

Chapter 2

Transfer Efficiency Analysis

Transfer efficiency is one of the most crucial aspects in a WPT system. High transfer efficiency means small energy loss, easy thermal design, etc. System parameters affect transfer efficiency and mismatched parameters may lead to low transfer efficiency. In-depth analysis of the impact factors on transfer efficiency should be offered.

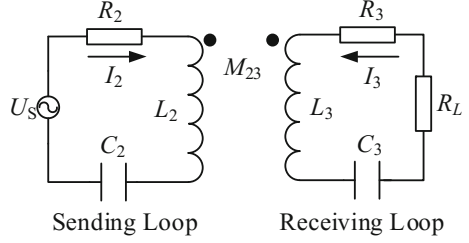
In a WPT system, the source can be categorized into two types: linear power supply (analog source) and switching power supply (power electronics source). The switches in the linear power supply work in the linear area, leading to large switching loss and thus low source efficiency. In comparison, the switches in the switching power supply work in the switching area, resulting in small switching loss and thus high source efficiency. Due to the restraint of current power electronics semiconductor technology, it is difficult for power electronics sources to output high power at high frequency. To employ power electronics sources, the resonant frequency of WPT systems should be reduced. The impact of decreasing the resonant frequency on transfer efficiency in a WPT system should be investigated.

In this chapter, two basic structures in a WPT systems, namely the two-coil structure and the four-coil structure, are modelled and analyzed. Load matching factor and transfer quality factor are introduced to unify the transfer efficiency expressions in these two basic structures. Load matching condition is studied. Two methods of reducing the resonant frequency are compared.

2.1 Modelling and Derivation

The two-coil structure and the four-coil structure, as shown in Fig. 1.3, are modelled and analyzed as follows.

Fig. 2.1 Equivalent circuit of two-coil structure



2.1.1 Two-Coil Structure

Based on CT, the equivalent circuit of the two-coil structure is shown in Fig. 2.1.

Assuming that the resonant frequencies of the sending loop and the receiving loop are the same, we have

$$f_0 = \frac{1}{2\pi\sqrt{L_2 C_2}} = \frac{1}{2\pi\sqrt{L_3 C_3}} \quad (2.1)$$

At the resonant frequency, the reflected impedance from the receiving loop to the sending loop is

$$Z_{\text{ref}23} = \frac{(\omega_0 M_{23})^2}{R_3 + R_L} \quad (2.2)$$

The system input impedance of the two-coil structure is

$$Z_{\text{in}} = R_2 + \frac{(\omega_0 M_{23})^2}{R_3 + R_L} \quad (2.3)$$

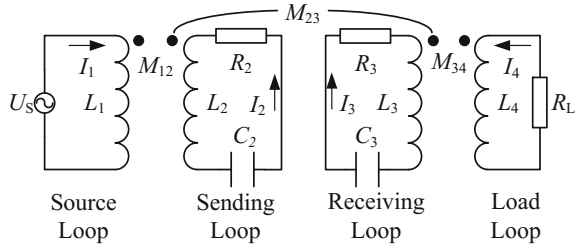
The transfer efficiency of the two-coil structure can be obtained by the product of the efficiencies on the sending loop and the receiving loop, expressed as

$$\eta_{\text{tm}} = \frac{Z_{\text{ref}23}}{R_2 + Z_{\text{ref}23}} \frac{R_L}{R_3 + R_L} = \frac{\left(\frac{\omega_0 M_{23}}{\sqrt{R_2 R_3}}\right)^2}{1 + \frac{R_L}{R_3} + \left(\frac{\omega_0 M_{23}}{\sqrt{R_2 R_3}}\right)^2} \frac{\frac{R_L}{R_3}}{1 + \frac{R_L}{R_3}} \quad (2.4)$$

2.1.2 Four-Coil Structure

Based on CT, the equivalent circuit of the four-coil structure is shown in Fig. 2.2.

