

Chapter 2

Topological Structure of Mechanisms and Its Symbolic Representation

2.1 Introduction

In order to reveal the mapping relations among topological structure, POC and DOF of mechanisms (i.e. to establish the basic equations for mechanism topology), topological structure of mechanisms and its symbolic representation are discussed.

2.1.1 *Traditional Description of Mechanism Topological Structure*

In the IFToMM's terminology, the mechanism topological structure is defined as [1]: “**STRUCTURE** (OF A MECHANISM): Number and kinds of elements in a mechanism (members and joints) and the sequence of their contact”.

According to this definition, topological structure of a mechanism include: (1) Number and type of pairs (such as P, R, H, C, S pairs, etc.). (2) Number and type of links (such as binary link, ternary link, etc.). (3) Connection relations among these kinematic pairs and links.

The traditional definition involves two key elements of topological structure: type of kinematic pairs and connection relations among links. But these two key elements are not enough to exactly describe topological structure of a mechanism. For the two mechanisms in Fig. 2.4, they have the same description if only these two key elements are used. But actually they have different topological structures.

2.1.2 Current Description of Mechanism Topological Structure

In this chapter, the following three key elements are used to describe the topological structure of a mechanism:

- (1) The first traditional key element: type of kinematic pair.
- (2) The second traditional key element: connection relations among links (or topological structure units).
- (3) The new key element: type of geometric constraints to pair axes imposed by links (hereafter called as geometric constraint type of axes), refer to Sect. 2.2.3 for detail.

The topological structure of any spatial mechanism can be exactly described using these three key elements of topological structure.

2.1.3 Topology Structure Design of Mechanisms

Topological structure design is to determine the three key elements of a mechanism’s topological structure and to draw the schematic diagram of the mechanism.


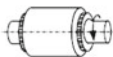
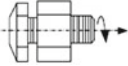
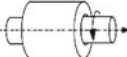






Topological structure design is hereafter called as topology design. The systematic theory and method for topology design of mechanisms is called as mechanism topology.

2.2 Three Key Elements of Mechanism Topological Structure

2.2.1 Type of Kinematic Pairs

The common used types of kinematic pair include single DOF kinematic pair (such as R, R and H pair) and multi-DOF kinematic pair (such as S and C pair), as shown in Table 2.1.

Table 2.1 Common types of kinematic pairs

P pair	R pair	H pair	C pair	S pair
				
				

A kinematic pair brings a certain type of geometric surface constraint to the connected two links. And thus determine the motion characteristics of this kinematic pair. For example, a P pair provides a prismatic surface constraint to the connected two links and thus determines that there is only one translation between these two links. An R pair brings in a cylindrical surface and end surface constraints to the connected two links and thus determines that there is only one rotation between these two links.

2.2.2 SOC Unit and Its Connection Relations

A mechanism can be regarded as being formed by connecting several topological structure units. According to different requirements, the topological structure unit may be of different types, such as link-pair unit, loop unit, Single-Open-Chain (SOC) unit, etc.

The connection relation among topological structure units can be described using different mathematical tools. For example, the connection relation among link-pair unit can be described using adjacency matrix [2–5] and the connection relation among loop units can be described using loop matrix [2, 6, 7].

A single open chain (SOC) refers to a simple open chain by links connected in series by kinematic pairs. Figure 2.1 shows an SOC which is expressed as $SOC\{-R_1 - R_2 - P_3 - \cdots - H_{(j-1)} - R_j-\}$ [8–16]. A sub-SOC refers to a part of an SOC. For example, sub- $SOC\{-R_1 - R_2 - P_3-\}$ and sub- $SOC\{-H_{(j-1)} - R_j-\}$ are two sub-SOCs of the SOC shown in Fig. 2.1.

There are only two types of connection relations between SOC, serial or parallel. Based on the SOC unit and the two types of connection relations, any serial mechanisms or multi-loop spatial mechanisms can be obtained [10–19]. So, the SOC unit and the connection relations can be used for generating any mechanisms, for establishing the three basic topological structure equations (Eq. (4.4) in Chap. 4, Eq. (5.3) in Chap. 5, and Eq. (6.8) in Chap. 6) and building up a systematic theory and method for unified modelling of mechanism topology, kinematics and dynamics based on the SOC unit (refer to Chap. 7 for detail).

