time varying. Therefore in this chapter, we will investigate the problem of robust control for a class of dynamic systems with multiple time-varying delays, and the uncertain sections are bounded by high-order polynomial functions with unknown gains. Employing our former idea [72], we propose two classes of controllers in this chapter. When the derivative of each time-varying time delay is less than one, a class of adaptive feedback controllers are proposed, and we prove the closed-loop systems are uniformly ultimately bounded stable based on Lyapunov stability theory and Lyapunov–Krasovskii function. If the time delay is bounded, by employing Razumikhin-type lemma, we construct a class of novel nonlinear feedback controllers, which can also render the corresponding closed-loop systems stable in the sense of uniform ultimate boundedness.

2.2 System Formulation and Preliminaries

Consider a class of dynamic systems described by the following differential-difference equations

$$\begin{cases} \frac{dx(t)}{dt} = Ax(t) + Bu(t) + \sum\limits_{j=1}^{r} E_j\left(x\left(t-h_j(t)\right), t\right) & \text{(2.1a)} \\ x(t) = \psi(t), \quad t \in [t_0 - \tau, t_0] & \text{(2.1b)} \end{cases}$$

where $t \in R$ is the time, $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, A and B are the known constant matrices of appropriate dimensions, $E_j(\cdot) : R^n \times R \to R^n$, $j \in \{1, 2, \ldots, r\}$, is nonlinear continuous vector function which represents delayed state perturbations for the system. In addition, the time delay $h_j(t)$, $j = 1, 2, \ldots, r$, are assumed to be time varying. The initial condition is given by (2.1b) where $\psi(t)$ is a continuous function on $[t_0 - \tau, t_0]$, and $\tau := \max\left\{h_j(t), j = 1, 2, \ldots r\right\}$. For proposing our controllers, we introduce the following standard assumptions which will be used in the following sections.

Assumption 2.1 The pair $\{A, B\}$ given in (2.1a) is completely controllable.

Assumption 2.2 There exists the continuous vector function $\eta_j(\cdot) : R^n \times R \to R^m$, $j \in \{1, 2, \ldots, r\}$ such that for all $(x, t) \in R^n \times R$

$$E_j\left(x\left(t-h_j(t)\right), t\right) = B\eta_j\left(x\left(t-h_j(t)\right), t\right) \tag{2.2}$$

and the following inequalities are satisfied

$$\left\|\eta_j\left(x\left(t-h_j(t)\right), t\right)\right\| \le \sum_{i=1}^{s} \beta_{ij} \left\|x\left(t-h_j(t)\right)\right\|^i \tag{2.3}$$

where $\|\cdot\|$ denotes the Euclidean norm, β_{ij} is unknown positive constant.

Remark 2.1 Assumption 2.1 is standard and denotes the internally stability of the nominal system. It is well known that Assumption 2.2 is a matching condition assumption. Different from the assumptions in existing literatures investigating robust control for time-delay systems, we assume that the uncertain section is not bounded by a linear function (for (2.3) $s = 1$), but bounded by a nonlinear function (2.3). Reference [72] proposed different control strategy to investigate the robust stabilization for system (2.1a)–(2.1b) with uncertainties bounded by a linear function, and the bounds were not known. In this chapter, we will investigate the control problem for a large class of time-delay systems and β_{ij} is also not required to be known. Before giving our main results, the following lemmas are given, which will be needed in this chapter.

Lemma 2.1 ([57]) *Consider the retarded functional differential equation*

$$\frac{dx\left(t\right)}{dt} = f\left(t, x_t\right) \tag{2.4}$$

with the initial condition

$$x\left(t\right) = \psi\left(t\right), t \in [t_0 - h, t_0]$$

Suppose that the functions $\gamma_i\left(\cdot\right), i = 1, 2, 3,$ *are of* K_∞*-class. If there is a continuous function* $V\left(\cdot\right) : [t_0 - h, \infty) \times R^n \rightarrow R^+$ *such that*
(1) $\gamma_1\left(\|x\|\right) \leq V\left(t, x\right) \leq \gamma_2\left(\|x\|\right), t \in [t_0 - h, \infty), x \in R^n$.
(2) There exists a continuous nonincreasing function $p\left(s\right) > s$ *for* $s > 0$