The equivalent resistances of the source coil and the load coil and the mutual inductances of non-adjacent coils are ignored. Normally the source coil and the load

Fig. 2.2 Equivalent circuit of four-coil structure

coil are small coils, whose turn numbers are far fewer than those of the sending coil and the receiving coil. Compared with the load resistance, the inductance of the load coil can be omitted. At the resonant frequency, the reflected impedance from the load coil to the receiving coil is

$$Z_{\text{ref}34} = \frac{(\omega_0 M_{34})^2}{R_L} \quad (2.5)$$

The reflected impedance from the receiving coil to the sending coil is

$$Z_{\text{ref}23} = \frac{(\omega_0 M_{23})^2}{R_3 + Z_{\text{ref}34}} = \frac{(\omega_0 M_{23})^2}{R_3 + \frac{(\omega_0 M_{34})^2}{R_L}} \quad (2.6)$$

The reflected impedance from the sending coil to the source coil is

$$Z_{\text{ref}12} = \frac{(\omega_0 M_{12})^2}{R_2 + Z_{\text{ref}23}} = \frac{(\omega_0 M_{12})^2}{R_2 + \frac{(\omega_0 M_{23})^2}{R_3 + \frac{(\omega_0 M_{34})^2}{R_L}}} \quad (2.7)$$

The system input impedance is

$$Z_{\text{in}} = j\omega_0 L_1 + \frac{(\omega_0 M_{12})^2}{R_2 + \frac{(\omega_0 M_{23})^2}{R_3 + \frac{(\omega_0 M_{34})^2}{R_L}}} \quad (2.8)$$

Because their equivalent resistances are ignored, the source coil and the load coil are assumed to be lossless. Similar to the two-coil structure, the transfer efficiency of the four-coil structure can be calculated by the product of the efficiencies of the sending loop and the receiving loop, expressed as

$$\eta_{\text{tm}} = \frac{Z_{\text{ref}23}}{R_2 + Z_{\text{ref}23}} \frac{Z_{\text{ref}34}}{R_3 + Z_{\text{ref}34}} = \frac{\left(\frac{\omega_0 M_{23}}{\sqrt{R_2 R_3}} \right)^2}{1 + \frac{(\omega_0 M_{34})^2}{R_3 R_L} + \left(\frac{\omega_0 M_{23}}{\sqrt{R_2 R_3}} \right)^2} \frac{\frac{(\omega_0 M_{34})^2}{R_3 R_L}}{1 + \frac{(\omega_0 M_{34})^2}{R_3 R_L}} \quad (2.9)$$

2.2 Impact Factor Analysis

From Eqs. (2.4) and (2.9) we can see that the two-coil structure and the four-coil structure have a similar expression of transfer efficiency. The transfer quality factor of both the two-coil structure and the four-coil structure is defined as

$$T_Q = \frac{\omega_0 M_{23}}{\sqrt{R_2 R_3}} = \rho_{23} \sqrt{Q_2 Q_3} \quad (2.10)$$

where Q_2 and Q_3 are defined, respectively, by

$$Q_2 = \frac{\omega_0 L_2}{R_2} \quad (2.11)$$

$$Q_3 = \frac{\omega_0 L_3}{R_3} \quad (2.12)$$

The load matching factors of the two-coil structure and the four-coil structure are defined, respectively, by

$$L_{M-2C} = \frac{R_L}{R_3} \quad (2.13)$$

$$L_{M-4C} = \frac{(\omega_0 M_{34})^2}{R_3 R_L} \quad (2.14)$$

Using the aforementioned factors, the transfer efficiency of these two structures can be unified as

$$\eta_{\text{un}} = \frac{T_Q^2}{1 + L_M + T_Q^2} \frac{L_M}{1 + L_M} \quad (2.15)$$

where L_M can be replaced by the corresponding load matching factor.

2.2.1 Load Matching Factor

From Eq. (2.15) we can see that the first term to the right of the equal sign decreases with L_M while the second term increases with L_M . Hence, there is an optimal L_M to achieve the maximum transfer efficiency. By letting $\partial \eta_{\text{un}} / \partial L_M = 0$, the optimal load matching factor is

$$L_{M-\text{opt}} = \sqrt{1 + T_Q^2} \quad (2.16)$$

The corresponding optimal load resistances can be calculated by replacing Eqs. (2.13) and (2.14) with Eq. (2.16) as

$$R_{L-2C-\text{opt}} = R_3 \sqrt{1 + T_Q^2} = \sqrt{R_3 + \frac{R_3}{R_2} (\omega_0 M_{23})^2} \quad (2.17)$$

$$R_{L-4C-\text{opt}} = \frac{(\omega_0 M_{34})^2}{R_3 \sqrt{1 + T_Q^2}} = \frac{(\omega_0 M_{34})^2}{\sqrt{R_3 + \frac{R_3}{R_2} (\omega_0 M_{23})^2}} \quad (2.18)$$

With the decreasing transfer distance or the increasing coupling, the mutual inductance increases, and thus the optimal load resistance of the two-coil structure increases, while that of the four-coil structure decreases. Furthermore, due to the fact that there is an extra load coil in the four-coil structure, there is a degree of freedom, namely M_{34} , to adjust the optimal load resistance in the four-coil structure.

