Planar Helix Driving Contact Curve Line Gear Mechanism

Yangzhi Chen and Xiongdun Xie

Abstract Based on the previous studies of line gear (LG), also named as space curve meshing wheel (SCMW), a planar helix driving contact curve line gear mechanism(PHDC LGM) is proposed in this article. Unlike the previous line gear mechanism which select circular helix as driving contact curve, planar helix is selected in this article in consider of the difficulty in micro processing. The formulae of both contact curves and center curves of driving and driven line gears were derived. Prototypes were designed and manufactured by 3D printing. Then kinematics experiments were conducted, and the results show that the mechanism proposed is capable of steady transmission in intersecting axis.

Keywords Line gear · Micro manufacture · Planar helix

1 Introduction

Line gear (LG), proposed by YZ Chen in 2007, is a novel gear mechanism based on space curve meshing theory [1]. Line gear mechanism commits in transmission via point contact by a pair of space conjugate curves. Circular helix is commonly used as driving contact curve for its manufacture convenience in machining process.

In the basic theory of LG, the motion at the meshing point should satisfy the following equation:

$$\boldsymbol{v}_{12} \cdot \beta = 0 \tag{1}$$

where v_{12} is the relative velocity at the meshing point between the driving and driven line teeth, and β is the unit normal vector of the driving contact curve at the meshing point [2].

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When circular helix is selected as driving contact curve, the significant of meshing equation should be: v_{12} is always tangent to the cylindrical surface enveloped the circular helix, so the driven contact curve will not disengage or interfere to the cylinder. Since 2007, YZ Chen et al. have made many progresses in researches of Line gear mechanism, including LG with parallel axis, intersect LG and skew LG, the contact ratio, failure criterion, strength design, manufacture technology and line gear box(LGB) [3–5]. Among the above researches, circular helix was selected as the driving contact curve.

To realize space transmission, at least one 3D spatial curve must be designed in a couple of conjugated contact curves. In this paper, planer helix instead of circular helix is selected as driving contact curve, and the vertical direction Z of the plane where the planar curve is on is selected as the normal vector. The meshing equation is written as below Eq. (2). In fact, this equation is suitable for any line gear pair that select planar curve as driving contact curve.

$$\boldsymbol{v}_{12} \cdot Z = 0 \tag{2}$$

Under this situation, the significant of the meshing equation is: v_{12} is always on the plane where the driving contact curve locates, without disengage. In other words, the z component of v_{12} is always be zero.

The new form of line gear proposed in this paper has advantages in the convenience of manufacture, especially in micro field.

On one hand, for the manufacture of LG in conventional scale, the numerical control processing is matured and can realize the machining of space helix easily [6]. For example, only two universal driving shafts are needed to LG manufacture in circular helix and planar helix processing. As a result, by using a special purpose machine for LG [7], machining process for space helix LG has a low cost and short processing time. In conclusion, planar helix can be machined by CNC milling machine, and circular helix can be machined by machining center. Both can be machined in the special purpose machine for LG.

On the other hand, for the manufacture of LG in micro scale, two optional micro manufacture ways are MEMS and non-MEMS manufacture technology. The MEMS manufacture technologies mainly include photo etching, chemical etching, electrochemical etching, LIGA, etc. The non-MEMS manufacture technologies include micro machining, laser etching, micro sculpture, micro injection, micro emboss, etc. [8]. Generally, non-MEMS manufacture technology is used for manufacturing 3D surface. There are many limiting conditions in micro scale manufacture, such as material properties in micro scale, stiffness and strength of micro structure, size effect that are shape and dimension, difficulties in clamping and releasing of the parts, residual stress and surface integrity [9]. 3D surface manufacture is harder than 2D surface manufacture in micro scale for the extra motion and its control. The manufacture technologies used to 2D or 2.5D structure are relatively mature, such as MEMS manufacture technology. Non-MEMS manufacture technology is also convenient to manufacture planar structure, For example, micro EDM milling was used to make micro turbine [10]; sacrificed layer and deep reactive ion etching is used to fabricate antenna shaped structure [11].

In conclusion, planar helix driving curve line gear mechanism(PHDC LGM) extends line gear to more optional design forms in conventional scale. While in micro scale, planar helix has the advantage in using of the mature micro manufacture. In micro scale, compared to 3D structure, planar structure can reach higher stiffness, strength and accuracy which will achieve accuracy transmission of line gear mechanism.

