Some Basic Concepts in Newtonian Mechanics

2

2.1 Nature of Motion and Space

Some of the deepest concepts in science include the nature of motion and space. From the very beginning of modern science, the matter has been examined by many great scientists and thinkers. One can safely remark that the pursuit of deeper understanding of the nature of motion and space continues even today and many points remain unresolved. The difficulty arises because these two aspects of the science of motion are at the very root of man's understanding of the universe. Motion is defined as the change of place as time progress, and this led the philosophers to ponder over the question 'what is space?' and 'what is time?' Though it is next to impossible to arrive at definite answers to the above questions, it is not very difficult to recognize 'space' as the 'arena of motion'. Any further philosophical discussion may not yield any fruitful results except framing of flowery sentences demanding a good vocabulary in English. Therefore, now onwards further discussions will be devoted to space and time relevant to better understanding of Newtonian dynamics.

2.1.1 Newton's Concept of Absolute Space and Time

Newton took a more straightforward route to the understanding of the nature of the space and time to describe his dynamics. As dynamics deals with motion that is defined as the change of position with time, it is obvious that the fundamental characteristics of space have to be linked with geometry. Newton believed in the existence of an entity called 'absolute space' that exists by itself on its own right independently. Similarly, he considered 'time' to be also an entity that is universal and flows uniformly independent of space. It may be worthwhile to quote Newton from his Scholium where he defined 'absolute time', 'absolute space' and 'absolute motion':

> Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse I do not define time, space, place, and motion, as being well known to all. Only I must observe that the common people conceive those quantities under no other motions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.
>
> I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably, without relation to anything external, and by another name is called duration: relative, apparent and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.
> II. Absolute space, in its own nature, without relation to anything external remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial or celestial space, determined by its position in respect of the earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be continually changed.
> III. Place is a part of space which a body takes up, and is according to the space, either absolute or relative.
> IV. Absolute motion is the translation of a body from one absolute place to another; and relative motion, the translation from one relative place into another.

Newton ascribed the absolute space with three-dimensional Euclidean geometry. It will be shown later how this geometrical structure of absolute space gives rise to the conservation principles. It is desirable that the primary characteristics of the absolute space and time of Newton are compiled here.

Important properties of Newtonian space and time: (i) Absolute space and time are passive; i.e., these do not get influenced by external agency. These do not depend on the presence of matter and energy in the universe. (ii) The

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behaviour of space remains same at all times. Similarly, time is also independent of the location where it is measured. (iii) Time interval between two events is same to all observers, and the order of events is independent of the observer and appears same to all. (iv) The absolute space follows Euclidean geometry.

Though all the above properties appear to be obvious to us as our senses have developed in a very small region of space surrounded by non-massive objects over a very short span of time, all the above points have been refuted by the philosophers and scientists of a later period. In more concrete terms, the following properties of absolute space lead to the observed rules of Newtonian dynamics. These are as follows: (i) the homogeneity of space, (ii) the isotropy of space, (iii) the continuity of space, (iv) the completeness of space, (v) the homogeneity of time, (vi) the continuity of time, (vii) the independence of space and time. These points will be again revisited in a later section to demonstrate the origin of the conservation principles of Newtonian dynamics.

The Newtonian dynamics hinges on one kinematic parameter, i.e. acceleration. Newton believed that this acceleration of an object, which is equal to the impressed force divided by the mass of the object, is with respect to the absolute space. So an observer who is moving with a uniform velocity with respect to the absolute space finds the acceleration to remain unchanged and the second law of motion works. Thus, the second law of motion remains valid in all frames of reference which are moving with uniform velocity with respect to the absolute space. It should be noted that the first and third laws also remain valid in these frames of reference which Newton called as 'inertial frames of reference'.

To demonstrate the fact that acceleration of a particle with respect to the absolute space is relevant in dynamics and its relative motion with respect to other objects does not play any role, Newton devised his famous bucket experiment described in the following section.

2.1.2 Newton's Bucket Experiment

This experiment, which was devised by Newton to prove his point that motion with respect to the absolute space is relevant, is quite simple and anyone can perform it. One has to hang a bucket, with some water in it, with the help of a rope (Fig. 2.1a). To start with, both the water and the bucket are stationary. Then, the bucket is spun as shown in Fig. 2.1b, but initially, the water is still stationary as it will take some time to pick up the rotational motion from the inner wall of the spinning bucket. Thus, it is observed that the bucket is rotating but the water is stationary with its surface flat as before. Next, the water in the bucket gradually picks up the rotational motion from the bucket, and at a stage, both the bucket and the water rotate and the water surface becomes concave as

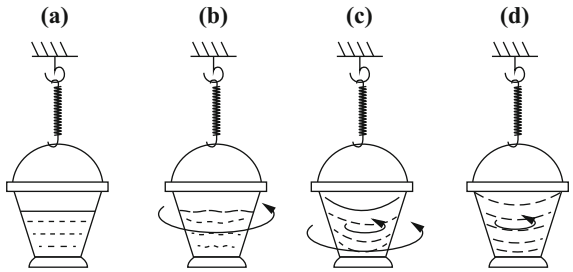


Fig. 2.1 Newton's bucket experiment

expected from our common knowledge (Fig. 2.1c). At the final stage of the experiment, the bucket is stopped by hand, but the water still continues to rotate due to inertia, and one finds that the surface of the rotating water is concave as shown in Fig. 2.1d. Newton's logic in support of the existence of absolute space was along the following lines.

Initially, the bucket and water are both stationary and the water body does not possess any acceleration with respect to the absolute space. Hence, the water surface is flat. In the next stage, the bucket body is rotating, but the water body is still stationary. So, the water body has motion relative to the bucket but has no acceleration with respect to the absolute space. Therefore, the water surface is flat. In the third stage, both the bucket and the water rotate and there is no motion of the water with respect to the bucket but has motion with respect to the absolute space. Hence, according to Newton's logic water surface is curved. Finally, again the water is moving with respect to the absolute space and the bucket. As expected, the water surface is curved because of its motion with respect to the absolute space.

The above experiment demonstrates (according to Newton) that the relative motion between the bucket and the water in it does not generate any force (due to relative acceleration of the water molecules) on the water. But when the water body is accelerating with respect to the absolute space, its molecules are subject to forces. Therefore, Newton concluded motion relative to absolute space has relevance to dynamics; relative motion between bodies develops no dynamical effect.

Newton suggests another experiment to establish his idea of absolute space using two globes in the following manner:

> For instance, if two globes, kept at a given distance one from the other by means of a cord that connects them, were revolved about their common centre of gravity, we might, from the tension of the cord, discover the endeavour of the globes to recede from the axis of this motion, and from thence we might compute the quantity of their circular motions. And then if any equal forces should be impressed at once on the alternate faces of the globes to augment or diminish their circular motions, from the increase or decrease of the tension of the cord, we might infer the increment or decrement of their motion; … And thus we might find both the quantity and the determination of this circular motion, even in an immense vacuum, where there was

> nothing external or sensible with which the globes could be compared.

However, it is not difficult to recognize the difficulty in conducting the above-mentioned globe experiment in an otherwise empty universe.

At this, it is quite pertinent to ask why only acceleration with respect to absolute space leads to dynamical effect, i.e. development of force. Uniform non-accelerated motion with respect to the absolute space should also lead to the development of force. This matter will be taken up for further discussion in the later sections. However, at this stage, it is enough to point out that all experiments conducted so far have not indicated the existence of any inertial resistive force to counter uniform motion with respect to the absolute space. Either it does not exist or the current level of technology is not capable of detecting such forces those being extremely small. The accuracy that can be achieved in the K-meson decay experiments may not be high enough to detect any ultra-small force, if there is any.

2.1.3 Newton's Bucket Experiment Follow up

Notwithstanding the logic and suggested experiments by Newton to establish the concept of absolute space and absolute nature of motion, his philosophy did not go unchallenged. Two contemporary scientist philosophers questioned the validity of the concept of absolute space and motion. G. W. Leibniz (1646–1716) was introduced to science of motion by none other than Huygens. Leibniz did not accept Newton's idea of absolute space and absolute time. His contention was that it is impossible to feel space without the presence of any matter, its existence can be identified as a 'system of relationships', and it is meaningless to endow 'space' with absolute existence. It is worthwhile to quote from his letter to S. Clarke, one of Newton's disciples, on this subject.

> As for my opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions. For space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together; without enquiring into their manner of existing. And when many things are seen together, one perceives that order of things among themselves.

But Leibniz was unable to provide a satisfactory explanation to Newton's bucket experiment considering his concept of reality of relative motion only. Interestingly, the seed of a truly relational mechanics was sowed by Newton's disciple, Clarke, in his last letter to Leibniz that reached him on 29 October 1716. No reply was ever written as Leibniz died on 14 November 1716. It is appropriate to quote the relevant part of the letter.

> It is affirmed,[1] that motion necessarily implies a relative change of situation in one body, with regard to other bodies; and yet no way is shown to avoid this absurd consequence, that then the mobility of one body depends on the existence of other bodies; and that any single body existing alone, would be incapable of motion; or that the parts of a circulating body (suppose the sun) would lose the vis centrifuga arising from their circular motion, if all the extrinsic matter around them were annihiliated.

Thus, without realizing, Clarke hit the bull's eye by conjecturing that if the matter in the rest of the universe is annihilated effects of motion, like curving of the water surface in a rotating bucket or the tension in the cord connected to the globes, would vanish. Hence, the relative motion of the water is not important as the matter content in the bucket is small to produce any visible effect. Though Clarke raised this point to establish the futility of a 'relative motion' theory, the hint got lost. Otherwise, both Leibniz and Berkeley, at a later time, could answer the question raised by Newton's bucket experiment.

Soon after Leibniz, Newton's concept of absolute space and absolute motion was criticized by Bishop G. Berkeley (1685–1753) following the line laid down by Leibniz. The criticism is found in his two books—'The Principle of Human Knowledge' published in 1710 and 'Of Motion'—published in 1721. His opinion on motion can be quoted from his first book as follows:

> But, notwithstanding what has been said, I must confess it does not appear to me that there can be any motion other than relative; so that to conceive motion there must be at least conceived two bodies, where of the distance or position in regard to each other is varied. Hence, if there was one only body in being could not possibly be moved. This seems evident, in that the idea I have of motion do the necessarily include relation.

Similarly, in his other book also the criticism is quoted as follows:

> No motion can be recognized or measured, unless through sensible things. Since this absolute space in no way affects the senses, it must necessarily be quite useless for the distinguishing of motions. Besides, determination or direction is essential to motion; but that consists in relation. Therefore, it is impossible that absolute motion should be conceived.

But, like Leibniz, Berkeley also failed to grasp the unintentional hint from Clarke that in the absence of matter in the rest of the universe, no effect of any motion can be sensed.

> Both Leibniz and Berkeley were somewhat confused but Berkeley took one step further in the right direction. He tried to replace Newton's absolute space by the conglomeration of the fixed stars that do not have motions relative to one another and can be conceived to form a frame of reference that can be used to describe motion. He mentions in his book "On Motions –", "- - - - for the removal of ambiguity and for the furtherance of the

[1] By Leibniz.

> mechanics of these philosophers who take the wider view of the system of things, it would be enough to bring in, instead of absolute space, relative space as confined to the heavens of the fixed stars, considered as at rest - - - - - - -"

Anyhow, the tremendous success of Newtonian mechanics in explaining all motions—celestial and terrestrial—did not allow much scope for further debate on the issue and Newton's ideas continued without any challenge for a century and a half.

2.1.4 Ernst Mach and Mach's Principle

The relative–absolute debate was resurrected by the nineteenth-century scientist philosopher Ernst Mach (1838–1916). He wanted to clear the nebulosity surrounding the nature of space and provide a solid concept devoid of any ambiguity. In the preface of his famous book 'The Science of Mechanics', he clearly stated in the preface that 'The present volume is not a treatise upon the application of the principles of mechanics. Its aim is to clear up ideas, expose the real significance of the matter and get rid of metaphysical obscurities'. He was the first to take up the challenge posed by Newton's thought experiment with two globes tied up by a chord and executing rotation and the other famous experiment with the bucket full of water. To explain the tension in the chord and the curvature of the water surface without bringing in the concept of absolute space was at a task with insurmountable difficulty for the Leibnizian relativists. Mach solved the difficulty with one simple stroke—by rejecting the experiments. According to him, one can say what will happen to the globe experiment when the matter from the rest of the universe vanishes. According to him it would be impossible to see any rotation of the pair of globes. So far as the bucket experiment is concerned, his view is quoted as follows:

> Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel are increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination.

Thus, according to Mach, dynamical effect is produced by relative motion between bodies and there is no role for the existence of a real absolute space. If the bucket would have been sufficiently massive, a rotating bucket could produce curved surface in the water even if it were not rotating with respect to the fixed stars (representing the absolute space). Hence, the problem is with the magnitude. Had our technology been advanced enough, such effects could be observed.

Mach did not prescribe a specific physical theory, but his proposition was also in the form of a principle without any quantitative model. Utilizing this principle, M. Schlick was the first to interpret in 1915 that 'the case of inertia must be assumed to be an interaction of masses'. Thus, the first hypothesis of the origin of inertia as the interaction of a body moving with respect to the matter present in the rest of the universe was born and became widely known as the 'Mach's principle'.

This concept greatly impressed Einstein, and in his paper of 1918, he clearly indicates 'Mach's principle' as a founding pillar of the General Theory of Relativity. To quote him

> c) Mach's Principle: The G-field is completely determined by the masses of the bodies. Since mass and energy are identical in accordance with the results of the Special Theory of Relativity and the energy is described formally by means of the symmetric energy tensor ($T_{\mu\nu}$), this means that the G-field is conditioned and determined by the energy tensor of the matter.

When Mach was 75 years old and became immobile because of a paralytic attack, Einstein's last letter from Zurich on 25 June 1913 included the following:

> Recently you have probably received my new publication on Relativity and Gravitation which I have at last finished after unending labour and painful doubt. Next year at the solar eclipse it will turn out whether the light rays are bent by the sun, in other words whether the basic and fundamental assumption of the equivalence of the acceleration of the reference frame and of the gravitational field really holds. If so, then your inspired investigations into the foundations of mechanics – despite Planck's unjust criticism – will receive a splendid confirmation. For it is a necessary consequence that inertia has its origin in a kind of mutual interactions of bodies, fully in the sense of your critique of Newton's bucket experiment.

Subsequently, many scientists and philosophers have provided support to the concept. The 'Mach's principle' is also considered important for providing explanation to the origin of inertial effect. According to Thomas Phipps, 'when the subway jerks, it's the fixed stars that throw you down'. With time, the term 'fixed stars' has been replaced by the term 'matter present in the rest of the universe'. Many scientists consider that inertia is nothing but the manifestation of a body accelerating with respect to the rest of the universe.

2.1.5 Quantification of Mach's Principle—Concept of Inertial Induction

Although the Mach's principle is very attractive as a concept, the primary difficulty was the absence of a quantitative model of the theory. Without such a quantitative model, not much further progress was possible. To remedy the situation, D.W. Sciama, the Cambridge University Professor of Physics, proposed the concept of 'inertial induction' as a

quantitative description of Mach's principle. According to Sciama, a major advantage of Mach's principle over the 'absolute space' concept of Newton is that the primary role played by 'acceleration' can be explained. According to the conventional doctrine of Mach's principle, the interactive force between two material objects depends on the relative 'acceleration' between the two bodies and the relative velocity does not lead to the development of any interactive force. To establish this point (that the velocity of a particle with respect to the matter present in the rest of the universe is not detectable), experiments have been conducted on the decay of K meson into π meson. The rate of K meson decay into two π mesons was expected to depend on its velocity with respect to the matter in the rest of the universe if the interactive force were dependent on velocity. But no such dependence was found from the experiments, and the existence of an interactive force depending on the relative velocity between two objects was disproved.

