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### Abstract

This chapter describes Global Climate Models (GCMs), limitations and uncertainties associated with the formulation of GCMs due to the effect of aerosols which are differently parameterized in GCMs, initial and boundary conditions for each GCM, parameters and model structure of GCMs, randomness, future greenhouse gas emissions, and scenarios leading to significant variability across model simulations of future climate. This chapter discussed the necessity of performance indicators for evaluating GCMs and explained mathematical description of these indicators. It also emphasized on normalization approach, weight computing techniques such as entropy and rating, ranking approaches, namely, compromise programming, cooperative game theory, TOPSIS, weighted average, PROMETHEE, and fuzzy TOPSIS. Spearman rank correlation which measures consistency in ranking pattern and group decision-making that aggregates individual rankings obtained by different techniques to form a single group preference is also part of this chapter. Ensembling methodology of GCMs is also discussed. Reader is expected to understand various uncertainties associated, role of decision-making techniques for ranking of GCMs by studying this chapter.

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### Keywords

Correlation • GCMs • Group decision • Normalization • Performance indicators  
Ranking • Uncertainty • Weight

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## 2.1 Introduction

Multicriterion decision-making (MCDM) techniques are capable of selecting the best global climate model (GCM). Selected GCMs can be explored for applications, e.g., downscaling and adaptation studies (Bogardi and Nachtnebel 1994). The present chapter discussed about GCMs, normalization, and weight estimation techniques, MCDM techniques, Spearman rank correlation coefficient, group decision-making, and methodology of ensembling of GCMs. The discussed techniques are demonstrated with numerical examples in climate modeling situations. In the present book, methods and techniques are used interchangeably.

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## 2.2 Global Climate Models

The Earth climate system is the result of interaction between various components such as atmosphere, snow, ice, land surface, ocean, and other water bodies, and living beings like humans and animals. Human-induced factors such as deforestation and burning of fossil fuels also lead to change in atmospheric composition in addition to various external factors. Due to this there will be climate change, which affects the variability of the parameters that continue to exist for a long period. For example, nitrous oxide, carbon dioxide, methane, ozone, hydrofluorocarbons, and sulfur hexafluoride are the greenhouse gases that are increasing and affecting global temperature over the past half century and are expected to follow the similar trend in future. These greenhouse gases affect the absorption, scattering, and emission of radiation in the Earth surface and atmosphere. These aspects necessitated to study the relationship between greenhouse gases and the global climate. This is possible through climate modeling or simply climate models. Various relevant climate models are described by Thompson and Perry (1997), Goudie and Cuff (2001), and Kendal and Henderson-Sellers (2013):

- Energy balance models are one dimensional in nature, which relate latitude and sea surface-level temperature variation.
- Radiative-convective models are one dimensional in nature. They analyze vertical temperature, explicit profile modeling of radiative process and convective adjustment.
- General circulation or global climate models (GCMs) are sophisticated numerical tools, which are three dimensional in nature. They simulate Earth's climate with different climate variables, initial and boundary conditions, and structure. GCMs are increasingly being employed to solve or to assess regional/local issues (What is a GCM 2013). Wilby et al. (2009) described GCMs as numerical solutions of a partial differential equation(s). GCMs are formulated on the principles of movement of energy, momentum of a particle, and conservation of mass.

- Coupled atmosphere–ocean global climate models combine the interactions of the atmospheric GCMs and oceanic GCMs.

Xu (1999) mentioned that GCMs are found to be capable of projecting average precipitation, temperature, etc., over future decades or centuries. He and numerous researchers, however, cautioned about the limitations of GCMs such as

- Accuracy of GCMs, which generally run at coarse grid resolution ( $\sim 3^\circ \times 3^\circ$ ), decreases with increasingly finer spatial and temporal scales, rendering them unable to represent sub-grid-scale features. In other words, GCMs are not able to effectively model sub-grid-scale processes which are of prime interest to hydrologists and water resources planners.
- Accuracy of GCMs decreases from free tropospheric to surface variables, whereas surface variables have significant application in water balance computations.

The uncertainties associated with the formulation of GCMs arise due to the effect of aerosols which are differently parameterized in GCMs, initial and boundary conditions for each GCM, parameter and model structure of GCMs, randomness, future greenhouse gas emissions, and Representative Concentration Pathways (RCPs) leading to significant variability across model simulations of future climate (Raje and Mujumdar 2010). These uncertainties accumulate from various levels such as GCM to downscaling level and may propagate to the local levels, which may affect the adaptation studies that would be used as the basis for implementation. In brief, the uncertainty begins at selection of suitable GCMs, selection of downscaling technique(s), and selection of suitable hydrologic model(s). Numerous authors suggested uncertainty minimization by employing ensembling of relevant models, which may provide increased confidence while projecting climate change. For example, instead of single GCM output, output of multiple GCMs can be used as the basis to feed inputs to downscaling techniques. Similarly outputs from some of the downscaling techniques such as Statistical Downscaling Modeling (SDSM) (Wilby et al. 2002), Multiple Linear Regression (MLR), and Artificial Neural Networks (ANN) can be ensembled (or these outputs can be studied) and can be passed on to hydrologic models. Similar experimentation can be performed in hydrologic modeling where few hydrologic models can be coupled or even hybridized (Semenov and Stratonovitch 2010).

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## 2.3 Performance Indicators for Evaluating GCMs

Pierce et al. (2009) in their studies raised the issues such as “What effect does picking different global models have on the regional climate study? If different global models give different downscaled results what strategy should be used for selecting the global models? Are there overall strategies that can be used to guide

the choice of models?” The above queries necessitated to evaluate the available GCMs for accuracy and their adaptability (Legates and McCabe 1999). Hence, the GCMs are to be evaluated to assess their performance by simulating the historic observations. This enables to choose GCMs of higher performance so that the relevant output obtained from the suitable/best GCMs can be used for further analysis (Mujumdar and Nagesh Kumar 2012).

A performance indicator is a measure of any GCM to determine how well it simulates the observed data. Simple, effective, and meaningful metrics are required to evaluate the GCMs across space and time and to evolve a subset of models that can be employed for hydrologic modeling applications. These indicators may provide the basis to assess the confidence level of outputs of GCMs (Helsel and Hirsch 2002; Gleckler et al. 2008; Johnson and Sharma 2009; Macadam et al. 2010; Wilks 2011; Sonali and Nagesh Kumar 2013; Ojha et al. 2014).

Different researchers used various performance indicators, such as, Sum of Squares of Deviation (SSD), Mean Square Deviation (MSD), Root Mean Square Deviation (RMSD), Normalized Root Mean Square Deviation (NRMSD), Absolute Normalized Mean Bias Deviation (ANMBD), Average Absolute Relative Deviation (AARD), Pearson Correlation Coefficient (CC), Nash–Sutcliffe Efficiency (NSE), and Skill Score (SS). Among all, SSD, MSD, RMSD, NRMSD, ANMBD, AARD are of deviation/error category. The mathematical descriptions of these indicators are as follows:

- (a) SSD is addition of the squared difference between the observed values and the GCM-simulated values.  $x_i$  and  $y_i$  are observed and simulated values respectively.  $T$  is number of datasets.

$$SSD = \sum_{i=1}^T (x_i - y_i)^2 \quad (2.1)$$

- (b) MSD is the average of squares of deviation.

$$MSD = \frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2 \quad (2.2)$$

- (c) RMSD is square root of mean square of deviation.

$$RMSD = \sqrt{\frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2} \quad (2.3)$$

- (d) NRMSD is the ratio of RMSD and mean of observed values. Less value of NRMSD is preferred.

$$NRMSD = \frac{\sqrt{\frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2}}{\bar{x}} \quad (2.4)$$

- (e) ANMBD is ratio of the mean of the differences between the observed and the GCM-simulated values to the mean of observed values. Less value of ANMBD is preferred.

$$ANMBD = \left| \frac{\frac{1}{T} \left( \sum_{i=1}^T (y_i - x_i) \right)}{\bar{x}} \right| \quad (2.5)$$

- (f) AARD is the mean of the absolute values of relative deviation. Less value of AARD is preferred.

$$AARD = \frac{1}{T} \sum_{i=1}^T |ARD_i|; \quad \text{where } ARD_i = \frac{(y_i - x_i)}{x_i} \quad (2.6)$$

- (g) CC relates strength of the linear relationship between the observed and the GCM-simulated values. Here,  $\bar{x}, \bar{y}$  are average of observed and simulated values, whereas  $\sigma_{obs}$  and  $\sigma_{sim}$  are the standard deviations. CC value near to 1.0 indicates good model performance. In all chapters wherever applicable, the word “correlation coefficient” is used as a generalization.

$$CC = \frac{\sum_{i=1}^T (x_i - \bar{x})(y_i - \bar{y})}{(T - 1)\sigma_{obs}\sigma_{sim}} \quad (2.7)$$

- (h) NSE is defined (Nash and Sutcliffe 1970) as:

$$NSE = 1 - \frac{\sum_{i=1}^T (x_i - y_i)^2}{\sum_{i=1}^T (x_i - \bar{x})^2} \quad (2.8)$$

NSE ranges from  $-\infty$  to 1. If a model simulates the observed conditions perfectly, NSE value will be 1 (Nash–Sutcliffe Efficiency 2017).

- (i) SS (Maximo et al. 2008) measures the similarity between the Probability Density Functions (PDFs) of the observed and simulated values across the entire PDF and expressed as

$$SS = \frac{1}{T} \sum_{i=1}^{nb} \min(f_m, f_o) \quad (2.9)$$

where nb is number of bins used to calculate the PDF for a given region.  $f_m, f_o$  are the frequencies of values in the given bin from the chosen GCM and of the observed values. Skill score varies between zero and one.

**Numerical Problem 2.1** Global climate model-simulated temperature data (in °K) in a given region in India along with the observed/historic data are presented in Table 2.1. Compute the performance of the GCM for its simulating capability with that of historic data in terms of SSD, MSD, RMSD, CC, NRMSD, ANMBD, AARD, NSE, and SS.

**Table 2.1** Historic/observed data and simulated data by GCM

Datasets	Historic/observed data (°K)	GCM-simulated data (°K)
1	243	244
2	244	248
3	245	251
4	246	258
5	247	248
6	248	264
7	248	253
8	249	252
9	249	253
10	250	264
11	250	256
12	251	256
13	253	265
14	254	245
15	255	258
16	256	267
17	257	259
18	258	261
19	259	260
20	260	260
21	262	266
22	263	270
23	265	266
24	269	264
25	270	273
26	271	270
27	273	272

**Solution:****Notation:**

$x_i$	Observed temperature value (°K)
$y_i$	Simulated value (°K)
$\bar{x}$	Mean of the observed values (°K)
$\bar{y}$	Mean of simulated values (°K)
$T$	Number of observations recorded
$\sigma_{obs}$	Standard deviation of the observed value set (°K)
$\sigma_{sim}$	Standard deviation of simulated value set (°K)

Estimated parameters

$$\begin{aligned}
 \bar{x} &= 255.37^\circ\text{K} & \sum_{i=1}^T (x_i - y_i)^2 &= 1232^\circ\text{K}^2 \\
 \bar{y} &= 259.37^\circ\text{K} & \sum_{i=1}^T (x_i - \bar{x})^2 &= 2016.29^\circ\text{K}^2 \\
 \sum_{i=1}^T (y_i - \bar{y})^2 &= 1750.29^\circ\text{K}^2 & \sum_{i=1}^T (x_i - \bar{x})(y_i - \bar{y}) &= 1483.30^\circ\text{K}^2
 \end{aligned}$$

Standard deviation of a set of observations is calculated by the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^T (x_i - \bar{x})^2}{(T - 1)}}$$

$$\text{Standard deviation of observed data} = \sigma_{obs} = \sqrt{\frac{2016.29}{26}} = 8.8062^\circ\text{K}$$

$$\text{Standard deviation of simulated data} = \sigma_{sim} = \sqrt{\frac{1750.29}{26}} = 8.2048^\circ\text{K}$$

Computation of performance indicators (Refer Table 2.2)

Table 2.2 Computation of relevant parameters

Dataset	$x_i$	$y_i$	$(x_i - y_i)$	$(x_i - y_i)^2$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x}) * (y_i - \bar{y})$	$\left  \frac{(y_i - x_i)}{x_i} \right $
1	243	244	-1	1	-12.3704	153.0261	-15.3704	236.2492	190.138	0.0041
2	244	248	-4	16	-11.3704	129.2853	-11.3704	129.286	129.286	0.0164
3	245	251	-6	36	-10.3704	107.5446	-8.3704	70.0636	86.8044	0.0245
4	246	258	-12	144	-9.3704	87.8038	-1.3704	1.8779	12.8412	0.0488
5	247	248	-1	1	-8.3704	70.0631	-11.3704	129.286	95.1748	0.0040
6	248	264	-16	256	-7.3704	54.3224	4.6296	21.4332	-34.122	0.0645
7	248	253	-5	25	-7.3704	54.3224	-6.3704	40.582	46.9524	0.0202
8	249	252	-3	9	-6.3704	40.5816	-7.3704	54.3228	46.9524	0.0120
9	249	253	-4	16	-6.3704	40.5816	-6.3704	40.582	40.582	0.0161
10	250	264	-14	196	-5.3704	28.8409	4.6296	21.4332	-24.8628	0.0560
11	250	256	-6	36	-5.3704	28.8409	-3.3704	11.3596	18.1004	0.0240
12	251	256	-5	25	-4.3704	19.1001	-3.3704	11.3596	14.73	0.0199
13	253	265	-12	144	-2.3704	5.6187	5.6296	31.6924	-13.3444	0.0474
14	254	245	9	81	-1.3704	1.8779	-14.3704	206.5084	19.6932	0.0354
15	255	258	-3	9	-0.3704	0.1372	-1.3704	1.8779	0.5075	0.0118
16	256	267	-11	121	0.6296	0.3964	7.6296	58.2108	4.8035	0.0430
17	257	259	-2	4	1.6296	2.6557	-0.3704	0.1371	-0.6036	0.0078
18	258	261	-3	9	2.6296	6.9150	1.6296	2.6555	4.2851	0.0116
19	259	260	-1	1	3.6296	13.1742	0.6296	0.3963	2.2851	0.0039
20	260	260	0	0	4.6296	21.4335	0.6296	0.3963	2.9147	0.0000
21	262	266	-4	16	6.6296	43.9520	6.6296	43.9516	43.9516	0.0153
22	263	270	-7	49	7.6296	58.2112	10.6296	112.9884	81.0996	0.0266
23	265	266	-1	1	9.6296	92.7298	6.6296	43.9516	63.8404	0.0038