2 Design Equations of Planar Helix Driving Curve Line Gear Mechanism

A line gear pair consists of a driving line gear and a driven line gear. They rotate with different constant angular velocity, respectively. As shown in Fig. 1, the driving line gear is fixed in $o_1 - x_1y_1z_1$ coordinate and a driven line gear is fixed in $o_2 - x_2y_2z_2$ coordinate, they rotate around the z axis of $o - xyz$ and $o_P - x_Py_Pz_P$ coordinates [12], respectively. a, b are the distance between O and O_P along x and z axes, respectively.

The transformation matrix from $o_1 - x_1y_1z_1$ to $o_2 - x_2y_2z_2$ is Eq. (3).

$$M_{21} = M_{2P} \cdot M_{PO} \cdot M_{O1} = \begin{bmatrix} -\cos\varphi_1\cos\varphi_2\cos\theta - \sin\varphi_1\sin\varphi_2 & -\sin\varphi_1\cos\varphi_2\cos\theta + \cos\varphi_1\sin\varphi_2 & -\cos\varphi_2\sin\theta & -a\cos\varphi_2\cos\theta + b\cos\varphi_2\sin\theta \\ \cos\varphi_1\sin\varphi_2\cos\theta - \sin\varphi_1\cos\varphi_2 & \sin\varphi_1\sin\varphi_2\cos\theta + \cos\varphi_1\cos\varphi_2 & \sin\varphi_2\sin\theta & a\sin\varphi_2\cos\theta - b\sin\varphi_2\sin\theta \\ \cos\varphi_1\sin\theta & \sin\varphi_1\sin\theta & -\cos\theta & a\sin\theta + b\cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The angular velocities of driving and driven ling gears are ω_1 and ω_2, respectively, supposing driving line gear and driven line gear turn angles φ_1 and φ_2 after t_P, respectively. The transmission ratio is i_{21}, the following relations are accessible: $\omega_2 = i_{21}\omega_1$; $\varphi_2 = i_{21}\varphi_1$

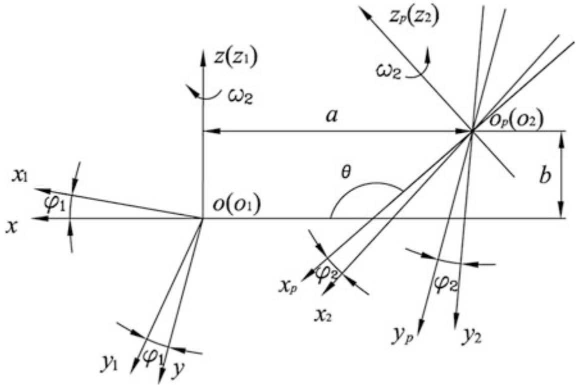

Fig. 1 Space curve meshing coordinates [3]

According to the definition of meshing equation, v_{12} is the relative velocity at the meshing point between the driving and driven line teeth, which is shown in Eq. (4).

$$\boldsymbol{v}_{12} = \begin{bmatrix} \left(-x_M^{(1)} \sin\varphi_1 + y_M^{(1)} \cos\varphi_1\right)(\varpi_1 - \varpi_2 \cos\theta) \\ -\left(x_M^{(1)} \cos\varphi_1 + y_M^{(1)} \sin\varphi_1\right)\varpi_1 + \left(z_M^{(1)} - b\right)\varpi_2 \sin\theta + \left(x_M^{(1)} \cos\varphi_1 + y_M^{(1)} \sin\varphi_1 + a\right)\varpi_2 \cos\theta \\ -\left(-x_M^{(1)} \sin\varphi_1 + y_M^{(1)} \cos\varphi_1\right)\varpi_2 \sin\theta \end{bmatrix} \tag{4}$$

According to the significant of the meshing equation, z component of v_{12} is zero, then Eq. (5) can be obtained, which is the kinematic restricted condition of (PHDC LGM) mechanism, and suitable to all planar-driving-contact-curve line gear mechanism.

$$-i_{21}\varpi_1 \sin\theta(y_M^{(1)} \cos\varphi_1 - x_M^{(1)} \sin\varphi_1) = 0 \tag{5}$$

In external gearing situation, the center curve of a driving line tooth is produced by translation of the driving contact curve a given r_1 along its radius of curvature; and the center curve of a driven line tooth is produced by translation of the driven contact curve a given r_2 along the negative direction of radius of curvature of the driving contact curve. In inner gearing situation, the center curves of driving line gear and driven line gear are produced by the similar way, but both translation directions of driving and driven contact curves are negative to external gearing situation. After two center curves being produced, two circulars are plotted acting as the cross section of the driving line tooth and driven line tooth which should be vertical to the center curve. Then the circle cross section moves along both center curves that form the driving line tooth and driven line tooth.