However, this could be a matter of accuracy. If the interactive force due to relative velocity be too small to be detected by the K meson decay experiment, then the question remains open. Anyhow Sciama considered the interactive force between two particles to depend only on their relative acceleration and named the phenomenon as 'inertial induction'. According to Sciama, the total force between two particles of mass[2] m_1 and m_2 has two parts—one static and the other dynamic as follows:

$$F = F_s + F_i \tag{2.1.1}$$

The static part of the force is the usual Newtonian gravitational attraction

$$G \times \frac{m_1 m_2}{r^2}$$

where r is the distance between the two particles and G is the constant of universal gravitation (Fig. 2.2a). The dynamic part of the total interactive force is due to the inertial induction effect due to the relative acceleration and is given by

$$G \times \frac{m_1 m_2}{cr^2} \times a$$

where c is the speed of light in a vacuum and a is the acceleration of m_2 with respect to m_1 as shown in Fig. 2.2b. Thus,

$$F = G\frac{m_1 m_2}{r^2} + G\frac{m_1 m_2}{cr^2}a \tag{2.1.2}$$

If a is reversed, the direction of F_i will also reverse. Of course, this is a very simplistic situation and the magnitude

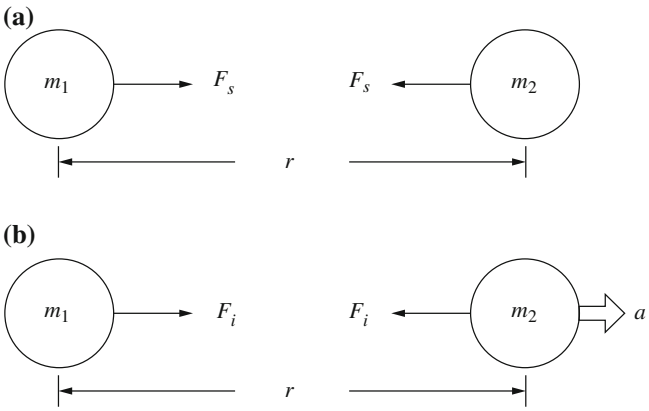


Fig. 2.2 Concept of inertial induction

of a can be either positive or negative. Sciama arrived at this concept after considering the force between two electric charges. Figure 2.3a shows two charges $+e_1$ and $-e_2$ at a distance r. Coulomb's law of electrostatic force suggests that the attractive force is given by

$$F = \varepsilon\frac{e_1 e_2}{r^2} \tag{2.1.3}$$

where ε is the dielectric constant of the free space. The similarity of the law with the law of universal gravitation is quite obvious. Now, if $-e_2$ is moved away with an acceleration a with respect to $+e_1$, then the total force between the two charges is given by

$$F' = \varepsilon\frac{e_1 e_2}{r^2} + \varepsilon\frac{e_1 e_2}{cr^2}a \tag{2.1.4}$$

Sciama attributed a similar effect of a in the case of gravitational interaction also in an intuitive manner. Of course, the model is quite simple and the question whether relative velocity generates any force or not remains unresolved. But this was the first attempt to quantify Mach's principle.

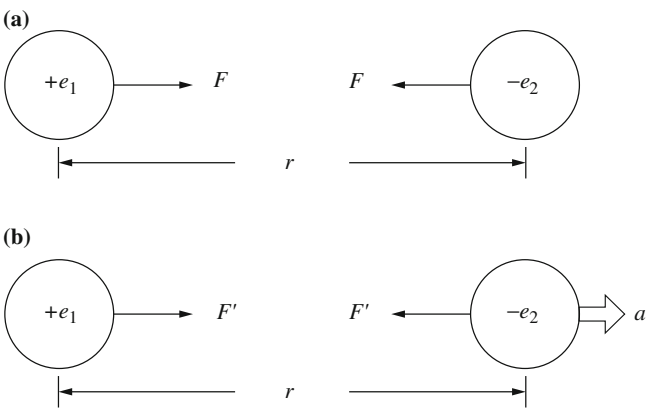


Fig. 2.3 Force between electric charges

[2]Actually, this is the gravitational mass.

2.1.6 Origin of Inertia

Once a quantitative model is available, it is possible to use it for finding the source of the inertial property of an object. Using Sciama's model of inertial induction, it is possible to have a rough estimate of the total force due to the interaction of a particle of (gravitational) mass m accelerating at the rate a with respect to the rest of the matter in the universe. If M_j be the mass of the jth object in the universe (Fig. 2.4)

$$F_i = \sum_{j}^{\text{ObservableUniverse}} \left(\frac{GM_j}{c^2 r}\right) ma \tag{2.1.5}$$

In the very large scale, it may be assumed that the matter is uniformly distributed all over and the matter in the universe does not possess any particular acceleration. Thus, for a stationary, homogenous and isotropic universe, the total force on the particle due to inertial induction from the rest of the universe can be expressed as follows:

$$F_i = \frac{Gm\rho}{c^2}\left[\iiint \frac{dv}{r}\right] a \tag{2.1.6}$$

where dv is the volume of a thin spherical elemental cell as shown in Fig. 2.5 and ρ is the uniform density of matter in the universe. The radius of the cell is r with the test particle at the centre.

Now, the elemental volume dv can be expressed as follows:

$$dv = 4\pi r^2 dr$$

Using this in (2.1.6), the total force on m due to inertial induction becomes

$$F_i = \frac{4\pi Gm\rho}{c^2}\left(\int_0^{R_0=c/H} \frac{1}{r} r^2 dr\right) a \tag{2.1.7}$$

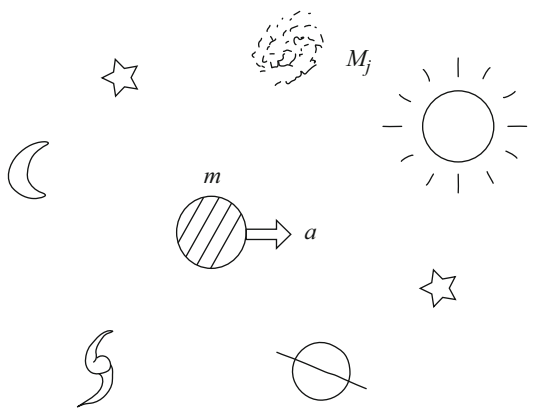


Fig. 2.4 A particle accelerating in the universe

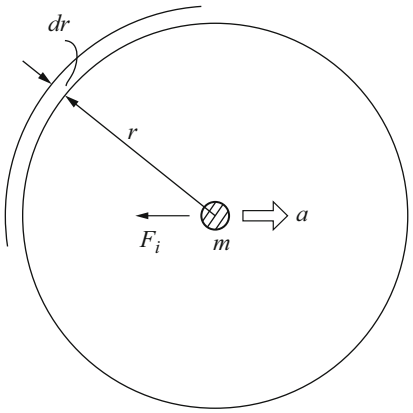


Fig. 2.5 Interaction of a particle with an element of the universe

where R_0 is the radius of the observable universe given by c/H with H as the Hubble's constant. From (2.1.7), one obtains

$$\begin{aligned} F_i &= \frac{2\pi G\rho R_0^2}{c^2} ma \\ &= \frac{2\pi G\rho}{H} ma \end{aligned} \tag{2.1.8}$$

Using the currently estimated density ρ as 10^{-26} kg m^{-3} and H as 50 km s^{-1} Mpc^{-1} ($\approx 1.5 \times 10^{-18}$ s^{-1}), (2.1.8) yields

$$F_i \approx 1.8ma \tag{2.1.9}$$

Ideally, F_i (i.e. the force to be exerted for accelerating a mass m at the rate a) should be equal to exactly ma. However, considering the approximate nature of the inertial induction model and the uncertainties involved in estimating the values of ρ and H, the close agreement is remarkable. This provides enough hint that Mach's principle may be the real cause of inertia as suggested by some scientists. It is felt that a more accurate modelling could lead to more accurate results.

It should be mentioned here that considering the theory of Weber's electrodynamic force law

$$F = \frac{e_1 e_2}{4\pi\varepsilon r^2}\left[1 - \frac{1}{c^2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{2r}{c^2}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right] \tag{2.1.10}$$

Tisserand proposed a modified law for gravitational interaction between two masses in the following form as early as 1872:

$$F = \frac{Gm_1 m_2}{r^2}\left[1 - \frac{1}{h^2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{2r}{h^2}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right] \tag{2.1.11}$$

where h was considered to be the speed of propagation of gravitation. It is interesting to note that besides the Newtonian static term, the force law contains a velocity-dependent term and an acceleration-dependent term very similar to that proposed by Sciama. However, Tisserand proposed the

model to explain the anomaly observed in the advance of the perihelion of Mercury. Since this was adequately explained by Einstein's General Theory of Relativity, Tisserand's model of dynamic gravitational interaction as a possible quantitative model of Mach's principle remained unnoticed till very recently.

Though it is expected that a more accurate modelling following Sciama's logic will give the desired result, there is a serious philosophical issue involved. The exactness of the equation $F = ma$ cannot be dependent on the fine tuning of the values of G, ρ, H and c. So, there must be some other matter of very fundamental nature to take care of such exactness. This problem will be taken up again in a later chapter.

2.2 Relative–Absolute Duality of Nature of Motion

The debate between the supporters of Newton's concept of motion (i.e. acceleration) with respect to an absolute space and the continental scientists' philosophy of relative (with respect to other objects in the universe) motion continued for a long time. However, as the progress of science did not get affected by the philosophical nature of the disagreement the debate is not given any serious consideration by the practising scientists; the subject has been gradually pushed to a dark corner of the world of philosophy of science.

One may think that the matter is of purely academic nature without any relevance to down-to-earth science but it is not exactly so. In the Newtonian concept of absolute space as the background for motion to take place 'inertia' is an intrinsic property of mass and the second law of motion is a 'law'. But it will be seen later in this volume that 'inertia' can be shown to be manifestation of an object's motion relative to the matter present in the rest of the universe. This solves also a number of paradoxes and mysteries in Newtonian scheme of things. These will be taken up in later sections of this book. In this section, it will be attempted to demonstrate that a sense of absolute motion can emerge from the results obtained through the observation of an object's relative motion with respect to other objects present in the universe. To keep the treatment uncomplicated, only a two-dimensional situation will be analysed though the results for a three-dimensional situation will remain the same.

2.2.1 The Nature of the Universe

It will be necessary first to identify the nature of the universe. From all observations, the universe appears to be infinite, homogenous, and isotropic. The objects in the universe have random motions, but there is no systematic universal motion. Locally, there are evolutions of systems and destructions are also of local nature. Such a universe is considered to satisfy the 'perfect cosmological principle'. In the nineteenth century, it was thought that the universe is filled up by stars. But now it is known that the main constituent members of the universe are the galaxies along with some other types of objects.

There are a number of cosmological models. According to the currently popular 'Big-Bang' theory, the universe is finite in size and possesses a universal expansion and the universe is also evolving as a whole. In the 'steady-state' theory the universe is infinite but possesses a universal expansion. The steady state is maintained by continuous creation of matter to keep the matter density unchanged. However, according to the less-known 'stationary universe' model the universe satisfies the 'perfect cosmological principle'. The expanding universe model is favoured because that provides a simple and straightforward explanation for the observed cosmological redshift without invoking new physics. But there is no direct evidence of universal expansion so far. So, to illustrate the absolute nature of motion using the relative motion of a body with respect to other bodies present in the universe, a model of infinite, homogenous, isotropic non-evolving stationary universe model will be chosen.

Figure 2.6 shows a plot of a million galaxies in the observable universe. It is clear that in the very large scale, the lumpiness of the matter content in the universe gradually reduces and the homogenous and isotropic nature of the universe becomes apparent. Apart from this, all contemporary observations also point towards the absence of any curvature of the universal space–time continuum and the geometry is strictly Euclidean. There is no systematic motion of the bodies in the universe though all objects possess some finite motion. In such a scenario, it is possible to conceive a mean rest frame of this quasistatic universe. When very large sections are considered, the centre of mass of each of these sections is almost without any perceptible motion. An imaginary network connecting these centres of mass constitutes a frame of reference embedded in the universe that may be treated as an absolute frame of the quasistatic infinite universe. Even the cosmic microwave radiation that fills the whole universe may be treated as a background for detecting absolute motions, i.e. motions with respect to the mean rest frame of the universe.

2.2.2 Absolute Motion in Terms of Relative Motion

In this section, an attempt will be made to show a methodology by which the displacement of an object in the universe can be provided with some scientific meaning.

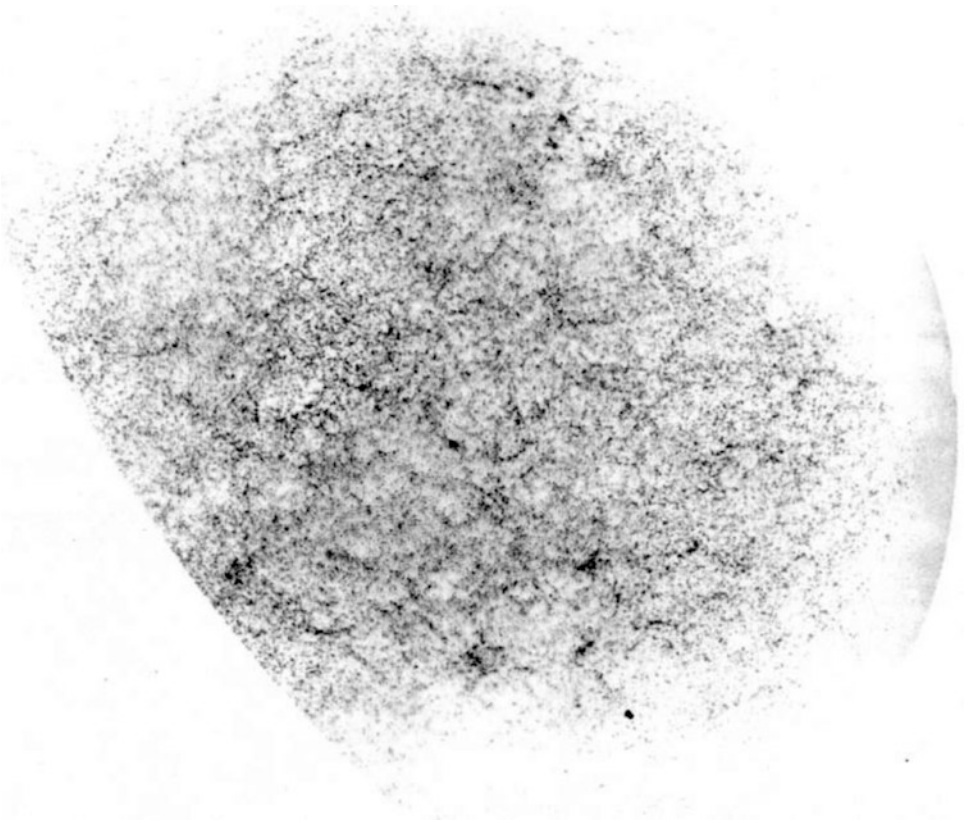


Fig. 2.6 A million galaxies

Figure 2.7a shows a section of the universe containing the targeted object P and other bodies. If another snapshot of the same region of the universe is taken at another instant of time, it may look as shown in Fig. 2.7b. Now as all objects in this universe are in motion, it is not possible to find out the amount of displacement of the object P. Even it may be, at a first glance, meaningless to talk about the displacement of the objects. This is the fundamental problem of motion; many philosophers and scientists feel that it is impossible to have any sense in talking about the motion of an object unless it is defined in relation to other objects. But as all objects are in motion, there cannot be any sense in contemplating motion of a body. But in an infinite, non-evolving homogenous and isotropic universe, the relative displacements of a body in relation to other bodies can lead to a sense of absolute motion.

