

Adaptive Observer Design for Quasi-one-sided Lipschitz Nonlinear Systems

Jun Huang, Lei Yu and Minjie Shi

Abstract This paper deals with the adaptive observer design problem for quasi-one-sided Lipschitz nonlinear systems. First, some useful assumptions are presented for the observer design purpose. Then, under the assumptions, an adaptive observer is constructed for the nonlinear system. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords Adaptive observer • Quasi-one-sided Lipschitz • Nonlinear systems

1 Introduction

With the development of theory and application, state estimation or observer for nonlinear systems has been studied widely in the field of control [1–4]. The literature of observer design problem mainly focuses on two aspects. One is that different types of observers for nonlinear systems have been investigated, such as adaptive observers [5, 6], H_∞ observers [7, 8], interval observers [9, 10] and so on. The other is that different design methods have been proposed for observers, [11] presented a linear matrix inequality (LMI) approach to design observer for one-sided Lipschitz nonlinear systems. [12] used Riccati equations to deal with full-order and reduced-order observers design problem for one-sided Lipschitz nonlinear systems. In view of different conditions on set-valued functions in Lur'e differential inclusion systems, [13] constructed the observer for the system under dissipative approach. [14, 15] designed the observer for the system by positive real method.

Recently, one-sided Lipschitz condition has been introduced for the control theory. [16, 17] firstly studied one-sided Lipschitz condition for observer design problem in nonlinear systems. The region of one-sided Lipschitz constant is much larger than that of traditional Lipschitz condition, because the one-sided Lipschitz constant can be zero or even negative while the Lipschitz constant must be positive.

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This means that one-sided Lipschitz condition is more general than Lipschitz condition. Following the line of [16–18] considered the observer design problem and presented the standard one-sided Lipschitz condition. [11, 12] gave different methods to design observers for one-sided Lipschitz nonlinear systems respectively. [19] improved the result of [16] and derived sufficient condition for the existence of observer of one-sided Lipschitz nonlinear systems.

It is worth noting that P -one-sided Lipschitz condition (modified one-sided Lipschitz condition) and quasi-one-sided Lipschitz condition have been introduced in [16, 17]. [16] illustrated that P -one-sided Lipschitz condition is less conservative than one-sided Lipschitz condition because a fixed symmetric definite matrix P broadens the range of nonlinear function $f(x)$ in the system. Meanwhile, [17] introduced a quasi-one-sided Lipschitz condition to estimate the influence of nonlinear function on the observer and verified that the quasi-one-sided Lipschitz condition is an extension of the P -one-sided Lipschitz condition. Under quasi-one-sided Lipschitz condition, [20] considered the robust stabilization problem of nonlinear networked control systems and established a less conservative sufficient condition. [21] addressed the problem of state feedback and output feedback control for a class of nonlinear systems, and the nonlinearity was assumed to satisfy global quasi-one-sided Lipschitz condition. For one-sided Lipschitz and quasi-one-sided Lipschitz systems, [22] presented fault detection algorithm based on sliding mode observer. [23] employed both Lyapunov function approach and linear matrix inequality technique to study finite-time H_∞ control problem of quasi-one-sided Lipschitz nonlinear systems with parameter uncertainties. It should be noted that the nonlinear function in [20–23] satisfies quasi-one-sided Lipschitz condition as well as quadratically inner-bounded condition. If the nonlinear function in the system is not quadratically inner-bounded, how can we design the observer for the system? There seems no available answer to it.

Motivated by above discussion, this paper investigates the adaptive observer design problem for quasi-one-sided Lipschitz nonlinear systems with uncertain parameters. The nonlinear function of the system does not satisfy the property of quadratic inner-boundedness. The contribution of this paper mainly lies in two aspects: One is that the adaptive observer design method is extended to quasi-one-sided Lipschitz nonlinear systems, and the quasi-one-sided Lipschitz term is more general than the standard one-sided Lipschitz term. The other is that a tractable method is provided to deal with the nonlinear function without quadratic inner-boundedness. The paper is organized as follows: Sect. 2 presents the problem formulation and some preliminaries. Section 3 designs the adaptive observer for quasi-one-sided Lipschitz nonlinear systems. Section 4 illustrates the effectiveness of the designed adaptive observer by numerical example.