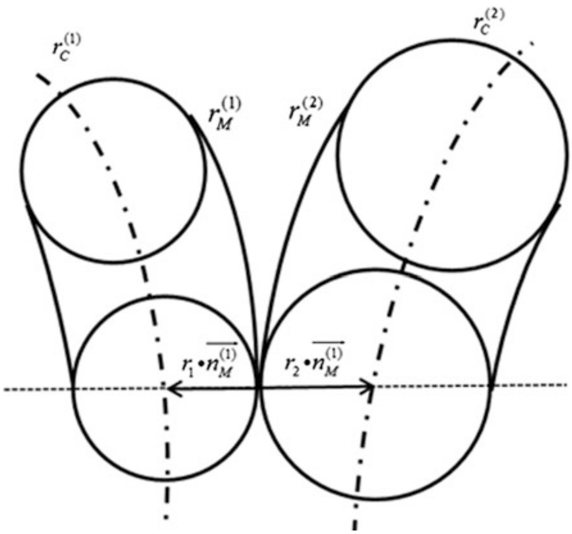

Fig. 2 Forming process of driving and driven line tooth

Figure 2 shows how the driving and driven line tooth are formed. Equation (6) shows the unit vector of radius of curvature of driving contact curve; Eqs. (7) and (8) show the center curve of driving and driven line teeth; Eq. (9) shows the results of center curve of driven line tooth after transform to $o_2 - x_2y_2z_2$ coordinate.

When r_1 and r_2 are given as zero, the center curve equations of driving and driven line teeth equal to the contact curves of driving and driven line tooth, respectively.

$$n_M^{(1)} = \left(\frac{y_M^{(1)\prime}}{\sqrt{\left(y_M^{(1)\prime}\right)^2 + \left(x_M^{(1)\prime}\right)^2}}, \frac{-x_M^{(1)\prime}}{\sqrt{\left(y_M^{(1)\prime}\right)^2 + \left(x_M^{(1)\prime}\right)^2}}, 0\right) \tag{6}$$

$$\begin{cases} x_C^{(1)} = x_M^{(1)} \mp r_1 n_{Mx}^{(1)} \\ y_C^{(1)} = y_M^{(1)} \mp r_1 n_{My}^{(1)} \\ z_C^{(1)} = z_M^{(1)} \end{cases} \tag{7}$$

$$(x_C^{(2)}, y_C^{(2)}, z_C^{(2)})^T = M_{21} \cdot (x_M^{(1)} \pm r_2 n_{Mx}^{(1)}, y_M^{(1)} \pm r_2 n_{My}^{(1)}, z_M^{(1)})^T \tag{8}$$

$$\begin{cases} x_C^{(2)} = (-\cos\varphi_1\cos\varphi_2\cos\theta - \sin\varphi_1\sin\varphi_2)\left(x_M^{(1)} \pm r_2 n_{Mx}^{(1)}\right) + \\ \qquad (-\sin\varphi_1\cos\varphi_2\cos\theta + \cos\varphi_1\sin\varphi_2)\left(y_M^{(1)} \pm r_2 n_{My}^{(1)}\right) - \\ \qquad \cos\varphi_2\sin\theta z_M^{(1)} + (-a\cos\varphi_2\cos\theta + b\cos\varphi_2\sin\theta) \\ y_C^{(2)} = (\cos\varphi_1\sin\varphi_2\cos\theta - \sin\varphi_1\cos\varphi_2)\left(x_M^{(1)} \pm r_2 n_{Mx}^{(1)}\right) + \\ \qquad (\sin\varphi_1\sin\varphi_2\cos\theta + \cos\varphi_1\cos\varphi_2)\left(y_M^{(1)} \pm r_2 n_{My}^{(1)}\right) + \\ \qquad \sin\varphi_2\sin\theta z_M^{(1)} + (a\sin\varphi_2\cos\theta - b\sin\varphi_2\sin\theta) \\ z_C^{(2)} = \cos\varphi_1\sin\theta\left(x_M^{(1)} \pm r_2 n_{Mx}^{(1)}\right) + \sin\varphi_1\sin\theta\left(y_M^{(1)} \pm r_2 n_{My}^{(1)}\right) \\ \qquad - \cos\theta z_M^{(1)} + (a\sin\theta + b\cos\theta) \end{cases} \tag{9}$$

The formula derivation process above is on basis of planar driving contact curve, theoretically, any planar curve can be selected as the driving contact curve. But planar helix is the optimal option for manufacture as presented in the introduction section.