(continued)



Table 2.2 (continued)

Dataset	$x_i$	$y_i$	$(x_i - y_i)$	$(x_i - y_i)^2$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x}) * (y_i - \bar{y})$	$\left  \frac{(y_i - \bar{y})}{x_i} \right $
24	269	264	5	25	13.6296	185.7668	4.6296	21.4332	63.0996	0.0186
25	270	273	-3	9	14.6296	214.0261	13.6296	185.766	199.3956	0.0111
26	271	270	1	1	15.6296	244.2853	10.6296	112.9884	166.1364	0.0037
27	273	272	1	1	17.6296	310.8038	12.6296	159.5068	222.6548	0.0037
Total	6895	7003	-108	1232		2016.2964		1750.296	1483.296	0.5541

$$SSD = \sum_{i=1}^T (x_i - y_i)^2 = 1232 \text{ } ^\circ\text{K}^2$$

$$MSD = \frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2 = \frac{1232}{27} = 45.63 \text{ } ^\circ\text{K}^2$$

$$RMSD = \sqrt{\frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2} = \sqrt{45.63} = 6.7549 \text{ } ^\circ\text{K}$$

$$CC = \frac{\sum_{i=1}^T (x_i - \bar{x})(y_i - \bar{y})}{(T-1)\sigma_{obs}\sigma_{sim}} = \frac{1483.29}{26 * 8.8062 * 8.2048} = 0.7896$$

$$NRMSD = \frac{\sqrt{\frac{1}{T} \sum_{i=1}^T (x_i - y_i)^2}}{\bar{x}} = \frac{6.7549}{255.37} = 0.02645$$

$$ANMBD = \left| \frac{\frac{1}{T} \sum_{i=1}^T (y_i - x_i)}{\bar{x}} \right| = \left| \frac{108}{27 * 255.37} \right| = 0.0156$$

$$AARD = \frac{1}{T} \sum_{i=1}^T |ARD_i| = \frac{1}{T} \sum_{i=1}^T \left| \frac{(y_i - x_i)}{x_i} \right| = \frac{0.5541}{27} = 0.02052$$

$$NSE = 1 - \frac{\sum_{i=1}^T (x_i - y_i)^2}{\sum_{i=1}^T (x_i - \bar{x})^2} = 1 - \frac{1232}{2016.29} = 1 - 0.6110 = 0.3890$$

### Computation of Skill Score (SS)

- The maximum and minimum temperatures of the complete dataset (observed and simulated as shown in Table 2.1) is found out. Here, in this case the maximum value is 273 °K, whereas the minimum value is 243 °K.
- Now an appropriate bin width is to be chosen. Here it is chosen as 5 °K.
- Hence, the number of bins (nb) is calculated as: (maximum – minimum)/bin width = (273 – 243)/5 = 6.

Then, the values are segregated into the bins, to find the frequencies  $f_o$  and  $f_m$  as follows (Table 2.3):

**Table 2.3** Computation of skill score

Bin	$f_o$ (frequency of observed datasets in the chosen bin)	$f_m$ (frequency of simulated datasets in the chosen bin)	Minimum of $f_o$ and $f_m$
243–248	7	4	4
249–253	6	4	4
254–258	5	4	4
259–263	4	4	4
264–268	1	7	1
269–273	4	4	4

Sum of the minimum of  $f_o$  &  $f_m = 4 + 4 + 4 + 4 + 1 + 4 = 21$

$$\text{Skill Score} = \frac{1}{T} \sum_{i=1}^{nb} \min(f_m, f_o) = \frac{21}{27} = 0.7777$$

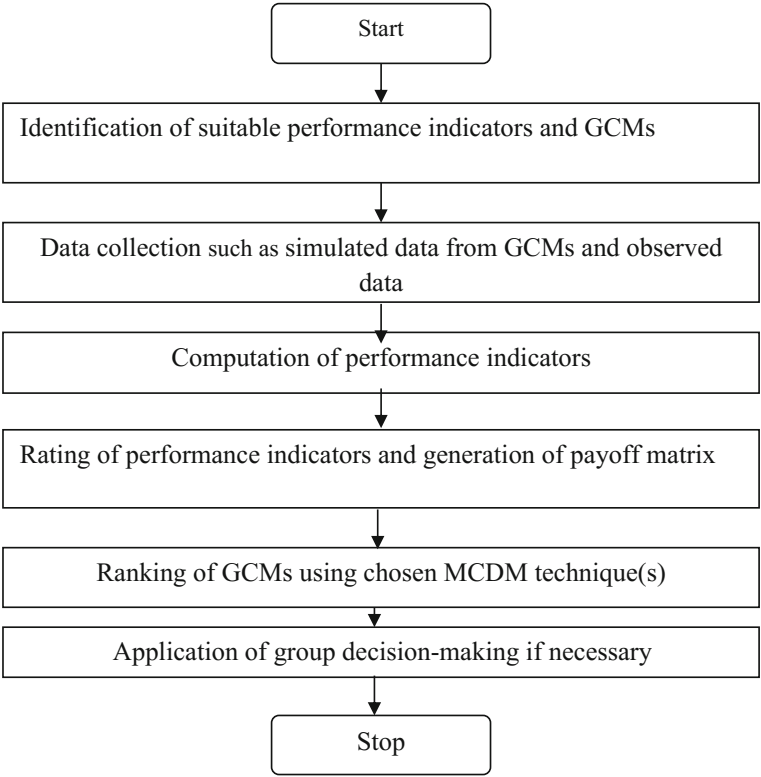
Table 2.4 presents summary of performance indicators

**Table 2.4** Summary of computed performance indicators

Indicator	Value	Remarks
Sum of squares of deviation (SSD)	1232 ( $^{\circ}\text{K}$ ) <sup>2</sup>	Lower value is preferable. Near to zero is ideal
Mean square deviation (MSD)	45.63 ( $^{\circ}\text{K}$ ) <sup>2</sup>	Lower value is preferable. Near to zero is ideal
Root mean square deviation (RMSD)	6.7549 ( $^{\circ}\text{K}$ )	Lower value is preferable. Near to zero is ideal
Pearson correlation coefficient (CC)	0.7896 (no unit)	Higher value is preferable. Near to one is ideal
Normalized root mean square deviation (NRMSE)	0.02645 (no unit)	Lower value is preferable. Near to zero is ideal
Absolute normalized mean bias deviation (ANMBD)	0.0156 (no unit)	Lower value is preferable. Near to zero is ideal
Average absolute relative deviation (AARD)	0.02052 (no unit)	Lower value is preferable. Near to zero is ideal
Nash–Sutcliffe efficiency (NSE)	0.3890 (no unit)	Higher value is preferable. Near to one is ideal
Skill score (SS)	0.7777 (no unit)	Higher value is preferable. Near to one is ideal

## 2.4 Ranking of Global Climate Models

Procedural steps for selection of the best GCM are presented in Fig. 2.1 (Raju and Nagesh Kumar 2014a; Duckstein et al. 1989).



**Fig. 2.1** Flowchart of procedural steps for selection of the best GCM

**2.4.1 Normalization Techniques**

Normalization facilitates the conversion of different non-commensurable indicators to the same space. In the present study, simple normalization technique (denoted as type 3) is presented (Table 2.5). More details of normalization techniques are available from Pomerol and Romero (2000) and Raju and Nagesh Kumar (2014a).

**Table 2.5** Description of chosen normalization technique (Type 3)

Description	Mathematical representation
Normalized value $k_{aj}$	$k_{aj} = \frac{K_j(a)}{\sum_{a=1}^T K_j(a)}$ <p>where <math>K_j(a)</math> is the value of indicator <math>j</math> for GCM <math>a</math>; <math>T</math> represents the total number of GCMs</p>

## 2.4.2 Weight Computing Techniques

A number of techniques are available for determination of weights. In the present chapter, only two techniques are described, namely, entropy and rating.

### 2.4.2.1 Entropy Technique

The methodology is explained in Table 2.6 (Pomerol and Romero 2000; Raju and Nagesh Kumar 2014a).

**Table 2.6** Methodology of entropy technique

Step	Description	Mathematical expression/remark
1	Normalize the payoff matrix if required	$k_{aj}$
2	Entropy for each indicator $j$	$En_j = -\frac{1}{\ln(T)} \sum_{a=1}^T k_{aj} \ln(k_{aj})$ for $j = 1, \dots, J$ $a$ is index for GCMs; ( $j = 1, 2, \dots, J$ ) where $J$ is number of indicators; $T$ represents total number of GCMs
3	Degree of diversification	$Dd_j = 1 - En_j$
4	Normalized weight of indicators	$r_j = \frac{Dd_j}{\sum_{j=1}^J Dd_j}$

**Numerical Problem 2.2** Eleven GCMs in Coupled Model Intercomparison Project (CMIP3), namely, BCCR-BCCM2.0, INGV-ECHAM4, GFDL2.0, GFDL2.1, GISS, IPSL-CM4, MIROC3, MRI-CGCM2, NCAR-PCMI, UKMO-HADCM3, and UKMO-HADGEM1 are analyzed for the variable, precipitation. Five indicators, namely, CC, NRMSD, ANMBD, AARD, SS are the performance indicators. Payoff matrix (11 GCMs vs. 5 indicators] is presented in Table 2.7. Apply entropy technique for determination of weights. Normalization technique 3 can be explored (Raju and Nagesh Kumar 2014b).

**Table 2.7** Values of performance indicators obtained for the 11 GCMs

GCM	CC	NRMSD	ANMBD	AARD	SS
BCCR	0.7751	0.7960	0.2744	1.7127	0.7717
ECHAM	0.7866	0.7573	0.1619	1.8639	0.6833
GFDL2.0	0.7868	0.8286	0.4157	0.8080	0.8150
GFDL2.1	0.7395	0.7871	0.1551	1.2731	0.8350
GISS	0.8275	0.8221	0.4786	0.7539	0.7783
IPSL	0.4740	1.2539	0.7082	1.0124	0.6583
MIROC3	0.8416	0.6224	0.0613	1.3811	0.8567
CGCM2	0.7708	0.9386	0.4985	0.6556	0.7550
PCMI	0.3553	1.1779	0.4899	1.6149	0.6283
HADCM3	0.8018	0.8793	0.5092	0.8002	0.8100
HADGEM1	0.8064	0.9422	0.5686	0.7010	0.7883

**Table 2.8** Transformed values of performance indicators obtained for the 11 GCMs

GCM	CC	NRMSD <sup>a</sup>	ANMBD <sup>a</sup>	AARD <sup>a</sup>	SS
BCCR	0.7751	−0.7960	−0.2744	−1.7127	0.7717
ECHAM	0.7866	−0.7573	−0.1619	−1.8639	0.6833
GFDL2.0	0.7868	−0.8286	−0.4157	−0.8080	0.8150
GFDL2.1	0.7395	−0.7871	−0.1551	−1.2731	0.8350
GISS	0.8275	−0.8221	−0.4786	−0.7539	0.7783
IPSL	0.4740	−1.2539	−0.7082	−1.0124	0.6583
MIROC3	0.8416	−0.6224	−0.0613	−1.3811	0.8567
CGCM2	0.7708	−0.9386	−0.4985	−0.6556	0.7550
PCMI	0.3553	−1.1779	−0.4899	−1.6149	0.6283
HADCM3	0.8018	−0.8793	−0.5092	−0.8002	0.8100
HADGEM1	0.8064	−0.9422	−0.5686	−0.7010	0.7883

<sup>a</sup>Minimum NRMSD, ANMBD, AARD are desirable. Negative sign is incorporated before values of indicators to represent in maximization perspective, i.e., (−min) = max

**Table 2.9** Entropy values, degree of diversification, and weight of indicators

Characteristic of indicator j	CC	NRMSD	ANMBD	AARD	SS
Entropy value $En_j$ (Step 2, Table 2.6)	0.9896	0.9922	0.9416	0.9719	0.9982
Degree of diversification $Dd_j$ (Step 3, Table 2.6)	0.0104	0.0078	0.0584	0.0281	0.0018
Weight $r_j$ (Step 4, Table 2.6)	0.0976	0.0732	0.5484	0.2639	0.0169

### Solution:

Applied normalization technique: 3 (Sect. 2.4.1, Table 2.5); Table 2.8 presents values of transformed payoff matrix. Table 2.9 presents entropy values, degree of diversification, and weight of indicators.