$$\dot{V}\left(t, x\right) \leq -\gamma_3\left(\|x\|\right) + \nu \tag{2.5}$$

if

$$V\left(\xi, x\left(\xi\right)\right) < p\left(\left(V\left(t, x\left(t\right)\right)\right)\right), \quad t - h \leq \xi \leq t, t \geq t_0 \tag{2.6}$$

where if $\nu > 0$ *is a constant, the solutions of* (2.5) *are uniformly ultimately bounded, and if* $\nu > 0$ *is a time-varying parameter and satisfies* $\int_0^\infty \nu\left(t\right) dt < \infty$, *the system is asymptotically stable.*

Lemma 2.2 *For any positive scalars* a, b, *and* c, *the following inequality holds*

$$ay^c - by^{c+1} \leq \frac{b}{c}\left(\frac{ac}{b\left(c+1\right)}\right)^{c+1}$$

where y *is a positive variable parameter.*

Proof Define function $f\left(y\right) = ay^c - by^{c+1}$, then we have

$$\frac{df\left(y\right)}{dy} = acy^{c-1} - b\left(c+1\right)y^c$$

and let $\frac{df(y)}{dy} = 0\,(y > 0)$, then we can obtain that

$$acy^{c-1} - b\,(c+1)\,y^{c} = 0$$

further that $y = \frac{ac}{b(c+1)} = y^*$, and we want to prove that $f\,(y^*)$ is the maximum value, so we need to examine that $\frac{d^2 f(y)}{dy^2}\left|_{y=y^*}\right. < 0$:

$$\begin{aligned}\frac{d^2 f\,(y)}{dy^2}\left|_{y=y^*}\right. &= \left[ac\,(c-1)\,y^{c-2} - b\,(c+1)\,cy^{c-1}\right]\left|_{y=y^*}\right. \\ &= \left(ac\,(c-1) - b\,(c+1)\,c\frac{ac}{b\,(c+1)}\right)\left(\frac{ac}{b\,(c+1)}\right)^{c-2} \\ &= -ac\left(\frac{ac}{b\,(c+1)}\right)^{c-2} \\ &< 0\end{aligned}$$

so $f\,(y^*)$ is the maximum value of $f\,(y)$, that is

$$ay^{c} - by^{c+1} \le f\left(y^*\right) = \frac{b}{c}\left(\frac{ac}{b\,(c+1)}\right)^{c+1}$$

The proof is completed.

2.3 Adaptive Robust State Feedback Controller

In this section, we will propose a class of adaptive robust state feedback controller based on L-K method. The following standard assumption is needed.

Assumption 2.3 The derivative of each time-varying delay is less than one, that is, $\dot{h}_j\,(t) \le \alpha_j < 1$. From Assumption 2.1, there exists any scalar μ and any positive symmetric matrix $Q \in R^{n\times n}$ that the following Riccati equation of the form

$$A^T P + PA - \mu PBB^T P = -Q \tag{2.7}$$

has a solution $P \in R^{n\times n}$ which is also a symmetric positive definite matrix.

Now for system (2.1a)–(2.1b), we will propose the following linear memoryless robust state feedback controller

$$u\,(t) = -\frac{\mu}{2}B^T Px - \overline{\theta}B^T\frac{\partial V}{\partial x}^T \tag{2.8}$$

where μ satisfies (2.7), function V is defined as follows:

$$V=\sum_{i=1}^{s}V_i \text{ and } V_i=\frac{1}{i}\left(x^TPx\right)^i \tag{2.9}$$

and $\overline{\theta}$ is the adaptive parameter with adaptive law

$$\frac{d\overline{\theta}(t)}{dt}=k\left\|\frac{\partial V}{\partial x}B\right\|^2-kl\overline{\theta} \tag{2.10}$$

where k and l are positive parameters which can be adjusted.