The parameter equation of a planar helix is expressed in Eq. (10), setting as the driving contact curve.

$$\begin{cases} x_M^{(1)} = (-mt_1 + n)\cos(kt_1) \\ y_M^{(1)} = (-mt_1 + n)\sin(kt_1) \\ z_M^{(1)} = 0 \end{cases} \tag{10}$$

Then, substituting Eq. (10) into Eq. (5), a particular solution of Eq. (5) is obtained as Eq. (11).

$$\varphi_1 = -kt_1 \tag{11}$$

Then, substituting Eq. (10) into Eqs. (6) and (7), the center curve of driving line tooth for PHDC LGM is obtained as Eq. (12).

$$\begin{cases} x_C^{(1)} = (-mt_1 + n)\cos(kt_1) \\ \mp r_1 \frac{k(-n + mt_1)\cos(kt_1) + m\sin(kt_1)}{\sqrt{k^2n^2 + m(m - 2k^2nt_1 + k^2mt_1^2)}} \\ y_C^{(1)} = (-mt_1 + n)\sin(kt_1) \\ \mp r_1 \frac{k(n - mt_1)\sin(kt_1) + m\cos(kt_1)}{\sqrt{k^2n^2 + m(m - 2k^2nt_1 + k^2mt_1^2)}} \\ z_C^{(1)} = 0 \end{cases} \tag{12}$$

Then, substituting Eqs. (10) and (11) into Eqs. (8) and (9), the center curve of driven line tooth for PHDC LGM is obtained as Eq. (13).

Table 1 The shape of driven contact curve related to θ

Driving contact curve	Driven contact curve		
	0°, 180°	90°, 270°	Other
Planar helix	Planar helix	Circular helix	Conical helix

$$\begin{cases} x_C^{(2)} = \cos(i_{21}kt_1)(-(a+n-mt_1)\cos\theta + b\sin\theta) \\ \pm r_2 \frac{k(n-mt_1)\cos(i_{21}kt_1)\cos\theta - m\sin(i_{21}kt_1)}{\sqrt{k^2n^2 + m(m-2k^2nt_1 + k^2mt_1^2)}} \\ y_C^{(1)} = \sin(i_{21}kt_1)(-(a+n-mt_1)\cos\theta + b\sin\theta) \\ \pm r_2 \frac{k(n-mt_1)\sin(i_{21}kt_1)\cos\theta + m\cos(i_{21}kt_1)}{\sqrt{k^2n^2 + m(m-2k^2nt_1 + k^2mt_1^2)}} \\ z_C^{(1)} = b\cos\theta + (a+n-mt_1)\sin\theta \\ \pm r_2 \frac{k(-n+mt_1)\sin\theta}{\sqrt{k^2n^2 + m(m-2k^2nt_1 + k^2mt_1^2)}} \end{cases} \tag{13}$$

The shape of the driven contact curve is related to the angle of the axes θ, as shown in Table 1.

3 A Design Example

3.1 The Equations of the Line Gear Teeth

The necessary parameters were given in Table 2, then the driving and driven contact curves were designed by Eqs. (14) and (15); and the driving and driven center curves were designed by Eqs. (16) and (17).

$$\begin{cases} x_M^{(1)} = (-10t_1 + 100)\cos(\frac{t_1}{4}) \\ y_M^{(1)} = (-10t_1 + 100)\sin(\frac{t_1}{4}) \end{cases} \tag{14}$$

$$\begin{cases} x_M^{(2)} = (-(200 - 10t_1)\cos\frac{3\pi}{4} + 100\sin\frac{3\pi}{4})\cos(0.0625t_1) \\ y_M^{(2)} = (-(200 - 10t_1)\cos\frac{3\pi}{4} + 100\sin\frac{3\pi}{4})\sin(0.0625t_1) \\ z_M^{(2)} = 100\cos\frac{3\pi}{4} + (200 - 10t_1)\sin\frac{3\pi}{4} \end{cases} \tag{15}$$

$$\begin{cases} x_C^{(1)} = (-10t_1 + 100)\cos(\frac{t_1}{4}) + \frac{\frac{1}{2}(-10t_1 + 100)\cos(\frac{t_1}{4}) + 20\sin(\frac{t_1}{4})}{\sqrt{625 + 10(10 - 12.5t_1 + 0.625t_1^2)}} \\ y_C^{(1)} = (-10t_1 + 100)\sin(\frac{t_1}{4}) + \frac{\frac{1}{2}(100 - 10t_1)\sin(\frac{t_1}{4}) + 20\cos(\frac{t_1}{4})}{\sqrt{625 + 10(10 - 12.5t_1 + 0.625t_1^2)}} \end{cases} \tag{16}$$