### 2.4.2.2 Rating Technique

Rating technique facilitates the rating of indicators on a numeral scale. However, there is likely chance of subjectivity while rating the indicators by individual experts and chosen numeral scales (Raju and Nagesh Kumar 2014a).

## 2.4.3 Multicriterion Decision-Making Techniques in Deterministic Scenario

Number of MCDM techniques can be applied to rank GCMs. However, in the present chapter only few techniques are discussed. Researchers are suggested to refer to Raju and Nagesh Kumar (2014a) for more details about various MCDM techniques.

### 2.4.3.1 Compromise Programming (CP):

It is established on distance measure  $L_p$  metric (Raju and Nagesh Kumar 2014a). The methodology is explained in Table 2.10.

**Table 2.10** Methodology of compromise programming

Step	Description	Mathematical expression/remark
1	Normalize the payoff matrix if required	Choose suitable normalization technique (Sect. 2.4.1)
2	Ideal value for each indicator $j$ among available GCMs	$k_j^*$ $j = 1, 2, \dots, J$ where $J$ is the number of indicators
3	$L_p$ metric value for each GCM $a$	$L_{pa} = \left[ \sum_{j=1}^J r_j^p  k_j^* - k_j(a) ^p \right]^{\frac{1}{p}}$ $k_j(a)$ = Value of indicator $j$ for GCM $a$ ; $r_j$ = Weight assigned to the indicator $j$ ; $p$ = Parameter (1, 2, ... $\infty$ )
4	Rank the GCMs built on the $L_{pa}$ values.	Lower $L_{pa}$ indicates suitable GCM

**Numerical Problem 2.3** Compute  $L_p$  metric values of GCMs and corresponding ranking pattern for the payoff matrix presented in Table 2.11 using compromise programming technique taking parameter  $p = 2$ . All the 36 GCMs of CMIP5 (Coupled Model Intercomparison Project 5), namely, ACCESS1.0, ACCESS1.3, BCC-CSM1.1, BCC-CSM1.1-m, BNU-ESM, CCSM4, CESM1-BGC, CESM1-CAM5, CESM1-FAST CHEM, CESM1-WACCM, CNRM-CM5, CSIRO-Mk3.6, CanESM2, FGOALS-s2, FIO-ESM, GFDL-CM3, GFDL-ESM2G, GFDL-ESM2M, GISS-E2-H, GISS-E2-R-CC, GISS-E2-R, HadCM3, HadGEM2-AO, INM-CM4, IPSL-CM5A-LR, IPSL-CM5A-MR, IPSL-CM5B-LR, MIROC4h, MIROC5, MIROC-ESM-CHEM, MIROC-ESM, MPI-ESM-LR, MPI-ESM-MR, MPI-ESM-P, MRI-CGCM3, and NorESM1-M are evaluated on three indicators, namely, SS, CC, and NRMSD on the climate variable maximum temperature. Weight of indicators for SS, CC, and NRMSD are 0.0483, 0.0435, and 0.9083 respectively (Raju et al. 2017).

**Solution:**

#### Sample calculation for ACCESS 1.0

Values of SS, CC, NRMSD = 0.8280, 0.9269, -0.1664

Ideal values of SS, CC, NRMSD = 0.9378, 0.9875, -0.1104 (Step 2, Table 2.10)

Weights of SS, CC, NRMSD = 0.0483, 0.0435, 0.9083

$L_p$  metric value for ACCESS 1.0 from ideal solution (Step 3, Table 2.10) is:

**Table 2.11** Indicator values for 36 GCMs (inputs),  $L_p$  Metric value, and rank (outputs)

GCM (1)	SS (2)	CC (3)	NRMSD <sup>a</sup> (4)	$L_p$ metric (5)	Rank (6)
ACCESS1.0	0.8280	0.9269	-0.1664	0.0512	6
ACCESS1.3	0.8036	0.8699	-0.2907	0.1640	30
BCC-CSM1.1	0.9230	0.9110	-0.2933	0.1662	31
BCC-CSM1.1-m	0.9093	0.9267	-0.1835	0.0665	10
BNU-ESM	0.8166	0.9731	-0.2621	0.1380	25
CCSM4	0.8326	0.9738	-0.2025	0.0838	18
CESM1-BGC	0.8519	0.9755	-0.1753	0.0591	9
CESM1-CAM5	0.8715	0.9647	-0.1893	0.0717	12
CESM1-FAST CHEM	0.8405	0.9730	-0.2004	0.0819	17
CESM1-WACCM	0.7304	0.9250	-0.2713	0.1466	27
CNRM-CM5	0.9218	0.9772	-0.1104	0.0009	1
CSIRO-Mk3.6	0.8160	0.9081	-0.2474	0.1246	24
CanESM2	0.8007	0.9360	-0.1910	0.0736	13
FGOALS-s2	0.9378	0.9875	-0.1364	0.0237	3
FIO-ESM	0.8660	0.9216	-0.2874	0.1609	29
GFDL-CM3	0.8264	0.9210	-0.3898	0.2539	34
GFDL-ESM2G	0.7614	0.9428	-0.4552	0.3133	35
GFDL-ESM2M	0.8895	0.9425	-0.3555	0.2226	33
GISS-E2-H	0.6387	0.8448	-0.1698	0.0562	7
GISS-E2-R-CC	0.7172	0.8151	-0.1994	0.0819	16
GISS-E2-R	0.6283	0.8036	-0.1976	0.0810	15
HadCM3	0.9078	0.5585	-0.6980	0.5340	36
HadGEM2-AO	0.8928	0.9455	-0.1952	0.0771	14
INM-CM4	0.9050	0.9054	-0.2813	0.1553	28
IPSL-CM5A-LR	0.7442	0.9132	-0.1736	0.0582	8
IPSL-CM5A-MR	0.7374	0.9242	-0.3548	0.2222	32
IPSL-CM5B-LR	0.7801	0.8562	-0.2184	0.0985	19
MIROC4h	0.8852	0.9704	-0.1880	0.0706	11
MIROC5	0.9052	0.9031	-0.1625	0.0475	5
MIROC-ESM-CHEM	0.9002	0.9789	-0.1313	0.0191	2
MIROC-ESM	0.8968	0.9749	-0.1417	0.0285	4
MPI-ESM-LR	0.8245	0.9573	-0.2286	0.1075	20
MPI-ESM-MR	0.8812	0.9356	-0.2310	0.1096	21
MPI-ESM-P	0.8350	0.9634	-0.2400	0.1179	22
MRI-CGCM3	0.7919	0.8674	-0.2460	0.1235	23
NorESM1-M	0.8199	0.9822	-0.2623	0.1381	26
Maximum value	0.9378	0.9875	-0.1104		

<sup>a</sup>Minimum NRMSD is desirable. Negative sign is incorporated before values of indicator to represent in maximization perspective, i.e.,  $(-\min) = \max$



$$\left[ \sqrt{\frac{[0.0483(0.9378 - 0.8280)]^2 + [0.0435(0.9875 - 0.9269)]^2}{[0.9083(-0.1104 + 0.1664)]^2}} \right] = 0.0512$$

Similarly,  $L_p$  metric value for other GCMs are computed. Suitable GCM is the one, which is having minimum  $L_p$  metric value from ideal solution (Step 4, Table 2.10). Columns 5 and 6 of Table 2.11 present the  $L_p$  metric values of GCMs and corresponding ranking pattern.

- Rank of 1 to the lowest  $L_p$  metric value and last rank to the highest  $L_p$  metric are to be given. The lower the rank, the better is the GCM, i.e., the GCM with rank 1 is the best and the GCM with rank 2 is the next best, and so on. The GCM with the highest rank is the least suitable for the case.
- $L_p$  metric value is varying between 0.0009 (first rank) and 0.5340 (least preferred) among 36 ranks.
- CNRM-CM5, MIROC-ESM-CHEM, and FGOALS-s2 are occupying the first three positions with  $L_p$  metric values of 0.0009, 0.0191, and 0.0237, respectively, and can be explored further for downscaling and adaptation studies (Table 2.11).
- GFDL-ESM2G and HadCM3 with  $L_p$  metric values of 0.3133 and 0.5340 occupied 35th and 36th positions (Table 2.11), which are the least suitable for the chosen data (Raju et al. 2017).

### 2.4.3.2 Cooperative Game Theory (CGT):

It is established on distance measure, i.e., as “far” as possible to “anti-ideal” solution (Gershon and Duckstein 1983; Raju and Nagesh Kumar 2014a). The methodology is explained in Table 2.12.

**Table 2.12** Methodology of CGT

Step	Description	Mathematical expression/remark
1	Normalize the payoff matrix if required	Choose suitable normalisation technique (Sect. 2.4.1)
2	Anti-ideal value for each indicator $j$ among available GCMs	$k_j^{**}$ $j = 1, 2, \dots, J$ where $J$ is the number of indicators
3	Geometric distance for GCM $a$	$D_a = \prod_{j=1}^J  k_j(a) - k_j^{**} ^{r_j}$ $k_j(a)$ = Value of indicator $j$ for GCM $a$ ; $r_j$ = Weight assigned to the indicator
4	Rank the GCMs built on the $D_a$ values	Higher $D_a$ indicates suitable GCM

**Numerical Problem 2.4** Solve Numerical Problem 2.3 using CGT. Use payoff matrix data in Table 2.11.

**Solution:**

**Sample calculation for ACCESS 1.0**

Values of SS, CC, NRMSD = 0.8280, 0.9269, -0.1664

Anti-ideal values of SS, CC, NRMSD = 0.6283, 0.5585, -0.698 (Step 2, Table 2.12)

Weights of SS, CC, NRMSD = 0.0483, 0.0435, 0.9083

Geometric distance value  $D_a$  for ACCESS 1.0 from anti-ideal solution (Step 3, Table 2.12) is:

$$\prod_{j=1}^3 \left| (0.8280 - 0.6283)^{0.0483} (0.9269 - 0.5585)^{0.0435} (-0.1664 - (-0.698))^{0.9083} \right|$$

$$D_a \text{ for ACCESS 1.0} = 0.9251 * 0.9574 * 0.5633 = 0.4989$$

Similarly, computation of  $D_a$  values for other GCMs are made. Suitable GCM is the one, which is having maximum value of  $D_a$  (Step 4, Table 2.12). Table 2.13 presents the  $D_a$  values of GCMs and corresponding ranking pattern.

**Table 2.13**  $D_a$  value and corresponding ranking pattern: CGT

GCM	$D_a$ value	Rank	GCM	$D_a$ value	Rank
ACCESS1.0	0.4990	6	GISS-E2-H	0.4254	22
ACCESS1.3	0.3865	30	GISS-E2-R-CC	0.4457	18
BCC-CSM1.1	0.3961	28	GISS-E2-R	0.0000	35
BCC-CSM1.1-m	0.4924	8	HadCM3	0.0000	36
BNU-ESM	0.4176	25	HadGEM2-AO	0.4819	11
CCSM4	0.4711	15	INM-CM4	0.4053	26
CESM1-BGC	0.4968	7	IPSL-CM5A-LR	0.4793	12
CESM1-CAM5	0.4861	10	IPSL-CM5A-MR	0.3256	32
CESM1-FAST CHEM	0.4737	14	IPSL-CM5B-LR	0.4443	19
CESM1-WACCM	0.3956	29	MIROC4h	0.4888	9
CNRM-CM5	0.5599	1	MIROC5	0.5088	5
CSIRO-Mk3.6	0.4272	21	MIROC-ESM-CHEM	0.5399	2
CanESM2	0.4751	13	MIROC-ESM	0.5303	4
FGOALS-s2	0.5393	3	MPI-ESM-LR	0.4468	17
FIO-ESM	0.3977	27	MPI-ESM-MR	0.4491	16
GFDL-CM3	0.3038	33	MPI-ESM-P	0.4383	20
GFDL-ESM2G	0.2406	34	MRI-CGCM3	0.4232	23
GFDL-ESM2M	0.3397	31	NorESM1-M	0.4182	24

- Rank of 1 to the highest  $D_a$  and last rank to the lowest  $D_a$  value are to be given. The lower the rank, the better is the GCM, i.e., the GCM with rank 1 is the best and the GCM with rank 2 is the second best, and so on. The GCM with the highest rank is the least suitable for the case.
- $D_a$  value is varying between 0.0 (last rank) and 0.5599 (first rank) among 36 ranks.
- CNRM-CM5, MIROC-ESM-CHEM, and FGOALS-s2 with  $D_a$  values of 0.5599, 0.5399, and 0.5393 are occupying the first three positions, respectively, (Table 2.13) and can be explored further for downscaling and adaptation studies.
- GISS-E2-R and HadCM3 with  $D_a$  values of 0.0 occupied 35th and 36th positions respectively.

### 2.4.3.3 Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS)

It is established on the distance measure between ideal and anti-ideal solutions (Opricovic and Tzeng 2004; Raju and Nagesh Kumar 2014a, 2015a). The methodology is explained in Table 2.14.