Theorem 2.1 *Consider the system* (2.1a)–(2.1b) *satisfying Assumptions 2.1–2.3, then the state feedback controller* (2.8) *with adaptive law* (2.10) *will render the closed-loop system uniformly ultimately bounded stable.*

Proof We first define a Lyapunov–Krasovskii functional candidate for closed-loop system as follows:

$$W\left(x,\overline{\theta}\right)=\sum_{i=1}^{s}V_i+\sum_{i=1}^{s}\sum_{j=1}^{r}\int_{t-h_j(t)}^{t}\digamma_{ij}\left\|x(z)\right\|^{2i}dz+\frac{1}{2k}\widetilde{\theta}^2 \tag{2.11}$$

where matrix P is the solution of algebraic Riccati differential equation (2.7), $\digamma_{ij}$ is positive scalar, $\widetilde{\theta}=\theta-\overline{\theta}$, θ is defined as

$$\theta=\sum_{j=1}^{r}\sum_{i=1}^{s}\frac{\beta_{ij}^2}{4\left(1-\alpha_j\right)\digamma_{ij}} \tag{2.12}$$

Then, by taking the time derivative of $W(\cdot)$ along the trajectories of closed-loop system, one can obtain that for any $t\geq t_0$

$$\begin{aligned}\frac{dW\left(x,\overline{\theta}\right)}{dt}=&\sum_{i=1}^{s}\left(x^TPx\right)^{i-1}x^T\left[A^TP+PA-\mu PBB^TP\right]x\\&+\frac{\partial V}{\partial x}B\sum_{j=1}^{r}\eta_j\left(x\left(t-h_j(t)\right),t\right)+\frac{1}{k}\widetilde{\theta}\frac{d\widetilde{\theta}}{dt}\\&-\overline{\theta}\left\|\frac{\partial V}{\partial x}B\right\|^2+\sum_{i=1}^{s}\sum_{j=1}^{r}\digamma_{ij}\left\|x(t)\right\|^{2i}\\&-\sum_{i=1}^{s}\sum_{j=1}^{r}\left(1-\dot{h}_j(t)\right)\digamma_{ij}\left\|x\left(t-h_j(t)\right)\right\|^{2i}\end{aligned} \tag{2.13}$$

we know

$$\begin{aligned}
&\frac{\partial V}{\partial x} B \sum_{j=1}^{r} \eta_j \left(x\left(t-h_j(t)\right), t\right) \\
&\leq \sum_{j=1}^{r}\sum_{i=1}^{s} \left\| \frac{\partial V}{\partial x} B \right\| \beta_{ij} \left\| x\left(t-h_j(t)\right)\right\|^{i} \\
&\leq \sum_{j=1}^{r}\sum_{i=1}^{s} \left(\frac{\beta_{ij}^2}{4\left(1-\alpha_j\right) F_{ij}} \left\| \frac{\partial V}{\partial x} B \right\|^2 + \left(1-\alpha_j\right) F_{ij} \left\| x\left(t-h_j(t)\right)\right\|^{2i} \right)
\end{aligned} \tag{2.14}$$

Substituting (2.14) into (2.13), one can obtain that

$$\begin{aligned}
\frac{dW\left(x,\overline{\theta}\right)}{dt} \leq & \sum_{i=1}^{s} \left(x^T P x\right)^{i-1} x^T(t) \left[A^T P + PA - \mu P B B^T P\right] x(t) \\
& + \sum_{j=1}^{r}\sum_{i=1}^{s} \frac{\beta_{ij}^2}{4\left(1-\alpha_j\right) F_{ij}} \left\| \frac{\partial V}{\partial x} B \right\|^2 \\
& + \frac{1}{k}\widetilde{\theta}\frac{d\widetilde{\theta}}{dt} - \overline{\theta} \left\| \frac{\partial V}{\partial x} B \right\|^2 + \sum_{j=1}^{r}\sum_{i=1}^{s} F_{ij} \left\| x(t)\right\|^{2i}
\end{aligned} \tag{2.15}$$