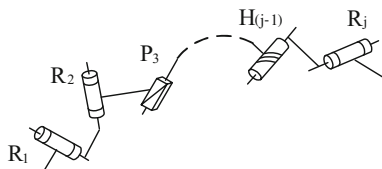


Fig. 2.1 Single open chain (SOC)

2.2.3 Geometric Constraint Type of Axes

Geometric constraint type of axes refers to the type of geometric constraint of a link to the axes of the pairs on this link. In commonly used mechanisms, there are only six types of geometric constraints of axes [8, 11–19]:

- (1) Axes of several adjacent R (or H) pairs are parallel, as shown in Fig. 2.2a. It is expressed as $SOC\{-R \parallel R \parallel \dots \parallel R-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2a.
- (2) Axes of two adjacent pairs are coincident, as shown in Fig. 2.2b. It is expressed as $SOC\{-R \mid H-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2b.
- (3) Axes of several adjacent R pairs intersect at one point, as shown in Fig. 2.2c. It is expressed as $SOC\{-\overbrace{RR \dots R}-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2c.
- (4) Axes of two adjacent pairs are perpendicular, as shown in Fig. 2.2d. It is expressed as $SOC\{-R \perp P-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2d.
- (5) Axes of several P pairs are parallel to one plane, as shown in Fig. 2.2e. It is expressed as $SOC\{-\diamond(P, P, \dots, P)-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2e.
- (6) Axes of kinematic pairs are allocated arbitrarily in the space, as shown in Fig. 2.2f. It is expressed as $SOC\{-R - R-\}$. There are many different sub-SOCs with this geometric constraint type of axes, as shown in Table 2.2f.

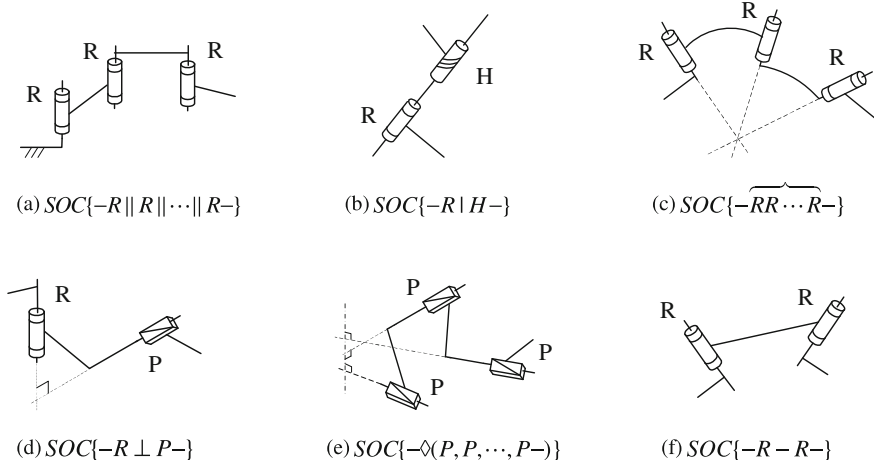


Fig. 2.2 Geometric constraint types of axes [11–15]

Table 2.2 Sub-SOCs of geometric constraint types

Geometric constraint type	Sub-SOCs
(a) Parallel to each other	$R \parallel R, R \parallel R \parallel R, R \parallel H, H \parallel H, H \parallel H \parallel H$, etc.
(b) Coincident axes	$R R, R P, H R, H H, H P$, etc.
(c) Intersecting at one point	$\widehat{RRR}, \widehat{RR}, \widehat{R\perp R}$, etc.
(d) Perpendicular to each other	$R\perp R, R\perp P, H\perp P, P\perp P$, etc.
(e) Parallel to a plane	$\diamond(P, P, P)$, etc.
(f) Arbitrary	$R - R, R - P, R - H, H - H$, etc.