2.2.2 Transfer Quality Factor

T_Q indicates the tightness of coupling between the sending coil and the receiving coil. From Eq. (2.15) we can see that the transfer efficiency of the two structures increases with the increasing T_Q . Therefore, to improve the transfer efficiency, T_Q should be as large as possible.

Replacing Eq. (2.15) with Eq. (2.16) yields the theoretical maximum transfer efficiency as

$$\eta_{\text{trn-max}} = \left(\frac{T_Q}{1 + \sqrt{1 + T_Q^2}} \right)^2 \quad (2.19)$$

The theoretical maximum transfer efficiency increases with T_Q as well, as shown in Fig. 2.3. When T_Q is small, the theoretical maximum transfer efficiency increases dramatically with T_Q . After T_Q reaches a threshold, the theoretical maximum transfer efficiency increases slowly with T_Q . The relationship between T_Q and some certain values of theoretical maximum transfer efficiency is tabulated in Table 2.1.

Fig. 2.3 Theoretical maximum transfer efficiency varying with T_Q

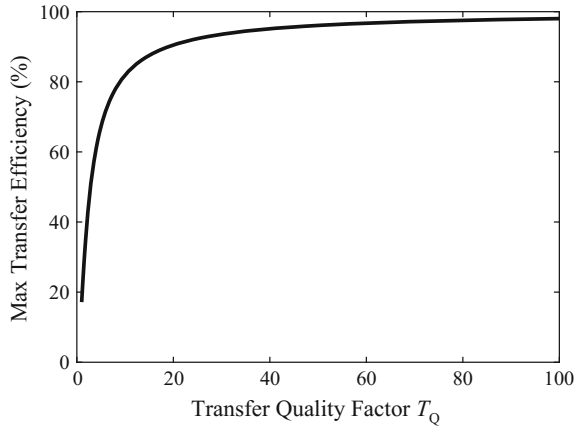


Table 2.1 Theoretical maximum transfer efficiency at different values of T_Q

T_Q	Theoretical maximum transfer efficiency (%)
2.11	40
2.83	50
3.87	60
5.58	70
8.95	80
18.98	90
39.00	95

2.3 Reducing Frequency

To employ high-efficiency power electronics inverters, which normally work at low frequencies, as the source of a WPT system, the resonant frequency should be reduced. Based on Eq. (2.1), there are two methods of reducing the resonant frequency: increasing the inductance or increasing the capacitance. According to the aforementioned analysis, T_Q determines the theoretical maximum transfer efficiency. Reducing the resonant frequency affects T_Q . Therefore, the impact of reducing the resonant frequency on T_Q should be investigated.

In a WPT system with stray capacitors as the resonant capacitors, the impact of reducing the resonant frequency on T_Q is varied case by case. No specific rules apply. Therefore, only the case with lumped capacitors as the resonant capacitors will be studied. To simplify analysis, the sending loop and the receiving loop are assumed to be identical, whose equivalent circuit is shown in Fig. 2.4.

T_Q can be expressed as

$$T_Q = \frac{\omega_0 M_{23}}{R_{\text{cap}} + R_{\text{ind}}} = \frac{M_{23}}{\sqrt{L_2 C_2} (R_{\text{cap}} + R_{\text{ind}})} \quad (2.20)$$

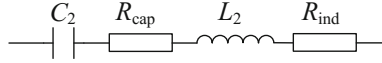


Fig. 2.4 Equivalent circuit of a resonant loop Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers

2.3.1 Increasing Inductance

If the turn numbers of the sending coil and the receiving coil are increased to N times the original value and the coils are still tightly wound, theoretically

$$L' = N^2 L_2 \quad (2.21)$$

$$C' = C_2 \quad (2.22)$$

$$M'_{23} = N^2 M_{23} \quad (2.23)$$

The resonant angular frequency is changed into

$$\omega' = \frac{1}{\sqrt{L' C'}} = \frac{1}{\sqrt{N^2 L_2 C_2}} = \frac{\omega_0}{N} \quad (2.24)$$

which is one N th of the original resonant frequency.