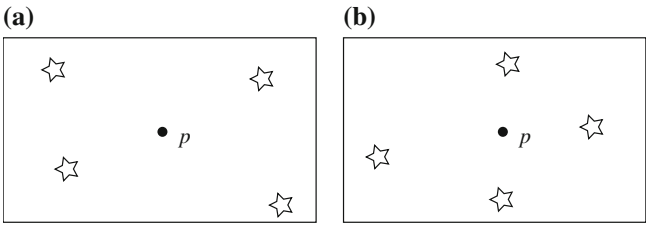


Fig. 2.7 Snapshots of a region at the two different instants

The problem will be dealt with in a reverse order. First, let it be assumed that there is an absolute frame, and finally, the validity of this assumption is established. At the first instant, when a snapshot is taken, let the position of the body P with respect to all the other bodies is given by

$$\boldsymbol{r}_1; (i = 1, 2, 3, \ldots, n, \ldots, \infty)$$

When the next snapshot is taken after an interval of time Δt, let these positions be $\boldsymbol{r}'_1$; (i = 1, 2, 3, …, n, …, ∞) when the body P has moved to position P' (with respect to the assumed mean rest frame of the universe). The other bodies at positions 1, 2, 3, … have moved to positions 1′, 2′, 3′, … (as indicated in the figure) in this assumed mean rest frame of the universe. Thus, the displacements of these bodies are given by $\Delta\boldsymbol{\rho}_1$; ($i$ = 1, 2, 3, …, n, …, ∞), whereas body P moves through the displacement $\Delta\boldsymbol{s}$ (all in the imagined mean rest frame of the universe). The quantities which are directly observed are the position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}'_1$; (i = 1, 2, 3, …, n, …, ∞). It can be shown that $\Delta\boldsymbol{s}$ can be determined from these quantities. From Fig. 2.8, the following equations are obtained:

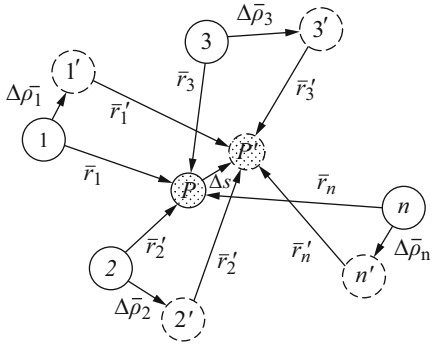


Fig. 2.8 Relative displacement of bodies

$$\begin{aligned}
\boldsymbol{r}_1 + \Delta\boldsymbol{s} &= \Delta\boldsymbol{\rho}_1 + \boldsymbol{r}_1' \\
\boldsymbol{r}_2 + \Delta\boldsymbol{s} &= \Delta\boldsymbol{\rho}_2 + \boldsymbol{r}_2' \\
\boldsymbol{r}_3 + \Delta\boldsymbol{s} &= \Delta\boldsymbol{\rho}_3 + \boldsymbol{r}_3' \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
\boldsymbol{r}_n + \Delta\boldsymbol{s} &= \Delta\boldsymbol{\rho}_n + \boldsymbol{r}_n' \\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

Summing up both sides of the above vector equations for N terms (when N is very large tending to infinity), the following equation is obtained:

$$\sum_{i=1}^{N} \boldsymbol{r}_i + N \times \Delta\boldsymbol{s} = \sum_{i=1}^{N} \Delta\boldsymbol{\rho}_i + \sum_{i=1}^{N} \boldsymbol{r}_i' \tag{2.2.1}$$

To proceed further, let the displacements $\Delta\boldsymbol{\rho}_i$ be considered to be equal in the magnitude, $\Delta\rho_{\mathrm{av}}$ but randomly oriented. In reality, the magnitudes of the displacements of the objects satisfy a normal distribution with an average value $\Delta\rho_{\mathrm{av}}$. But the final outcome of this argument remains the same. From the random walk theorem,

$$\sum_{i=1}^{N} \Delta\boldsymbol{\rho}_i = \Delta\boldsymbol{\rho}$$

where $|\Delta\boldsymbol{\rho}| = \sqrt{N} \times \Delta\rho_{\mathrm{av}}$. If $\hat{\boldsymbol{k}}$ be a unit vector along the direction of the resultant vector $\Delta\boldsymbol{\rho}$,

$$\sum_{i=1}^{N} \Delta\boldsymbol{\rho}_i = \hat{\boldsymbol{k}}\sqrt{N\Delta\rho_{\mathrm{av}}} \tag{2.2.2}$$

Substituting this resultant in (2.2.1) and dividing both sides by N

$$\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{r}_i + \Delta\boldsymbol{s} = \frac{\hat{\boldsymbol{k}}}{\sqrt{N}}\Delta\rho_{\mathrm{av}} + \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{r}_i'$$

when N becomes very large, $\frac{\Delta\rho_{\mathrm{av}}}{\sqrt{N}}$ tends to zero, and the above equation becomes after rearrangement

$$\Delta\boldsymbol{s} = \frac{1}{N}\left(\sum_{i=1}^{N} \boldsymbol{r}_i' - \sum_{i=1}^{N} \boldsymbol{r}_i\right) \tag{2.2.3}$$

Therefore, it is seen that if the initial and final positions of an object with respect to a very large number of other objects in the universe are known, the displacement of the object with respect to the mean rest frame of the universe can be determined from (2.2.3). Thus, relative position can yield the absolute motion. This is a fine example of relative–absolute duality in the science of motion.

As already mentioned earlier in this section, the concept of absolute motion is not absurd as it sounds. In fact, the cosmic microwave background radiation provides such a notion to absolute nature of motion. The motion of the earth through this 'soupe' of background radiation can be determined by observing the dipole anisotropy in the Doppler effect. The magnitude of the earth's absolute velocity (with respect to this radiation) has been found out to be 260 km s^{-1}.

2.3 Inertial and Gravitational Mass

The concept of mass was first proposed by Newton to quantify the quantity of matter. The definition of mass given by Newton was 'quantity of matter', and he quantified it by the product of density and volume. In earlier times, the 'bulk' or the 'quantity of matter' used to be recognized by the weight of an object. Newton recognized mass as an intrinsic property of matter and different from weight which is nothing but the force of gravitational pull by the earth. So weight is not a property of an object as the gravitational pull does not remain constant. In Newton's scheme of things, 'mass' comes into picture in two ways. In the first case, the mass represents the inertia to change of motion. Quantitatively, this is manifested in the second law

$$\boldsymbol{F} = m\boldsymbol{a}$$

On the other hand, the law of universal gravitation proposed by Newton also incorporates a quantity called 'mass'. The force of gravitational attraction between two bodies with masses m_1 and m_2 is given by

$$|\boldsymbol{F}| = G\frac{m_1 m_2}{r^3}|\boldsymbol{r}|$$

where $\boldsymbol{r}$ is the position vector of one body with respect to the other. It should be noted that in Newtonian mechanics, the phenomenon of accelerating a body with an impressed force has no relation to the gravitational attraction between two bodies. So the properties designated by the term 'mass' in the two cases need not be the same. In modern times, two different names are used to denote this difference: in the case

of a body's resistance (or inertia), change in the state of motion is called 'inertial mass' (m_i), and in the case of gravitational attraction, the property is defined as the 'gravitational mass' (m_g). Over and above, to make further finer distinction, the gravitational mass of a body on which the gravitational pull acts is often termed as 'passive gravitational mass' and the mass of the body exerts the pull is called the 'active gravitational mass'. However, according to Newton's third law, the situation is symmetric and there is no important difference between the 'active' and 'passive' gravitational mass of a body.

Newton did not make any special effort, to differentiate between the 'inertial' and 'gravitational' mass though he was aware of the difference. Perhaps, the primary reason for this was the traditional way of considering the weight of an object as its bulk (representing the mass). But, nevertheless, he proposed to demonstrate the equality by more accurate experiments than the standard 'free fall' experiments of Galileo. This point will be taken up later in this section.

2.3.1 Inertial Mass

To concretize the concept of inertial mass, a methodology needs to be developed. A thought experiment can be conceived in which different bodies are impressed upon the same force.

The scheme is indicated in Fig. 2.9. To ensure the constancy of the impressed force F, the compression of the spring is used. A definite force compresses a spring by a definite predetermined amount. If the experiments are conducted in an inertial frame of reference, the second law of motion stipulates that

$$F = m_1 a_1 = m_2 a_2 \tag{2.3.1}$$

where m_1 and m_2 are the inertial masses of two bodies being accelerated at the rates a_1 and a_2, respectively. Conducting the experiments in an inertial frame also ensures that the results will be agreed upon by all inertial observers. Equation (2.3.1) yields

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} \tag{2.3.2}$$

This is an unambiguous way of defining the ratio of inertial masses of two bodies. So, if one of the bodies, say body 2, is assumed to have mass equal to 1 (as the standard), then the mass of any other body is given by

m = m/1 = (acceleration of the standard mass)/(acceleration of the body)

where the above relation is also used to define the unit of force. In the SI unit, the mass of the standard mass is 1 kg, and when a force acting on the body produces unit acceleration, 1 m s^{-2}, the force is defined to be 1 N. In Newtonian mechanics, the inertial (and also gravitational) mass remains conserved. The mass also shows additive property. If two objects have masses m_1 and m_2, they accelerate at the rate $\boldsymbol{a}$ when subjected to forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$,

$$\boldsymbol{F}_1 = m_1 \boldsymbol{a} \quad \text{and} \quad \boldsymbol{F}_2 = m_2 \boldsymbol{a}$$

When the two bodies are combined and subjected to a force ($\boldsymbol{F}_1 + \boldsymbol{F}_2$), the following result is obtained:

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 = (m_1 + m_2)\boldsymbol{a}$$

So the combined bodies as a single body have mass $m_1 + m_2$.

2.3.2 Gravitational Mass

The fact that all bodies fall with equal acceleration when no resistance acts was well established by Galileo first and other subsequent scientists later. If an object has a gravitational mass m_g and inertial mass m_i, the force of attraction due to gravity on the earth's surface is given by Newton's law of universal gravity[3] as follows:

$$F = \frac{G m_{E_g} m_g}{R_E^2} \tag{2.3.3}$$

when m_{E_g} represents the gravitational mass of the earth and R_E is the earth's radius. This force acting on the object produces an acceleration according to the second law as follows:

$$a = \frac{F}{m_i} = \left(\frac{G m_{E_g}}{R_E^2}\right)\frac{m_g}{m_i} = g \tag{2.3.4}$$

where g is the acceleration due to gravity on earth's surface. Since 'g' is found to be same for two objects 1 and 2,

$$\frac{m_{g_1}}{m_{i_1}} = \frac{m_{g_2}}{m_{i_2}} = \alpha \text{ (const)} \tag{2.3.5}$$

which implies that the ratio of the gravitational mass and the inertial mass is same for all objects. Using the known value

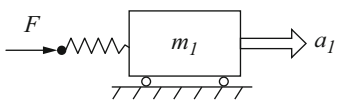


Fig. 2.9 Experiment to determine inertial mass

[3]In fact, Newton did not formulate his law of universal gravitation in the familiar form.

$m_{E_g} = 5.976 \times 10^{24}$ kg, $R_E = 6.378 \times 10^6$ m and $g = 9.80665$ m s^{-2}

$$\frac{m_g}{m_i} = \frac{1}{G} \times 9.80665 \times \frac{6.378^2 \times 10^{12}}{5.976 \times 10^{24}} = \frac{6.6754 \times 10^{-11}}{G} = \alpha$$

Taking $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, α becomes equal to unity rendering the gravitational and inertial masses equal. This equality of gravitational and inertial masses intrigued all scientists as there has been no theory relating the phenomena of inertia and gravitation till Mach propose his very qualitative philosophy towards the end of the nineteenth century.

2.3.3 Equivalence of Inertial and Gravitational Mass

Traditionally, all users of Newtonian mechanics generally do not ponder over the matter of using 'mass' without making any distinction whether it is inertial or gravitational. This is so because of the exact equivalence of the inertial and gravitational mass of an object (i.e. the ratio of these two properties is a constant for all objects). Though the observed results of the free fall experiments lead to this understanding, the accuracy of these results was not high and many other experiments have been devised to test this equivalence. Even at present some billion-dollar experiments are being planned to test the accuracy of this equivalence—the matter is of so profound importance to scientists and philosophers.

Huygens was the first to derive in 1673 the expression for the period of oscillation of a simple pendulum as follows:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Using this formula, the value of g was determined with a higher degree of accuracy than that obtained from the free fall experiments. By conducting experiments with the length l, as 11 ft and using the oscillating mass made of different materials, Newton was able to prove

$$(m_i - m_g)/m_i \approx \pm 10^{-3}$$

meaning that the inertial and gravitational masses of a body are equal within one part in thousand. Using a spherical pendulum, Newton was able to improve the accuracy further.

Figure 2.10 shows the arrangement. The equilibrium configuration of the pendulum, when rotated at an angular speed ω, is given by the balancing act of the deflecting moment $m_i\omega^2 l \sin\theta$. $l \cos\theta$ and the restoring moment $m_g g l \sin\theta$. The obtained relation is

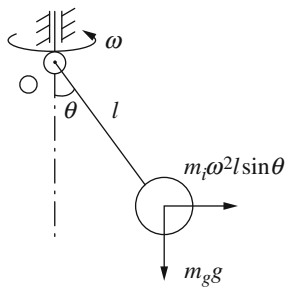


Fig. 2.10 Newton's conical pendulum experiment

$$m_i\omega^2 l^2 \sin\theta\cos\theta = m_g g l \sin\theta$$

or,

$$(m_i/m_g) = g/(\omega^2 l\cos\theta)$$

As the pendulum mass rotates at speed ω, the period of rotation is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\left[g/\{(m_i/m_g)l\cos\theta\}\right]^{1/2}}$$

Newton equated this period with the time period of a pendulum with length $l \cos\theta$ given by

$$T = \frac{2\pi}{\left[g/\{(m_i/m_g)l\cos\theta\}\right]^{1/2}}$$

and used the time period expression for a simple pendulum to determine the speed ω of a conical pendulum. Using the experiments, Newton established the equivalence of m_i and m_g with a much higher level of accuracy.

In 1880, Baron Eötvös from Budapest conducted an experiment to demonstrate the equivalence with an accuracy level of 10^{-8}. In 1964, Dicke and, in 1971, Braginski and Panov verified the equivalence to a limit of 10^{-12}. In more recent experiments using space technology, the accuracy of one part is 10^{18} has been achieved! The basic principle followed in Eötvös experiment is explained through the simplified diagram in Fig. 2.11.