Note The special geometric relations among pair axes are first proposed by Dizioglu [20]. We concluded these special geometric relations to six geometric constraint types in 1983 and applied them in structure synthesis of serial mechanisms and overconstrained loops [8, 9]. In 1992, these geometric constraint types were used in structure synthesis of multi-loop spatial mechanisms [10]. In 2001, the geometric constraint type was regarded as one of the three key elements to describe the topological structure of mechanisms [11–15]. Now, the geometric constraint types are widely used in structure analysis and synthesis of robot mechanisms [11–19]

2.3 Symbolic Representation of Mechanism Topological Structure

2.3.1 Symbolic Representation of Multi-DOF Pair

A multi-DOF pair can be regarded as being formed by several single-DOF pairs connected in series. So, a multi-DOF pair can be expressed as an SOC composed of several single-DOF pairs. For example, the S pair can be expressed as

$SOC\{-\widehat{RRR}-\}$, the C pair can be expressed as $SOC\{-R|P-\}$, the E pair can be expressed as $SOC\{-P\perp R\perp P-\}$ and the U pair can be expressed as $SOC\{-\widehat{R\perp R}-\}$.

2.3.2 Symbolic Representation of Topological Structure

Based on the three key elements of mechanism topological structure, the topological structure of any spatial mechanism can be exactly described using three types of pairs (P, R and H), six geometric constraint types of axes and two connection relations between SOC units (serial or parallel) and can be expressed symbolically.

For example, topological structure of the serial mechanism in Fig. 2.3a can be expressed symbolically as $SOC\{-R_1(\perp P_2) \parallel R_3 - \widehat{R_4 R_5} -\}$. In this symbolic expression, $R_1(\perp P_2) \parallel R_3$ means $R_1 \perp P_2$ and $R_1 \parallel R_3$.

The second example is the parallel mechanism shown in Fig. 2.3b. Topological structure of this parallel mechanism can be expressed symbolically as follows:

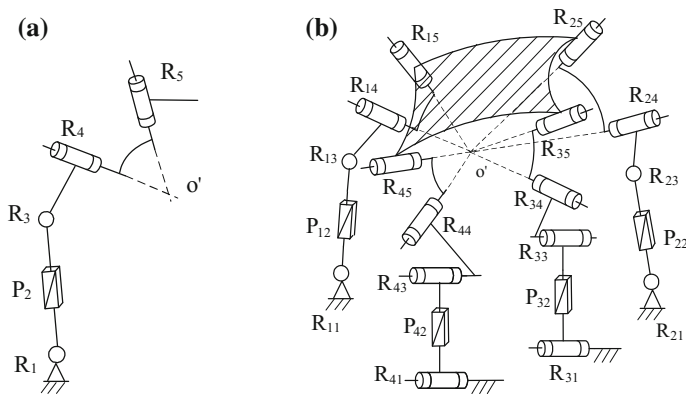


Fig. 2.3 Mechanism topological structures

- (a) Topological structure of four branches: $4 - SOC\{-R_{i1}(\perp P_{i2}) \parallel R_{i3} - \overbrace{R_{i4}R_{i5}} -\}(i = 1-4)$.
- (b) Topological structure of the moving platform: R_{i4}, R_{i5} ($i = 1-4$) intersect at point o' .
- (c) Topological structure of the fixed platform: $R_{11} \parallel R_{21}, R_{31} \parallel R_{41}$, R_{11} and R_{31} are skewed arbitrarily.

Another example includes the two single-loop chains in Fig. 2.4. The single-loop chain in Fig. 2.4a can be expressed symbolically as $SLC\{\overbrace{-R_1R_2R_3 - R_4R_5R_6 -}\}$ and the single-loop chain in Fig. 2.4b can be expressed symbolically as $SLC\{-R_1 \parallel R_2 \parallel R_3 - \overbrace{R_4R_5R_6} -\}$. From this example, we find that it is necessary to use geometric constraint type of axes to describe mechanism topological structure. Otherwise, it will be very hard for us to show the different topological structures of these two single-loop chains since they contain the same type and number of pairs.