Letting $\sum_{j=1}^{r} F_{ij} = r_i$, we know that F_{ij} can always be selected to be sufficiently small such that the following inequality holds

$$\lambda_{\min}^{i-1}(P)\,\lambda_{\min}(Q) - r_i = -\xi_i < 0$$

where $\lambda_{\min}(P)$ and $\lambda_{\min}(Q)$ represent the minimum eigenvalues of P and Q respectively, ξ_i is a positive scalar, so one can obtain

$$\begin{aligned}
&\sum_{i=1}^{s} \left(x^T P x\right)^{i-1} x^T(t) \left[A^T P + PA - \mu P B B^T P\right] x(t) + \sum_{j=1}^{r}\sum_{i=1}^{s} F_{ij} \left\| x(t)\right\|^{2i} \\
&\leq \sum_{i=1}^{s} - \left(\lambda_{\min}^{i-1}(P)\,\lambda_{\min}(Q) - r_i\right) \left\| x(t)\right\|^{2i} \\
&= -\sum_{i=1}^{s} \xi_i \left\| x(t)\right\|^{2i}
\end{aligned}$$

Substituting above inequality and adaptive law (2.10) into (2.15), we further get

$$\begin{aligned}
\frac{dW\left(x,\overline{\theta}\right)}{dt} &\le -\sum_{i=1}^{s}\xi_i\left\|x\left(t\right)\right\|^{2i}+\sum_{j=1}^{r}\sum_{i=1}^{s}\frac{\beta_{ij}^2}{4\left(1-\alpha_j\right)F_{ij}}\left\|\frac{\partial V}{\partial x}B\right\|^2\\
&\quad+\frac{1}{k}\widetilde{\theta}\frac{d\widetilde{\theta}}{dt}-\overline{\theta}\left\|\frac{\partial V}{\partial x}B\right\|^2\\
&=-\sum_{i=1}^{s}\xi_i\left\|x\left(t\right)\right\|^{2i}+\widetilde{\theta}\left\|\frac{\partial V}{\partial x}B\right\|^2+\frac{1}{k}\widetilde{\theta}\frac{d\widetilde{\theta}}{dt}\\
&=-\sum_{i=1}^{s}\xi_i\left\|x\left(t\right)\right\|^{2i}+l\left(\theta-\overline{\theta}\right)\overline{\theta}\\
&=-\sum_{i=1}^{s}\xi_i\left\|x\left(t\right)\right\|^{2i}-\frac{1}{2}l\overline{\theta}^2-\frac{1}{2}l\left(\theta-\overline{\theta}\right)^2+\frac{1}{2}l\theta^2\\
&<-\sum_{i=1}^{s}\xi_i\left\|x\left(t\right)\right\|^{2i}+\frac{1}{2}l\theta^2
\end{aligned}\tag{2.16}$$

As we know that θ is a constant and l is an adjustable parameter, it is easy for us to obtain that the closed-loop system is robust uniformly ultimately bounded stable in light of Lyapunov stability theory.

Remark 2.2 From (2.13), we can know that one can obtain the upper bound on the steady state as small as desired by decreasing the parameter l. So the system designer can tune the size of the residual set by adjusting properly this parameter l in the adaptive law.

Remark 2.3 In this section, we have proposed a memoryless state feedback controller with the adaptive parameter of one dimension, which renders the close-loop system robust stable in the sense of uniform ultimate boundedness. If we choose $s = 1$, that is, the uncertain time-delay sections satisfy first-order functions with unknown parameters, and from (2.7) we can obtain the corresponding controller which is similar to that of [72], so [72] is a special case of this chapter and the results in this chapter extend the results in [72].

2.4 Novel Nonlinear Feedback Controller

In this section, we will propose a nonlinear feedback controller for system (2.1a)–(2.1b) based on Lypunov–Razumikhin method.

Theorem 2.2 *Consider the system* (2.1a)–(2.1b) *with Assumptions 2.1 and 2.2, we can construct the following nonlinear state feedback controller*

$$\begin{aligned} u &= u_1 + u_2 \\ u_1 &= -\frac{1}{2}(\mu + \upsilon) B^T P x \\ u_2 &= -\rho(x) \frac{B^T P x \rho(x)}{\left\| B^T P x \rho(x) \right\| + f(t)} \end{aligned} \tag{2.17}$$

which renders the closed-loop system stable in the sense of uniform ultimate boundedness, where μ and P satisfy (2.7), υ is a positive parameter which can be adjusted and

$$\rho(x) = \sum_{i=1}^{s} (v_i + \zeta_i \|x(t)\|) \|x(t)\|^i, \; f(t) = \vartheta e^{-\epsilon t}$$

where v_i, ζ_i, ϑ, and ϵ are also adjustable positive parameters.