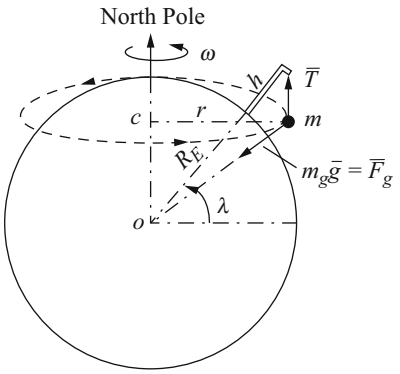


Fig. 2.11 Principle of Eötvös experiment

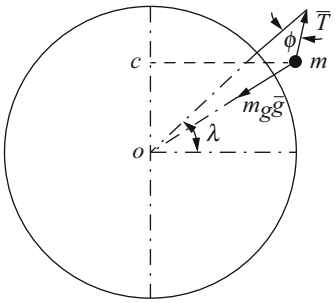


Fig. 2.12 Forces on mass in Eötvös experiment

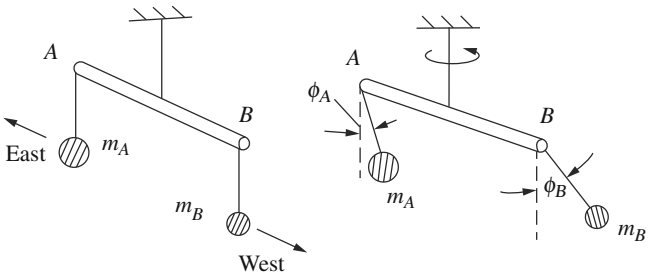


Fig. 2.13 Development of an unbalance moment when the equivalence is not valid

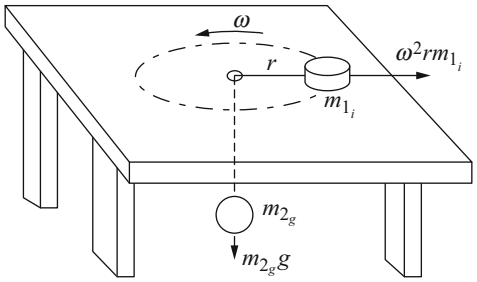


Fig. 2.14 Balancing of gravitational pull by inertial centrifugal force

Because of the earth's rotation, the mass has acceleration towards the centre C of the circle. The magnitude of the centripetal acceleration directed towards C is $\omega^2 r \approx {}^2R\cos\lambda$ as $h \lll R$, where h is the height of the post from which the pendulum suspended. The force required for this acceleration is equal to $m\omega^2 R\cos\lambda$. This force is the resultant of the tension T and the gravitational pull F_g on the mass. The force towards C acting on the bob (Fig. 2.12) is given by

$$m_g g\cos\lambda - T\cos(\lambda+\varphi) = m_i\omega^2 R\cos\lambda$$

or,

$$(m_g g - m\omega^2 R)\cos\lambda = T\cos(\lambda+\varphi) \tag{2.3.6}$$

Since the mass is not subjected to any acceleration in the direction of the local vertical

$$m_g g\sin\lambda = T\sin(\lambda+\varphi) \tag{2.3.7}$$

Eliminating T from the above two equations

$$\left(1 - \frac{m_i\omega^2 R}{m_g g}\right)\cot\lambda = \cot(\lambda+\varphi)$$

Simplifying after approximation

$$\varphi \approx \left(\frac{m_i}{m_g}\right)\frac{\omega^2 R}{g}\sin\lambda\cos\lambda \tag{2.3.8}$$

Since φ depends on the ratio (m_i/m_g), it will not be same for different materials if the equivalence of m_i and m_g is not true. To check this, two bobs of same weight but made of different materials are hung from a suspended lever AB as shown in Fig. 2.13. If the angles φ_A and φ_B are different, then the suspended lever will be subjected to a resultant moment causing it to rotate as indicated. However, if $m_i \equiv m_g$, φ will be same for different material bobs and no rotation of the suspended lever will be noticed. In the experiments conducted by Eötvös, no rotation of the suspended lever was found.

Before Newton, Huygens recognized the equivalent nature of the inertial centrifugal force and gravitational pull. He suggested an experiment through which this could be verified.

Figure 2.14 shows the scheme of this experiment. When the mass m_1 is rotated at the appropriate angular velocity ω keeping the length of the rotating part of the rope r, the centrifugal force $m_{1_i}\omega^2 r$ due to the inertial mass can balance the hanging weight. By suitable adjustment, it can be shown that $m_i = m_g$ when $m_1 = m_2$. Only two and half centuries later this equivalence of gravity and inertia became the foundation for Einstein's General Theory of Relativity. It is also interesting to note this is at the root of all planetary motions.

2.4 Space–Time and Symmetry in Newtonian Mechanics

It may occur to careful readers if there are definite reasons for the laws of motion to be as they are. As the laws of the game of chess are closely linked to the structure of a chessboard, the laws of motion are also linked to certain properties of the space and time. In fact while presenting the laws of motion such special properties are implicitly taken for granted. Of course the laws of mechanics cannot be directly derived only from the structure of space and time,

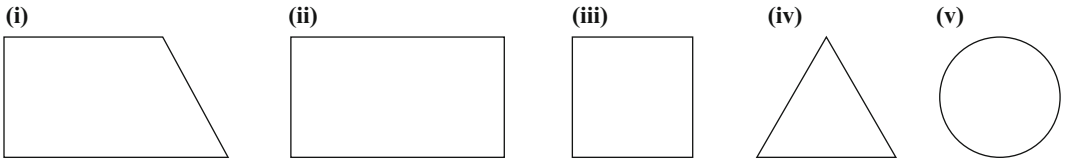


Fig. 2.15 Symmetry of geometric shapes

but understanding their close inter dependence provides a deeper understanding of the concepts.

Three important properties have been *a priory* assumed while formulating the laws of motion. These are as follows:

i. Homogeneity of space,
ii. Isotropy of space and
iii. Homogeneity of time

The homogeneity of space implies that the results of a dynamics experiment do not depend on the location where the experiment is conducted. Similarly, the isotropic nature of space ensures that the results of a dynamics experiment are not dependent on the orientation. Similarly, the outcome of a dynamical lest is independent of the time when it is conducted suggesting the homogeneity of time.

Another important aspect that plays a very profound role in mechanics is 'symmetry'. To understand the meaning, it may be helpful to present the concept of symmetry through geometrical shapes.

Figure 2.15 shows five geometric shapes with varying degrees of symmetry. The shape shown in (i) does not possess any symmetry as the appearance changes when it is rotated in the range 0 < angle of rotation < 360°. The rectangle shown in (ii) preserves its appearance when rotated through 180°. The square and the equilateral triangle shown in (iii) and (iv) possess increasingly higher degree of symmetry. The square and the triangle maintain the shape through a rotation of 90° and 60°, respectively. Finally, the circle shown in (v) is perfectly symmetric as any amount of rotation does not alter the shape. Thus, symmetry is signified by the 'absence of change', and in case of geometrical shapes, it is recognized by the lack of change through rotation or reflection.

Figure 2.16 shows the reflections in the cases of the shapes (i) and (ii). It is shown that the shape is preserved in the case of a rectangle but not for the shape shown in (i). Any amount of rotation of the reflected shape does not produce the original shape.

In case of geometrical figures, the symmetry is depicted by the absence of change when it is rotated or reflected. In the case of dynamics, the symmetry is identified by the lack of change in the results of an experiment when one goes from one inertial frame to another or when such an experiment is moved through space or conducted at a different time

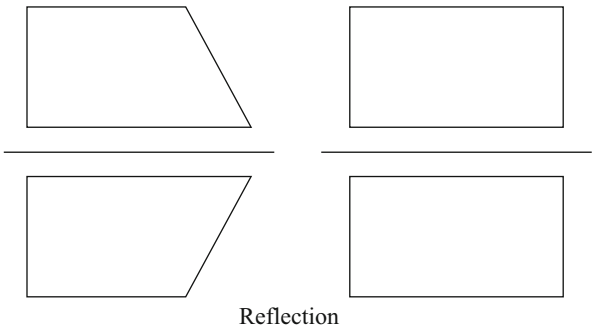


Fig. 2.16 Reflection of shapes

or it is rotated through space. How the symmetry properties are linked to various laws of motion and conservation principle will be discussed in the later section.

2.5 Early Concept of Energy

At this point, it is desirable to focus the attention to the development in science of motion that was taking place in the continent at a time when Newton was developing his mechanics. After Rene Descartes and Christian Huygens, it was Gottfried Wilhelm Leibniz (1646–1716) who was busy in developing this branch of science. One should not forget that it was Leibniz who first introduced the word 'dynamics' to describe this branch of natural philosophy. Newton used to refer to this discipline as 'rational mechanics' and considered it to be a mathematical discipline. However, Leibniz considered his 'dynamics' to be a physical discipline. Of course, Leibniz's treatment of the subject was predominantly metaphysical and, quite often, lost its meaning in the modern context.

The primary difference in the approach followed by Leibniz was because of his refusal to accept the concept of force as per the Newtonian doctrine. At the same time, being a follower of Descartes and Huygens, Leibniz did not agree to the 'action-at-a-distance' approach for gravitational interaction. Apart from this the basic idea, behind his dynamics were conceived some years before the publication of Principia. So, the explanations for the planetary motion given by Leibniz depended heavily on metaphysics. To him, 'motion' has no reality and only relative motion is meaningful. He also totally rejected Newton's concept of 'absolute space'.

Leibniz proposed two types of forces—'passive force' and 'active force'. The 'passive force' is somewhat akin to the inertial effect in bodies, and the 'active force' is directly related to the motion of bodies. Thus, passive force represents the mass of a body, whereas the active force relates to the mass and speed of a body. In Leibniz's opinion, the motion of a body has no reality; it is possible to conceive motion as change of position relative to other bodies. Active forces, according to the scheme proposed by Leibniz, again could be classified into two types—'living' forces and 'dead' forces. Those which are associated with only the tendency of motion were considered as 'dead' forces, and those which were associated with the motion were called 'living'. Without entering into the metaphysical discussion on Leibniz's science of motion, in modern language, one can understand 'dead' forces as the product of mass and an infinitesimal change is speed, i.e. $m{\cdot}dv$. This is nothing but an infinitesimal change in momentum and is same as the concept of impetus or impulse used by Newton in his analyses. (It should be remembered that Newton did not use the second law in a form it is used in modern times. He solved his mechanics problems using the concept of impulse that is equal to the change in momentum in very short durations.) The 'living' force in Leibniz's mechanics is proportional to the product of mass and the square of the speed, i.e. mv^2. Leibniz used the term 'vis viva' for this quantity. A considerable degree of nebulosity exists in Leibniz's writings, and maybe the true nature of these quantities was not very clear to Leibniz himself. When translated into the modern language, 'dead' force bears a close relationship with the concept of force or, more strictly speaking, with the idea of 'virtual work'. But the living force is merged with the modern concept of kinetic energy with some subtle differences. According to Leibniz, the living force had both direction and magnitude, whereas kinetic energy is a purely scalar quantity.

Using examples from collision problems, Leibniz also claimed that his 'motive action' (meaning living force, most probably) remains conserved. Using this principle, he solved collision problems with elastic bodies. Thus, the concept of an entity that is conserved took shape in continental mechanics that laid the foundation of the modern concept of mechanical energy. More elaborate discussion on energy and related issues will be taken up in the later section.

2.6 The Principle of Relativity and Galilean Transformation

It was Galileo who first removed the biggest hurdle in resolving the problem of a moving earth by introducing the concept of relativity and the indistinguishability of inertial frames from one another. At the present time, it is next to impossible to understand the tremendous difficulty in incorporating this idea into the thought process of not only the common men but even the philosophers.

Since mass is an invariant quantity in Newtonian mechanics and the acceleration of a body is observed to be the same by the observers in all inertial frames, the second law ensures that the observers will also agree on the force to be the same. Thus, two inertial observers agree about the cause of a given motion to be the same though the description of motion will be different. To elaborate this point, let the following example be considered.

An observer A sitting in a moving train throws a ball in the vertical direction and catches it back. Another observer B stands on the platform and observes the motion of the ball. Their descriptions are indicated in Fig. 2.17a, b. But both can predict their respective observations by using the second law of motion employing the respective initial conditions. The implication of the above experiment is that the laws of motion are the same in all inertial frames. This is an example of the Principle of Relativity, the general statement of which is that the laws of physics do not distinguish one inertial frame from another. Though this idea was used by Einstein for developing his Special Theory of Relativity, it was first proposed by Galileo in his famous book 'Dialogue Concurring Two Chief World Systems—Ptolemaic and Copernican'.

Though the text is widely read, it may be useful to present here the dialogue of two imaginary characters (one was, of course, Galileo himself), Sagredo and Salviati.

> Salviati: Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need to throw it no more

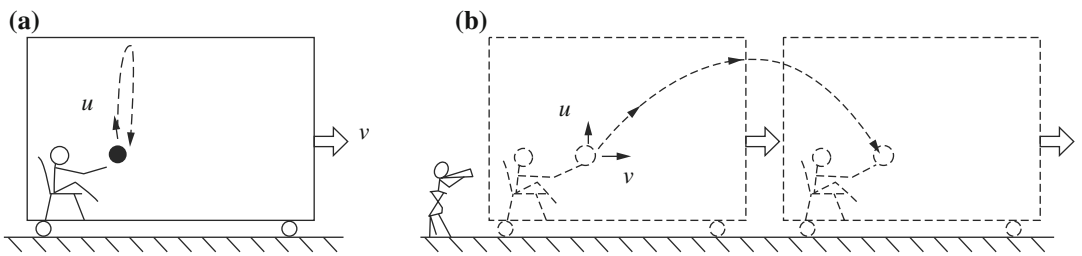


Fig. 2.17 **a**, **b** A ball's motion observed by two observers, **c** Two inertial observers looking at the motion of a particle

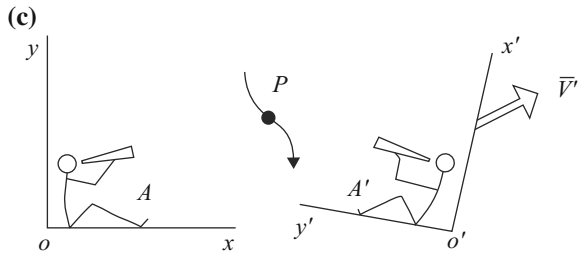


Fig. 2.17 (continued)

> strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still….

The above statement of Salviati suggests that it is not possible to detect uniform motion from observing internal motion phenomena. Thus, Galileo was the first proponent of 'The Principle of Relativity' that states that 'all inertial frames are equivalent so far as the laws of motion are concerned'. This principle which is within the domain of Newtonian mechanics is also termed as Galilean Relativity to distinguish it from Einstein's Special Theory of Relativity. The transformation of kinematic quantities from one inertial frame observer to another follows the rules which are known as Galilean transformation. If A and A' be two observers in inertial frames S and S', respectively, and, if the O_x and $O'_{x'}$ are aligned with the direction of the velocity V with which S' moves with respect to frame S, then the velocity components of a particle follow the transformation rules as given below:

$$\begin{aligned} t' &= t \\ u'_x &= u_x - V \\ u'_y &= u_y \\ u'_z &= u_z \end{aligned}$$

The transformation of coordinates follows the following rules:

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned}$$

It is also clear that acceleration components transform as follows:

$$a_{x'} = \frac{\mathrm{d}u'_x}{\mathrm{d}t'} = \frac{\mathrm{d}(u_x - V)}{\mathrm{d}t} = \frac{\mathrm{d}u_x}{\mathrm{d}t} = a_x$$

Similarly,

$$a_{y'} = a_y \quad \text{and} \quad a_{z'} = a_z$$

Hence,

$$\boldsymbol{a}' = \boldsymbol{a}$$

It will be seen in what follows is that many interesting features of Newtonian mechanics can be derived through the application of the symmetry concepts and the Principle of Relativity.

2.6.1 The Principle of Relativity

Before proceeding further, it is desirable to examine the Principle of Relativity in a more mathematical form to concretize the concepts.

Figure 2.17c shows two different inertial observers A and A' taking note of the motion of a particle P as indicated. The inertial frame o'–x'–y' has a uniform velocity $\boldsymbol{V}'$ with respect to the other inertial frame o–x–y as shown in the figure. Although, A and A' will have different descriptions of the motion of P as $y(x)$ and $y'(x')$, both of them will agree on the acceleration of P as $\boldsymbol{a} = \boldsymbol{a}'$. The Principle of Relativity in Newtonian Mechanics can be expressed in a more elaborate manner as described below.