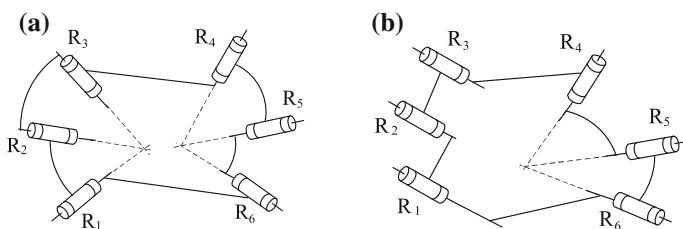


Fig. 2.4 Topological structure of single-loop chains

2.3.3 Composition of an SOC

- (1) An SOC can be formed by connecting several pairs in Table 2.1 in series, as shown in Fig. 2.1.
- (2) An SOC can be formed by connecting several sub-SOCs in Table 2.2 in series.

For example, $SOC\{-R \parallel R \parallel R - \widehat{RR} -\}$ can be regarded as being formed by connecting sub- $SOC\{-R \parallel R \parallel R -\}$ and sub- $SOC\{-\widehat{RR} -\}$ (refer to Table 2.2) in series.

- (3) An SOC can be formed by connecting several SOCs in series. For example, $SOC\{-R_{11}(\perp P_{12}) \parallel R_{13} - \widehat{R_{14}R_{15}R_{25}R_{24}} - R_{23}(\perp P_{22}) \parallel R_{21} -\}$ corresponding to first loop of the parallel mechanism (Fig. 2.3b) is formed by connecting $SOC\{-R_{i1}(\perp P_{i2}) \parallel R_{i3} - \widehat{R_{i4}R_{i5}} -\}$ ($i = 1, 2$) corresponding to the first two branches in series.

2.4 Topological Structure Invariance

Generally, topological structure of a mechanism is determined by design, processing and assembling. Topological structure described by the three key elements is invariant during motion process of the mechanism (excluding singular positions). In other words, topological structure of a mechanism is independent of motion position of the mechanism and the fixed coordinate system (i.e. it is not necessary to establish the fixed coordinate system).

For example, during motion process of the mechanism in Fig. 2.5a, the singular position refers to such a position that axes of non-adjacent kinematic pairs (R_1 , R_2 , R_4 and R_5) intersect temporarily at one point $o_1(o_2)$, as shown in Fig. 2.5b. In other nonsingular positions, the topological structure is invariant, as shown in Fig. 2.5a and Fig. 2.6.

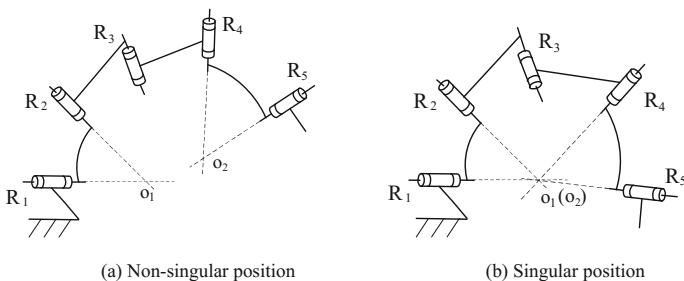


Fig. 2.5 Singular position of non-adjacent pair axes

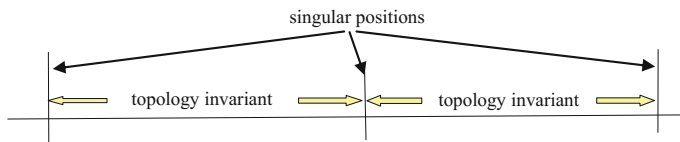


Fig. 2.6 Invariance property of mechanism topological structure

Correspondingly, symbolic representation of the mechanism topological structure based on three key elements is also motion process invariant (excluding singular positions). So this symbolic representation is a kind of geometric representation [11–19].

