

# Chapter 2

## Modeling and Hardware Components of Control System of the OTV

### 2.1 Introduction

The design of navigation and guidance system of the OTV is carried out on the basis of the kinetic equations and dynamic equations of the control object. This chapter takes the OTV as the control object first, defines the coordinate system, analyzes the forces acting on the flight vehicle, and establishes the kinetics motion equation. In addition, it discusses the hardware components of control system of flight vehicle as well.

### 2.2 System of Coordinate Systems

#### 2.2.1 System of Proprio-coordinate

Definition of the system of proprio-coordinate  $O_bX_bY_bZ_b$ : the system of proprio-coordinate is one type of dynamic coordinate system that is connected to flight vehicle and moves along with the vehicle. Its origin is at the centroid of the flight vehicle. The  $O_bX_b$ -axis is parallel to the fuselage axis and points forward within the symmetrical plane of flight vehicle. The  $O_bZ_b$ -axis is also inside the symmetrical plane and perpendicular to the  $O_bX_b$ -axis and points downward. The  $O_bY_b$ -axis is perpendicular to the symmetrical plane and points rightward.

#### 2.2.2 Geocentric Inertial Coordinate System

Definition of geocentric inertial coordinate system  $O_eX_iY_iZ_i$ : the origin of such coordinate system is at the geocentric, the  $O_eO_eX_i$ -axis in equatorial plane points at

the mean equinox, the  $O_e Z_i$ -axis is perpendicular to the equatorial plane and coincides with the Earth rotation direction, and the  $O_e Y_i$ -axis is determined according to right-handed coordinate system.

### 2.2.3 Launching Coordinate System

Definition of launching coordinate system  $OXYZ$ : the coordinate origin coincides with the launch point  $O$ , that is, the  $OX$ -axis in the horizontal plane of launching point points at the launching direction. The  $OY$ -axis points at upward, that is, perpendicular to the horizon of launching point. The  $OZ$ -axis is perpendicular to the  $OXY$  plane and forms the right-handed coordinate system. Since the launching point  $O$  rotates with the Earth, the launching coordinate system is the one moving coordinate system.

### 2.2.4 Launching Inertial System

Definition of launching inertial coordinate system  $O_a X_a Y_a Z_a$ : At the instant of flight vehicle takes off, the origin  $O_a$  coincides with the launching point, and the axles are coinciding with that of the launching coordinate system. After the flight vehicle takes off, the origin  $O_a$  and the axial directions of the coordinate system are kept static in inertial space.

### 2.2.5 Orbital Coordinate System of Injection Point

The orbital coordinate system of injection point  $O_{ocf} X_{ocf} Y_{ocf} Z_{ocf}$ : the origin is selected at the geocentric, the  $OY_{ocf}$ -axis is the joint line between the geocentric and injection point upward positive (that is away from the Earth),  $OX_{ocf}$ -axis is perpendicular to  $OY_{ocf}$ -axis, and parallel to the local level of the injection point, pointing at the motional direction of the flight vehicle. The  $OZ_{ocf}$ -axis,  $OX_{ocf}$ -axis, and  $OY_{ocf}$ -axis make up the right-hand rule.

### 2.2.6 Relations of Coordinate System Transformation

(1) From Launching Coordinate System to Launching Inertial Coordinate System

The transformational matrix from launching coordinate system to launching inertial coordinate system is shown with  $A_{lcf}^{acf}$ ,

$$A_{lcf}^{acf} = D^T \bullet M_Z^T(\omega_0 t) \bullet D \quad (2.2.1)$$

Here, the superscript T represents the transposition,  $M_Z$  represents the rotation along Z-axis (Similarly,  $M_Y$  and  $M_X$  represent rotation along Y-axis and X-axis respectively,  $\omega_0$  is the angular velocity of the Earth,  $D$  is the matrix related to launching azimuth  $A_0$  and launching latitude  $B_0$ , and the specific mode refers to the documentation [14]).