Let S be an isolated system in which the particles interact with one another but not with the rest of the universe. Observer A marks the motion of all particles (with specific initial positions and velocities at $t = 0$) as time progresses. The positions and velocities of all particles are recorded as

$$\begin{aligned} \boldsymbol{r}_i &= \boldsymbol{r}_i(t); \quad i = 1, 2, \ldots, N \\ \boldsymbol{v}_i &= \boldsymbol{v}_i(t); \quad i = 1, 2, \ldots, N \end{aligned}$$

Now, observer A' conducts an experiment with the same particles with the same initial positions and same initial velocities in A' frame of reference, and then, the description of motion with the progress of time will be the same as that marked by A. Thus, with

$$\boldsymbol{r}_i(o) = \boldsymbol{r}'_i(o); \quad i = 1, 2, \ldots, N$$

and

$$\boldsymbol{v}_i(o) = \boldsymbol{v}'_i(o); \quad i = 1, 2, \ldots, N$$

the subsequent motion's description will be the same or mathematically

$$\boldsymbol{r}_i(t) = \boldsymbol{r}'_i(t'); \quad i = 1, 2, \ldots, N$$

and

$$\boldsymbol{v}_i(t) = \boldsymbol{v}'_i(t'); \quad i = 1, 2, \ldots, N$$

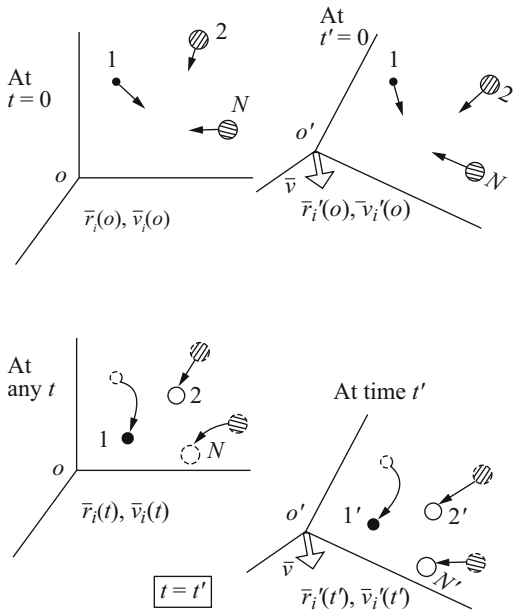


Fig. 2.18 Explicit illustration of the Principle of Relativity

with a further stipulation that

$$t = t'$$

Figure 2.18 illustrates the above diagrammatically.

One important point needs to be carefully noted. The Principle of Relativity also ensures three important properties of the space and time. The homogeneity of space is ensured as the inertial frames are located at different locations. The orientation of the inertial frames does not affect the laws of motion indicating the isotropy of space. Since the principle is valid not depending on when the experiments are conducted, it ensures the homogeneity of time.

The profoundness of the principle is evident in the fact that according to the Principle of Relativity, all inertial frames are equivalent for *all* physical phenomena not only the dynamical ones.

2.6.2 Symmetry and Relativity

The Principle of Relativity, when applied to mechanics problems involving some symmetry, can lead to many important results without solving the problems. Some examples presented below illustrate the point.

The first example is based on a collision problem of two isolated identical particles. Before the collision, the situation is 'perfectly' symmetric when viewed by an inertial observer A as shown in Fig. 2.19a. The possibilities of the state of motion after the collision could be as indicated in Fig. 2.19b. To determine the possibility without solving the collision problem, let the same phenomenon be observed by A' as shown in Fig. 2.19a. At the beginning, both A and A' found the situation to be symmetric. According to the Principle of Relativity, the two observers must find the description of motion after the collision to be indistinguishable. A little careful study shows that the situation indicated in (i) of Fig. 2.19b is the only case that satisfies the Principle of Relativity. Cases (ii) and (iii) look different to the two observers, and the descriptions differ. If a third observer A'' is considered in the frame $x''-y''-z''$, the possibilities shown in (iv) and (v) look different and the descriptions differ after the collision. Therefore, the only possible situation is as shown in (i). In general, if a situation is symmetric, after a dynamical interaction the situation must maintain the symmetry according to the Principle of Relativity.

If the case of a head-on symmetric collision of two identical particles appears to be too restrictive, a little more generalized case can be considered where two identical particles collide in an unsymmetrical fashion as indicated in Fig. 2.20.

Figure 2.20 shows the phenomenon. The figure shows the initial positions and velocities of the particles, and let the particles be initially free to move (as they are far off from each other and the system is an isolated one from the rest of the universe). Therefore, in the inertial frame x–y–z, both will move with no acceleration. Next, let the collision be

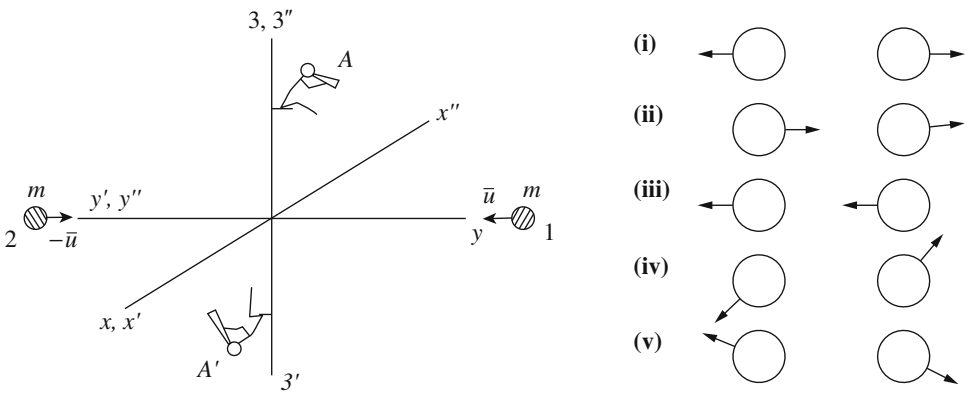


Fig. 2.19 Presentation of symmetry

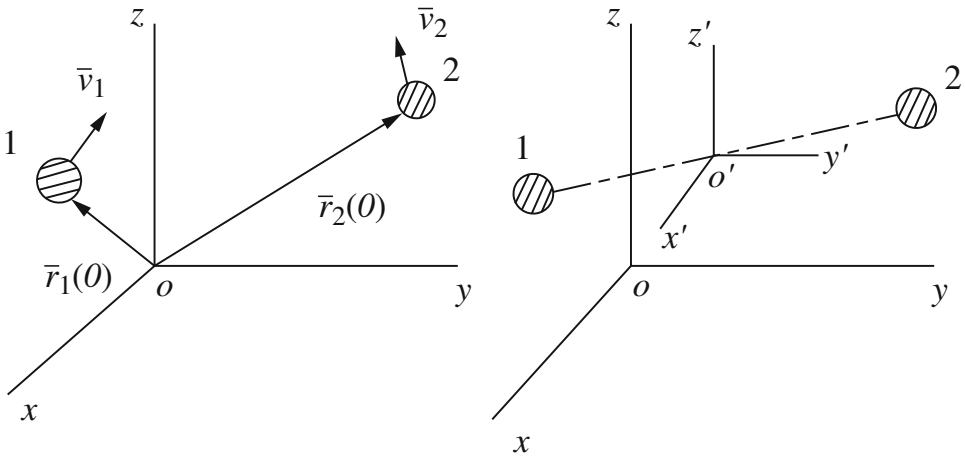


Fig. 2.20 Unsymmetrical collision of two identical particles

observed from a frame $x'-y'-z'$ whose origin is at the midpoint of the line joining the two particles, O'. Initially, the velocity of the point O' is

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}[\boldsymbol{r}_1(o)+\boldsymbol{r}_2(o)]=\frac{1}{2}[\boldsymbol{v}_1(o)+\boldsymbol{v}_2(o)]=\boldsymbol{v}_{o'}(o)$$

i.e. a constant. Therefore, if the axes of the primed frame of reference be always kept parallel to those in the inertial frame x–y–z, the primed frame moves with respect to the inertial frame with a constant velocity. Thus, the frame $x'-y'-z'$ is also an inertial frame if the velocity is kept constant. Now, when observed from this second inertial frame, the collision is a symmetrical one. The initial velocities of the two particles in this frame are found out as follows:

$$\begin{aligned}\boldsymbol{v}_1'(o)&=\boldsymbol{v}_1(o)-\boldsymbol{v}_{o'}(o)=\boldsymbol{v}_1(o)-\frac{1}{2}[\boldsymbol{v}_1(o)+\boldsymbol{v}_2(o)]\\&=\frac{1}{2}[\boldsymbol{v}_1(o)+\boldsymbol{v}_2(o)]=\boldsymbol{u}(o)\end{aligned}$$

and

$$\begin{aligned}\boldsymbol{v}_2'(o)&=\boldsymbol{v}_2(o)-\boldsymbol{v}_{o'}(o)=\boldsymbol{v}_2(o)-\frac{1}{2}[\boldsymbol{v}_1(o)+\boldsymbol{v}_2(o)]\\&=-\frac{1}{2}[\boldsymbol{v}_1(o)+\boldsymbol{v}_2(o)]=-\boldsymbol{u}(o)\end{aligned}$$

As this frame is also an inertial frame, the Principle of Relativity has to be satisfied; i.e., the initial symmetry has to be maintained. So the positions of particles 1 and 2 in the new frame must also remain symmetrical, implying that during the subsequent motion also $\boldsymbol{r}_1(t) = -\boldsymbol{r}_2(t)$. It ensures that even when the particles start interacting the origin O' remains at the middle of the two particles. This also ensures that the velocities and accelerations of the particles are always equal and opposite when the collision is viewed from the frame $x'-y'-z'$. Thus,

$$\boldsymbol{v}_1'(t)=-\boldsymbol{v}_2'(t)\quad\text{and}\quad\boldsymbol{a}_1'(t)=\boldsymbol{a}_2'(t)$$

Therefore, $\boldsymbol{a}_1'(t)+\boldsymbol{a}_2'(t)=0$ in the $x'-y'-z'$ frame, and since accelerations are same in all inertial frames, all inertial observers conclude that

$$\boldsymbol{a}_1'(t)+\boldsymbol{a}_2'(t)=0$$

or,

$$\frac{\mathrm{d}}{\mathrm{d}t}[m\boldsymbol{v}_1(t)+m\boldsymbol{v}_2(t)]=0$$

or,

$$m\boldsymbol{v}_1(t)+m\boldsymbol{v}_2(t)=\text{constant}$$

meaning that the total momentum of the system is conserved. Therefore, a considerable amount of information can be extracted by applying the symmetry concept and the Principle of Relativity.

2.6.3 Form Invariance of Physical Laws

Physical laws represent the generalized patterns of physical phenomena. The Principle of Relativity states that no distinctions can be made among the inertial frames by conducting internal experiments only. It means that the observers in different inertial frames must arrive at identical conclusions about the laws of physics from the results of the internally conducted experiments. Or in other words, the form of the laws of physics must be invariant. This very important property dictated by the Principle of Relativity is called the 'form invariance of physical laws'.

This property of physical laws puts very severe restrictions on the allowed forms. A very suitable example can be taken to illustrate this as follows. Let two particles 1 and 2 interact gravitationally, and let the phenomenon be observed

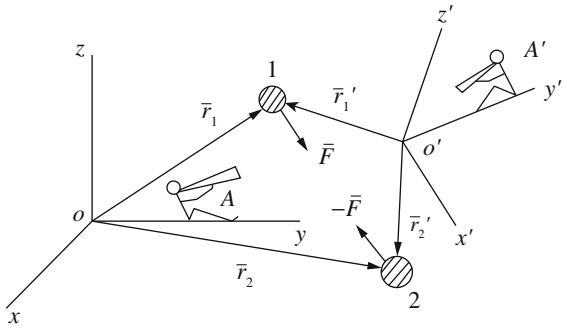


Fig. 2.21 Gravitational interaction as viewed from two inertial frames

by two inertial observers A and A' sitting in the inertial frames $x-y-z$ and $x'-y'-z'$ as indicated in Fig. 2.21.

Observer A notices the positions of the two particles 1 and 2 as $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. Similarly the observer A' records the positions of the same particles with respect to the primed frame of reference $x'-y'-z'$ as $\boldsymbol{r}_1'$ and $\boldsymbol{r}_2'$. Now let a law of gravitation is suggested in the following form. The gravitational force is given by

$$|\boldsymbol{F}| = \frac{Gm_1m_2}{|\boldsymbol{r}_1+\boldsymbol{r}_2|^2}$$

Hence, the accelerations of the particles should be

$$\boldsymbol{a}_1 = \frac{\boldsymbol{F}}{m_1} \quad \text{and} \quad \boldsymbol{a}_2 = -\frac{\boldsymbol{F}}{m_2}$$

The force of acceleration according to the observer A', according to this suggested force law, will be

$$|\boldsymbol{F}'| = \frac{Gm_1m_2}{|\boldsymbol{r}_1'+\boldsymbol{r}_2'|^2}$$

as mass is an invariant quantity. Thus, the acceleration of the particles according to A' will be

$$\boldsymbol{a}_1' = \frac{\boldsymbol{F}'}{m_1} \quad \text{and} \quad \boldsymbol{a}_2' = -\frac{\boldsymbol{F}'}{m_2}$$

But both A and A' being inertial observers they must agree that $\boldsymbol{a}_1 = \boldsymbol{a}_1'$ and $\boldsymbol{a}_2 = \boldsymbol{a}_2'$. This requires the two quantities

$$\frac{Gm_2}{|\boldsymbol{r}_1+\boldsymbol{r}_2|^2} \quad \text{and} \quad \frac{Gm_2}{|\boldsymbol{r}_1'+\boldsymbol{r}_2'|^2}$$

must be equal which is not true as $|\boldsymbol{r}_1+\boldsymbol{r}_2| \neq |\boldsymbol{r}_1'+\boldsymbol{r}_2'|$. Hence, the suggested form is not admissible. On the other hand, a force law

$$|\boldsymbol{F}| = \frac{Gm_1m_2}{|\boldsymbol{r}_1-\boldsymbol{r}_2|^2}$$

is admissible because $|\boldsymbol{r}_1-\boldsymbol{r}_2| = |\boldsymbol{r}_1'-\boldsymbol{r}_2'|$. Therefore, in general, the laws of physics must remain invariant under transformation from one inertial frame to another.

2.6.4 Energy and Energy Function

One of the most profound concepts in the science of motion is the concept of energy. However, an attempt to define energy is futile (like many other very important quantities in physics such as charge, force). Qualitatively, one can state that energy is something that is required to do work or get things done. In the whole of Principia, Newton did not refer to this concept even once. It was the contemporary continental scientists like Huygens and Leibniz who considered a quantity like energy (Leibniz's vis viva) to play an important role in dynamics.

The significance of 'energy' is in its conservation property. The 'energy', i.e. the capacity to do work in a dynamical system, is conserved. Of course, energy can be stored (or possessed by something) in various forms, viz. chemical, heat, sound, electromagnetic, nuclear and mechanical. The sum total of energy in an isolated system is always conserved though there can be transformation from one form into another. So far as dynamics is concerned, the only form of energy that is relevant is mechanical energy. In dynamics, if mechanical energy is transformed into another non-mechanical form like heat or chemical, then it is considered to be lost. Thus, in mechanics, energy conservation of a system implies that the total mechanical energy remains constant.

Mechanical energy can be in two forms. When the capacity to do work of a body is by virtue of its motion (like a bullet), the energy is called 'kinetic energy' (it is very similar to Leibniz's 'vis viva'). Mechanical energy can exist in another form, i.e. 'potential energy'. A body with potential energy can do work not by virtue of its motion but because of its internally stored energy. Best examples of a body possessing potential energy are that which is raised to a height against gravity, or a spring that is deformed.

Figure 2.22 shows the various examples of a mechanical system (or body) to possess mechanical energy in different manners. In the first case, the capacity to do work comes from the speed of the body, whereas in the second and third examples, the capacity to do work is in potential form. Quantitative measures of energy can be done by equating it to the amount of work done. So it is important to understand what is meant by work. Work is done either by a force or against a force, and the amount of work is the product of the force and the displacement of the point of application of the force A as illustrated in Fig. 2.23.