**Table 2.14** Methodology of TOPSIS

Step	Description	Mathematical expression/remark
1	Normalize the payoff matrix if required	Choose suitable normalisation technique (Sect. 2.4.1)
2	Ideal value and anti-ideal value for each indicator $j$ among available GCMs	$k_j^*, k_j^{**}$ $j = 1, 2, \dots, J$ where $J$ is number of indicators
3	Separation measure $DS_a^+$ of GCM $a$ from the ideal solution	$DS_a^+ = \sqrt{\sum_{j=1}^J r_j (k_j(a) - k_j^*)^2}$
4	Separation measure $DS_a^-$ of GCM $a$ from the anti-ideal solution	$DS_a^- = \sqrt{\sum_{j=1}^J r_j (k_j(a) - k_j^{**})^2}$
5	Relative closeness $CR_a$	$CR_a = \frac{DS_a^-}{(DS_a^- + DS_a^+)}$
6	Rank the GCMs built on the $CR_a$ values	Higher $CR_a$ indicates suitable GCM

**Numerical Problem 2.5** Eleven GCMs in Coupled Model Intercomparison Project (CMIP3), namely, BCCR-BCCM2.0, INGV-ECHAM4, GFDL2.0, GFDL2.1, GISS, IPSL-CM4, MIROC3, MRI-CGCM2, NCAR-PCMI, UKMO-HADCM3, and UKMO-HADGEM1 are evaluated for climate variables, precipitation (PR), and temperature at 3 levels, i.e., 500, 700, 850 mb (and referred from now as T500, T700, T850) on performance indicator SS. Equal weight of 0.25 is considered for each skill score indicator, SPR, ST500, ST700, and ST850. Determine ranking of GCMs using TOPSIS. Relevant data is presented in Table 2.15. Use abbreviations

**Table 2.15** Skill score values for the chosen 11 GCMs

GCM	SPR	ST500	ST700	ST850
BCCR	0.7717	0.5533	0.2583	0.3300
ECHAM	0.6833	0.3483	0.2800	0.3917
GFDL2.0	0.8150	0.5533	0.4000	0.4183
GFDL2.1	0.8350	0.5533	0.2633	0.3783
GISS	0.7783	0.2833	0.2033	0.3150
IPSL	0.6583	0.2833	0.2017	0.4167
MIROC3	0.8567	0.3583	0.3867	0.3800
CGCM2	0.7550	0.4517	0.2367	0.5250
PCMI	0.6283	0.3217	0.1783	0.3367
HADCM3	0.8100	0.3483	0.4100	0.4100
HADGEM1	0.7883	0.2850	0.2067	0.3983

BCCR, ECHAM, GFDL2.0, GFDL2.1, GISS, IPSL, MIROC3, CGCM2, PCMI, HADCM3, and HADGEM1 for GCMs for easy computation (Raju and Nagesh Kumar 2015a). Assume ideal values of each indicator as 1 and anti-ideal values of each indicator as 0.

### Solution:

#### Sample calculation for BCCR-BCCM 2.0

Values of SPR, ST500, ST700, ST850: 0.7717, 0.5533, 0.2583, 0.3300

Ideal values of SPR, ST500, ST700, ST850 = 1 each (Step 2, Table 2.14)

Anti-ideal values of SPR, ST500, ST700, ST850 = 0 each (Step 2, Table 2.14)

Weight of SPR, ST500, ST700, ST850 = 0.25 each

Separation measure of BCCR from ideal solution, i.e.,  $DS^+$  for BCCR is (Step 3, Table 2.14):

$$\left[ 0.25 * \sqrt{\frac{(0.7717 - 1.00)^2 + (0.5533 - 1.00)^2 + (0.2583 - 1.00)^2}{(0.3300 - 1.00)^2}} \right]$$

$$= 0.2796$$

Separation measure of BCCR from anti-ideal solution, i.e.,  $DS^-$  for BCCR is (Step 4, Table 2.14):

$$0.25 * \left[ \sqrt{\frac{(0.7717 - 0.00)^2 + (0.5533 - 0.00)^2 + (0.2583 - 0.00)^2}{(0.3300 - 0.00)^2}} \right]$$

$$= 0.2595$$

Relative closeness of BCCR with reference to anti-ideal solution, i.e.,  $CR_a$  for BCCR is (Step 5, Table 2.14):  $\frac{DS_{BCCR}^-}{(DS_{BCCR}^- + DS_{BCCR}^+)} = \frac{0.2595}{(0.2595 + 0.2796)} = 0.4814$

**Table 2.16**  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$  values and corresponding ranking pattern: TOPSIS

GCM	$DS_a^+$	$DS_a^-$	$CR_a$	Rank
BCCR	0.2796	0.2595	0.4814	6
ECHAM	0.2972	0.2264	0.4324	8
GFDL2.0	0.2414	0.2856	0.5420	1
GFDL2.1	0.2688	0.2757	0.5063	2
GISS	0.3228	0.2273	0.4132	9
IPSL	0.3170	0.2133	0.4022	10
MIROC3	0.2730	0.2688	0.4961	3
CGCM2	0.2703	0.2629	0.4931	5
PCMI	0.3272	0.2005	0.3800	11
HADCM3	0.2689	0.2638	0.4952	4
HADGEM1	0.3110	0.2377	0.4332	7

Similarly,  $CR_a$  values for other GCMs are computed. Suitable GCM is the one with maximum relative closeness  $CR_a$  value from anti-ideal solution (Step 6, Table 2.14). Table 2.16 presents the  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$  value of GCMs and corresponding ranking pattern.

- GFDL2.0, GFDL2.1, MIROC3 with  $CR_a$  values of 0.5420, 0.5063, and 0.4961, respectively, occupied the first three positions (Table 2.16).
- IPSL, PCMI with  $CR_a$  values of 0.4022 and 0.3800 occupied the last the two positions (Table 2.16).

#### 2.4.3.4 Weighted Average Technique

It is utility-related technique (Raju and Nagesh Kumar 2014a). The methodology is explained in Table 2.17.

**Table 2.17** Methodology of weighted average technique

Step	Description	Mathematical expression/remark
1	Normalize the payoff matrix if required	Choose suitable normalization technique (Sect. 2.4.1)
2	Utility of GCM $a$	$V_a = \left[ \sum_{j=1}^J r_j k_j \right]$ $k_j$ = Value of indicator $j$ for GCM $a$ ; $r_j$ = Weight assigned to the indicator $j$
3	Rank the GCMs built on the $V_a$ values	Higher $V_a$ indicates suitable GCM

**Numerical Problem 2.6** Solve Numerical Problem 2.3 using weighted average technique. Use payoff matrix data in Table 2.11.

**Solution:****Sample calculation for ACCESS 1.0**

Values of SS, CC, NRMSD = 0.8280, 0.9269, -0.1664

Weight of SS, CC, NRMSD = 0.0483, 0.0435, 0.9083

Weighted Average value  $V$  for ACCESS 1.0 (Step 2, Table 2.17) is:

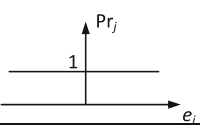
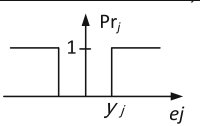
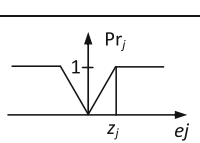
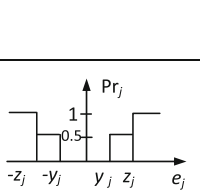
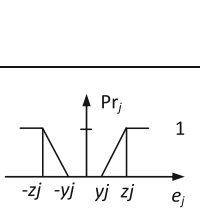
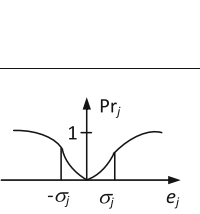
$$\begin{aligned} V \text{ for ACCESS 1.0} &= 0.8280 \times 0.0483 + 0.9269 \times 0.0435 + (-0.1664) \\ &\quad \times 0.9083 \\ &= -0.0708 \end{aligned}$$

Similarly, weighted average values for other GCMs are computed. Best GCM is the one with maximum weighed average value  $V_a$  (Step 3, Table 2.17). Table 2.18 presents the  $V_a$  values of GCMs and corresponding ranking pattern.

- Utility value of GCMs are varying between -0.0132 (first rank) and -0.5658 (least preferred) among 36 ranks.
- CNRM-CM5, MIROC-ESM-CHEM, and FGOALS-s2 with utility values of -0.0132, -0.0332, and -0.0356, respectively, are occupying the first three positions.
- GFDL-ESM2G and HadCM3 with utility values of -0.3356 and -0.5658 occupied 35th and 36th positions respectively.

**Table 2.18**  $V_a$  value and corresponding ranking pattern: weighted average technique

GCM	$V_a$	Rank	GCM	$V_a$	Rank
ACCESS1.0	-0.0708	6	GISS-E2-H	-0.0866	11
ACCESS1.3	-0.1873	31	GISS-E2-R-CC	-0.1110	17
BCC-CSM1.1	-0.1821	30	GISS-E2-R	-0.1141	18
BCC-CSM1.1-m	-0.0824	9	HadCM3	-0.5658	36
BNU-ESM	-0.1562	26	HadGEM2-AO	-0.0930	13
CCSM4	-0.1013	16	INM-CM4	-0.1724	28
CESM1-BGC	-0.0756	7	IPSL-CM5A-LR	-0.0820	8
CESM1-CAM5	-0.0878	12	IPSL-CM5A-MR	-0.2464	33
CESM1-FAST CHEM	-0.0991	15	IPSL-CM5B-LR	-0.1234	19
CESM1-WACCM	-0.1709	27	MIROC4h	-0.0857	10
CNRM-CM5	-0.0132	1	MIROC5	-0.0645	5
CSIRO-Mk3.6	-0.1458	23	MIROC-ESM-CHEM	-0.0332	2
CanESM2	-0.0941	14	MIROC-ESM	-0.0429	4
FGOALS-s2	-0.0356	3	MPI-ESM-LR	-0.1261	20
FIO-ESM	-0.1791	29	MPI-ESM-MR	-0.1265	21
GFDL-CM3	-0.2740	34	MPI-ESM-P	-0.1357	22
GFDL-ESM2G	-0.3356	35	MRI-CGCM3	-0.1474	24
GFDL-ESM2M	-0.2389	32	NorESM1-M	-0.1559	25

Types of generalized indicator functions			Preference function values for various types of indicator functions
1	Usual		$Pr_j = \begin{cases} 0 & \text{if } e_j \leq 0 \\ 1 & \text{if } e_j > 0 \end{cases}$
2	Quasi		$Pr_j = \begin{cases} 0 & \text{if } e_j \leq y_j \\ 1 & \text{if } e_j > y_j \end{cases}$
3	Linear preference and no indifference area		$Pr_j = \begin{cases} \frac{e_j}{z_j} & \text{if } e_j \leq z_j \\ 1 & \text{if } e_j > z_j \end{cases}$
4	Level		$Pr_j = \begin{cases} 0 & \text{if } e_j \leq y_j \\ 0.5 & \text{if } y_j < e_j \leq z_j \\ 1 & \text{if } e_j > z_j \end{cases}$
5	Linear preference and indifference area		$Pr_j = \begin{cases} 0 & \text{if } e_j \leq y_j \\ \frac{(e_j - y_j)}{(z_j - y_j)} & \text{if } y_j < e_j \leq z_j \\ 1 & \text{if } e_j > z_j \end{cases}$
6	Gaussian		$Pr_j = [1 - e^{\frac{-e_j^2}{2\sigma_j^2}}]$

**Fig. 2.2** Types of various indicator functions and relevant preference function values in PROMETHEE-2

### 2.4.3.5 Preference Ranking Organization Method of Enrichment Evaluation (PROMETHEE-2)

It is built-in preference function concept (Pomerol and Romero 2000; Raju and Nagesh Kumar 2014a, b; Brans et al. 1986). Preference function  $Pr_j(a, b)$  depends on the pairwise difference  $e_j$  between the evaluations  $k_j(a)$  and  $k_j(b)$  of GCMs  $a$  and  $b$  for indicator  $j$ , chosen indicator function and corresponding parameters such as indifference and preference thresholds  $y_j$  and  $z_j$ . Six types of indicator functions are available (Fig. 2.2). The methodology is explained in Table 2.19.

**Table 2.19** Methodology of PROMETHEE-2

Step	Description	Mathematical expression/remark
1	Multi indicator preference index (MIPI)	$\pi(a, b) = \frac{\sum_{j=1}^J r_j \text{Pr}_j(a, b)}{\sum_{j=1}^J r_j}$
2	Outranking index of GCM $a$ in the $T$ GCMs [ $T$ is number of GCMs]	$\phi^+(a) = \frac{\sum_A \pi(a, b)}{(T-1)}$
3	Outranked index of GCM $a$ in the $T$ GCMs	$\phi^-(a) = \frac{\sum_A \pi(b, a)}{(T-1)}$
4	Net ranking of GCM $a$ in the $T$ GCMs	$\phi(a) = \phi^+(a) - \phi^-(a)$
5	Rank the GCMs built on the $\phi(a)$ values	Higher $\phi(a)$ indicates suitable GCM

**Numerical Problem 2.7** Eleven GCMs in Coupled Model Intercomparison Project (CMIP3), namely, BCCR-BCCM2.0, INGV-ECHAM4, GFDL2.0, GFDL2.1, GISS, IPSL-CM4, MIROC3, MRI-CGCM2, NCAR-PCMI, UKMO-HADCM3, and UKMO-HADGEM1, are evaluated on indicators CC, NRMSD, ANMBD, AARD, and SS for climate variable precipitation. Payoff matrix (GCMs vs. indicators) is presented in Table 2.20. Weights of the indicators obtained by entropy technique are 0.0976, 0.0729, 0.5481, 0.2640, and 0.0174 respectively. Using PROMETHEE-2 technique, determine the ranking of GCMs using the above weights. Assume usual indicator function for all indicators. Use abbreviations BCCR, ECHAM, GFDL2.0, GFDL2.1, GISS, IPSL, MIROC3, CGCM2, PCMI, HADCM3, and HADGEM1 for GCMs for easy computations (Raju and Nagesh Kumar 2014b).