- (2) Transformational matrix from launching inertial coordinate system to the orbital coordinate system of injection point is shown with  $A_{acf}^{ocf}$ ,

$$A_{acf}^{ocf} = M_Z(-U) \bullet M_Y(i) \bullet M_Z(-\Delta\Omega) \bullet M_Y(-90^\circ) \bullet M_Z(B_0) \bullet M_Y(A_0) \quad (2.2.2)$$

Here,  $\Delta\Omega$  is the difference between the right ascension of ascending node  $\Omega$  and the right ascension in  $OY$ -axis of the launching inertial coordinate system,  $i$  is the orbit inclination, and  $U$  is the geocentric angle, that is, the argument of latitude.

- (3) Transformational matrix from launching inertial coordinate system to the system of proprio-coordinate is shown with  $A_{acf}^{bcf}$ .

$$A_{acf}^{bcf} = M_X(\gamma) \bullet M_Y(\psi) \bullet M_Z(\varphi) \quad (2.2.3)$$

Here,  $\varphi$  is the pitch angle,  $\psi$  is the yaw angle, and  $\gamma$  is the roll angle.

- (4) Transformational matrix from geocentric inertial coordinate system to launching inertial coordinate system is shown with  $A_{ecf}^{acf}$ ,

$$A_{ecf}^{acf} = M_Y(-90^\circ - A_0) \bullet M_X(B_0) \bullet M_Z(\lambda_0 - 90^\circ) \bullet M_Z(\Omega_G) \quad (2.2.4)$$

Here,  $\Omega_G$  is the included angle between the  $O_EX_i$ - and  $O_EX_E$ -axes of the geocentric coordinate system, related to the moment of launching (the definition of the geocentric coordinate system refers to the documentation [14]).

## 2.3 Mechanical Model

In analyzing the load of the OTV, take it as the mass point. The mass-center dynamics equation is

$$m \frac{dv_a}{dt} = G + G_2 + P \quad (2.3.1)$$

Here,  $m$  is the mass of flight vehicle,  $v_a$  is the absolute velocity vector of the mass-center of flight vehicle, the resultant external force acted are the engine thrust  $P$ , gravitational attraction  $G$ , and solar–lunar perturbation gravity  $G_2$ . When the motor does not work, the external forces acting on the flight vehicle are the gravitational attraction and solar–lunar perturbation gravity.

### 1. Gravitational attraction

Within the range of flight height of the OTV, the gravitational attraction is the important external force, whose volume is related to the orbital height.

### 2. Solar–lunar gravity

When the flight vehicle revolves around the Earth, it is affected not only by the central body—the Earth— but also by the gravity of the Moon and the Sun. Here, other celestial bodies beyond the central celestial body are called perturbed celestial body. The flight vehicle is called disturbed body. The central celestial body and disturbed body are taken as the mass point.

### 3. Engine thrust

The control force of the OTV is provided by the sustainer motor, which fulfills ignition action according to the control command, produces thrust to change the flight track of the OTV, realizes orbital maneuver, and tracks the brake. Before the sustainer motor is ignited, it needs to adjust the attitude of vehicle to correct the thrust direction of the motor. The task of adjusting attitude is fulfilled by attitude control nozzle.

Besides this, the flight vehicle is also loaded by the external forces including thin atmospheric drag, solar radiation pressure, tide perturbation, and the third problem gravitational perturbation, since the magnitudes of these external forces are basically between  $10^{-15} \sim 10^{-12} km/s^2$ , whose influence is usually omitted.

## 2.4 Equations of Motion

### 2.4.1 Mass-Center Dynamics and Kinematics Equation

Set up the mass-center dynamic equation in the launching inertial system and present the following formulas directly without derivation:

$$m \begin{bmatrix} \frac{dv_{xicf}}{dt} \\ \frac{dv_{yicf}}{dt} \\ \frac{dv_{zicf}}{dt} \end{bmatrix} = \left( A_{icf}^{bcf} \right)^T \begin{bmatrix} F + F_{cicfx} \\ F_{cicfy} \\ F_{cicfz} \end{bmatrix} + m \frac{g'_r}{r} \begin{bmatrix} x_{icf} \\ y_{icf} \\ z_{icf} \end{bmatrix} + m \frac{g_{\omega e}}{\omega_0} \begin{bmatrix} \omega_{0xicf} \\ \omega_{0yicf} \\ \omega_{0zicf} \end{bmatrix}. \quad (2.4.1)$$