Proof Applying controller (2.17) to (2.1a)–(2.1b) yields

$$\frac{dx(t)}{dt} = Ax(t) + B(u_1 + u_2) + \sum_{j=1}^{r} E_j\left(x\left(t - h_j(t)\right), t\right)$$

We first define the following Lyapunov function candidate

$$W(x(t), t) = x^T(t) P x(t)$$

where P is the solution of (2.7). Then, the derivative of $W(x(t), t)$ along the above system is

$$\begin{aligned} \frac{dW(x(t), t)}{dt} &= x^T(t)\left(PA + A^T P - \mu P B B^T P\right) x(t) - \upsilon \left\| B^T P x \right\|^2 \\ &\quad + 2x^T(t) P \sum_{j=1}^{r} E_j\left(x\left(t - h_j(t)\right), t\right) - 2x^T P B u_2 \\ &= -x^T(t) Q x(t) + 2x^T(t) P B \sum_{j=1}^{r} \eta_j\left(x\left(t - h_j(t)\right), t\right) \\ &\quad - 2x^T P B u_2 - \upsilon \left\| B^T P x \right\|^2 \end{aligned}$$

We further obtain

$$\begin{aligned} \frac{dW(x(t), t)}{dt} &\leq -x^T Q x - \upsilon \left\| B^T P x \right\|^2 - 2x^T P B u_2 \\ &\quad + 2 \left\| x^T P B \right\| \sum_{j=1}^{r} \sum_{i=1}^{s} \beta_{ij} \left\| x\left(t - h_j(t)\right) \right\|^i \end{aligned} \tag{2.18}$$

Following the Razumikhin-type theorem, we assume that for some positive scalar $q > 1$, such that the following inequality holds

$$V\left(x\left(\omega\right),\omega\right) < q^2 V\left(x\left(t\right),t\right), t-\tau \le \omega \le t.$$

For $V = x^T P x$, further we have

$$\left\|x\left(\omega\right)\right\| < q\overline{\rho}\left\|x\left(t\right)\right\|, t-\tau \le \omega \le t$$

where $\overline{\rho} = \sqrt{\lambda_{\max}\left(P\right)/\lambda_{\min}\left(P\right)}$, so

$$\begin{aligned}
&2\left\|x^T P B\right\| \sum_{j=1}^{r}\sum_{i=1}^{s}\beta_{ij}\left\|x\left(t-h_j\left(t\right)\right)\right\|^i - 2x^T P B u_2 \\
&\le 2\left\|x^T P B\right\| \sum_{j=1}^{r}\sum_{i=1}^{s}\left(q\overline{\rho}\right)^i\beta_{ij}\left\|x\left(t\right)\right\|^i - 2x^T P B\rho\left(x\right)\frac{B^T P x\rho\left(x\right)}{\left\|B^T P x\rho\left(x\right)\right\| + f\left(t\right)} \\
&\le 2\left\|x^T P B\right\|\left(\sum_{j=1}^{r}\sum_{i=1}^{s}\left(q\overline{\rho}\right)^i\beta_{ij}\left\|x\right\|^i - \sum_{i=1}^{s}\upsilon_i\left\|x\right\|^i\right) \\
&\quad -2\left\|B^T P x\right\|\sum_{i=1}^{s}\zeta_i\left\|x\left(t\right)\right\|^{i+1} + 2f\left(t\right) \qquad (2.19)
\end{aligned}$$

then let

$$d_i = \sum_{j=1}^{r}\left(q\overline{\rho}\right)^i\beta_{ij} \qquad (2.20)$$

Substituting (2.19), (2.20) into (2.18), we have

$$\begin{aligned}
\frac{dW}{dt} &\le -x^T Q x + 2f\left(t\right) - \upsilon\left\|B^T P x\right\|^2 + 2\left\|x^T P B\right\| \\
&\quad \times \sum_{i=1}^{s}\left[\left(d_i - \upsilon_i\right)\left\|x\left(t\right)\right\|^i - \zeta_i\left\|x\left(t\right)\right\|^{i+1}\right] \qquad (2.21)
\end{aligned}$$

