

# Preface

This book was born as the class notes of the course *Anisotropic structures: theory and design* that I hold at Ecole Polytechnique for the students of Master *M4S—Multiphysics and Multiscale Modeling of Materials and Structures*, a specialization curriculum of the second year of *Master in Mechanics* of the University Paris-Saclay, a federation of Universities in the Paris region.

As such, this book, intended as an introduction to anisotropic elasticity, is specially addressed to graduate students of courses in theoretical mechanics and engineering, as well as to Ph.D. students and researchers in mechanics. The book is composed of some classical parts and some more advanced topics.

Chapter 1 is a rapid introduction to the matter: the mathematical consequences of anisotropy, the true topic of this book, are briefly introduced along with some typical anisotropic phenomena. Also, a quick nod to anisotropy classifications is given. Some basic elements of elasticity of crystals are proposed too and the chapter ends with a recall of some fundamental equations of elasticity, needed in the following.

Chapter 2 concerns general anisotropic elasticity, that is, the three-dimensional elasticity of anisotropic bodies. Starting from the classical Hooke's law for anisotropic materials, the subject is treated in depth: the classical Voigt's and Kelvin's notations are introduced and the choice of Kelvin's one is argued. After having explained the mechanical meaning of the anisotropic elastic constants, the elastic symmetries are treated extensively. Then, the technical constants are introduced and the problem of determining the bounds on the elastic and technical constants is tackled. This chapter ends with the determination of the symmetry planes and with the notion of curvilinear anisotropy. Finally, a list of anisotropic materials is given, with an original way of plotting the technical constants.

Chapter 3 is devoted to a classical subject: plane anisotropic elasticity. The classical cases of plane strain, plane stress and generalized plane stress are developed and compared. Then, the Lekhnitskii theory is treated in detail and the chapter ends with an introduction to the Stroh's problem.

Chapter 4 is an introduction to the polar formalism. This mathematical technique for plane anisotropy was introduced by Prof. G. Verchery in 1979 and I worked

extensively on it during a long period, first with Prof. Verchery, then with different collaborators. The polar method is an effective tool for analyzing plane anisotropic phenomena and it is especially useful in the resolution of design problems. This part of the book is hence a complete presentation of the polar formalism; classical results are revisited and presented in a new way, along with some more recent results concerning special cases of elastic bodies as well as other properties, like piezoelectricity, anisotropy induced by damage, strength. Some examples of plane anisotropic materials end the chapter. The polar formalism is applied extensively in the last two chapters of the book.

Chapter 5 is an application of the previous results and concepts to the mechanics of anisotropic laminates, the main anisotropic bodies used in modern structural applications. The classical theory of laminated plates is first introduced and developed also for the polar formalism. Some general results, easily obtained applying this last, are highlighted. Then, the most used types of laminates are described and some sufficient rules for obtaining general elastic properties are given. Also, a study on the sensitivity of uncoupling and quasi-homogeneity to ply angles errors is detailed. This chapter ends with an extension of the classical theory to thermal and hygral properties and with a brief recall of higher order theories for laminated plates, given here also in the framework of the polar formalism.

Chapter 6 is an excursion into engineering design problems. Based upon the use of the polar formalism, that in design problems gives its best results, the chapter proposes first the theoretical problem of the optimal material orientation and then an original classification of anisotropic structures design problems and a presentation of classical and modern methods of design. More recent and modern problems of design are also considered: the optimization of anisotropy fields, that is, of structures with anisotropic properties depending upon the position, the case of modular systems and some multiphysics problems, concerning the thermal and piezoelectric response of an anisotropic structure. Several examples and details on modern numerical approaches complete the chapter.

The book makes use of modern mathematical tools: matrix and tensorial algebra are widely used throughout all the text, and the reader is considered to be familiar with standard tensor algebra.

The topic being very wide, this text is necessarily incomplete, as an introductory text usually is. Nevertheless, the author hopes sincerely that it will help the reader to progress in the field of anisotropic elasticity, a so particular world, unusual to mechanicians normally used to deal with isotropic materials.

I am indebted to many people for this book, namely Prof. Verchery, who introduced me to the polar method and to the research in this field when we were together at the University of Burgundy. Collaborating with him has been a privilege and a true opportunity for me. I wish to thank also Prof. M. Potier-Ferry, of the University of Lorraine, who has been a key encounter in my professional life, and to remember Prof. P. Villaggio, who has been much more than my Ph.D. supervisor at the University of Pisa: he remains for me an invaluable example of scientist, scholar, intellectual and the person who more than anyone else made me passionate about science and research.

I have had the opportunity to work with different colleagues on several topics presented in this book, especially A. Vincenti, E. Valot, M. Montemurro, A. Catapano, B. Desmorat, R. Barsotti, J. Pouget. I wish sincerely to thank them all: without their help, this book would not exist.

Finally, I need to thank my family: my wife Carla and my children Bianca and Alessandro. Their patience and support has been important during these last years.

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