Here,  $F$  is the engine thrust. Assuming it as constant,  $[F_{cicfx} \ F_{cicfy} \ F_{cicfz}]^T$  is the thrust of attitude control engine,  $m \frac{g'_r}{r} [x_{icf} \ y_{icf} \ z_{icf}]^T + m \frac{g_{\omega e}}{\omega_0} [\omega_{0xicf} \ \omega_{0yicf} \ \omega_{0zicf}]^T$  is the component of gravitational acceleration in launching inertial coordinate system when takes the Earth as the ellipsoid, and  $[F'_{kx1icf} \ F'_{ky1icf} \ F'_{kz1icf}]^T$  is the additional quantity caused by the second consumption of engine.

The mass-center motional equation is

$$\begin{cases} \frac{dx_{icf}}{dt} = v_{xicf} \\ \frac{dy_{icf}}{dt} = v_{yicf} \\ \frac{dz_{icf}}{dt} = v_{zicf} \end{cases} \quad (2.4.2)$$

Thus, all the mass-center translation dynamics and kinetics equations have been given.

### 2.4.2 Around Mass-Center Dynamics and Kinematics Equation

Establish the rotation around mass-center dynamics equation in body coordinate system and present the following directly without derivation:

$$\begin{bmatrix} I_{xbcf} & 0 & 0 \\ 0 & I_{ybcf} & 0 \\ 0 & 0 & I_{zbcf} \end{bmatrix} \begin{bmatrix} \dot{\omega}_{xbcf} \\ \dot{\omega}_{ybcf} \\ \dot{\omega}_{zbcf} \end{bmatrix} = - \begin{bmatrix} (I_{z1} - I_{y1}) \omega_{zbcf} \omega_{ybcf} \\ (I_{x1} - I_{z1}) \omega_{xbcf} \omega_{zbcf} \\ (I_{y1} - I_{x1}) \omega_{ybcf} \omega_{xbcf} \end{bmatrix} + \begin{bmatrix} M_{xcbcf} \\ M_{ybcf} \\ M_{zbcf} \end{bmatrix}. \quad (2.4.3)$$

Here, the definitions of the expressions are as follows:

$\begin{bmatrix} I_{xbcf} & 0 & 0 \\ 0 & I_{ybcf} & 0 \\ 0 & 0 & I_{zbcf} \end{bmatrix}$  is the inertia matrix of the OTV in body coordinate system.

$[\dot{\omega}_{xbcf} \ \dot{\omega}_{ybcf} \ \dot{\omega}_{zbcf}]^T$  is the component of angular acceleration of the OTV in body coordinate system.

$[\omega_{xbcf} \ \omega_{ybcf} \ \omega_{zbcf}]^T$  is the component of angular acceleration of the OTV in body coordinate system.

$[M_{xcbcf} \ M_{ycbcf} \ M_{zcbcf}]^T$  is the control torque produced by the control actuator of the OTV.

The rotation around mass-center kinematics equation is as follows:

$$\begin{bmatrix} \dot{\gamma} \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & \sin \gamma \tan \psi & \cos \gamma \tan \psi \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma / \cos \psi & \cos \gamma / \cos \psi \end{bmatrix} \begin{bmatrix} \omega_{nbx}^b \\ \omega_{nby}^b \\ \omega_{nbz}^b \end{bmatrix}. \quad (2.4.4)$$

Here,  $\gamma$ ,  $\psi$ , and  $\theta$  are the three Eulerian angles.