In the case that $\upsilon_i \ge d_i$, it is easy to obtain that

$$\frac{dW}{dt} \le -x^T Q x + 2f\left(t\right) \qquad (2.22)$$

Based on Lemma 2.1, the nonlinear state feedback controller will render the closed-loop system asymptotically stable. In the case that $\upsilon_i < d_i$, based on Lemma 2.2 the following inequality holds

$$\sum_{i=1}^{s}\left[\left(d_i-\upsilon_i\right)\|x(t)\|^i-\zeta_i\|x(t)\|^{i+1}\right]$$
$$\leq \sum_{i=1}^{s}\frac{\zeta_i}{i}\left(\frac{i\left(d_i-\upsilon_i\right)}{\zeta_i\,(i+1)}\right)^{i+1}=b \tag{2.23}$$

Substituting (2.23) into (2.21), one can get

$$\begin{aligned}\frac{dW}{dt}&\leq -x^TQx+2f(t)-\upsilon\left\|B^TPx\right\|^2+2b\left\|x^TPB\right\|\\&\leq -x^TPx+\frac{b^2}{\upsilon}+2f(t)\end{aligned}$$

As we know b, υ are constants, it is easy to obtain that the closed-loop system is uniformly ultimately bounded stable. Therefore, the controller (2.17) can render the closed-loop system uniformly ultimately bounded stable in the whole.

Remark 2.4 From (2.21) and (2.23), one can simply get the following inequality

$$\frac{dV(x(t),t)}{dt}\leq -sV(x(t),t)+\varpi$$

where s and ϖ are positive parameters. For $V=x^TPx$, it is easy to obtain that system (2.1a)–(2.1b) is exponentially uniformly ultimately stable with the controller (2.17).

Remark 2.5 If υ_i is selected sufficiently large, (2.22) will hold and the closed-loop system will be asymptotically stable. If $\upsilon_i < d_i$, we can get (2.21), and from (2.21) one can obtain the upper bound on the steady state $x(t)$ as small as desired by increasing parameter υ. In practical systems, the designer can choose proper parameters υ, υ_i, and ζ_i to render the controlled system satisfying special performance level required and also the consuming energy less. In this section, the controller is obtained based on Lyapunov–Razumikhin lemma. So we need not assume any limitation on the time derivative of time delay.

2.5 Simulations

We will give two examples to verify the validity of the both controllers in this section.

Example 1 Logistic chaotic system

$$\dot{x}=-26x+bx(t-d)+cx(t-d)^2$$

when $b=-c=104$ and the time delay $d=0.5$, the system is chaotic and the chaotic behavior is depicted in Fig. 2.1.

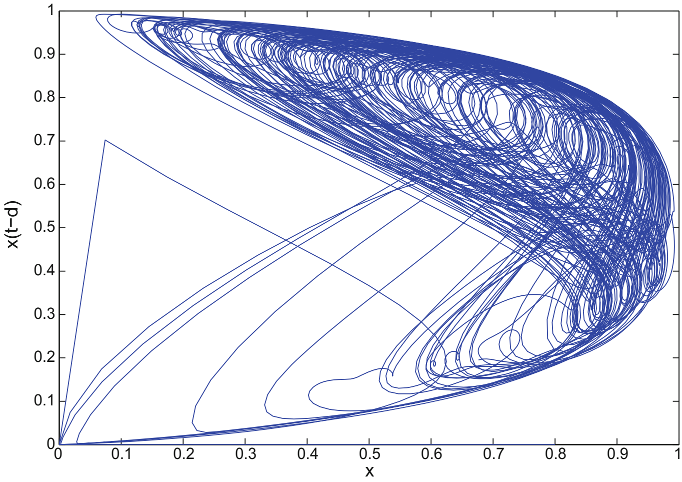

Fig. 2.1 Behaviour of logistic time-delay chaotic system

We will consider controlling this class of systems by using adaptive controllers and novel nonlinear feedback controllers proposed in this chapter. Adaptive feedback controller: Based on Sect. 2.3, for controller (8–10) we let $\mu = 0$, $V = 50\sum_{i=1}^{2}\frac{1}{i}x^{2i}$, then the feedback controller is

$$u\left(t\right) = -100\overline{\theta}\left(x + x^{3}\right)$$

with corresponding adaptive law

$$\dot{\overline{\theta}} = 100\left(x + x^{3}\right)^{2} - 0.01\overline{\theta}$$

for simulation the initial value is selected as $x\left(0\right) = 1$ and $\overline{\theta}\left(0\right) = 0$, then sample time is $T = 0.001$ s the response of the state of closed-loop system is shown in Fig. 2.2, and we can see that the solution of the closed-loop system reaches the small region of equilibrium point 0 in limited time.