Because the length of the coil is increased to N times the original value and taken the skin effect and proximity effect into consideration when the frequency is reduced, the equivalent resistance of the inductor is changed into

$$R'_{\text{ind}} < N R_{\text{ind}} \quad (2.25)$$

Assuming that the equivalent resistance of the capacitor remains constant after reducing the frequency, we have

$$R'_{\text{cap}} = R_{\text{cap}} \quad (2.26)$$

Therefore, the total equivalent resistance is

$$R' < N R_{\text{ind}} + R_{\text{cap}} \quad (2.27)$$

The transfer quality factor after increasing the inductance is

$$T'_Q = \frac{\omega' M'_{23}}{R'} > \frac{\omega_0}{N} \frac{N^2 M_{23}}{N R_{\text{ind}} + R_{\text{cap}}} > T_Q \quad (2.28)$$

If reducing the resonant frequency is achieved by increasing the inductance (or rather increasing the turn number of the coils), ideally the transfer quality factor

could even be larger than the original value. The transfer quality factor is not reduced. However, increasing the inductance leads to bulky size of the coils, which should be taken into consideration.

2.3.2 Increasing Capacitance

When the number of capacitors connected in parallel is increased to N^2 times the original value, the parameters are changed into

$$L'' = L_0 \quad (2.29)$$

$$C'' = N^2 C_0 \quad (2.30)$$

$$M''_{23} = M_{23} \quad (2.31)$$

The resonant angular frequency is changed into

$$\omega'' = \frac{1}{\sqrt{L''C''}} = \frac{1}{\sqrt{N^2 L_0 C_0}} = \frac{\omega_0}{N} \quad (2.32)$$

Assuming that the equivalent resistance of a single capacitor remains constant after reducing the resonant frequency, the total equivalent resistance of all the capacitors connected in parallel is

$$R''_{\text{cap}} = \frac{R_{\text{cap}}}{N^2} \quad (2.33)$$

Taken the skin effect and proximity effect into consideration, we have

$$R''_{\text{ind}} < R_{\text{ind}} \quad (2.34)$$

Therefore, the total equivalent resistance is

$$R'' < R_{\text{ind}} + \frac{R_{\text{cap}}}{N^2} \quad (2.35)$$

The transfer quality factor after increasing the capacitance is

$$T''_Q = \frac{\omega'' M''_{23}}{R''} \approx \frac{\omega_0}{N} \frac{M_{23}}{R_{\text{ind}} + \frac{R_{\text{cap}}}{N^2}} = \frac{\omega_0 M_{23}}{N R_{\text{ind}} + \frac{R_{\text{cap}}}{N}} \quad (2.36)$$

In most practical applications, the equivalent resistance of the inductor is far larger than that of the capacitor, namely

$$R_{\text{ind}} \gg R_{\text{cap}} \quad (2.37)$$

When ignoring the equivalent resistance of the capacitor, Eq. (2.36) can be changed into

$$T_Q'' \approx \frac{T_Q}{N} \quad (2.38)$$

When increasing the number of the capacitors connected in parallel to reduce the resonant frequency, the transfer quality factor decreases dramatically. It can be predicted that the transfer efficiency drops sharply in this case. Therefore, considering the transfer efficiency, it is not a good way to reduce the resonant frequency by increasing the capacitance.

2.4 Calculations and Experiments

To validate the aforementioned analysis, an experimental prototype is implemented, as shown in Fig. 2.5. The parameters are tabulated in Table 2.2.

The mutual inductance of the sending coil and the receiving coil, the corresponding transfer quality factor and the theoretical maximum transfer efficiency varying with the transfer distance are depicted in Fig. 2.6.

2.4.1 Load Matching

According to Eqs. (2.17) and (2.18), the optimal load resistances of the two-coil structure and the four-coil structure are shown in Fig. 2.7. The optimal load

Fig. 2.5 Photograph of experimental prototype
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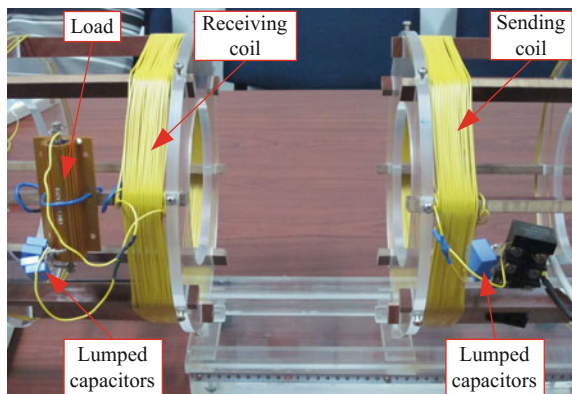