The topological structure invariance can be used for establishing the three basic topological structure equations (Eq. (4.4) in Chap. 4, Eq. (5.3) in Chap. 5 and Eq. (6.8) in Chap. 6).

2.5 Definition of Mechanism Topological Structure

In order to realize prescribed functions and performance requirements, pairs and links of a mechanism must be connected in a certain way. This connection way is called topological structure of the mechanism. In the topological structure, the size of pairs or links and the space between them are not considered. But the geometric constraint type of axes must be kept unchanged [19].

2.6 Classification of Mechanisms Based on Topological Structure

According to definition of the topological structure in Sect. 2.5, there are the following three types of mechanisms:

(1) Non-overconstrained mechanisms

A non-overconstrained mechanism refers to such a mechanism that number of its independent displacement equations is six and the $\text{DOF} \geq 1$.

Existence condition of non-overconstrained mechanism is related only to types of pairs and connection relations among links. The non-overconstrained mechanism is also called trivial kinematic chain [21].

For example, the general 7R mechanism and the 6-SPS parallel mechanism are non-overconstrained mechanisms.

(2) General overconstrained mechanism

A general overconstrained mechanism refers to such a mechanism that number of its independent displacement equations is less than six and the $\text{DOF} \geq 1$.

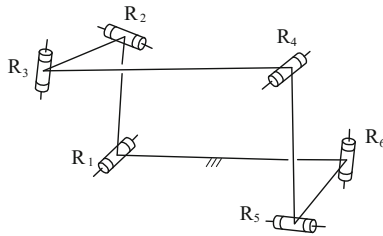


Fig. 2.7 Bricard mechanism

Existence condition of general overconstrained mechanism is related to types of pairs, connection relations among links and the geometric constraint types of axes. The general overconstrained mechanism is also called exceptional kinematic chain [21].

For example, the mechanisms shown in Figs. 2.3b and 2.4 are general overconstrained mechanisms.

(3) Special overconstrained mechanism

A special overconstrained mechanism refers to such a mechanism that number of its independent displacement equations is less than six and the $\text{DOF} = 1$.

Existence condition of special overconstrained mechanism is related to types of pairs, connection relations among links and a certain functional relationship with link parameters (link length and skew angle comply with a certain functional relationship). The special overconstrained mechanism is also called paradoxical kinematic chain [21].

For example, Bricard mechanism is a six-bar special overconstrained mechanism [22, 23], as shown in Fig. 2.7. Its existence conditions include: $d_i = 0$ ($i = 1-6$), $\alpha_i = \pi/2$ ($i = 1-6$) and $a_1^2 + a_3^2 + a_5^2 = a_2^2 + a_4^2 + a_6^2$.

The systematic theory and method for topology design covers only non-overconstrained mechanisms (the trivial kinematic chains) and general overconstrained mechanisms (the exceptional kinematic chains). Researches on special overconstrained mechanisms (the paradoxical kinematic chains) may be found in other references [22–24].

2.7 Summary

- (1) Mechanism topological structure contains three key elements: types of pairs (Table 2.1), connection relations among topological structure units (such as link-pair unit, loop unit and SOC unit) and geometric constraint types of axes (Fig. 2.2). The topological structure of any spatial mechanism can be exactly described using these three elements.
- (2) Any serial mechanisms or multi-loop spatial mechanisms can be generated by connecting SOC units parallels or in series.

- (3) Mechanism topological structure has invariance property and is independent of motion position of the mechanism (excluding singular positions) and the fixed coordinate system (i.e. it is not necessary to establish the fixed coordinate system) (Fig. 2.6).
- (4) Symbolic representation of mechanism topological structure is a geometric expression and is independent of motion position of the mechanism and the fixed coordinate system. So, it provides a theoretical basis for operations of mechanism topology (refer to Chaps. 3–5) for detail.
- (5) According to definition of mechanism topological structure in Sect. 2.5, there are three types of mechanisms: non-overconstrained mechanisms (trivial kinematic chains), general overconstrained mechanisms (exceptional kinematic chains) and special overconstrained mechanisms (paradoxical kinematic chains).

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