The notations used in this paper are as follows: $\|x\|$ and $\|A\|$ mean the Euclidean norm of the vector x and the Euclidean norm of the matrix A respectively, x^T is the transposition of the vector x , and A^T denotes the transposition of the matrix A . I is the identity matrix. $P > (<)0$ represents the positive (negative) definite matrix P with $P = P^T$. $\langle \cdot, \cdot \rangle$ stands for the inner product, i.e., given $x, y \in R^n$, then $\langle x, y \rangle = x^T y$.

2 Problem Formulation and Preliminaries

Consider the following nonlinear system

$$\begin{cases} \dot{x} = Ax + Bf_1(Hx, u) + Df_2(x, u)\theta, \\ y = Cx, \end{cases} \quad (1)$$

where $x \in R^n$ is the state of the system, $u \in R^r$ is the control input, and $y \in R^q$ is the measurable output. $\theta \in R^l$ is the unknown constant vector, $f_1(\cdot, \cdot) : R^n \times R^r \rightarrow R^m$ and $f_2(\cdot, \cdot) : R^n \times R^r \rightarrow R^{k \times l}$ are given nonlinear functions. A, B, H, D, C are known real matrices with appropriate dimensions. Without loss of generality, it is assumed that C is of full row rank.

Firstly, let us recall some basics of one-sided Lipschitz function, more details can be referred to [16, 17].

Definition 1 (*Standard one-sided Lipschitz condition*) $f(v, u)$ is said to be one-sided Lipschitz with respect to v if there exists a constant $\rho \in R$ such that

$$\langle f(v, u) - f(w, u), v - w \rangle \leq \rho \|v - w\|^2, \quad (2)$$

holds for any $v, w \in R^m$, where ρ is called the one-sided Lipschitz constant. The real number ρ can be positive, zero, or even negative.

Definition 2 (*P-one-sided Lipschitz condition*) $f(v, u)$ is said to be P -one-sided Lipschitz with respect to v if there exists a matrix $P > 0$ such that

$$\langle \Phi(v, u) - \Phi(w, u), v - w \rangle \leq v_p \|v - w\|^2, \quad (3)$$

holds for any $v, w \in R^m$, where $\Phi(v, u) = Pf(v, u)$. The real number v_p is called the P -one-sided Lipschitz constant, which can be positive, zero, or even negative.

Definition 3 (*Quasi-one-sided Lipschitz condition*) $f(v, u)$ is said to be quasi-one-sided Lipschitz with respect to v if there exist matrices $P > 0$ and M such that

$$\langle \Phi(v, u) - \Phi(w, u), v - w \rangle \leq (v - w)^T M (v - w), \quad (4)$$

holds for any $v, w \in R^m$, where $\Phi(v, u) = Pf(v, u)$. The real symmetric matrix M is called the quasi-one-sided Lipschitz constant matrix.

In order to obtain the main result, the following assumptions and lemmas are needed.

Assumption 1 $f_1(v, u)$ is quasi-one-sided Lipschitz function, i.e., there exist matrices $P > 0$ and M such that

$$\langle \Phi_1(v, u) - \Phi_1(w, u), v - w \rangle \leq (v - w)^T M (v - w), \quad (5)$$

holds for any $v, w \in R^m$, where $\Phi_1(v, u) = Pf_1(v, u)$.

Assumption 2 $f_2(x, u)$ is Lipschitz function, i.e., there exists a constant $\eta > 0$ such that

$$\|f_2(x, u) - f_2(\hat{x}, u)\| \leq \eta \|x - \hat{x}\|, \quad (6)$$

holds for any $x, \hat{x} \in R^n$, and the constant η is Lipschitz constant.