Table 2.2 Parameters of experimental prototype

Parameter	Symbol	Value
Inductance of sending coil	L_2	60.80 μH
Inductance of receiving coil	L_3	60.80 μH
Capacitance of sending coil	C_2	4.70 nF
Capacitance of receiving coil	C_3	4.70 nF
Inductance of source coil	L_1	1.20 μH
Inductance of load coil	L_4	1.20 μH
Mutual inductance of source coil and sending coil	M_{12}	4.66 μH
Mutual inductance of load coil and receiving coil	M_{34}	4.66 μH
Equivalent resistance of sending loop	R_2	0.77 Ω
Equivalent resistance of receiving loop	R_3	0.77 Ω
Resonant frequency	f_0	298 kHz

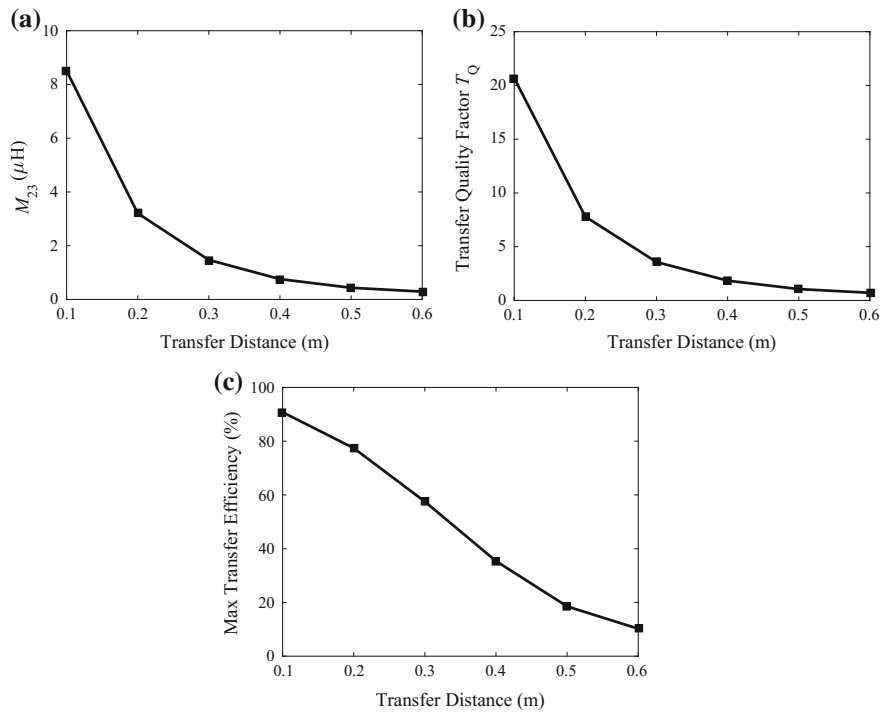
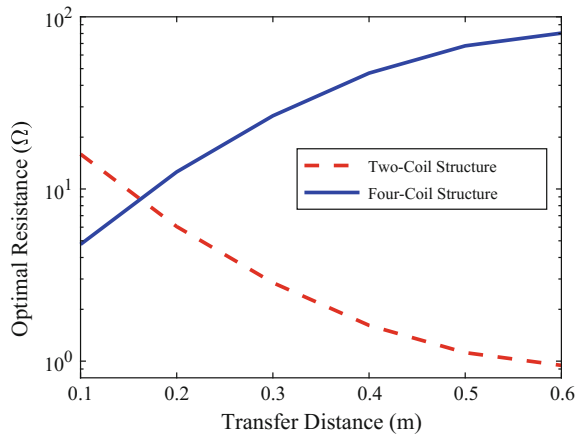


Fig. 2.6 **a** M_{23} , **b** T_Q , and **c** theoretical maximum transfer efficiency varying with transfer distance
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Fig. 2.7 Optimal load resistances of two-coil and four-coil structures varying with transfer distance
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resistance of the two-coil structure increases with the decreasing transfer distance, while that of the four-coil structure decreases with the decreasing transfer distance.

The calculations and the experiments of the transfer efficiency of the two-coil structure varying with the load resistance under different transfer distances are demonstrated in Fig. 2.8, and those of the four-coil structure in Fig. 2.9. The experiments agree with the calculations, verifying the aforementioned analysis.

2.4.2 Reducing Frequency

Based on the experimental prototype in Fig. 2.5, the two methods of reducing the resonant frequency, namely increasing the inductance and increasing the capacitance, are investigated. To increase the inductance, the turn numbers of the sending coil and the receiving coil are changed from 10 to 20. To increase the capacitance, another three capacitor are added to be connected to the original capacitor in parallel. The parameters are tabulated in Table 2.3.

After increasing the inductance, the mutual inductance between the sending coil and the receiving coil varying with the transfer distance is shown in Fig. 2.10.