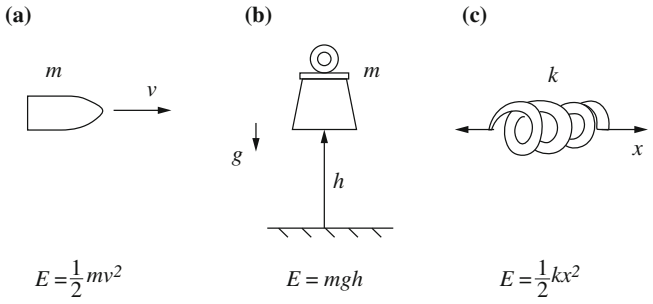


Fig. 2.22 Examples of mechanical energy

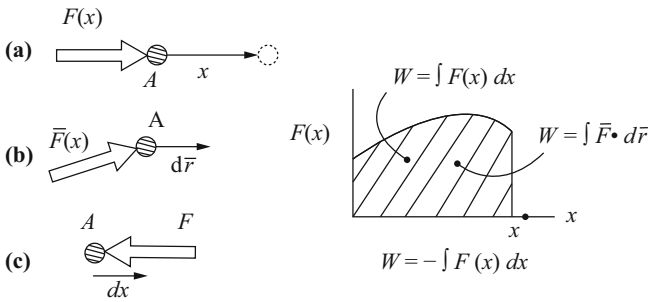


Fig. 2.23 Work done by a force

If the force and displacement of the point of application of the force A are vectors, then the scalar product summed over the whole process is taken as the work done by the force. The scalar quantity, work, goes into the system (to which the point A belongs) to be stored as energy. When the displacement of the point A is against the direction of application of the force (Fig. 2.23c), the work is done by the system against the force. Hence, the work done is negative and is achieved through the expenditure of energy stored in the system. In case of the vector nature of the force and the displacement, the sign of the scalar product automatically decides the situation. Using this concept, it is seen that the kinetic energy is given by $½mv^2$ and the gravitational or strain potential energy is given by mgh and $½kx^2$ where 'h' is the height raised and x is the deformation of a spring from its natural length, k being the stiffness of the spring given by the force required to produce unit deformation (elongation or compression). The law of conservation of mechanical energy of a system states that the sum total of the kinetic and potential energy remains constant. Figure 2.24 shows a number of different types of mechanical systems and their respective total energy expression.

2.6.5 Energy Function

The concept of energy function is very important. A particular mechanical system can be disturbed by putting it in motion. A system undergoes infinite states of motion depending on the initial disturbance. But is there something that describes all possible states of motion of the system and not a particular state? If such a 'thing' exists, then that describes the 'system' consisting of all the 'possible' states of motion. In fact, there is such a thing that describes a 'system' instead of a particular motion executed by it. This 'something' can be called the 'energy function' of a system. Each mechanical system possesses a unique 'energy function'. The positions and velocities of the individual particles constituting a system determine the value of its 'energy function'. An example can illustrate the importance of energy function concept.

Let the first case be that of a simple spring mass system. Its energy function is given as follows:

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

Since it is a conservative isolated system, E must be conserved:

$$\begin{aligned} \text{or,} \quad & \frac{dE}{dt} = 0 \\ \text{or,} \quad & m\dot{x}\ddot{x} + kx\dot{x} = 0 \\ \text{or,} \quad & \dot{x}(m\ddot{x} + kx) = 0 \end{aligned}$$

Hence, two possible situations are permitted. Either the body is at rest, i.e. $\dot{x} = 0$, or the following relation is satisfied:

$$m\ddot{x} + kx = 0$$

The example is simple but shows how the dynamical behaviour expressed in the form of a relationship among various quantities (in modern language this is called the equation of motion). It can be stated that the dynamical characteristics of a mechanical system are governed by its 'energy function'. It should be carefully noted that the 'energy function' of a system is not explicitly dependent on time. For an isolated conservative system, the energy function remains constant implying

$$E[\boldsymbol{r}_i(t); i = 1, 2, \ldots, N, \boldsymbol{v}_i(t); i = 1, 2, \ldots, N] = \text{constant}$$

or

$$\frac{dE}{dt} = 0$$

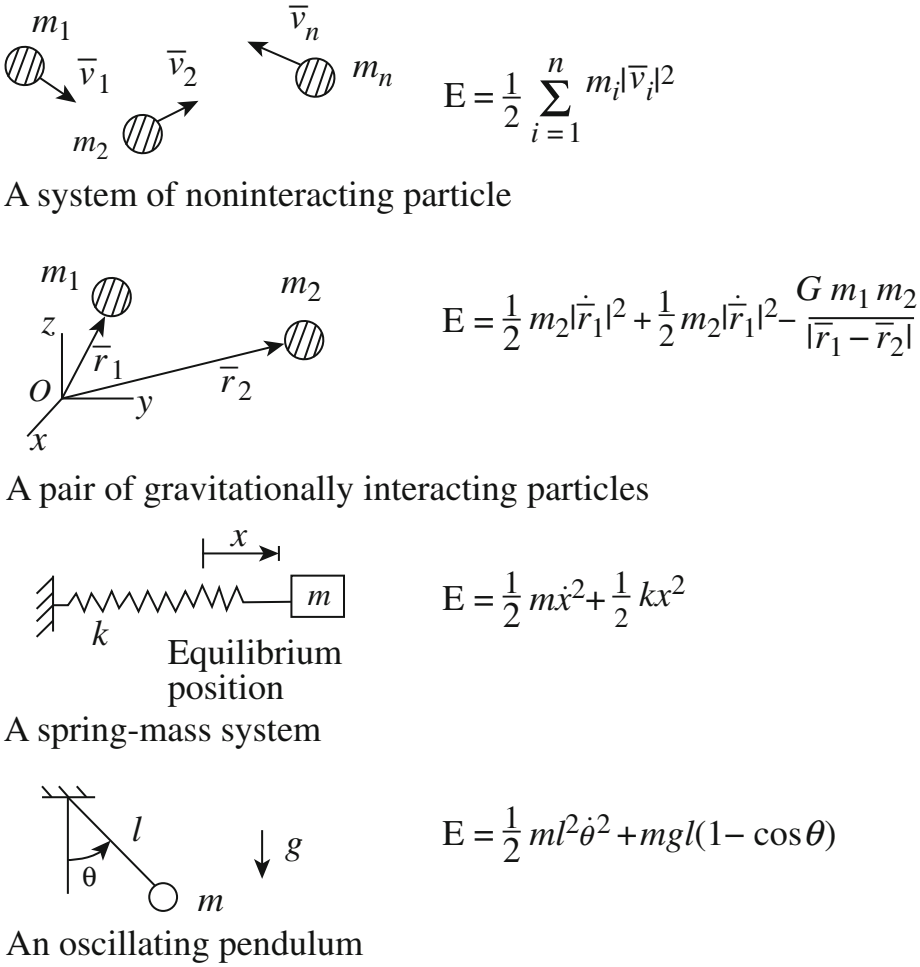


Fig. 2.24 Examples of mechanical system and their total energy

which signifies that the results of an experiment are independent of the time when it is conducted. Thus, homogeneity of time is a direct consequence of energy conservation.

It is interesting to study what is the situation when a mechanical phenomenon is observed in a non-inertial frame. It is quite obvious that the velocities of the bodies seen from a non-inertial frame do not remain constant. Thus, the energy of a mechanical system does not remain conserved. Since energy conservation is not valid in a non-inertial frame of reference, homogeneity of time is not satisfied in such frames. As just mentioned, the kinetic energy of a particle (or, a system of particles) is not same in different inertial frames (though it remains conserved in any particular frame) as the velocities are different in different inertial frames of reference. However, the potential energy of a system depends on the interaction among particles. Considering a case of a two-particle system, it is clear that the potential energy depends on the distance between two particles. Hence, as the distance remains unchanged to different observers, the potential energy is an invariant quantity independent of the observer's frame of reference.

2.7 Laws of Motion and the Properties of Space and Time

In this section, the laws of motion will be studied in association with the properties of space and time, and from the Principle of Relativity. Let the case of an isolated single particle moving freely in an inertial frame be taken up. The energy function of this particle will be of the form $E|\boldsymbol{v}|^2$. It cannot depend on time as time is homogenous; similarly, the energy function cannot depend on either the position $\boldsymbol{r}$ or the direction of $\boldsymbol{v}$ since the space is both homogenous and isotropic. Next, let the y and y' axes of two inertial observers A and A' be aligned with the direction of $-\boldsymbol{v}$ as indicated in Fig. 2.25 without affecting the generality of the result.

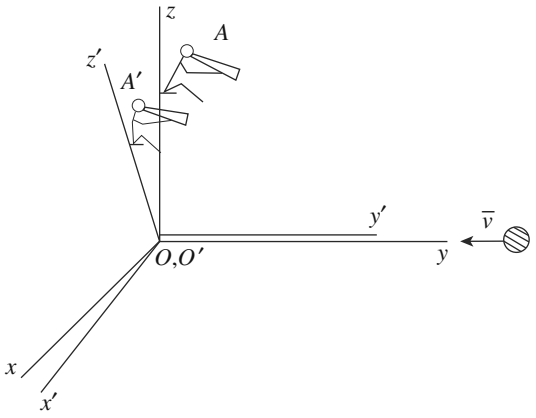


Fig. 2.25 Motion of a free particle with respect to two inertial frame with different orientation

To begin with the motion possesses symmetry in both the frames as x, z, x' and z' components of $\boldsymbol{v}$ are zero. Following the Principle of Relativity, both observers must not be able to detect any break of symmetry during the subsequent motion. So all the above four components of $\boldsymbol{v}$ must continue to remain zero. Hence, the particle moves along a straight line. Furthermore as $E|\boldsymbol{v}|^2$ remains constant, the speed $|\boldsymbol{v}|$ must remain constant. These are the two predictions of the first law.

2.7.1 The Second Law of Motion

Extracting the second law of motion from the properties of the space and time along with the application of the Principle of Relativity can be achieved as demonstrated below.

Figure 2.26 shows two particles 1 and 2 that are interacting with each other but are isolated from the rest of the universe. Let the particles be in motion with velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ as shown, their instantaneous locations being $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. Let U be the potential energy of the system because of the interaction of the particles, and it is a function of their instantaneous distance $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$ only. If the kinetic energies of the individual particles be T_1 and T_2, depending on the square of their speeds v_1 and v_2, the total mechanical energy of the whole system can be expressed as follows:

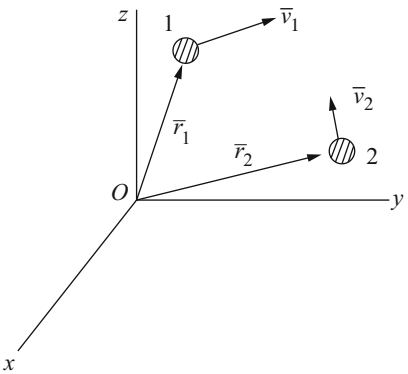


Fig. 2.26 System of two interacting particles

$$E = T_1 v_1^2 + T_2 v_2^2 + U[(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)]$$

Since the system is an isolated one and neither of the above energy terms being explicitly dependent on time (as time is homogenous as mentioned earlier), the total energy is conserved. Hence, writing $v_1^2 = \boldsymbol{v}_1 \cdot \boldsymbol{v}_1$ and $v_2^2 = \boldsymbol{v}_2 \cdot \boldsymbol{v}_2$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}[T_1(\boldsymbol{v}_1 \cdot \boldsymbol{v}_1) + T_2(\boldsymbol{v}_2 \cdot \boldsymbol{v}_2) + U[(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)] = 0 \tag{2.7.1}$$

Using the identities

$$\frac{\mathrm{d}(\boldsymbol{v}_1 \cdot \boldsymbol{v}_1)}{\mathrm{d}t} = 2\boldsymbol{v}_1\boldsymbol{a}_1, \quad \frac{\mathrm{d}(\boldsymbol{v}_2 \cdot \boldsymbol{v}_2)}{\mathrm{d}t} = 2\boldsymbol{v}_2\boldsymbol{a}_2$$

and

$$\frac{\mathrm{d}[(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)]}{\mathrm{d}t} = 2(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2)$$

(2.7.1) can be written in the following form:

$$\frac{\mathrm{d}T_1}{\mathrm{d}v_1^2}(2\boldsymbol{v}_1 \cdot \boldsymbol{a}_1) + \frac{\mathrm{d}T_2}{\mathrm{d}v_2^2}(2\boldsymbol{v}_2 \cdot \boldsymbol{a}_2) + \frac{\mathrm{d}U}{\mathrm{d}[(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)]} 2(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = 0 \tag{2.7.2}$$

Let the following quantities are defined as given below:

$$m_1(v_1)^2 = 2\frac{\mathrm{d}T_1}{\mathrm{d}v_1^2}$$

$$m_2(v_2)^2 = 2\frac{\mathrm{d}T_2}{\mathrm{d}v_2^2}$$

and,

$$f(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) = 2\frac{\mathrm{d}U}{\mathrm{d}[(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)]}$$

Incorporating these definitions in (2.7.2), one obtains

$$\frac{\mathrm{d}E}{\mathrm{d}t} = m_1\left(v_1^2\right)\boldsymbol{v}_1 \cdot \boldsymbol{a}_1 + m_2\left(v_2^2\right)\boldsymbol{v}_2 \cdot \boldsymbol{a}_2 + f(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = 0$$

Separating the kinetic and potential energy terms

$$m_1\left(v_1^2\right)\boldsymbol{v}_1 \cdot \boldsymbol{a}_1 + m_2\left(v_2^2\right)\boldsymbol{v}_2 \cdot \boldsymbol{a}_2 = -f(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) \tag{2.7.3}$$

Now, it should be remembered that all equations are form invariant and valid in all inertial frames. The right-hand side of (2.7.3) has the same value in all inertial frames as the quantities $(|\boldsymbol{r}_1-\boldsymbol{r}_2|)$ and $(|\boldsymbol{v}_1-\boldsymbol{v}_2|)$ have the same values in all inertial frames. Hence, the quantity

$$f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=\frac{\mathrm{d}U}{\mathrm{d}t}$$

is an invariant quantity[4] and, therefore, the term

$$m_1\left(v_1^2\right)\boldsymbol{v}_1\cdot\boldsymbol{a}_1+m_2\left(v_2^2\right)\boldsymbol{v}_2\cdot\boldsymbol{a}_2=\frac{\mathrm{d}T}{\mathrm{d}t}$$

is also invariant. Therefore,

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}t}&=m_1\left(v_1^2\right)\boldsymbol{v}_1\cdot\boldsymbol{a}_1+m_2\left(v_2^2\right)\boldsymbol{v}_2\cdot\boldsymbol{a}_2\\&=m_1\left(v_1'^2\right)\boldsymbol{v}_1'\cdot\boldsymbol{a}_1'+m_2\left(v_2'^2\right)\boldsymbol{v}_2'\cdot\boldsymbol{a}_2'=\frac{\mathrm{d}T'}{\mathrm{d}t}\end{aligned}$$

This invariance restricts the function $m_1\left(v_1^2\right)$ and $m_2\left(v_2^2\right)$. It can be shown that one choice of m_1 and m_2 satisfies the condition of invariance as demonstrated below. Let the following forms be assumed

$$m_1\left(v_1^2\right)=m_1(\text{constant}),\quad m_2\left(v_2^2\right)=m_2(\text{constant})$$

and,

$$m_1\boldsymbol{a}_1=-m_2\boldsymbol{a}_2 \tag{2.7.4}$$

with above choices

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}t}&=m_1\boldsymbol{v}_1\cdot\boldsymbol{a}_1+m_2\boldsymbol{v}_2\cdot\boldsymbol{a}_2=m_1\boldsymbol{v}_1\cdot\boldsymbol{a}_1-m_2\boldsymbol{v}_2\cdot\boldsymbol{a}_2\\&=m_1\boldsymbol{a}_1\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=m_1\boldsymbol{a}_1'\cdot\left(\boldsymbol{v}_1'-\boldsymbol{v}_2'\right)=\frac{\mathrm{d}T'}{\mathrm{d}t}\end{aligned} \tag{2.7.5}$$

by using the Galilean transformation rules. It can be proved that the choices given in (2.7.4) are the only way to maintain invariance of $\frac{\mathrm{d}T}{\mathrm{d}t}$. It is left as an exercise for the reader. Next, let the following definitions be introduced:

$$\begin{aligned}\boldsymbol{F}_1&=-f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_1-\boldsymbol{r}_2)\quad\text{and}\\\boldsymbol{F}_2&=-f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_2-\boldsymbol{r}_1)\end{aligned} \tag{2.7.6}$$