Assumption 3 The unknown parameter vector θ is bounded with $\mu > 0$, i.e.,

$$\|\theta\| \leq \mu. \quad (7)$$

Assumption 4 Let $\beta = \eta\mu\|D\|$, where η and μ are defined in (6) and (7). There exist constant $\varepsilon > 0$, matrices $Q > 0$, L , F , N such that

$$\begin{aligned} & Q(A - LC) + (A - LC)^T Q + \beta Q^2 \\ & + 2(H - FC)^T M(H - FC) + (\beta + \varepsilon)I \leq 0, \end{aligned} \quad (8)$$

$$B^T Q = P(H - FC), \quad (9)$$

$$D^T Q = NC. \quad (10)$$

Lemma 1 [24] Let $\varphi : R^+ \rightarrow R$ be a known function. If φ is uniformly continuous and the integral $\int_0^\infty \varphi(s)ds$ exists, then $\lim_{t \rightarrow \infty} \varphi(t) = 0$.

Lemma 2 For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ with $S_{11}^T = S_{11}$ and $S_{22}^T = S_{22}$, the following conditions are equivalent:

- (1) $S < 0$,
- (2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,
- (3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

3 Main Result

The adaptive observer for the system (1) is designed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bf_1(H\hat{x} + F(y - C\hat{x}), u) \\ \quad + Df_2(\hat{x}, u)\hat{\theta} + L(y - C\hat{x}), \\ \dot{\hat{y}} = C\hat{x}, \end{cases} \quad (11)$$

with

$$\dot{\hat{\theta}} = f_2^T(\hat{x}, u)N(y - C\hat{x}). \quad (12)$$

Denote that $e = x - \hat{x}$, $\tilde{\theta} = \theta - \hat{\theta}$, $v = Hx$, $w = H\hat{x} + F(y - C\hat{x})$ and $\tilde{f}_1 = f_1(v, u) - f_1(w, u)$, $\tilde{f}_2 = f_2(x, u) - f_2(\hat{x}, u)$, $\hat{f}_2 = f_2(\hat{x}, u)$. Subtracting (11) from (1) results in the error system:

$$\dot{e} = (A - LC)e + B\tilde{f}_1 + D\tilde{f}_2\theta + D\hat{f}_2\tilde{\theta}. \quad (13)$$

We can now state the theorem as follows.

Theorem 1 *If Assumptions 1–4 hold, then (11) is an adaptive observer for the system (1), i.e., $e(t)$ of the error system (13) satisfies $\lim_{t \rightarrow \infty} e(t) = 0$.*

Proof Let us consider the following Lyapunov function candidate

$$V = e^T Q e + \tilde{\theta}^T \tilde{\theta}. \quad (14)$$

Along the trajectories of the error system (13), the derivative of V can be calculated as

$$\begin{aligned} \dot{V} &= 2e^T Q \dot{e} + 2\tilde{\theta}^T \dot{\tilde{\theta}} \\ &= 2e^T Q(A - LC)e + 2e^T Q B \tilde{f}_1 + 2e^T Q D \tilde{f}_2 \theta \\ &\quad + 2e^T Q D \hat{f}_2 \tilde{\theta} + 2\tilde{\theta}^T \dot{\tilde{\theta}}. \end{aligned} \quad (15)$$

By Assumption 1 and (9), we have

$$\begin{aligned} 2e^T Q B \tilde{f}_1 &= 2e^T (H - FC)^T P \tilde{f}_1 \\ &= 2\langle P \tilde{f}_1, (H - FC)e \rangle \\ &\leq 2e^T (H - FC)^T M (H - FC)e. \end{aligned} \quad (16)$$

By Assumptions 2 and 3, the following holds

$$\begin{aligned} 2e^T Q D \tilde{f}_2 \theta &\leq 2\|e^T Q\| \|D\| \|\tilde{f}_2\| \|\theta\| \\ &\leq 2\eta\mu \|D\| \|e^T Q\| \|e\| \\ &\leq \beta(e^T Q^2 e + e^T e). \end{aligned} \quad (17)$$

From (10) and (12), we can obtain that

$$\begin{aligned} 2e^T Q D \hat{f}_2 \tilde{\theta} + 2\tilde{\theta}^T \dot{\tilde{\theta}} &= 2e^T Q D \hat{f}_2 \tilde{\theta} - 2\tilde{\theta}^T \dot{\hat{\theta}} \\ &= 2e^T Q D \hat{f}_2 \tilde{\theta} - 2\tilde{\theta}^T \hat{f}_2^T D^T Q e = 0. \end{aligned} \quad (18)$$