**Table 2.20** Values of performance indicators obtained for the 11 GCMs

GCM	CC	NRMSD <sup>a</sup>	ANMBD <sup>a</sup>	AARD <sup>a</sup>	SS
BCCR	0.7751	0.7960	0.2744	1.7127	0.7717
ECHAM	0.7866	0.7573	0.1619	1.8639	0.6833
GFDL2.0	0.7868	0.8286	0.4157	0.8080	0.8150
GFDL2.1	0.7395	0.7871	0.1551	1.2731	0.8350
GISS	0.8275	0.8221	0.4786	0.7539	0.7783
IPSL	0.4740	1.2539	0.7082	1.0124	0.6583
MIROC3	0.8416	0.6224	0.0613	1.3811	0.8567
CGCM2	0.7708	0.9386	0.4985	0.6556	0.7550
PCMI	0.3553	1.1779	0.4899	1.6149	0.6283
HADCM3	0.8018	0.8793	0.5092	0.8002	0.8100
HADGEM1	0.8064	0.9422	0.5686	0.7010	0.7883



**Solution:**

Pairwise difference between values of GCMs and preference functions for indicator CC:

Pairwise difference between values of GCMs for each indicator (five in this case) are to be computed. For example, for indicator CC, the pairwise difference of values in Table 2.21, between BCCR and GFDL2.0 is  $0.7751 - 0.7868 = -0.0117$  (Table 2.22a) and so the corresponding value of preference function under usual indicator function is 0 (as  $-0.0117 < 0$ ). Vice versa, pairwise difference between GFDL2.0 and BCCR for CC is 0.0117 (Table 2.22a) and corresponding value of preference function is 1 (as  $0.0117 > 0$ ) as in the case of usual indicator function (Table 2.23a), elements of preference function matrix are either 0 or 1. Pairwise preference function values are computed for each indicator in the similar format. Table 2.22a–e presents pairwise difference matrix for CC, NRMSD, ANMBD, AARD, SS whereas Table 2.23a–e present preference function values for CC, NRMSD, ANMBD, AARD, and SS.

**Table 2.21** Transformed values of performance indicators obtained for the 11 GCMs

GCM	CC	NRMSD <sup>a</sup>	ANMBD <sup>a</sup>	AARD <sup>a</sup>	SS
BCCR	0.7751	−0.7960	−0.2744	−1.7127	0.7717
ECHAM	0.7866	−0.7573	−0.1619	−1.8639	0.6833
GFDL2.0	0.7868	−0.8286	−0.4157	−0.8080	0.8150
GFDL2.1	0.7395	−0.7871	−0.1551	−1.2731	0.8350
GISS	0.8275	−0.8221	−0.4786	−0.7539	0.7783
IPSL	0.4740	−1.2539	−0.7082	−1.0124	0.6583
MIROC3	0.8416	−0.6224	−0.0613	−1.3811	0.8567
CGCM2	0.7708	−0.9386	−0.4985	−0.6556	0.7550
PCMI	0.3553	−1.1779	−0.4899	−1.6149	0.6283
HADCM3	0.8018	−0.8793	−0.5092	−0.8002	0.8100
HADGEM1	0.8064	−0.9422	−0.5686	−0.7010	0.7883

<sup>a</sup>Minimum NRMSD, ANMBD, AARD are desirable. Negative sign is incorporated before values of indicators to represent in maximization perspective, i.e.,  $(-\text{min}) = \text{max}$

Multi Indicator Preference Index,  $\pi(\text{ECHAM}, \text{BCCR})$  for pairwise GCMs (ECHAM, BCCR) is computed as follows (Step 1, Table 2.19):

Preference function values for ECHAM and BCCR for indicators CC, NRMSD, ANMBD, AARD, SS are 1, 1, 1, 0, 0. Corresponding weights of indicators are 0.0976, 0.0729, 0.5481, 0.2640, and 0.0174 respectively.

Multi Indicator Preference Index for pair of GCMs (ECHAM and BCCR)

$$\frac{[0.0976 \times 1 + 0.0729 \times 1 + 0.5481 \times 1 + 0.2640 \times 0 + 0.0174 \times 0]}{[0.0976 + 0.0729 + 0.5481 + 0.2640 + 0.0174]} = \frac{0.7186}{1} = 0.7186$$

Computations are repeated for all possible pairs for all indicators resulting in Table 2.24.

Table 2.22 Pairwise difference matrix

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIROC3	CGCM2	PCMI	HADCM3	HADGEM1
(a) Pairwise difference matrix for indicator CC											
BCCR	0	-0.0115	-0.0117	-0.0356	0.0524	-0.3011	0.0665	0.0043	0.4198	-0.0267	0.0313
ECHAM	0.0115	0	-0.0002	0.0471	-0.0409	0.3126	-0.055	0.0158	0.4313	-0.0152	-0.0198
GFDL2.0	0.0117	0.0002	0	0.0473	-0.0407	0.3128	-0.0548	0.016	0.4315	-0.015	-0.0196
GFDL2.1	-0.0356	-0.0471	-0.0473	0	-0.088	0.2655	-0.1021	-0.0313	0.3842	-0.0623	-0.0669
GISS	0.0524	0.0409	0.0407	0.088	0	0.3535	-0.0141	0.0567	0.4722	0.0257	0.0211
IPSL	-0.3011	-0.3126	-0.3128	-0.2655	-0.3535	0	-0.3676	-0.2968	0.1187	-0.3278	-0.3324
MIROC3	0.0665	0.055	0.0548	0.1021	0.0141	0.3676	0	0.0708	0.4863	0.0398	0.0352
CGCM2	-0.0043	-0.0158	-0.016	0.0313	-0.0567	0.2968	0	0	0.4155	-0.031	-0.0356
PCMI	-0.4198	-0.4313	-0.4315	-0.3842	-0.4722	-0.1187	-0.4863	-0.4155	0	-0.4465	-0.4511
HADCM3	0.0267	0.0152	0.015	0.0623	-0.0257	0.3278	-0.0398	0.031	0.4465	0	-0.0046
HADGEM1	0.0313	0.0198	0.0196	0.0669	-0.0211	0.3324	-0.0352	0.0356	0.4511	0.0046	0
(b) Pairwise difference matrix for indicator NRMSE											
BCCR	0	-0.0387	0.0326	-0.0089	0.0261	0.4579	0.1736	0.1426	0.3819	0.0833	0.1462
ECHAM	0.0387	0	0.0713	0.0298	0.0648	0.4966	-0.1349	0.1813	0.4206	0.122	0.1849
GFDL2.0	-0.0326	-0.0713	0	-0.0415	-0.0065	0.4253	-0.2062	0.11	0.3493	0.0507	0.1136
GFDL2.1	0.0089	-0.0298	0.0415	0	0.035	0.4668	-0.1647	0.1515	0.3908	0.0922	0.1551
GISS	-0.0261	-0.0648	0.0065	-0.035	0	0.4318	-0.1997	0.1165	0.3558	0.0572	0.1201
IPSL	-0.4579	-0.4966	-0.4253	-0.4668	-0.4318	0	-0.6315	-0.3153	-0.076	-0.3746	-0.3117
MIROC3	0.1736	0.1349	0.2062	0.1647	0.1997	0.6315	0	0.3162	0.5555	0.2569	0.3198
CGCM2	-0.1426	-0.1813	-0.11	-0.1515	-0.1165	0.3153	-0.3162	0	0.2393	-0.0593	0.0036
PCMI	-0.3819	-0.4206	-0.3493	-0.3908	-0.3558	0.076	-0.5555	-0.2393	0	-0.2986	-0.2357
HADCM3	-0.0833	-0.122	-0.0507	-0.0922	-0.0572	0.3746	-0.2569	0.0593	0.2986	0	0.0629
HADGEM1	-0.1462	-0.1849	-0.1136	-0.1551	-0.1201	0.3117	-0.3198	-0.0036	0.2357	-0.0629	0

(continued)

Table 2.22 (continued)

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIROC3	CGCM2	PCMI	HADCM3	HADGEM1
(c) Pairwise difference matrix for indicator ANMBD											
BCCR	0	-0.1125	0.1413	-0.1193	0.2042	0.4338	-0.2131	0.2241	0.2155	0.2348	0.2942
ECHAM	0.1125	0	0.2538	-0.0068	0.3167	0.5463	-0.1006	0.3366	0.328	0.3473	0.4067
GFDL2.0	-0.1413	-0.2538	0	-0.2606	0.0629	0.2925	-0.3544	0.0828	0.0742	0.0935	0.1529
GFDL2.1	0.1193	0.0068	0.2606	0	0.3235	0.5531	-0.0938	0.3434	0.3348	0.3541	0.4135
GISS	-0.2042	-0.3167	-0.0629	-0.3235	0	0.2296	-0.4173	0.0199	0.0113	0.0306	0.09
IPSL	-0.4338	-0.5463	-0.2925	-0.5531	-0.2296	0	-0.6469	-0.2097	-0.2183	-0.199	-0.1396
MIROC3	0.2131	0.1006	0.3544	0.0938	0.4173	0.6469	0	0.4372	0.4286	0.4479	0.5073
CGCM2	-0.2241	-0.3366	-0.0828	-0.3434	-0.0199	0.2097	-0.4372	0	-0.0086	0.0107	0.0701
PCMI	-0.2155	-0.328	-0.0742	-0.3348	-0.0113	0.2183	-0.4286	0.0086	0	0.0193	0.0787
HADCM3	-0.2348	-0.3473	-0.0935	-0.3541	-0.0306	0.199	-0.4479	-0.0107	-0.0193	0	0.0594
HADGEM1	-0.2942	-0.4067	-0.1529	-0.4135	-0.09	0.1396	-0.5073	-0.0701	-0.0787	-0.0594	0
(d) Pairwise difference matrix for indicator AARD											
BCCR	0	0.1512	-0.9047	-0.4396	-0.9588	-0.7003	-0.3316	-1.0571	-0.0978	-0.9125	-1.0117
ECHAM	-0.1512	0	-1.0559	-0.5908	-1.11	-0.8515	-0.4828	-1.2083	-0.249	-1.0637	-1.1629
GFDL2.0	0.9047	1.0559	0	0.4651	-0.0541	0.2044	0.5731	-0.1524	0.8069	-0.0078	-0.107
GFDL2.1	0.4396	0.5908	-0.4651	0	-0.5192	-0.2607	0.108	-0.6175	0.3418	-0.4729	-0.5721
GISS	0.9588	1.11	0.0541	0.5192	0	0.2585	0.6272	-0.0983	0.861	0.0463	-0.0529
IPSL	0.7003	0.8515	-0.2044	0.2607	-0.2585	0	0.3687	-0.3568	0.6025	-0.2122	-0.3114
MIROC3	0.3316	0.4828	-0.5731	-0.108	-0.6272	-0.3687	0	-0.7255	0.2338	-0.5809	-0.6801
CGCM2	1.0571	1.2083	0.1524	0.6175	0.0983	0.3568	0.7255	0	0.9593	0.1446	0.0454
PCMI	0.0978	0.249	-0.8069	-0.3418	-0.861	-0.6025	-0.2338	-0.9593	0	-0.8147	-0.9139
HADCM3	0.9125	1.0637	0.0078	0.4729	-0.0463	0.2122	0.5809	-0.1446	0.8147	0	-0.0992
HADGEM1	1.0117	1.1629	0.107	0.5721	0.0529	0.3114	0.6801	-0.0454	0.9139	0.0992	0

(continued)

Table 2.22 (continued)

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIRO C3	CGCM2	PCMI	HADCM3	HAD GEM1
(e) Pairwise difference matrix for indicator SS											
BCCR	0	0.0884	-0.0433	-0.0633	-0.0066	0.1134	-0.085	0.0167	0.1434	-0.0383	-0.0166
ECHAM	-0.0884	0	-0.1317	-0.1517	-0.095	0.025	-0.1734	-0.0717	0.055	-0.1267	-0.105
GFDL2.0	0.0433	0.1317	0	-0.02	0.0367	0.1567	-0.0417	0.06	0.1867	0.005	0.0267
GFDL2.1	0.0633	0.1517	0.02	0	0.0567	0.1767	-0.0217	0.08	0.2067	0.025	0.0467
GISS	0.0066	0.095	-0.0367	-0.0567	0	0.12	-0.0784	0.0233	0.15	-0.0317	-0.01
IPSL	-0.1134	-0.025	-0.1567	-0.1767	-0.12	0	-0.1984	-0.0967	0.03	-0.1517	-0.13
MIROC3	0.085	0.1734	0.0417	0.0217	0.0784	0.1984	0	0.1017	0.2284	0.0467	0.0684
CGCM2	-0.0167	0.0717	-0.06	-0.08	-0.0233	0.0967	-0.1017	0	0.1267	-0.055	-0.0333
PCMI	-0.1434	-0.055	-0.1867	-0.2067	-0.15	-0.03	-0.2284	-0.1267	0	-0.1817	-0.16
HADCM3	0.0383	0.1267	-0.005	-0.025	0.0317	0.1517	-0.0467	0.055	0.1817	0	0.0217
HADGEM1	0.0166	0.105	-0.0267	-0.0467	0.01	0.13	-0.0684	0.0333	0.16	-0.0217	0