Table 2 Given parameters

θ	m	n	k	a	b	i_{21}	Z_1	Z_2	r_1	r_2
135°	10	100	1/4	100	100	4	10	*40*	*2*	*2*

$$\begin{cases} x_C^{(2)} = \cos(0.0625t_1)(-(200-10t_1)\cos 135^\circ + 100\sin 135^\circ) \\ \quad -\frac{0.5(100-10t_1)\cos(0.0625t_1)\cos\frac{3\pi}{4}-20\sin(0.0625t_1)}{\sqrt{625+10(10-12.5t_1+0.625t_1^2)}} \\ y_C^{(2)} = \sin(0.0625t_1)(-(200-10t_1)\cos\frac{3\pi}{4} + 100\sin\frac{3\pi}{4}) \\ \quad -\frac{0.5(100-10t_1)\sin(0.0625t_1)\cos\frac{3\pi}{4}+20\cos(0.0625t_1)}{\sqrt{625+10(10-12.5t_1+0.625t_1^2)}} \\ z_C^{(2)} = 100\cos\frac{3\pi}{4} + (200-10t_1)\sin\frac{3\pi}{4} \\ \quad -\frac{0.5(-100+10t_1)\sin\frac{3\pi}{4}}{\sqrt{625+10(10-12.5t_1+0.625t_1^2)}} \end{cases} \tag{17}$$

3.2 *Prototypes Design*

In LG transmission, only the precision of the contact curves are required to ensure the accuracy of meshing. The specific shapes of line teeth can be designed according to actual situation. The center curve and the contact curve define the position of a line tooth and the curvature of meshing point, the shape of other part of line tooth and the wheel body can be designed in consideration of strength and manufacture condition [13].

The model of the line gear pair by Solidworks is shown in Fig. 3, considering the material saving in 3D printing and strength requirement. The wheel is spoke type and the shape of a line tooth was designed by backside support, as shown in Fig. 4.

4 A Kinematics Experiment of a Planar Helix Driving Curve LG Pair

Stereo lithography Appearance(SLA) is adopted to manufacture the sample of line gear pair. the surface profile accuracy of the process is 0.025 mm. As shown in Fig. 5, the prototype of the designed PHDC LGM; the driving line gear is on the right and the driven line gear is on the left.

To measure the transmission ratio of the designed LG pair, a kinematics experiment was conducted by use of a test rig with a 4-DOF positioning table, as shown in Fig. 6. The driving and driven line gears are fixed with encoders and the Decklink SDI to collect the angular rotations of driving line gear and driven line