Nonlinear feedback controller: From Sect. 2.4, we can construct the following controller

$$u = -\rho\left(x\right)\frac{x\rho\left(x\right)}{\left|x\right|\rho\left(x\right) + f\left(t\right)} - 30x$$

where

$$\rho\left(x\right) = 100\left|x\right| + 150x^{2} + \left|x\right|^{3}, \; f\left(t\right) = e^{-t}$$

and also the simulation result is shown in Fig. 2.3, from which we can obtain that the controller renders the closed-loop system uniformly ultimately bounded stable.

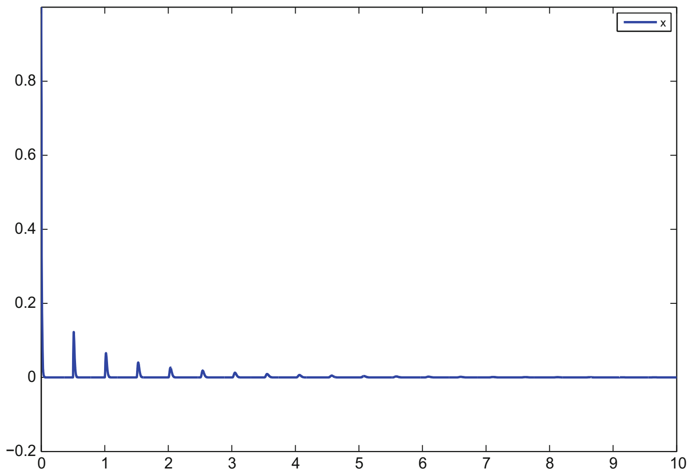

Fig. 2.2 State response for Example 1 with adaptive controller

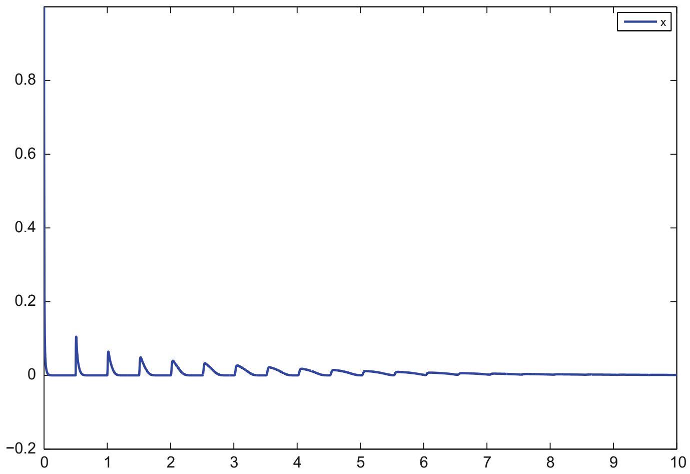

Fig. 2.3 State response for Example 1 with nonlinear controller

Example 2 Consider the following system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 0 \\ \delta x_1 \left(t - 0.8 \left|\cos t\right|\right) x_2 \left(t - 0.5 \left|\sin t\right|\right) \end{pmatrix}$$

where δ is an unknown parameter and we find that the unknown sections are not bounded by first-order function, so the existing results can not apply to control for this class of system. Let us consider the controllers proposed in this chapter, for simulation we let $\delta = 2\sin t$; let $\mu = 8$, $V(x) = \sum_{i=1}^{2} \frac{1}{i} \left(x^T x\right)^i$, further we