The transfer quality factors and the corresponding theoretical maximum transfer efficiency of the original system and the systems after increasing the inductance and the capacitance are depicted in Fig. 2.11. The transfer quality factor of the system after increasing the inductance is the highest, while that after increasing the capacitance is the lowest, consistent with the theoretical analysis. Therefore, the theoretical maximum transfer efficiency of the system after increasing the inductance is the highest, and that after increasing the capacitance is the lowest.

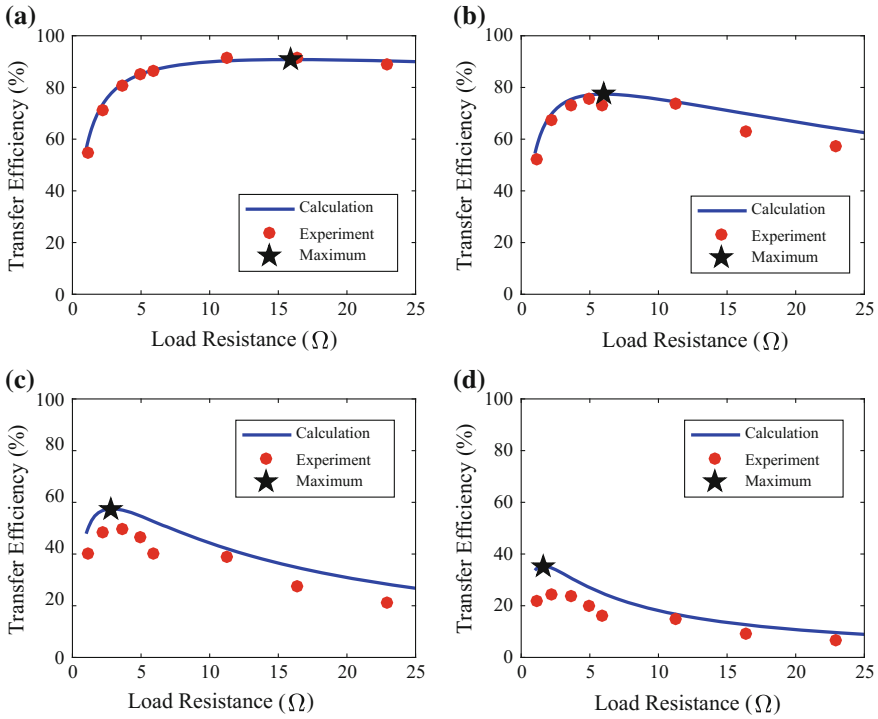


Fig. 2.8 Calculations and experiments of transfer efficiency of two-coil structure varying with load resistance under **a** 0.1 m; **b** 0.2 m; **c** 0.3 m; **d** 0.4 m Reprinted from Ref. [2] with permission from Institute of Electrical and Electronics Engineers

Based on Eqs. (2.17) and (2.18), the optimal load resistances of the three systems in the two-coil structure and the four-coil structure varying with the transfer distance are displayed in Fig. 2.12. The optimal load resistance of the system after increasing the inductance is the highest, while that after increasing the capacitance is the lowest.

The load resistance is set to be 10 Ω . The calculations and experiments of the transfer efficiency of these three systems in the two-coil structure and the four-coil structure are shown in Figs. 2.13 and 2.14, respectively. For both the two-coil and four-coil structures, compared with increasing the capacitance, increasing the inductance is proved to be a better method of reducing the resonant frequency while maintaining good transfer efficiency.

In Fig. 2.13, there are minor differences of the three system at 0.1 m and this is caused by load matching. The load resistance (10 Ω in this setup) is closest to the optimal load resistance of the system after increasing the capacitance, as can be read