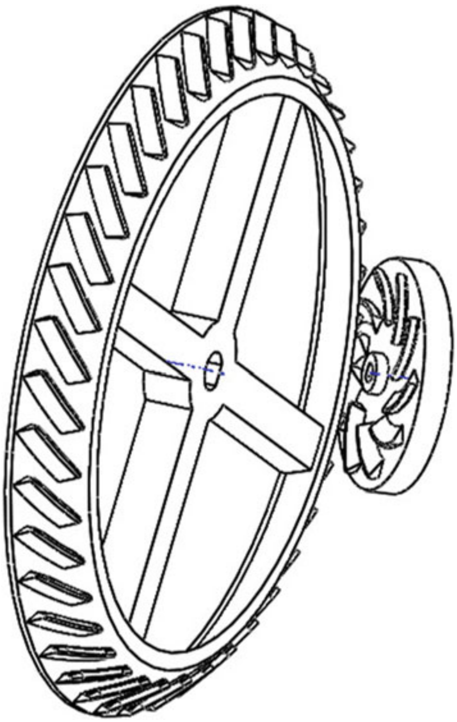

Fig. 3 Line gear pair model

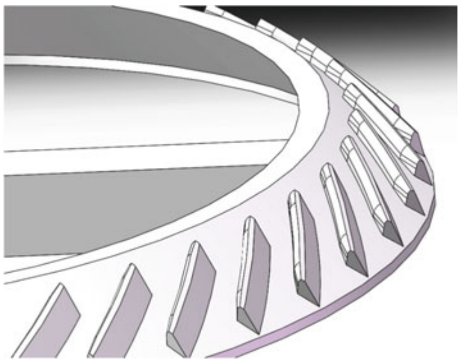

Fig. 4 Design of the wheel body and line teeth

gear in the same time interval, then the instant angular velocity is calculated. Finally, the result of the division of the instant angular velocity of the driven and driving LG is the instant transmission ratio of the prototype of designed line gear mechanism. The sampling interval in this experiment is 150 ms. After median filtering, the results of the instant transmission ratio is shown in Fig. 7.

Fig. 5 Prototypes of the PHHDC LGM

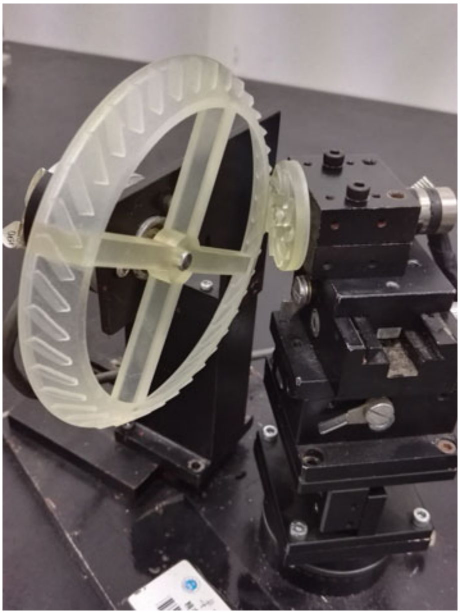

Fig. 6 Prototypes installed in test rig

As shown in Fig. 7, the instant transmission ratios are fluctuating around the design theoretical value of 4. which means the kinematics experiment result is consistent with the desired value. Some error indicators are listed in Table 3. The errors come from the following aspects: manufacturing error of the prototypes, installing and positioning error and errors in data collecting and calculating including original data error, truncation error and round-off error.

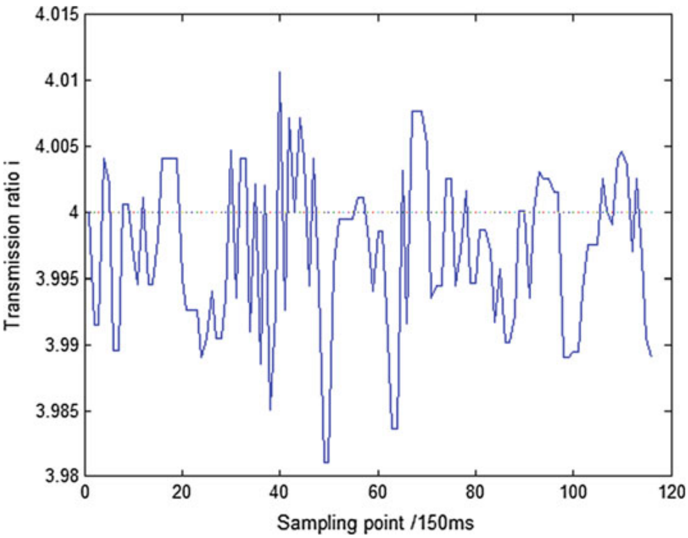

Fig. 7 Instant transmission ratio after median filtering

Table 3 Error indicators of kinematics experiment data

	AVG	Dev of AVG (%)	Max Dev (%)	Std Dev	Dev of Std Dev (%)
Original data	4.0001	0.0025	1.6	0.0212	0.53
Median filtering	3.9971	0.0725	0.5	0.0061	0.15

5 Conclusion

This paper proposed a novel form of LG pair named as PHDC LGM in which planar helix is selected as driving contact curve, in consideration of its advantage in the convenience of micro manufacture. The formulae of both contact curves and center curves of driving and driven line gears were derived. A prototype of a LG mechanism is designed according to the proposed equations and manufactured by 3D printing. Then a kinematics experiment was conducted on a 4-DOF positioning table. The instant transmission data fluctuation is little with the deviation percentage of standard deviation 0.53%. The result shows that PHDC LGM can be capable of committing of a steady transmission in a given angle intersecting axis.

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