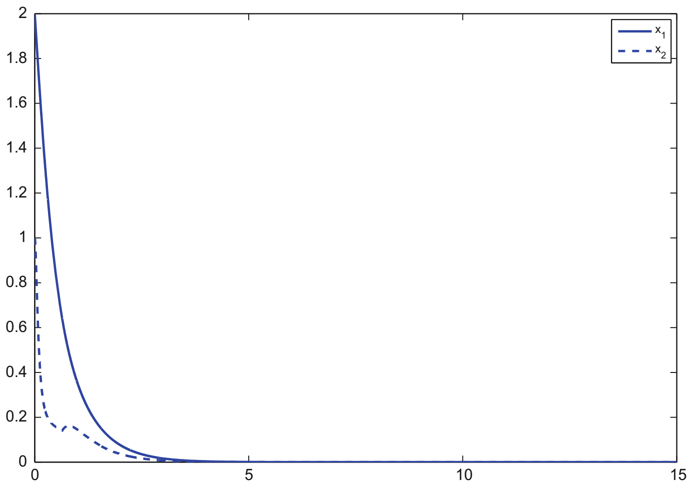

Fig. 2.4 State responses for Example 2 with adaptive controller

consider the following controller based on (2.8)–(2.10)

$$u(t) = -4x_2 - 2\overline{\theta}\left(x_2 + x_1^2 x_2 + x_2^3\right)$$

with adaptive law

$$\dot{\overline{\theta}} = \left(x_2 + x_1^2 x_2 + x_2^3\right)^2 - 0.1\overline{\theta}$$

the simulation result is as follows: Fig. 2.4 ($x_1(0) = 2$, $x_2(0) = 1$).

Nonlinear feedback controller: Consider the following feedback controller based on Sect. 2.4

$$u(t) = -6x_2 - \rho(x)\frac{x_2\rho(x)}{|x_2|\rho(x) + e^{-t}}$$

where

$$\rho(x) = 4\|x\| + 4\|x\|^2 + \|x\|^3$$

and the simulation result is shown in Fig. 2.5 ($x_1(0) = 2$, $x_2(0) = 1$).

One can also see that the closed-loop system is stable in the sense of uniform ultimate boundedness. From the simulation results of two examples, we can see that the both controllers can render the closed-loop system stable in the sense of uniform ultimate boundedness. So the proposed control schemes are valid and feasible.

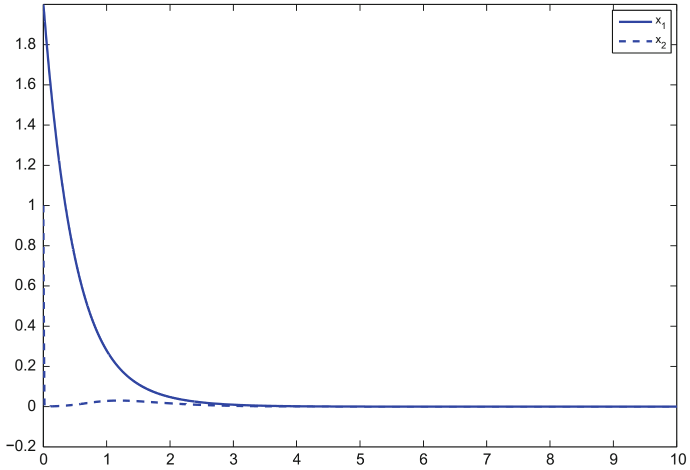

Fig. 2.5 State responses for Example 2 with nonlinear controller

2.6 Conclusion

In this chapter, the problem of robust stabilization for a class of dynamic systems with multiple delayed state perturbations has been considered. It is assumed that perturbations are bounded by high-order polynomial with unknown gains, and such gains are assumed to be unknown positive constants. Moreover, the delayed state perturbations are assumed to satisfy the so-called matching condition. Two different continuous state feedback controllers have been proposed based on different methods, respectively. Firstly, a robust adaptive feedback controller is proposed under the assumption that the derivative of time delay is less than one. Only requiring the time delay is bounded, we also construct a novel nonlinear feedback controller. Both the controllers developed in this section are continuous and can globally stabilize the uncertain closed-loop time-delay dynamic system in the sense of uniform ultimate boundedness. Finally, numerical examples are given to demonstrate the synthesis procedure for the two classes of continuous state feedback controllers developed in this chapter. It is shown from the examples that the results obtained are effective and feasible.

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