Table 2.23 Preference function matrix

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIROC3	CGCM2	PCMI	HADCM3	HADGEM1
(a) Preference function matrix for indicator CC											
BCCR	0	0	0	1	0	1	0	1	1	0	0
ECHAM	1	0	0	1	0	1	0	1	1	0	0
GFDL2.0	1	1	0	1	0	1	0	1	1	0	0
GFDL2.1	0	0	0	0	0	1	0	0	1	0	0
GISS	1	1	1	1	0	1	0	1	1	1	1
IPSL	0	0	0	0	0	0	0	0	1	0	0
MIROC3	1	1	1	1	1	1	0	1	1	1	1
CGCM2	0	0	0	1	0	1	0	0	1	0	0
PCMI	0	0	0	0	0	0	0	0	0	0	0
HADCM3	1	1	1	1	0	1	0	1	1	0	0
HADGEM1	1	1	1	1	0	1	0	1	1	1	0
(b) Preference function matrix for indicator NRMSED											
BCCR	0	0	1	0	1	1	0	1	1	1	1
ECHAM	1	0	1	1	1	1	0	1	1	1	1
GFDL2.0	0	0	0	0	0	1	0	1	1	1	1
GFDL2.1	1	0	1	0	1	1	0	1	1	1	1
GISS	0	0	1	0	0	1	0	1	1	1	1
IPSL	0	0	0	0	0	0	0	0	0	0	0
MIROC3	1	1	1	1	1	1	0	1	1	1	1
CGCM2	0	0	0	0	0	1	0	0	1	0	1
PCMI	0	0	0	0	0	1	0	0	0	0	0
HADCM3	0	0	0	0	0	1	0	1	1	0	1
HADGEM1	0	0	0	0	0	1	0	0	1	0	0

(continued)

Table 2.23 (continued)

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIROC3	CGCM2	PCMI	HADCM3	HAD GEM1
(c) Preference function matrix for indicator ANMBD											
BCCR	0	0	1	0	1	1	0	1	1	1	1
ECHAM	1	0	1	0	1	1	0	1	1	1	1
GFDL2.0	0	0	0	0	1	1	0	1	1	1	1
GFDL2.1	1	1	1	0	1	1	0	1	1	1	1
GISS	0	0	0	0	0	1	0	1	1	1	1
IPSL	0	0	0	0	0	0	0	0	0	0	0
MIROC3	1	1	1	1	1	1	0	1	1	1	1
CGCM2	0	0	0	0	0	1	0	0	0	1	1
PCMI	0	0	0	0	0	1	0	1	0	1	1
HADCM3	0	0	0	0	0	1	0	0	0	0	1
HADGEM1	0	0	0	0	0	1	0	0	0	0	0
(d) Preference function matrix for indicator AARD											
BCCR	0	1	0	0	0	0	0	0	0	0	0
ECHAM	0	0	0	0	0	0	0	0	0	0	0
GFDL2.0	1	1	0	1	0	1	1	0	1	0	0
GFDL2.1	1	1	0	0	0	0	1	0	1	0	0
GISS	1	1	1	1	0	1	1	0	1	1	0
IPSL	1	1	0	1	0	0	1	0	1	0	0
MIROC3	1	1	0	0	0	0	0	0	1	0	0
CGCM2	1	1	1	1	1	1	1	0	1	1	1
PCMI	1	1	0	0	0	0	0	0	0	0	0
HADCM3	1	1	1	1	0	1	1	0	1	0	0
HADGEM1	1	1	1	1	1	1	1	0	1	1	0

(continued)

Table 2.23 (continued)

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIROC3	CGCM2	PCMI	HADCM3	HADGEM1
(e) Preference function matrix for indicator SS											
BCCR	0	1	0	0	0	1	0	1	1	0	0
ECHAM	0	0	0	0	0	1	0	0	1	0	0
GFDL2.0	1	1	0	0	1	1	0	1	1	1	1
GFDL2.1	1	1	1	0	1	1	0	1	1	1	1
GISS	1	1	0	0	0	1	0	1	1	0	0
IPSL	0	0	0	0	0	0	0	0	1	0	0
MIROC3	1	1	1	1	1	1	0	1	1	1	1
CGCM2	0	1	0	0	0	1	0	0	1	0	0
PCMI	0	0	0	0	0	0	0	0	0	0	0
HADCM3	1	1	0	0	1	1	0	1	1	0	1
HADGEM1	1	1	0	0	1	1	0	1	1	0	0

Table 2.24 Multi indicator preference index

GCM	BCCR	ECHAM	GFDL 2.0	GFDL 2.1	GISS	IPSL	MIRO C3	CGCM2	PCMI	HADCM3	HAD GEM1
BCCR	0	0.2814	0.621	0.0976	0.621	0.736	0	0.736	0.736	0.621	0.621
ECHAM	0.7186	0	0.621	0.1705	0.621	0.736	0	0.7186	0.736	0.621	0.621
GFDL2.0	0.379	0.379	0	0.3616	0.5655	1	0.264	0.736	1	0.6384	0.6384
GFDL2.1	0.9024	0.8295	0.6384	0	0.6384	0.736	0.264	0.6384	1	0.6384	0.6384
GISS	0.379	0.379	0.4345	0.3616	0	1	0.264	0.736	1	0.9826	0.7186
IPSL	0.264	0.264	0	0.264	0	0	0.264	0	0.379	0	0
MIROC3	1	1	0.736	0.736	0.736	0.736	0	0.736	1	0.736	0.736
CGCM2	0.264	0.2814	0.264	0.3616	0.264	1	0.264	0	0.4519	0.8121	0.885
PCMI	0.264	0.264	0	0	0	0.621	0	0.5481	0	0.5481	0.5481
HADCM3	0.379	0.379	0.3616	0.3616	0.0174	1	0.264	0.1879	0.4519	0	0.6384
HADGEM1	0.379	0.379	0.3616	0.3616	0.2814	1	0.264	0.115	0.4519	0.3616	0



Computation of  $\phi^+$  (Step 2, Table 2.19)

$$\begin{aligned}\phi^+ \text{ for BCCR} &= \frac{[0 + 0.2814 + 0.6210 + 0.0976 + 0.6210 + 0.7360 + 0 + 0.7360 + 0.7360 + 0.6210 + 0.6210]}{10} \\ &= 0.5071\end{aligned}$$

Computation of  $\phi^-$  (Step 3, Table 2.19)

$$\begin{aligned}\phi^- \text{ for BCCR} &= \frac{[0 + 0.7186 + 0.3790 + 0.9024 + 0.3790 + 0.2640 + 1 + 0.2640 + 0.2640 + 0.3790 + 0.3790]}{10} \\ &= 0.4929\end{aligned}$$

Computation of net  $\phi$  (Step 4, Table 2.19)

$$\text{Net } \phi \text{ for BCCR} = \phi^+ \text{ for BCCR} - \phi^- \text{ for BCCR} = 0.5071 - 0.4929 = 0.0142$$

$\phi^+$ ,  $\phi^-$ , net  $\phi$  values for GCMs are computed and presented in Table 2.25. Suitable GCM is the one, which is having the highest net  $\phi$  value (Step 5, Table 2.19). Table 2.25 presents  $\phi^+$ ,  $\phi^-$ , net  $\phi$ , and corresponding ranking pattern of all GCMs.

**Table 2.25** Values of  $\phi^+$ ,  $\phi^-$ , net  $\phi$  and ranks of GCMs

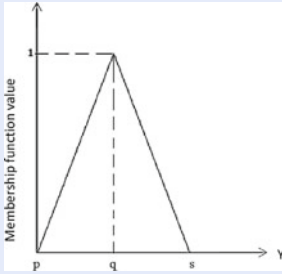
GCM	$\phi^+$	$\phi^-$	Net $\phi$	Rank
BCCR	0.5071	0.4929	0.0142	6
ECHAM	0.5564	0.4436	0.1127	5
GFDL2.0	0.5962	0.4038	0.1924	4
GFDL2.1	0.6924	0.3076	0.3848	2
GISS	0.6255	0.3745	0.2511	3
IPSL	0.1435	0.8565	-0.7130	11
MIROC3	0.8152	0.1848	0.6304	1
CGCM2	0.4848	0.5152	-0.0304	7
PCMI	0.2793	0.7207	-0.4413	10
HADCM3	0.4041	0.5959	-0.1918	8
HADGEM1	0.3955	0.6045	-0.2090	9

- MIROC3 and GFDL2.1 with  $\phi$  values of 0.6304, 0.3848 occupied the first two positions.
- IPSL with  $\phi$  value of -0.7130 occupied last position (Raju and Nagesh Kumar 2014b).

### 2.4.4 Multicriterion Decision-Making Technique in Fuzzy Scenario

Imprecision in indicator values that may arise due to interpolation, averaging procedures, and approximations can be tackled through fuzzy logic. Extension of TOPSIS in fuzzy named as Fuzzy TOPSIS is presented as an initiation. Table 2.26 presents the methodology (Opricovic and Tzeng 2004; Raju and Nagesh Kumar 2014a, 2015b).

**Table 2.26** Methodology of Fuzzy TOPSIS

Step	Description	Mathematical expression/remark
1	Input payoff matrix and specification of membership function; for triangular membership function $\tilde{Y}_{ij}(p_{ij}, q_{ij}, s_{ij})$ where p, q, s are lower, middle, and upper values	Payoff matrix will change depending on the chosen membership function  Typical triangular membership function
2	Ideal value and anti-ideal value for each indicator $j$ among available GCMs	$\tilde{Y}_j^*$ (ideal), $\tilde{Y}_j^{**}$ (anti-ideal) represent with elements $(p_j^*, q_j^*, s_j^*), (p_j^{**}, q_j^{**}, s_j^{**}); j = 1, 2, \dots, J$ where $J$ is the number of indicators
3	Separation measure of each GCM $a$ from the ideal solution	$DS_a^+ = \sum_{j=1}^J d(\tilde{Y}_{aj}, \tilde{Y}_j^*) = \sqrt{\frac{[(p_{aj}-p_j^*)^2 + (q_{aj}-q_j^*)^2 + (s_{aj}-s_j^*)^2]}{3}}$
4	Separation measure of each GCM $a$ from the anti-ideal solution	$DS_a^- = \sum_{j=1}^J d(\tilde{Y}_{aj}, \tilde{Y}_j^{**}) = \sqrt{\frac{[(p_{aj}-p_j^{**})^2 + (q_{aj}-q_j^{**})^2 + (s_{aj}-s_j^{**})^2]}{3}}$
5	Relative closeness $CR_a$	$CR_a = \frac{DS_a^-}{(DS_a^- + DS_a^+)}$
6	Rank the GCMs built on the $CR_a$ values	Higher $CR_a$ indicates suitable GCM

**Numerical Problem 2.8** Eleven GCMs in CMIP3 scenario mentioned in Table 2.27 are analyzed for the variable, precipitation. CC, NRMSD, and SS are the performance indicators. Payoff matrix (11 GCMs vs. 5 indicators) is presented in Table 2.27. Rank the GCMs using fuzzy TOPSIS. Assume equal weights for indicators. Ideal values of CC, NRMSD, SS as (1, 1, 1) each whereas anti-ideal values of CC, NRMSD, SS as (0, 0, 0) each (Raju and Nagesh Kumar 2015b).