Substituting (16)–(18) into (15) yields

$$\begin{aligned} \dot{V} \leq & e^T [Q(A - LC) + (A - LC)^T Q + \beta(Q^2 + I) \\ & + 2(H - FC)^T M(H - FC)]e. \end{aligned} \quad (19)$$

It follows from (8) that

$$\dot{V} \leq -\varepsilon e^T e. \quad (20)$$

Integrating both sides from 0 to t yields

$$V(t) \leq V(0) - \int_0^t \varepsilon e^T(s) e(s) ds. \quad (21)$$

Since $V(t) > 0$, (21) implies $\int_0^t \varepsilon e^T(s) e(s) ds \leq V(0)$, which deduces to

$$\lim_{t \rightarrow \infty} \int_0^t \varepsilon e^T(s) e(s) ds \leq V(0) < \infty. \quad (22)$$

By Lemma 1, we obtain

$$\lim_{t \rightarrow \infty} \varepsilon e^T(t) e(t) = 0, \quad (23)$$

which means that

$$\lim_{t \rightarrow \infty} e(t) = 0. \quad (24)$$

Thus, the proof is completed.

Remark 1 The function $f_1(v, u)$ does not satisfy the quadratically inner-bounded condition, thus the methods proposed in the former works such as [20–23] can not be applied in this paper.

Remark 2 Different from the system with P -one-sided Lipschitz term studied in [25], we consider the system with quasi-one-sided Lipschitz term and uncertain parameters in this paper. Besides, the pair (P, M) can be obtained by solving quasi-one-sided Lipschitz condition constraint (5).

Remark 3 By using elimination of matrix variables, the observer gain L can be given as $L = \frac{\sigma}{2} Q^{-1} C^T$ in Theorem 1.

Remark 4 As we know, how to find a suitable one-sided Lipschitz function which can include standard one-sided Lipschitz function, P -one-sided Lipschitz function and quasi-one-sided Lipschitz function is still an open problem, and the function that we use is affected by the method of converting nonlinear matrix inequality to linear matrix inequality, the practical model, the generality of one-sided Lipschitz nonlinear system and so on.

4 Numerical Example

Consider the system (1) with

$$A = \begin{bmatrix} -1 & -2 \\ 1 & -1.2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \quad u = \sin t,$$

$$f_1(Hx, u) = \begin{bmatrix} 0.5u \\ \sin(x_1 + x_2) - x_2^{\frac{1}{3}} \end{bmatrix}, \quad f_2(x, u) = \sin x_1.$$

It was verified that $f_1(Hx, u)$ is not quadratically inner-bounded in [25]. Meanwhile, according to the mean-value theorem, there exists non-zero $\xi_0 \in (\min(x_2, \hat{x}_2), \max(x_2, \hat{x}_2))$ and $\xi = (\xi_1, \xi_2) \in Co(x, \hat{x})$ such that

$$\begin{aligned} & \langle Pf_1(Hx, u) - Pf_1(H\hat{x}, u), H(x - \hat{x}) \rangle \\ &= \alpha[(\cos(H_1\xi), -\cos(H_1\xi))(x - \hat{x})(x_2 - \hat{x}_2) \\ &\quad - \frac{\alpha}{3}\xi_0^{-\frac{2}{3}}(x_2 - \hat{x}_2)^2] \\ &\leq \alpha|(\cos(H_1\xi), -\cos(H_1\xi))| \|(x - \hat{x})(x_2 - \hat{x}_2)\| \\ &\leq (H(x - \hat{x}))^T \begin{bmatrix} 1.42\alpha & 0 \\ 0 & 1.42\alpha \end{bmatrix} H(x - \hat{x}), \end{aligned} \tag{25}$$

where $H_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $f_1(Hx, u)$ is quasi-one-sided Lipschitz and $M = v_P$
 $I = \begin{bmatrix} 1.42\alpha & 0 \\ 0 & 1.42\alpha \end{bmatrix}$ for any $P = \text{diag}\{\lambda, \alpha\}$.