Therefore,

$$\boldsymbol{F}_1\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=-f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=-\frac{\mathrm{d}U}{\mathrm{d}t}$$

and

$$\boldsymbol{F}_2\cdot(\boldsymbol{v}_2-\boldsymbol{v}_1)=-f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_2-\boldsymbol{r}_1)\cdot(\boldsymbol{v}_2-\boldsymbol{v}_1)=-\frac{\mathrm{d}U}{\mathrm{d}t}$$

so,

$$\frac{\mathrm{d}U}{\mathrm{d}t}=-\boldsymbol{F}_1\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=-\boldsymbol{F}_2\cdot(\boldsymbol{v}_2-\boldsymbol{v}_1) \tag{2.7.7a}$$

Again from (2.7.5)

$$\frac{\mathrm{d}T}{\mathrm{d}t}=m_1\boldsymbol{a}_1\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=m_2\boldsymbol{a}_2\cdot(\boldsymbol{v}_2-\boldsymbol{v}_1) \tag{2.7.7b}$$

Combining (2.7.7a) and (2.7.7b)

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}t}+\frac{\mathrm{d}U}{\mathrm{d}t}&=(m_1\boldsymbol{a}_1-\boldsymbol{F}_1)\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)\\&=(m_2\boldsymbol{a}_2-\boldsymbol{F}_2)\cdot(\boldsymbol{v}_2-\boldsymbol{v}_1)=\frac{\mathrm{d}E}{\mathrm{d}t}=0\end{aligned} \tag{2.7.8}$$

Unless $\boldsymbol{v}_1=\boldsymbol{v}_2$ or the vector $(\boldsymbol{v}_1-\boldsymbol{v}_2)$ is perpendicular to the vector $(m_1\boldsymbol{a}_1-\boldsymbol{F}_1)$, (2.7.8) can be satisfied if and only if

$$\boldsymbol{F}_1=m_1\boldsymbol{a}_1$$

and,

$$\boldsymbol{F}_2=m_2\boldsymbol{a}_2 \tag{2.7.9}$$

which are nothing but the second law of motion and $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are the forces. Thus, the second law of motion can be derived from the properties of the space, time and the Principle of Relativity.

2.7.2 The Third Law of Motion

This law is a contribution of Newton alone unlike the first and second laws which had major contributions from others before Newton. The profound importance of this law is often missed. The basic content of the law has two forms—weak form and strong form. The weak form of the third law states that the forces between two interacting particles are always equal and opposite. Thus if the forces on particles 1 and 2 be $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, then

$$\boldsymbol{F}_1=-\boldsymbol{F}_2$$

Figure 2.27 shows two particles and the forces acting on those. As per the third law, the forces are equal and opposite. However, the strong form of the third law states that these

[4]This can be demonstrated in a more direct way using the laws of Galilean transformation. Let x–y–z be two inertial frames with the following transformation equations $\boldsymbol{r}'=\boldsymbol{r}-\boldsymbol{u}t$, $\boldsymbol{v}'=\boldsymbol{v}-\boldsymbol{u}$, $\boldsymbol{a}'=\boldsymbol{a}, t'=t$; then, the following equalities are obtained: $\boldsymbol{r}_1'-\boldsymbol{r}_2'=\boldsymbol{r}_1-\boldsymbol{r}_2$ and $\boldsymbol{v}_1'-\boldsymbol{v}_2'=\boldsymbol{v}_1-\boldsymbol{v}_2$; and so, $\frac{\mathrm{d}U}{\mathrm{d}t}=f(|\boldsymbol{r}_1-\boldsymbol{r}_2|)(\boldsymbol{r}_1-\boldsymbol{r}_2)\cdot(\boldsymbol{v}_1-\boldsymbol{v}_2)=f\left(|\boldsymbol{r}_1'-\boldsymbol{r}_2'|\right)\left(\boldsymbol{r}_1'-\boldsymbol{r}_2'\right)\cdot\left(\boldsymbol{v}_1'-\boldsymbol{v}_2'\right)=\frac{\mathrm{d}U'}{\mathrm{d}t}$.

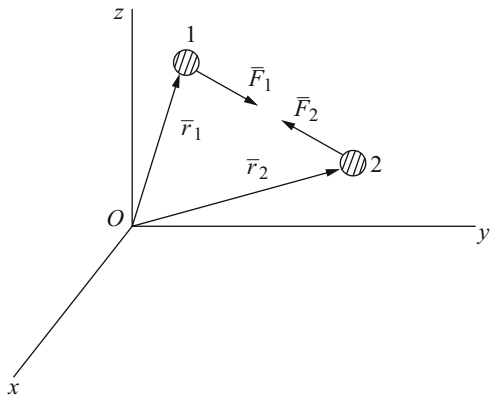


Fig. 2.27 Third law of motion

forces are not only equal in magnitude and opposite in direction, but act along the same line of action.

Thus, using the quantities shown in Fig. 2.27

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2$$

and

$$\boldsymbol{F}_1 \times (\boldsymbol{r}_1 - \boldsymbol{r}_2) = \boldsymbol{F}_2 \times (\boldsymbol{r}_1 - \boldsymbol{r}_2) = 0$$

and

$$\boldsymbol{F}_1 \times \boldsymbol{r}_1 = -\boldsymbol{F}_2 \times \boldsymbol{r}_2 \tag{2.7.10}$$

In fact from the previous section, it has been found that

$$\frac{\mathrm{d}U}{\mathrm{d}t} = f(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = -\boldsymbol{F}_1 \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = -\boldsymbol{F}_2 \cdot (\boldsymbol{v}_2 - \boldsymbol{v}_1)$$

or,

$$-\boldsymbol{F}_1 \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = \boldsymbol{F}_2 \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2)$$

or,

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2$$

Again,

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\boldsymbol{F}_1 \cdot (\boldsymbol{v}_1 - \boldsymbol{v}_2) = -\boldsymbol{F}_1 \cdot \boldsymbol{v}_1 - \boldsymbol{F}_2 \cdot \boldsymbol{v}_2 = -\boldsymbol{F}_1 \cdot \frac{\mathrm{d}\boldsymbol{r}_1}{\mathrm{d}t} - \boldsymbol{F}_2 \cdot \frac{\mathrm{d}\boldsymbol{r}_2}{\mathrm{d}t}$$

Hence,

$$\partial U = -\boldsymbol{F}_1 \cdot \partial \boldsymbol{r}_1 - \boldsymbol{F}_2 \cdot \partial \boldsymbol{r}_2 \tag{2.7.11}$$

Using $\partial U = 0$, since $\mathrm{d}U/\mathrm{d}t = 0$

$$\boldsymbol{F}_1 \cdot \partial \boldsymbol{r}_1 - \boldsymbol{F}_2 \cdot \partial \boldsymbol{r}_2 = 0 \tag{2.7.12}$$

Now, considering the homogeneity of space a pure translation of the pair of particles should not result in any change. Under pure translation, both the particles undergo equal displacement

$$\partial \boldsymbol{r}_1 = \partial \boldsymbol{r}_2 = \partial \boldsymbol{r}$$

Hence,

$$\boldsymbol{F}_1 = -\boldsymbol{F}_2$$

as a result of the homogeneity of space. When a pure rotation is imparted, the displacement of the two particles are equal and opposite, as shown in Fig. 2.28.

So,

$$\partial \boldsymbol{r}_1 = -\partial \boldsymbol{r}_2 = \partial \boldsymbol{r}$$

Using the above condition in (2.7.12), one obtains

$$\boldsymbol{F}_1 \cdot \partial \boldsymbol{r}_1 - \boldsymbol{F}_2 \cdot \partial \boldsymbol{r}_2 = 0$$

or,

$$2\boldsymbol{F}_1 \cdot \partial \boldsymbol{r}_1 = 0$$

But for infinitesimal displacement, $\delta \boldsymbol{r}$ is perpendicular to $(\boldsymbol{r}_1 - \boldsymbol{r}_2)$. Hence, $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are parallel to the line joining the particles. This is a consequence of the isotropy of space. The above two results complete the third law of motion.

The third law of motion also leads to two important conservation principles as explained below. For the pair of particles under consideration, the two important quantities are linear momentum and the angular momentum. These are defined as follows:

$\boldsymbol{P}$ = Linear momentum $= m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$

$\boldsymbol{l}_o$ = Angular momentum about $O = \boldsymbol{r}_1 \times m_1\boldsymbol{v}_1 + \boldsymbol{r}_2 \times m_2\boldsymbol{v}_2$

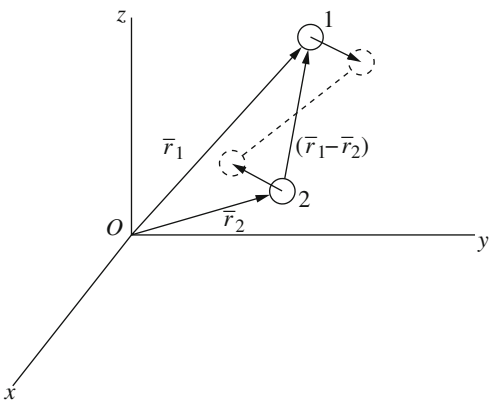


Fig. 2.28 Rotation of a pair of particles

When the third law is employed, the following results are obtained:

$$\frac{d\boldsymbol{P}}{dt} = \frac{d}{dt}(m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2) = m_1\boldsymbol{a}_1 + m_2\boldsymbol{a}_2 = \boldsymbol{F}_1 + \boldsymbol{F}_2 = 0$$

So $\boldsymbol{P}$ is conserved. Again

$$\frac{d\boldsymbol{l}_o}{dt} = \frac{d}{dt}(\boldsymbol{r}_1 \times m_1\boldsymbol{v}_1 + \boldsymbol{r}_2 \times m_2\boldsymbol{v}_2)$$
$$= \boldsymbol{v}_1 \times m_1\boldsymbol{v}_1 + \boldsymbol{r}_1 \times m_1\boldsymbol{a}_1 + \boldsymbol{v}_2 \times m_2\boldsymbol{v}_2 + \boldsymbol{r}_2 \times m_2\boldsymbol{a}_2$$

as O is a point fixed in the frame of reference meaning $\dot{\boldsymbol{r}}_1 = \boldsymbol{v}_1$ and $\dot{\boldsymbol{r}}_2 = \boldsymbol{v}_2$. So,

$$\frac{d\boldsymbol{l}_o}{dt} = \boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 = \boldsymbol{r}_1 \times \boldsymbol{F}_1 - \boldsymbol{r}_2 \times \boldsymbol{F}_2$$

by the first part of the third law of motion.

Thus,

$$\frac{d\boldsymbol{l}_o}{dt} = (\boldsymbol{r}_1 - \boldsymbol{r}_2) \times \boldsymbol{F}_1 = 0$$

because $\boldsymbol{F}_1$ acts along the vector $(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ by the second part of the third law.

Finally, the properties of the space and time, i.e. the homogeneity of space, the isotropy of space and the homogeneity of time, result in:

- The conservation of linear momentum,
- The conservation of angular momentum and
- The conservation of energy.

So, these conservation principles and the laws of motion are the manifestations of the properties of the space and time and the Principle of Relativity.

2.8 Action-at-a-Distance and Spatiotemporal Locality

Accepting the existence of non-contact force in the science of motion was a serious problem to a large number of scientists in the seventeenth century. Frankly speaking, it is not very easy to grasp the reality behind such 'action-at-a-distance' phenomenon. It is not very easy even at the present time. This section is devoted to a discussion on this very important issue.

2.8.1 Early Work on Non-contact Forces

The earlier diffused ideas and nebular concepts regarding the motion of planets led to the introduction of metaphysical treatment like providing the planets with 'soules'. The first work on physical force without requiring physical contact between the interacting bodies was the book 'De Magnete, Magnetieisque Corporibus, et de Magno Magnete Tellure' by W. Gilbert published from London in 1600. Kepler, who was the prime person working on the motion of planet, could free himself from J.C. Scaliger's (1484–1558) concept of 'souls' and 'spirits' once he came across Gilbert's book. In the earlier days, he used to call the gravitational force also as a 'magnetic force'. The matter has been already discussed in Sect. 1.7.3.

It was not very easy to accept the gravitational interaction as in the continent the influence was of Descartes' physics that was based upon the impact between particles. Kepler was an exception; he not only conceived of a force like gravitation by the sun on the planets controlling their motions, but he correctly attributed the attraction by the moon to explain the phenomenon of tide. But Galileo was also a disbeliever in such 'occult' non-contact forces. Galileo proposed a theory of the tides which was obviously wrong. According to Galileo's theory, the accumulation of ocean water in certain regions of earth was related to the speed of the earth relative to the distant stars based on a combination of the orbital motion and the daily rotation. Even during the later period, scientists like Descartes and Huygens were totally opposed to the idea of non-contact 'gravitational force'. A meeting to discuss the nature of 'gravity' was organized in 1669 at Paris Académie. Two scientists, Robervel and Frenicle proposed the gravity to be a force of attraction by the earth on objects near the earth's surface—a mutual force between like bodies. But Huygens opposed the proposition vehemently as is clear from a translation of his statement given below:

> To discover a cause of weight that is intelligible, it is necessary to investigate how weight can come about while assuming the existence only of bodies made of one common matter in which one admits no quality or inclination to approach each other but solely different sizes, figures and motions.

In fact, according to many historians of science, this total rejection of the concept of a non-contact gravitational attraction denied Huygens in developing a complete synthesis of the science of motion. Although he developed most of the basic elements much before Newton.

To begin with, Newton also attempted many models to explain gravitational attraction through mechanical means. The story of apple falling giving the idea of explaining the motion of the moon to young Newton in 1665 may contain a certain element of truth, but Newton's attempt at that time was to try for a mechanical explanation. Such models based on Cartesian principles of collision of fundamental particles filling the universe could somehow generate a force towards the earth. But such models could not result in an equal and

opposite force on the earth that attracted it to the other interacting bodies, viz. the apple and the moon. The correct idea of universal gravitation took shape in Newton's mind much later, perhaps, in 1680. Thus, Newton transformed dynamics from its restricted domain of 'contact force' only to forces extending over long distances. Thus, the concept of the 'action-at-a-distance' achieved acceptability among the scientists.

2.8.2 Spatiotemporal Locality and Action-at-a-Distance

In general, it is easy to conceive that a 'cause' can give rise to a 'result' only at that location in space and precisely at that instant of time. Of course, there are instances where, apparently, the result takes some time to show up, but a little thinking reveals that in all such cases there are innumerable 'mini-intermediate results' which connect the original 'cause' with the final 'result'.