**Table 2.27** Indicators obtained for 11 GCMs

Model	CC			NRMSD			SS		
	$p_{ij}$	$q_{ij}$	$s_{ij}$	$p_{ij}$	$q_{ij}$	$s_{ij}$	$p_{ij}$	$q_{ij}$	$s_{ij}$
UKMO-HAD GEM1	0.649	0.806	0.964	0.390	0.466	0.578	0.714	0.788	0.863
GISS	0.670	0.828	0.985	0.436	0.534	0.687	0.704	0.778	0.853
GFDL2.0	0.629	0.787	0.945	0.433	0.529	0.680	0.741	0.815	0.889
BCCR-BCCM 2.0	0.617	0.775	0.933	0.448	0.551	0.717	0.697	0.772	0.846
IPSL-CM4	0.316	0.474	0.632	0.305	0.350	0.410	0.584	0.658	0.733
UKMO-HADCM3	0.644	0.802	0.960	0.413	0.499	0.631	0.736	0.810	0.884
GFDL2.1	0.582	0.740	0.897	0.452	0.557	0.727	0.761	0.835	0.909
INGV-ECHAM 4	0.629	0.787	0.944	0.466	0.579	0.765	0.609	0.683	0.758
MIROC3	0.684	0.842	0.999	0.544	0.705	1.000	0.782	0.857	0.931
MRI-CGCM2	0.613	0.771	0.929	0.391	0.467	0.581	0.681	0.755	0.829
NCAR- PCMI	0.198	0.355	0.513	0.322	0.372	0.441	0.554	0.628	0.703

**Solution:****Sample calculation for GISS**

Values of CC, NRMSD, SS in triangular membership function: (0.670, 0.828, 0.985), (0.436, 0.534, 0.687), (0.704, 0.778, 0.853)

Ideal values of CC, NRMSD, SS = (1, 1, 1) each

Anti-ideal values of CC, NRMSD, SS = (0, 0, 0) each

- (i) Separation measure of GISS from ideal solution (Step 3, Table 2.26):

$$\begin{aligned}
 DS_{GISS}^+ &= \sqrt{\frac{[(p_{aj} - p_j^*)^2 + (q_{aj} - q_j^*)^2 + (s_{aj} - s_j^*)^2]}{3}} \\
 &= \sqrt{\frac{[(0.670 - 1)^2 + (0.828 - 1)^2 + (0.985 - 1)^2]}{3}} \text{ for correlation coefficient} \\
 &\quad + \sqrt{\frac{[(0.436 - 1)^2 + (0.534 - 1)^2 + (0.687 - 1)^2]}{3}} \text{ for normalized root mean square deviation} \\
 &\quad + \sqrt{\frac{[(0.704 - 1)^2 + (0.778 - 1)^2 + (0.853 - 1)^2]}{3}} \text{ for skill score} \\
 &= 0.2150 + 0.4594 + 0.2299 = 0.9043
 \end{aligned}$$

- (ii) Separation measure of GISS from anti-ideal solution (Step 4, Table 2.26):

$$\begin{aligned}
DS_{GISS}^- &= \sqrt{\frac{[(p_{aj} - p_j^{**})^2 + (q_{aj} - q_j^{**})^2 + (s_{aj} - s_j^{**})^2]}{3}} \\
&= \sqrt{\frac{[(0.670 - 0)^2 + (0.828 - 0)^2 + (0.985 - 0)^2]}{3}} \text{ for correlation coefficient} \\
&\quad + \sqrt{\frac{[(0.436 - 0)^2 + (0.534 - 0)^2 + (0.687 - 0)^2]}{3}} \text{ for normalized root mean square deviation} \\
&\quad + \sqrt{\frac{[(0.704 - 0)^2 + (0.778 - 0)^2 + (0.853 - 0)^2]}{3}} \text{ for skill score} \\
&= 0.8376 + 0.5619 + 0.7807 = 2.1802
\end{aligned}$$

- (iii) Relative closeness of GISS with reference to anti-ideal measure (Step 5, Table 2.26):

$$CR_{GISS} = \frac{DS_{GISS}^-}{(DS_{GISS}^- + DS_{GISS}^+)} = \frac{2.1802}{(2.1802 + 0.9043)} = 0.7068$$

Table 2.28 presents  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$ , and ranking pattern for 11 global climate models.

**Table 2.28** Ranking pattern of global climate models

Model	$DS_a^+$	$DS_a^-$	$CR_a$	Rank
UKMO-HADGEM1	0.9804	2.0914	0.6808	7
GISS	0.9043	2.1802	0.7068	3
GFDL2.0	0.9076	2.1714	0.7052	4
BCCR-BCCM 2.0	0.9378	2.1424	0.6955	6
IPSL-CM 4	1.5351	1.5100	0.4959	10
UKMO-HADCM3	0.9296	2.1466	0.6978	5
GFDL2.1	0.9024	2.1776	0.7070	2
INGV-ECHAM 4	0.9869	2.0989	0.6802	8
MIROC3	0.6732	2.4833	0.7867	1
MRI-CGCM2	1.0413	2.0251	0.6604	9
NCAR-PCMI	1.6576	1.3906	0.4562	11

- MIROC3 occupied first position with  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$  values, 0.6732, 2.4833, 0.7867, respectively, followed by GFDL 2.1 with  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$  values, 0.9024, 2.1776, 0.7070 respectively. Third position is occupied by GISS with,  $DS_a^+$ ,  $DS_a^-$ ,  $CR_a$  values, 0.9043, 2.1802, 0.7068 respectively.
- Relative closeness is almost same with slight difference of 0.0002 for GFDL 2.1 and GISS for both second and third positions (Raju and Nagesh Kumar 2015b).

### 2.4.5 Spearman Rank Correlation Coefficient

Spearman rank correlation coefficient (R) measures the correlation (Gibbons 1971) between ranks (Table 2.29).

**Table 2.29** Methodology of Spearman rank correlation coefficient

Step	Description	Mathematical expression/remark
1	Spearman rank correlation coefficient	$R = 1 - \frac{6 \sum_{a=1}^T e_a^2}{T(T^2 - 1)}$ $e_a$ is difference between ranks for the same GCM $a$ ; T is number of GCMs; R value varies between -1 and 1

**Numerical Problem 2.9** Ranking of 11 GCMs obtained by compromise programming and TOPSIS are presented in Table 2.30. Compute Spearman rank correlation coefficient (R).

**Table 2.30** Ranking pattern obtained by compromise programming and TOPSIS

GCM	Compromise programming	TOPSIS
1	6	8
2	5	7
3	4	4
4	2	3
5	3	2
6	11	11
7	1	1
8	7	9
9	10	10
10	8	5
11	9	6

**Solution:**

$e_a$  values are -2, -2, 0, -1, 1, 0, 0, -2, 0, 3, 3;  $e_a^2$  values are 4, 4, 0, 1, 1, 0, 0, 4, 0, 9, 9

$\sum e_a^2$  value is 32 (Step 1, Table 2.29)

$$R = 1 - \frac{6 * 32}{11(11^2 - 1)} = 0.8545$$

Spearman rank correlation coefficient value is 0.8545 (Step 1, Table 2.29).

### 2.4.6 Group Decision-Making

Group decision-making is a procedure in which ranking pattern with reference to individual ranking techniques are integrated to form a single group preference. Table 2.31 presents the methodology (Morais and Almeida 2012; Raju et al. 2017):

**Table 2.31** Computation of strength, weakness, and net strength of each GCM

Step	Description	Mathematical expression/remark
1	Division of the ranks	The descending order rankings are divided into upper and lower portions: $X = T/2$ for even number of GCMs and $T/2 + 1$ for odd number of GCMs and $Y = X + 1$ where $T$ is the number of the GCMs. The GCMs with rankings from 1 to $X$ constitute the upper portion
2	Strength of each GCM $a$	$ST_a = \sum_{k=1}^m \sum_z^X (X - z + 1) q_{az}^k \quad \forall a, k \quad \forall z = 1, \dots, X$ <p>where <math>q_{az}^k = 1</math> if GCM <math>a</math> is in the position <math>z</math> for the ranking technique <math>k</math> and 0 otherwise. <math>a</math> corresponds to the GCMs in the upper portion; <math>z</math> is the position in upper portion ranging from the first position to the <math>X</math>th position (<math>z = 1</math>st, ... <math>x</math>th) and <math>k</math> represents a ranking technique (<math>k = 1, 2, \dots, m</math>)</p>
3	Weakness of the GCM $a$	$WE_a = \sum_{k=1}^m \sum_{z=y}^T (z - Y + 1) q_{az}^k \quad \forall a, k \quad \forall z = y, \dots, T$ <p>where, <math>q_{az}^k = 1</math> if GCM <math>a</math> is in the position <math>z</math> for the MCDM technique <math>k</math> and 0 otherwise. <math>a</math> corresponds to the GCMs in the lower portion; <math>z</math> is the position in lower portion ranging from the first position to the lower portion (<math>Y</math>th) up to the last ranking in the lower portion</p>
4	Net strength of GCM $a$	$NS_a = ST_a - WE_a$
5	Rank the GCMs built on the $NS_a$ values	Higher $NS_a$ indicates suitable GCM

**Numerical Problem 2.10** Four MCDM techniques, namely, CP, TOPSIS, WA, and PROMETHEE ranked 20 GCMs, G1 to G20 (Table 2.32). Compute the group ranking of GCMs on strength and weakness perspective (Raju et al. 2017).

**Table 2.32** Ranking pattern by CP, TOPSIS, WA, PROMETHEE

GCMs	CP	TOPSIS	WA	PROMETHEE
G1	9	9	9	4
G2	7	7	7	9
G3	12	12	12	8
G4	2	2	2	5

(continued)

Table 2.32 (continued)

GCMs	CP	TOPSIS	WA	PROMETHEE
G5	6	6	5	14
G6	13	13	13	15
G7	5	5	6	12
G8	1	1	1	6
G9	8	8	8	1
G10	11	11	11	13
G11	16	16	16	16
G12	9	9	9	4
G13	10	10	10	2
G14	15	15	15	7
G15	4	4	4	3
G16	11	11	11	13
G17	9	9	9	4
G18	14	14	14	11
G19	14	14	14	11
G20	3	3	3	10

Solution:

Table 2.33 presents GCMs in the descending order of ranking. Here  $X = T/2$  where  $T$  is number of GCMs; accordingly  $X$  is fixed as 10 (However,  $X$  can be fixed any other value considering the intuition of decision maker). In this regard, upper portion consists of GCMs having ranks from 1 to 10.

Table 2.33 Ranking of GCMs in the descending order

Rank	CP	TOPSIS	WA	PROMETHEE
1	G8	G8	G8	G9
2	G4	G4	G4	G13
3	G20	G20	G20	G15
4	G15	G15	G15	G1, G12, G17
5	G7	G7	G5	G4
6	G5	G5	G7	G8
7	G2	G2	G2	G14
8	G9	G9	G9	G3
9	G1, G12, G17	G1, G12, G17	G1, G12, G17	G2
10	G13	G13	G13	G20
11	G10,G16	G10, G16	G10,G16	G18, G19
12	G3	G3	G3	G7
13	G6	G6	G6	G10, G16
14	G18, G19	G18, G19	G18, G19	G5
15	G14	G14	G14	G6
16	G11	G11	G11	G11

Strength of a GCM can be stated as the sum of the positional count of a GCM (Step 2, Table 2.31). For example, strength of GCM G5 is explained as follows (Table 2.34): G5 occupied fifth rank ( $z = 5$ ) for WA which means that  $q_{az}^k = 1$  and  $(X - z + 1)$  is  $(10 - 5 + 1) = 6$ ; Sixth rank ( $z = 6$ ) for CP and TOPSIS;  $q_{az}^k = 1$  and  $(X - z + 1)$  is  $(10 - 6 + 1) = 5$ : Note that G5 has occupied fifth and sixth ranks in upper portion. According to Step 2, Table 2.31, strength of G5 is computed as:  $6 * 1 + 5 * 2 = 16$  (Table 2.34). Similar procedure is repeated for lower portion (weakness perspective; Step 3, Table 2.31). G5 occupied fourteenth rank ( $z = 14$ ) for PROMETHEE which means that  $q_{az}^k = 1$  and  $(z - Y + 1)$  is  $(14 - 11 + 1) = 4$ . Accordingly, weakness of G5 is  $4 * 1 = 4$  (Table 2.35).

**Table 2.34** Computation of strength of GCM G5( $X = 10$ )

Rank/ $z$	$q_{az}^k$				$(X - z + 1)$	Strength ( $ST_a$ ) = $(X - z + 1) * q_{az}^k$
	CP	TOPSIS	WA	PROMETHEE		
1	0	0	0	0	$(10 - 1 + 1) = 10$	0
2	0	0	0	0	$(10 - 2 + 1) = 9$	0
3	0	0	0	0	$(10 - 3 + 1) = 8$	0
4	0	0	0	0	$(10 - 4 + 1) = 7$	0
5	0	0	1	0	$(10 - 5 + 1) = 6$	$6 * 1 = 6$
6	1	1	0	0	$(10 - 6 + 1) = 5$	$5 * 2 = 10$
7	0	0	0	0	$(10 - 7 + 1) = 4$	0
8	0	0	0	0	$(10 - 8 + 1) = 3$	0
9	0	0	0	0	$(10 - 9 + 1) = 2$	0
10	0	0	0	0	$(10 - 10 + 1) = 1$	0
Total strength of G5						16

**Table 2.35** Computation of weakness of GCM G5 ( $X = 10$ ;  $Y = X + 1 = 11$ )

Rank/ $j$	$q_{az}^k$				$(z - Y + 1)$	Weakness ( $WE_a$ ) = $(z - Y + 1) * q_{az}^k$
	CP	TOPSIS	WA	PROMETHEE		
11	0	0	0	0	$(11 - 11 + 1) = 1$	0
12	0	0	0	0	$(12 - 11 + 1) = 2$	0
13	0	0	0	0	$(13 - 11 + 1) = 3$	0
14	0	0	0	1	$(14 - 11 + 1) = 4$	4
15	0	0	0	0	$(15 - 11 + 1) = 5$	0
16	0	0	0	0	$(16 - 11 + 1) = 6$	0
Total weakness of G5						4

Net strength of G5 is  $16 - 4 = 12$  (Step 4, Table 2.31). Similarly, strength, weakness, and net strength of other GCMs are computed and presented in Table 2.36. G8, G4, G15, G20 occupied first four positions with net strengths of 35, 33, 29, 25. However, the last position (fifteenth rank) is occupied by G11 with a net strength of  $-24$ . GCM with high net strength is desirable. Accordingly, all the GCMs were ranked.