In the meantime, $f_2(x, u)$ is globally Lipschitz with the Lipschitz constant $\eta = 1$. Selecting the unknown parameter $\theta = 0.5$, we can obtain that $\mu = 0.5$. Let $P = I$, this means $v_P = 1.42$. Based on the above results, we solve (8)–(10) by Scilab, then $Q = \begin{bmatrix} 41.2694 & 1 \\ 1 & 1 \end{bmatrix}$, $L = \begin{bmatrix} 171.6597 \\ -111.1394 \end{bmatrix}$, $F = \begin{bmatrix} -40.2694 \\ -1 \end{bmatrix}$, $N = 4.0269$, $\varepsilon = 0.0475$. We now use the Simulink in Matlab to complete the simulation. As shown in Figs. 1, 2, the estimated state trajectories of the adaptive observer (11) converge to the state trajectories of the system (1). Figure 3 shows the estimated parameter $\hat{\theta}$, and it does not converge to the nominal value $\theta = 0.5$. From the simulation results, we can conclude the observer design method is valid in this paper.

Fig. 1 The state x_1 of system (1) and the estimated state $\hat{x}_1$ of adaptive observer (11)

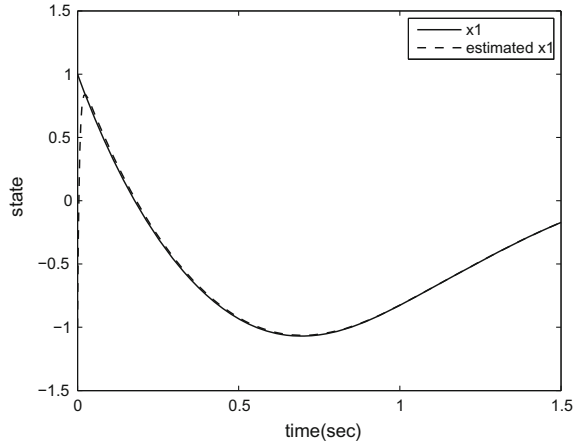


Fig. 2 The state x_2 of system (1) and the estimated state $\hat{x}_2$ of adaptive observer (11)

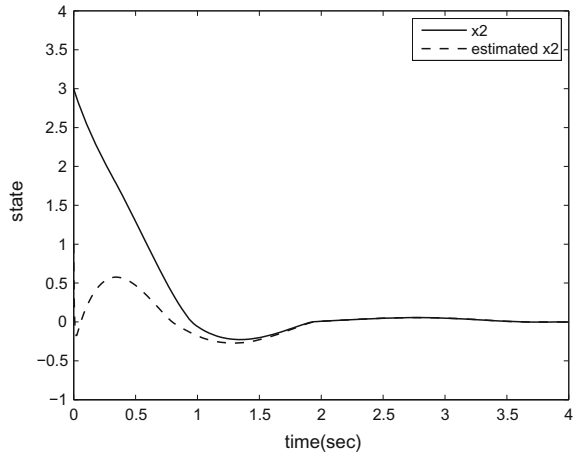
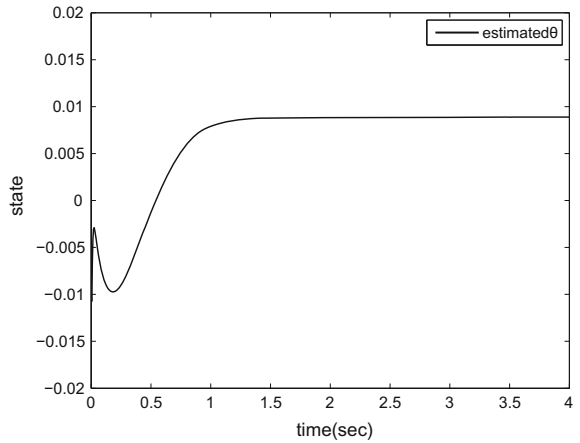


Fig. 3 The estimated parameter $\hat{\theta}$ of adaptive observer (11)



5 Conclusion

We consider the adaptive observer design problem for quasi-one-sided Lipschitz nonlinear systems in this paper. Based on some assumptions, we construct an adaptive observer for the system. Then, we verify that the error system is asymptotically stable by Lyapunov stability theory. Finally, we also simulate the numerical example to show the effectiveness of the proposed method.

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