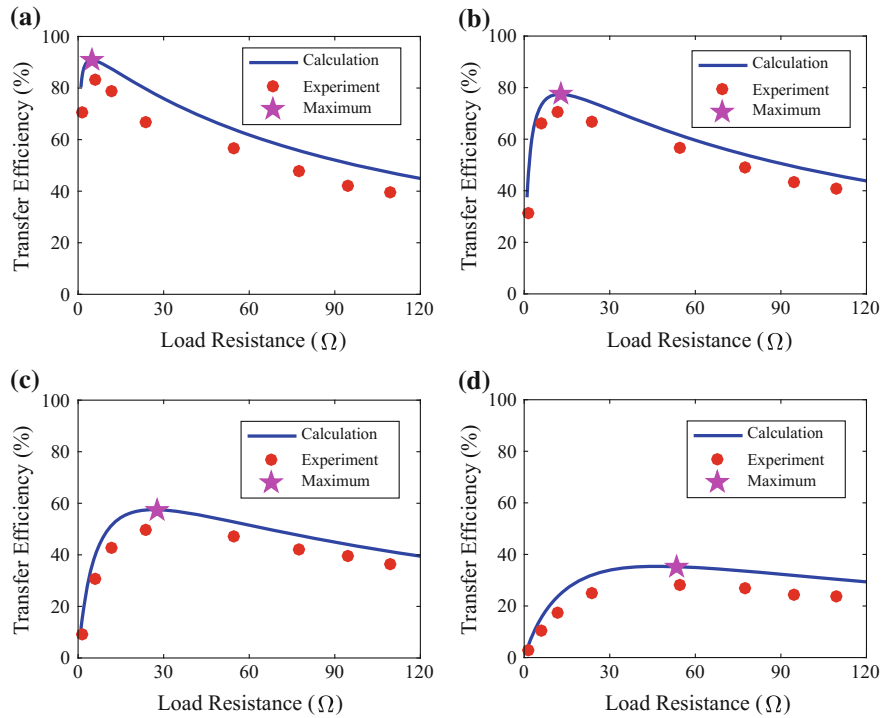


Fig. 2.9 Calculations and experiments of transfer efficiency of four-coil structure varying with load resistance under **a** 0.1 m; **b** 0.2 m; **c** 0.3 m; **d** 0.4 m Reprinted from Ref. [2] with permission from Institute of Electrical and Electronics Engineers

Table 2.3 Parameters of experimental prototype with reducing frequency

Parameter	Original system	Increasing inductance	Increasing capacitance
Inductance of sending coil L_2	60.80 μH	199.16 μH	60.80 μH
Inductance of receiving coil L_3	60.80 μH	199.16 μH	60.80 μH
Capacitance of sending coil C_2	4.70 nF	4.70 nF	18.80 nF
Capacitance of receiving coil C_3	4.70 nF	4.70 nF	18.80 nF
Inductance of source coil L_1	1.20 μH	1.20 μH	1.20 μH
Inductance of load coil L_4	1.20 μH	1.20 μH	1.20 μH
Mutual inductance of source coil and sending coil M_{12}	4.66 μH	15.00 μH	4.66 μH
Mutual inductance of load coil and receiving coil M_{34}	4.66 μH	15.00 μH	4.66 μH
Equivalent resistance of sending loop R_2	0.77 Ω	1.30 Ω	0.60 Ω
Equivalent resistance of receiving Loop R_3	0.77 Ω	1.30 Ω	0.60 Ω
Resonant frequency f_0	298 kHz	165 kHz	149 kHz

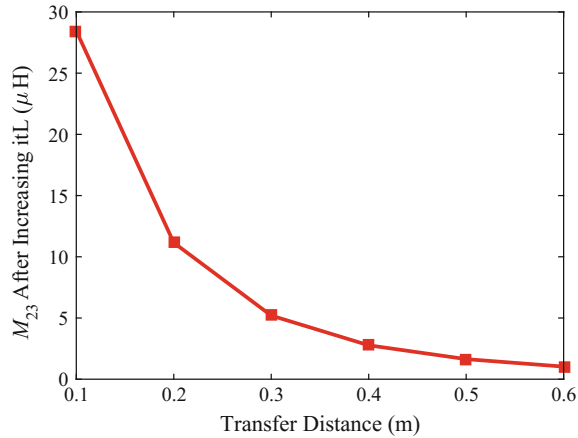


Fig. 2.10 M_{23} varying with transfer distance after increasing inductance Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers

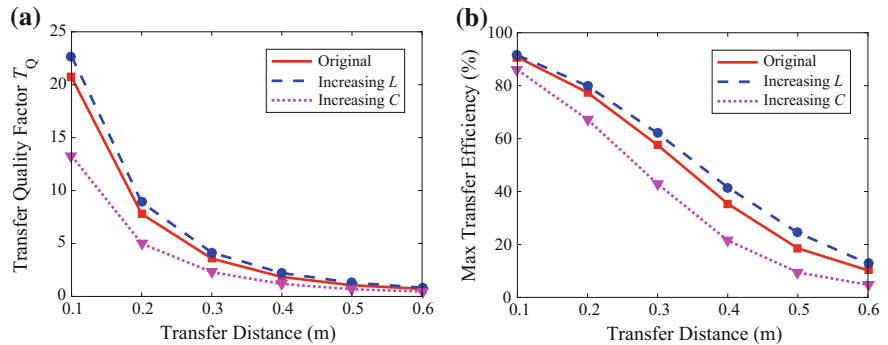


Fig. 2.11 **a** Transfer quality factor and **b** theoretical maximum transfer efficiency of original system and systems after increasing inductance and capacitance Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers

from Fig. 2.12a, while farthest to that after increasing the inductance, whose theoretical maximum transfer efficiency is the highest. Therefore, the transfer efficiency of the system after increasing the capacitance can be roughly the same as those of the original system and the system after increasing the inductance. When the transfer distance is over 0.2 m, the optimal load resistance of the system after increasing the inductance is closest to the actual load resistance. Combined with the fact that the transfer quality factor of the system after increasing the inductance is the highest, the transfer efficiency of the system after increasing the inductance is the highest when the transfer distance is over 0.2 m. Similar analysis can be applied to Fig. 2.14, where the transfer efficiency difference is large at short transfer distance and small at long transfer distance.

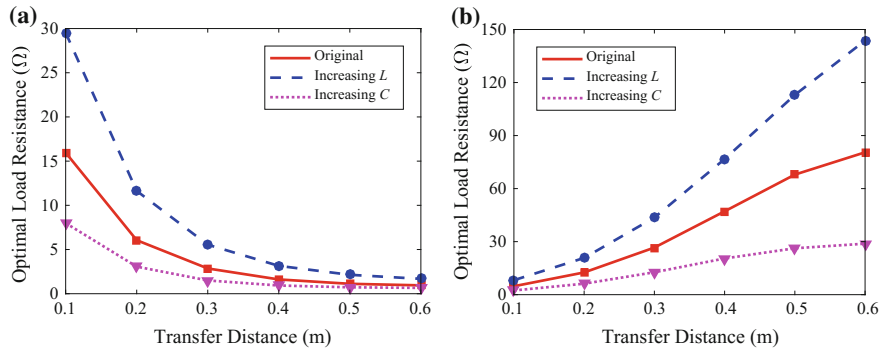


Fig. 2.12 Optimal load resistances of original system and systems after increasing inductance and capacitance in a two-coil and b four-coil structures Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers

Fig. 2.13 Calculations and experiments of transfer efficiency of original system and systems after increasing inductance and capacitance in two-coil structure Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers

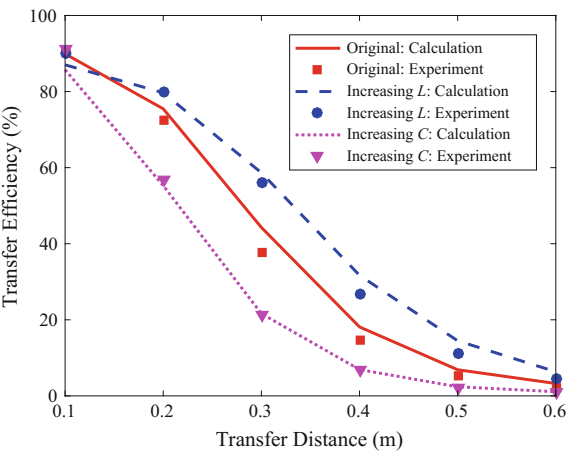
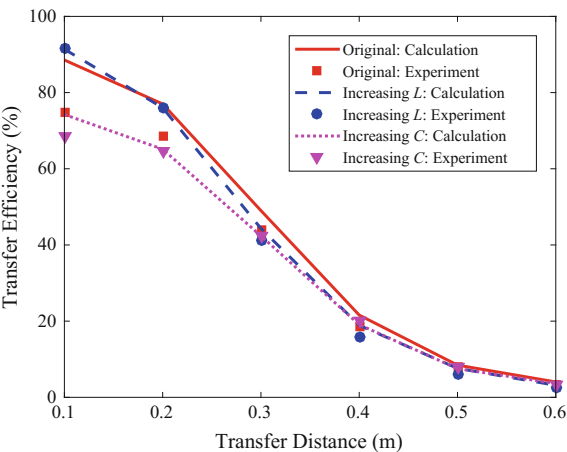


Fig. 2.14 Calculations and experiments of transfer efficiency of original system and systems after increasing inductance and capacitance in four-coil structure Reprinted from Ref. [1] with permission from Institute of Electrical and Electronics Engineers



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Key Technologies of Magnetically-Coupled Resonant
Wireless Power Transfer

Zhang, Y.

2018, XVIII, 117 p. 82 illus., 45 illus. in color., Hardcover

ISBN: 978-981-10-6537-8