**Table 2.36** Strength, weakness and net strength of GCMs

GCMs	Strength ( $ST_a$ )	sum	Weakness ( $WE_a$ )	sum	$NS_a = ST_a - WE_a$	Rank
G1	10(0) + 9(0) + 8 (0) + 7(1) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (1 + 1 + 1) + 1(0)	13	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10(0)	0	13	8
G2	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(1 + 1 + 1) + 3 (0) + 2(1) + 1(0)	14	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	14	7
G3	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(1) + 2 (0) + 1(0)	3	1(0) + 2(1 + 1 + 1) + 3 (0) + 4(0) + 5(0) + 6 (0) + 7(0) + 8(0) + 9 (0) + 10(0)	6	-3	10
G4	10(0) + 9 (1 + 1 + 1) + 8(0) + 7 (0) + 6(1) + 5(0) + 4 (0) + 3(0) + 2(0) + 1 (0)	33	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	33	2
G5	10(0) + 9(0) + 8 (0) + 7(0) + 6(1) + 5 (1 + 1) + 4(0) + 3 (0) + 2(0) + 1(0)	16	1(0) + 2(0) + 3(0) + 4 (1) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	4	12	9
G6	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(0) + 2(0) + 3 (1 + 1 + 1) + 4(0) + 5 (1) + 6(0) + 7(0) + 8 (0) + 9(0) + 10(0)	14	-14	14
G7	10(0) + 9(0) + 8 (0) + 7(0) + 6(2) + 5 (1) + 4(0) + 3(0) + 2 (0) + 1(0)	17	1(0) + 2(1) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	2	15	6
G8	10(1 + 1 + 1) + 9 (0) + 8(0) + 7(0) + 6 (0) + 5(1) + 4(0) + 3 (0) + 2(0) + 1(0)	35	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	35	1
G9	10(1) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3 (1 + 1 + 1) + 2(0) + 1 (0)	19	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	19	5
G10	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(1) + 2(0) + 3(1) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	6	-6	11
G11	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6 (1 + 1 + 1 + 1) + 7 (0) + 8(0) + 9(0) + 10 (0)	24	-24	15

(continued)

**Table 2.36** (continued)

GCMs	Strength ( $ST_a$ )	sum	Weakness ( $WE_a$ )	sum	$NS_a = ST_a - WE_a$	Rank
G12	10(0) + 9(0) + 8 (0) + 7(1) + 6(0) + 5 (0) + 4(0) + 3 (1 + 1 + 1) + 2(0) + 1 (0)	13	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	13	8
G13	10(0) + 9(1) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(1 + 1 + 1)	12	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	12	9
G14	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(1) + 3(0) + 2 (0) + 1(0)	4	1(0) + 2(0) + 3(0) + 4 (0) + 5(1 + 1 + 1) + 6 (0) + 7(0) + 8(0) + 9 (0) + 10(0)	15	-11	12
G15	10(0) + 9(0) + 8 (0) + 7(1 + 1 + 1) + 6 (0) + 5(0) + 4(0) + 3 (0) + 2(0) + 1(0)	29	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	29	3
G16	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(3) + 2(0) + 3(1) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	6	-6	11
G17	10(0) + 9(0) + 8 (0) + 7(1) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (1 + 1 + 1) + 1(0)	13	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9(0) + 10 (0)	0	13	8
G18	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(1) + 2(0) + 3(0) + 4 (1 + 1 + 1) + 5(0) + 6 (0) + 7(0) + 8(0) + 9 (0) + 10(0)	13	-13	13
G19	10(0) + 9(0) + 8 (0) + 7(0) + 6(0) + 5 (0) + 4(0) + 3(0) + 2 (0) + 1(0)	0	1(1) + 2(0) + 3(0) + 4 (1 + 1 + 1) + 5(0) + 6 (0) + 7(0) + 8(0) + 9 (0) + 10(0)	13	-13	13
G20	10(0) + 9(0) + 8 (1 + 1 + 1) + 7(0) + 6 (0) + 5(0) + 4(0) + 3 (0) + 2(0) + 1(1)	25	1(0) + 2(0) + 3(0) + 4 (0) + 5(0) + 6(0) + 7 (0) + 8(0) + 9 (0) + 10 (0)	0	25	4

\*3(1) means product of 3 and 1

Numerous authors worked on various aspects of MCDM, entropy technique, group decision-making, and Spearman rank correlation with relevance to GCM selection and weight of performance indicators (Anandhi et al. 2011; Johnson et al. 2011; Taylor et al. 2012; Fu et al. 2013; Su et al. 2013; Perkins et al. 2013; Raju and Nagesh Kumar 2014a, b, 2015a, b, 2016; Raju et al. 2017; Hughes et al. 2014).

### 2.4.7 Ensemble of GCMs

Forecasting future climate projections will be helpful for efficient planning in order to mitigate and adapt to changing the climate. GCMs are widely used for this purpose but the common practice is to employ output of a single GCM or single scenario which ultimately results in various uncertainties. Policy decisions formulated on these results cannot be agreed upon because they reflect only a partial assessment of the risk involved.

Uncertainties in these projections have to be assessed to provide higher quality and more quantitative climate change information. In order to address the underlying uncertainties in climate modeling, a number of GCMs and emission scenarios are employed and termed as Multi Model Ensemble (MME). The process of integrating and ensembling of models can be done by taking simple arithmetical average or by following a weighting procedure developed on the performance of the GCMs simulating historic climate data. The models considered in the ensemble process should be reliable, i.e., they should represent the present-day climate factually and involves comparing GCM simulations with observed climate by considering performance measures. The best performing models can be employed for formulating MME.

The present chapter dealt with a description of climate models, performance evaluation, MCDM methodology. Forthcoming chapter discusses various down-scaling techniques.

#### Software (Information as on 30.12.2016)

Researchers can write their computer programs in any of the programming environment after ascertaining the structure of the algorithm discussed in various chapters of the book. However, the following information is provided for better understanding of the representative tools that may be employed.

PROMETHEE: Visual PROMETHEE 1.4: <http://www.promethee-gaia.net/software.html>.

Spearman Rank Correlation Coefficient: SPSS (Statistical Package for Social Sciences) (<http://www-03.ibm.com/software/products/en/spss-statistics>).

#### Revision Questions and Exercise Problems

- 2.1 What are different types of available climate models?
- 2.2 What is one-dimensional radiative-convective (RC) model?

- 2.3 What are global climate models or general circulation models and their purpose?
- 2.4 What is the opinion of researcher Xu on the GCMs?
- 2.5 What is a coupled atmosphere–ocean GCM?
- 2.6 What are the various components of GCMs?
- 2.7 What are the various uncertainties involved while handling GCMs and related aspects?
- 2.8 What are the expansions of RCP and SDSM?
- 2.9 What is the purpose of SDSM?
- 2.10 What is the purpose of performance indicators? What are the ideal requirements to be a performance indicator?
- 2.11 What are the available performance indicators?
- 2.12 Differentiate skill score, correlation coefficient, normalized root mean square deviations, and Nash–Sutcliffe efficiency.
- 2.13 Name three deviation/error-related performance measures.
- 2.14 What are the limitations of GCMs? How can these be tackled for effective application of GCMs?
- 2.15 What are issues raised by researchers Pierce and his team on climate models?
- 2.16 What are greenhouse gases (GHG) and how changes in greenhouse gases are attributed to natural and anthropogenic factors?
- 2.17 Mention four researchers who contributed extensively to evaluation of GCMs.
- 2.18 Precipitation data simulated from a GCM is 2, 3, 6, 7, 8, 10, 11, 12.3, 16.3, 17.2, 18.3, 18.7, and 19.1 whereas observed data is 11.2, 15.8, 13.2, 17.2, 19.3, 8.2, 6.7, 17.3, 16.2, 9.3, 12.1, 13.2, and 23.1. Compute the sum of squares of deviation, mean square deviation, root mean square deviation, Pearson correlation coefficient, normalized root mean square deviation, absolute normalized mean bias deviation, average absolute relative deviation, skill score, and Nash–Sutcliffe efficiency. Use four bins while computing skill score. Discuss the outcome in detail.
- 2.19 Relative humidity simulated from a GCM is 0.34, 0.56, 0.32, 0.23, 0.14, 0.10, and 0.23 whereas observed data is 0.23, 0.45, 0.42, 0.76, 0.33, 0.12, and 0.32. Compute skill score, normalized root mean square deviations and correlation coefficient.
- 2.20 What are the procedural steps for selection of best GCM?
- 2.21 Differentiate between compromise programming and cooperative game theory. How are they efficient in ranking GCMs?
- 2.22 How can strength and weakness of each GCM be computed in group decision-making?
- 2.23 How do the weights of indicators affect the ranking of GCMs? Is it necessary to have different weights for different indicators?
- 2.24 Solve Numerical Problem 2.2 using entropy technique. Consider data from Table 2.7 and consider only CC, ANMBD, AARD, NRMSD for analysis.

- 2.25 Solve Numerical Problem 2.3 using compromise programming. Use data in Table 2.11. Assume equal weights for all the indicators.
- 2.26 Solve Numerical Problem 2.3 using cooperative game theory. Use data in Table 2.11. Assume equal weights for all the indicators.
- 2.27 Solve Numerical Problem 2.5 using TOPSIS. Use data in Table 2.15. Assume weights of 0.2, 0.2, 0.5, 0.1 for the indicators respectively.
- 2.28 Solve Numerical Problem 2.3 using weighted average technique. Use data in Table 2.11. Assume equal weights for all the indicators.
- 2.29 Solve Numerical Problem 2.7 using PROMETHEE. Use data in Table 2.20. Assume equal weights for all the indicators. Analyze the numerical problem assuming (a) usual indicator (b) quasi-indicator with indifference value as 0.2.
- 2.30 Solve Numerical Problem 2.8 using fuzzy TOPSIS. Use data in Table 2.27. Assume weights of indicators CC, NRMSD, SS as (0.1, 0.1, 0.1), (0.2, 0.2, 0.2), (0.3, 0.3, 0.3).
- 2.31 Nine GCMs in CMIP3 environment as mentioned in Table 2.37 are analyzed for C1 and C2. Payoff matrix is trapezoidal membership function (Nine GCMs vs. two indicators) is presented in Table 2.37. Rank the GCMs using fuzzy TOPSIS. Assume equal weights for indicators. Take ideal values of C1, C2 as (1, 1, 1, 1) each whereas anti-ideal values of C1, C2 as (0, 0, 0, 0) each.

**Table 2.37** Data matrix of 9 GCMs and random assignment of clusters

GCMs	C <sub>1</sub>	C <sub>2</sub>
(1)	(2)	(3)
A1	(0.2, 0.3, 0.4, 0.5)	(0.4, 0.55, 0.66, 0.88)
A2	(0.5, 0.6, 0.7, 0.8)	(0.3, 0.5, 0.6, 0.8)
A3	(0.3, 0.6, 0.9, 1.0)	(0.2, 0.4, 0.6, 0.8)
A4	(0.6, 0.7, 0.8, 0.9)	(0.1, 0.2, 0.4, 0.8)
A5	(0.5, 0.7, 0.8, 0.9)	(0.4, 0.6, 0.8, 1)
A6	(0.2, 0.4, 0.6, 0.8)	(0.22, 0.44, 0.66, 0.88)
A7	(0.2, 0.3, 0.5, 0.8)	(0.44, 0.8, 0.9, 1)
A8	(0.2, 0.4, 0.8, 1.0)	(0.11, 0.44, 0.8, 0.9)
A9	(0.1, 0.4, 0.7, 0.9)	(0.3, 0.5, 0.9, 1)

**Hint:** Distance between two trapezoidal fuzzy numbers is

$$\sqrt{\frac{[(p_{aj}-p_j)^2 + (q_{aj}-q_j)^2 + (s_{ia}-s_j)^2 + (t_{ia}-t_j)^2]}{4}}$$
 Where (p, q, s, t) are elements of Trapezoidal fuzzy number.

- 2.32 Solve Numerical Problem 2.9 using Spearman rank correlation coefficient. Use data in Table 2.30. Consider the first six GCMs for the analysis.

- 2.33 Solve Numerical Problem 2.10 using group decision-making. Consider ranking of CP and PROMETHEE only for group decision-making calculations. Use data in Table 2.32. Consider first ten ranks of GCMs for the analysis. Compute strength, weakness, and net strength of each GCM.

### Advanced Review Questions

- 2.34 Why the word global is affixed before climate while naming global climate models?
- 2.35 Why the word circulation is suffixed after general while naming general circulation models?
- 2.36 Name various laws on which GCMs are developed?
- 2.37 If given data is imprecise, it is expected that computed performance measures may not be accurate. How can this be encountered?
- 2.38 In your opinion, can any other performance measures be initiated or developed? Discuss in detail the limitations of the existing performance measures mentioned in the present chapter and possible improvements?
- 2.39 List the GCMs that are available under CMIP3 and CMIP5.
- 2.40 Mention two case studies where researchers used any of the performance measures mentioned in the present chapter.
- 2.41 What are the similarities between CP and TOPSIS? Is there any way PROMETHEE-2 and CP can be related?
- 2.42 How may ranking pattern differ for different  $p$  values in compromise programming? Discuss mathematically?
- 2.43 How do ideal and anti-ideal values affect the outcome? Is it necessary to normalize the indicator values?
- 2.44 How is Spearman rank correlation coefficient related to MCDM techniques? Explain same in the present context?
- 2.45 Is there any relation between group decision-making and Spearman rank correlation coefficient?
- 2.46 Is it possible to correlate output of entropy with rating techniques with reference to weight estimation?
- 2.47 How may group decision-making affect if MCDM techniques are not of equal importance? How can this be considered in the group decision-making analysis?

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