In a loose sense, one can logically accept that a particle can influence the motion of another one when it is in physical contact. It can be stated that two bodies can be considered to be in physical contact when no region of space, however small, can be placed between them. Similarly when two events are separated by a gap of interval, however small, can be considered to be temporally local. Very precise and scientific definitions of 'spatial', 'temporal' and 'spatiotemporal locality' have been given by Marc Lange in 'An Introduction to the Philosophy of Physics'. These are quoted below:

> Spatial locality: For any event E and for any finite distance $\delta > 0$, no matter how small, there is a complete set of causes of E such that for each event C in this set, there is a location at which it occurs that is separated by a distance no greater than δ from a location at which E occurs.
>
> Temporal locality: For any event E and for any finite temporal interval $\tau > 0$, no matter how short, there is a complete set of E's causes such that for each event C (a cause) in this set, there is a moment at which it occurs that is separated by an interval no greater than τ from a moment at which E occurs.
>
> Spatiotemporal locality: For any event E, any finite temporal interval $\tau > 0$, and any finite distance $\delta > 0$, there is a complete set of causes of E such that for each event C in this set, there is a location at which it occurs that is separated by a distance no greater than δ form a location at which E occurs, and there is a moment at which C occurs at the former location that is separated by an interval no greater than τ from a moment at which E occurs at the latter location.

Even without the precise and robust definitions quoted above, it is not difficult to conceive the violation of spatiotemporal locality in the physical world. 'Action-at-a-distance' is something that violates this condition; precisely because of this, many philosophers and scientists in the past could not accept the concept of gravitational interaction between two bodies at a distance. It is also the reason why Mach's principle has remained unacceptable to many. Gravitational action violates 'spatial locality' condition as the interacting bodies are separated by finite space. On the other hand, Mach's principle violates 'spatiotemporal locality' condition as illustrated through Fig. 2.29.

Two bodies A and B, as shown in Fig. 2.29 are located at a distance r. At $t = t_1$, body A is given an acceleration a with respect to body B. According to Mach's principle, A experiences a force F resisting the acceleration because of body B at the same instant. The information about A's acceleration takes a finite time to reach B, and its effect should take a finite time to reach back A in the form of the resisting force F_i. This is a kind of a 'reverse' violation of spatiotemporal locality. The bodies at separate locations cannot develop 'instantaneous' relationship for any 'cause–result' phenomenon. It is an insurmountable hurdle in accepting and understanding such 'action-at-a-distance' phenomenon.

2.8.3 The Concept of Field

Apart from the phenomenon of gravitation, very serious problems involving violation of spatiotemporal locality condition are exhibited by the phenomena of electricity and magnetism. Therefore, from the mediaeval period, philosophers and scientists continuously attempted to provide acceptable physical solutions to the observed 'action-at-a-distance' phenomena.

The first clear and concrete evidence of a body influencing another at a distance was the phenomenon of tide. In the ninth century AD, an Arabian philosopher, Abu Mashar published a book in 850 AD in which he clearly emphasized the relationship between the moon's position in the sky and the state of the tide. Of course, his primary aim was to establish 'astrology' on scientific basis. But it was Kepler who first realized the need for an action without physical contact to explain the motions of planets. Gilbert's

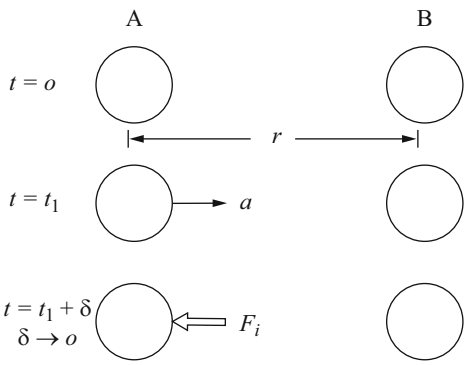


Fig. 2.29 Violation of spatiotemporal locality by Mach's principle

book also brought the existence of magnetic force as another example of action-at-a-distance. Though Galileo and Huygens were against any such 'action-at-a-distance' phenomena and associated gravitational attraction, Newton's development of the celestial mechanics stopped all further hostilities. However, both Newton and Huygens had to explain the phenomena involving transfer of influence through open space. In case of Newton, it was the phenomenon of gravitation and Huygens had to explain his theory of light in which the light propagated as the wave-like oscillations in a medium called 'aether'.[5] Thus, the concept of a continuous medium filling all space became essential to physically explain the observed phenomena involving 'action-at-a-distance'. The presence of a medium helped the scientists to accept such phenomena, and the violation of 'spatiotemporal locality' condition was eliminated. Roger Boscovitch (1711–1787), who worked upon the mechanics of both Newton and Leibniz, was perhaps the first to propose a theory in which the intermediate space between two interacting bodies (without any direct physical contact) is filled up by points associated with local properties that could interact with other points and develop either attraction or repulsion. This kind of a hypothetical model may be considered as the forerunner of the later mathematically developed theory of field. Though science historians like Mary Hesse attribute 'field' concept to the hydrodynamics developed by Leonhard Euler in the true sense, the 'velocity field' belonging to a flowing fluid is very different from the concept of gravitational or electromagnetic fields. The velocity field represents the property of the material particles occupying the space. The concept of an aether filling all space was also used by the philosopher Kant (Immanuel Kant: 1723–1804).

The foundation stone for the modern field concept was first laid down by Michael Faraday (1791–1867). He could understand the phenomenon of electrical induction only by assuming the propagation of the effect was by forces filling the intervening space between the interacting bodies. Initially, he was hesitant to assert his assumption as a real physical entity; however, gradually he became more confident about the existence of the field and the associated lines of forces. Finally, it was James Clerk Maxwell who published his seminal paper in 1864 and put the field concept on a rigorous mathematical foundation. In the earlier times, the field at a point used to be considered to be a condition of the all pervading 'aether' at that location. The subject received more impetus from the urge to understand electrostatic and electromagnetic forces. The force between two stationary electric charges $+q_1$ and $+q_2$ is given by Coulomb's law as follows:

$$F = \frac{\varepsilon q_1 q_2}{r^2}$$

when r is the distance between the charges and ε is a universal constant. Augustin de Coulomb (1736–1806) framed this law in 1785. Though the law of electrostatic force is very similar to that of universal gravitation proposed by Newton almost a century before Coulomb, the gravitational force is extremely weak in comparison with electrostatic force. Thus, the gravitational attractive force between two electrons is 10^{-42} times that of the electrostatic force between them. If $+q_1$ be a stationary charge and $+q_2$ be the test charge, then the electrostatic interaction is represented by Fig. 2.30.

The electrostatic field is represented by the radially outflowing 'lines of forces' from $+q_2$, and the intensity of the field at a location drops as inversely proportional to the square of the distance (qualitatively indicated by the size of the arrow heads). Force on the test particle is given by the product of the test charge and the intensity of the field at the location of the test charge. Thus, the intensity of the electrostatic field at location $\boldsymbol{r}$ is given by

$$\boldsymbol{E}_e = \frac{\varepsilon q_1}{r^3}\boldsymbol{r} \tag{2.8.1}$$

In a similar way, the gravitational field of a stationary mass m_1 can be expressed as follows:

$$\boldsymbol{E}_g = -\frac{Gm_1}{r^3}\boldsymbol{r} \tag{2.8.2}$$

The diagrammatic representation of (2.8.2) is shown in Fig. 2.31.

Thus, the 'field lines' or the 'lines of forces' characterize the field and may be considered to be the diagrammatic representation of a field. If the test charge be considered to be always positive, then the field due to two equal charges with opposite signs becomes as shown in Fig. 2.32.

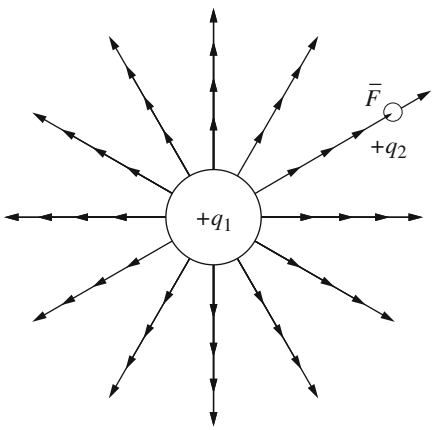


Fig. 2.30 Electrostatic field of a stationary charge

[5]Also spelled as 'ether'.

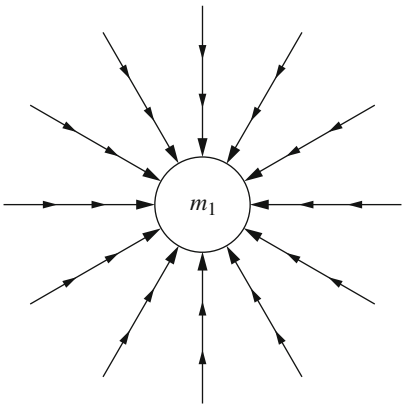


Fig. 2.31 Gravitational field of a stationary mass

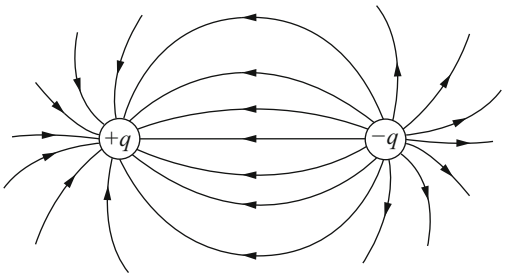


Fig. 2.32 Electrostatic field due to two equal and opposite charges

It should be noted that unlike electric charge, gravitational mass does not have any polarity and the interaction between two mass particles is always attractive. The gravitational field due to two equal masses can be represented by the 'field lines' or 'lines of forces' as shown in Fig. 2.33.

The idea of 'lines of forces' first came to Faraday's mind from his observation of the patterns generated by iron filings around a magnet (familiar with most students of science in high schools). Just like the electrostatic case, magnets also produce magnetic fields. As magnets also possess two poles —north and south—the lines of forces around a bar magnet (or a solenoid with current passing through the coil) are similar to those developed due to two equal and opposite charges as shown in Fig. 2.32. This scheme satisfactorily explains 'action-at-a-distance' phenomena. However, it is still unclear if the 'lines of forces' are real entities or not.

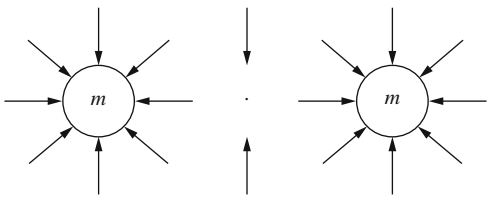


Fig. 2.33 Gravitational field of two mass particles

Experiments with gravitational field in a laboratory are difficult. But extensive research has been conducted on the electric lines of forces. If one takes a bar magnet (or a solenoid with current passing through the coil), its surrounding space is expected to be filled by the lines of forces of the associated magnetic field. If a conductor is moved through these hypothesized lines of forces keeping the magnet stationary, a current is generated due to electromagnetic induction. The cutting of the lines of forces by the conductor is considered to be the cause of the current developed. Now, if the magnet is moved keeping the conductor stationary, a current is introduced in the conductor. It looks obvious if one assumes that the lines of forces are rigidly attached to the source, i.e. the magnet. However, the results from experiments with 'Faraday's disc' based on a report by Faraday in 1832 lead to a serious problem as explained below.

Figure 2.34 shows the scheme diagrammatically. A metallic disc is attached to the end of a bar magnet with the help of a pin so that the disc can be rotated keeping the magnet still. The lines of forces coming out of the bar magnet are also indicated to pass through the metallic disc. A galvanometer can detect any current developed in the wire whose one end is connected to the central pin and the other end touches the disc's periphery. If the magnet (and the conductor with the galvanometer) is kept still and the disc is rotated, a current develops in the conducting wire. This is easy to understand as the rotating conducting disc cuts the lines of forces which are stationary along with their source, the bar magnet. But when the disc and the magnet are rotated together (so that there is no relative motion between the disc and the magnet), a current is induced. Again if the magnet is rotated keeping the rest of the system stationary, no current is induced. If the lines of the forces are assumed to be rigidly connected to their source, both the above results defy explanation. This poses a serious question on the reality of the lines of forces. However if the conducting wire of the galvanometer circuit is considered as a part of the current generating system, the phenomenon is understood. However, there are other situations where an explanation is difficult

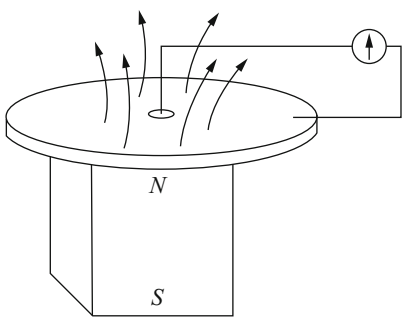


Fig. 2.34 Experiment with Faraday's disc

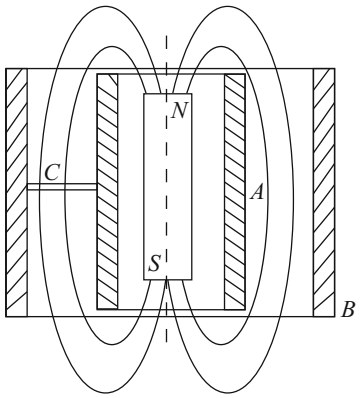


Fig. 2.35 Charged cylinder experiment

considering the lines of forces as physically real entities. Another experiment that decidedly proves that the lines of forces are not entities rigidly attached to the source magnet is briefly discussed below.

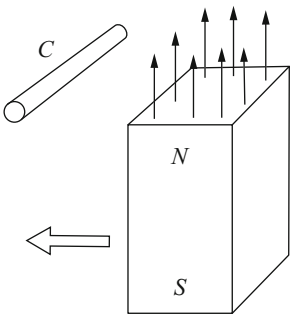
A cylindrical magnet is surrounded by two coaxial metallic cylinders *A* and *B* as shown in Fig. 2.35.

A and *B* are connected by a conducting rod *C*. Whenever the conductor *C* cuts the lines of forces, electrons flow making one of the cylinders to have an extra charge. The conductor can be cut off during the process, and the extra charge in one of the cylinders will be maintained and can be detected. When the cylinders are rotated and the magnet is kept stationary, a charge difference between *A* and *B* is developed as expected. When the cylinders are stationary and the magnet is rotated about its axis, no charge difference develops, implying that the lines of forces are not rigidly connected to the magnet. But when the cylinders and the magnet are rotated together, a charge difference is developed, again implying that the lines of forces are not rigidly connected with the magnet.

2.8.4 Field and Absolute Space

The experimental observations discussed in the previous section again bring back the question of the existence of 'absolute space'. The concept of fields, represented by the lines of forces, is one of the two possible ways to understand the phenomenon of 'action-at-a-distance' without violating spatiotemporal locality condition. The idea is that the effect, represented by a field, propagates through the universal medium 'aether'. Though there has been no direct evidence of the existence of aether, propagation of waves in the form of a disturbance in the aether can be conceived as a means of

Fig. 2.36 Conductor and a magnet with translation



transferring effects through a vacuum. Even the light is considered to propagate as waves in the aether medium. The other possible way to understand transfer of effects is through the flow of particles. Corpuscular theory of light gained acceptance because of this reason.

Now, whatever may be the medium filling all space, it is expected that it can also represent an entity like absolute space. Thus when any disturbance in this absolute space propagates, its motion is independent of the motion of other bodies including the source of the disturbance. That is why the speed of propagation of electromagnetic waves through a vacuum is a constant. The fields, described in the previous section, are also nothing but disturbances in the empty medium (may be called 'aether') . Thus, the existence of any field is characterized by the disturbance in the absolute space and, once created, has nothing to do with its source. Hence, the lines of forces described in the experiments in Sect. 1.8.3 have no connection with the source magnet and, therefore, do not share any motion. But an experiment with a conductor and a magnet with uniform translation, shown in Fig. 2.36, yields an unexpected result. A current develops! One possible explanation for this difference in results could be that rotational motion involves acceleration, whereas translation with constant speed does not.

However, there could be some link between the non-attachments of the field lines with the source and the existence of absolute space. Further discussion may tend to become more philosophical, and so the matter is closed at this point.

Doing experiments in the similar lines with gravitation is not possible. The similarity between gravitation and electric fields is derived from the similarity between Newton's law of gravitation and Coulomb's law of electrostatic force. The best testing laboratory for Newtonian gravitation is the solar system, and studying the motion of the celestial bodies deeper understanding of Newtonian gravitation can be achieved.

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