### 2.4.3 Orbital Elements

The position and velocity of flight vehicle, besides shown with the numerical integration of mass-center kinematics equation, could also be expressed by the analytic method of orbital elements. The six orbital elements corresponding to the position and velocity vectors are the major semi-axis  $a$ , eccentricity  $e$ , inclination of orbit  $i$ , right ascension of ascending node  $\Omega$ , argument of perigee  $\omega$ , and eccentric anomaly  $E$ . The basic equations are

$$\begin{cases} \frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} [e(S \sin f + T \cos f) + T] \\ \frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} [(S \sin f + T \cos f) + T \cos E] \\ \frac{di}{dt} = \left( \frac{rW}{na^2\sqrt{1-e^2}} \right) \cos(f + \omega) \\ \frac{d\omega}{dt} = -\cos i \frac{d\Omega}{dt} + \frac{1}{nae} \left[ \sqrt{1-e^2} (-S \cos f + T \sin f) + T \sin E \right] \\ \frac{dE}{dt} = \frac{a}{r} \left[ n - \sqrt{1-e^2} \left( \frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} \right) + \sin E \left( \frac{de}{dt} \right) \right] - \frac{2}{na} S \end{cases} \quad (2.4.5)$$

Here,  $n = \sqrt{\mu a^{-3/2}}$ ,  $\mu$  is the gravitational constant,  $f$  is the true anomaly, while  $\sin f$  and  $\cos f$  may be given by  $\sin E$  and  $\cos E$ , that is,

$$\begin{cases} r \sin f = a\sqrt{1-e^2} \sin E \\ r \cos f = a(\cos E - e) \\ r = a(1 - e \cos E) \end{cases} \quad (2.4.6)$$

Here,  $S$ ,  $T$ , and  $W$  are the three components of acceleration vectors. The relations with the resultant external force are

$$S = F \bullet \hat{r}, T = F \bullet \hat{t}, W = F \bullet \hat{w}. \quad (2.4.7)$$

Here,  $\hat{r}$ ,  $\hat{t}$ , and  $\hat{w}$  are the respective unit vectors of radial, horizontal, and normal orbit, and there is

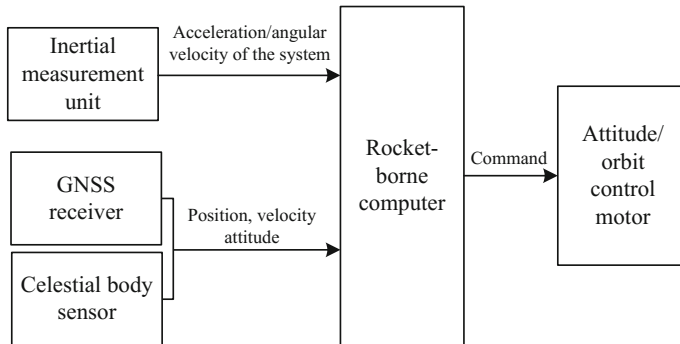
$$\begin{cases} \hat{r} = \cos u \hat{P}_* + \sin u \hat{Q}_* \\ \hat{t} = -\sin u \hat{P}_* + \cos u \hat{Q}_* \\ \hat{w} = \hat{r} \times \hat{t} \end{cases} \quad (2.4.8)$$

Here,  $u = f + w$ ; the expressions of unit vector  $\hat{P}_*$  and  $\hat{Q}_*$  are

$$\hat{P}_* = \begin{pmatrix} \cos \Omega \\ \sin \Omega \\ 0 \end{pmatrix}, \hat{Q}_* = \begin{pmatrix} -\sin \Omega \cos i \\ \cos \Omega \cos i \\ \sin i \end{pmatrix}. \quad (2.4.9)$$

## 2.5 Hardware Components of Control System

The hardware of OTV control system commonly consists of inertial measurement unit, GNSS signal receiver, celestial sensor, space-borne computer, and attitude control motor. Its structure is shown in Fig. 2.1. The inertial measurement unit forms the state measurement system with inertial instrument (gyroscope and accelerometer) and determines the information of position, velocity, and attitude of flight vehicle during the flight autonomously. The GNSS signal receiver receives the navigation information provided by satellites. The celestial sensor observes celestial body and obtains navigation information of the flight vehicle. The space-borne computer analyzes, discriminates, and solves the measurement information in various channels; carries out analysis; calculates the navigation parameters of attitude, velocity, and position of the vehicle; and forms control command,



**Fig. 2.1** Hardware components of the OTV

to control the attitude/orbital control motor to realize the missions of scheduled attitude adjustment and orbital motion.

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