Small Polar Codes Concatenating to STBC Applied to MIMO Systems

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Abstract This paper uses a concatenation of small Polar Codes length (N=32) and Space Time Block Code, this Polar-STBC is applied to no diversity (SISO), SIMO, MISO and MIMO systems. Minimum Mean Square Error using Successive Interference Cancellation (MMSE-SIC) is a soft output used to the receiver in order to improve Bit Error Rate (BER) and finally Successive Cancellation Decoder (SCD) is placed to the decoder in order to improve the BER and Frame Error Rate (FER). Comparison between several STBC without concatenation schemes and this small Polar-STBC shown that the proposed allows minimizing the BER and FER performances.

Keywords Polar codes · STBC · MIMO · MMSE-SIC · BER

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1 Introduction

The diversity spatial is one of methodologies using the capacity in MIMO systems to combat channel fading. The principle is to convey several replicas of the same information through each antenna. By doing this, the probably of losing the information decreases exponentially [1].

STBC provides full spatial diversity in the collocated MIMO systems, but it doesn't have the coding gain over fading channels. In the documentation many approach of concatenate STBC techniques to other codes have been proposed [2–4]. In [5, 6], a concatenation scheme of good encoding and decoding named Polar Codes in with STBC called Polar-STBC of long length have been discussed and achieved sufficient gain due to this concatenation. In [7] the authors propose the antennas detection and reduce the complexity of the receiver by offering Maximum Likelihood detection algorithm. In [8] we proposed propose a linear filter detection MMSE-SIC using a small Polar Code which allowed to reduce the complexity while maintaining the BER performance.

This paper proposes a small length Polar-STBC scheme presented in [6, 9]. We applied it with a soft output MMSE-SIC at the receiver at first time followed by a Successive Cancellation decoder (SCD) to compare the BER and FER performance between Polar-STBC and STBC only with small Polar length.

The rest of the paper is organized as follows. Section 2 gives a brief review of Polar codes. The system model Polar-STBC and the Soft Output detector are presented in Sect. 3. Section 4 gives firstly numerical simulations of the Polar-STBC and STBC only systems using MMSE-SIC at the receiver, and secondly the SCD algorithm results using Polar-STBC and several STBC are applied to the detector. While Sect. 5 conclude the document.

2 Polar Codes

Polar codes are defined as the first codes that achieve the channel capacity [5]. *PC* (*N*, *K*) denotes a polar code of block length $N = 2^n$ and dimension K, W is a Binary-input Discrete Memoryless Channel (B-DMC), I(W) is the symmetric capacity [5] and Z(W) is the reliability parameter. Let A and its complementary in $\{1, \ldots, N\}$ A^c respectively denote information and frozen bits sets. The construction of polar codes is based on channel polarization [5, 8].

Polar coding

The relations of polar coding are

$$x_1^N = u_1^N G_N \tag{1}$$

$$G_N = B_N \begin{bmatrix} G_N/2 & 0 \\ G_N/2 & G_N/2 \end{bmatrix} \tag{2}$$

B_N denotes a permutation matrix, $G_1 = [1]$, u_1^N an information set, x_1^N a code word, y_1^N the received word, G_N a generator matrix, and $u_1^N = \{u_1, u_2, \ldots, u_N\}$. In polar coding if u_1^N follows a uniform distribution then $W_N^{(i)}$ is the channel really seen by u_i [5]. The most reliable channels $W_N^{(i)}$ are used to carry the information bits and the least reliable contain frozen bits $Z\left(W_N^{(i)}\right) \leq Z\left(W_N^{(j)}\right)$, with $i \in A$ and $j \in A^c$[10].

3 System Model

In our system model, we propose double encoding, a small polar coding following to STBC, after their concatenation. The items are sent to the MIMO systems. To cooperative diversity system, Rayleigh channel and Additive White Gaussian Noise (AWGN) are also use. We used the same encoding offer to the first section. Alamouti MIMO system is shown in Fig. 1, where N_t represents transmit antennas and N_r receiver antennas, BPSK modulated data stream is used in this model.

After polar encoding, these polar code words are STBC encoded and fed to the N_t transmitting antennas by using

$$\begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} \tag{3}$$

where $x_i\{i = 1, \ldots, N_r\}$ are the ith transmit antenna and $\aleph_t^{N_r}$ represents AWGN which is modeled by independent and identically distributed (i.i.d) samples with variance $\sigma_0/2$. The received signal at the time t is $y_t^{N_r}$ such that:

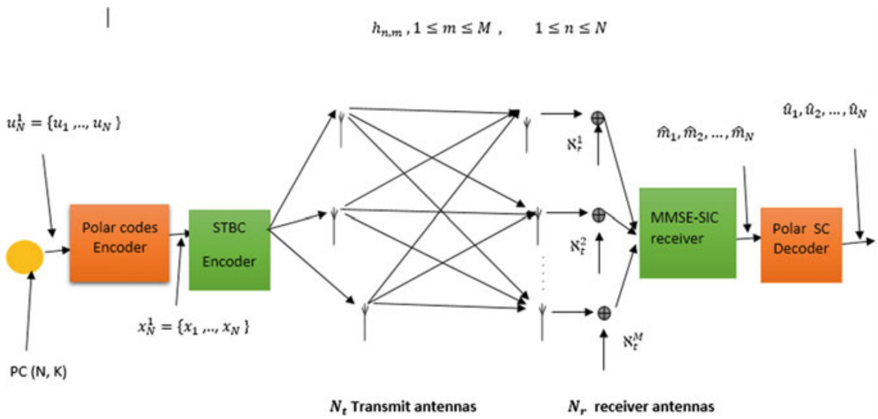

Fig. 1 The system detailed diagram block

$$y_t^{N_r} = \sum_{i=1}^{N_r} h_{j,i} x_i + \aleph_t^{N_r} \quad (4)$$

$h_{j,i}$ is the gain of frequency between the ith transmit and the jth receive antenna, and H is the MIMO matrix channel.

We adopt in this paper this strategy, the rows of each coding scheme represents a different time instant, the columns are the transmitted symbol. Assuming that each symbol has duration T, then at time $t+T$, the symbols $-x_2^*$ and x_1^* where $(.)^*$ denotes the complex conjugate, are transmitted from antenna 1 and antenna 2 respectively.

Suppose that we have two receive antennas, the receivers symbols are developed in [9, 10].

$$y_t^{N_r} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} + \aleph_t^{N_r} \quad (5)$$

The STBC decoder provides the followings signals:

$$\hat{x}_1 = h_{1,1}^* y_1^1 + h_{2,1} y_2^{1*} + h_{1,2}^* y_1^2 + h_{2,2} y_2^{2*} \quad (6)$$

$$\hat{x}_2 = h_{2,1}^* y_1^1 - h_{1,1} y_2^{1*} - h_{1,2} y_2^{2*} + h_{2,2}^{2*} y_1^2 \quad (7)$$

which after changing in (14) and (15) we obtained:

$$\hat{x}_1 = \left(|h_{1,1}|^2 + |h_{2,1}|^2 + |h_{1,2}|^2 + |h_{2,2}|^2\right) x_1 + h_{1,1}^* \aleph_1^1 + h_{2,1} \aleph_2^{1*} + h_{1,2}^* \aleph_1^2 + h_{2,2} \aleph_2^2 \quad (8)$$

$$\hat{x}_2 = \left(|h_{1,1}|^2 + |h_{2,1}|^2 + |h_{1,2}|^2 + |h_{2,2}|^2\right) x_2 - h_{1,1} \aleph_2^{1*} + h_{2,1}^* \aleph_1^1 + h_{1,2} \aleph_2^{2*} + h_{2,2}^* \aleph_1^2 \quad (9)$$

After these two encoding, a soft output as MMSE-SIC is placed to the output, the proposed algorithm is presented to the following section

4 Proposed MMSE-SIC Receiver

The MMSE-SIC principle is to detect signal in one iteration by nulling out other co-Channel Interference. The idea is to use MMSE detector in order to exploit this combining weight matrix [11]. If the signal is detected, it is immediately fed back to the linear combining process and its contribution is cancelled from the received signal in the next detection iteration and found the next the minimum MSE as shown [8].

In [8], the matrix W satisfies $WH = 1$. The MMSE detector is replaced by:

$$W^{i} = \left(H^{(i)H}H^{(i)} + \sigma_n^2 I\right)^{-1} H^{(i)H} \tag{10}$$

The final decision is given by:

$$MSE^{(l)} = k_l^2\left(1 - (H_l^{(i)H} w_l^{(i)}\right) \tag{11}$$

The decision for each antenna is given by

$$\widehat{x}_l^{(i)} = w_l^H y_l \tag{12}$$

$$\widehat{x}_l^{(i)} = \mathrm{sign}\left\{\mathrm{Real}\left(w_l^H y_l\right)\right\} \tag{13}$$

where $H_1^{(l)}$ represents the first column of the channel matrix $H^{(l)}, w_1^{(l)}$ denotes the weight matrix of first antenna, k_l^2 is the transmit signal of antenna l.

The MMSE-SIC algorithm during a STBC encoding (at time T and T+1) can be described as a recursive procedure as follows.

```
Algorithm 1: the recursive MMSE-Algorithm
```

```
Initialization: set i = L
```

`for i = 1 to` $L^{(l)}$

Step 1. `compute the weight matrix using (10)`

Step 2. `Determine MSE for each antenna` l `using (12)`

Step 3. `Determine` $l_i = argmin_{l_i \in L_i}\{MSE_{L_i}\}$

Step 4. `Determine the estimate (12)`

Step 5. `Update the receiver by cancelling the contribution of estimate from received signal`

Step 6. $y_{i+1} = y_i - H_l^{(i)H}\hat{x}_l^{(i)}$

Step 7. $i = i + 1$`, return to Step1.`

This algorithm allow us to determine the first BER performance of MMSE-SIC on SISO, SIMO, MISO and MIMO in Simulation 1.

Table 1 Polar-STBC and STBC only Parameter with SC Decoding

Parameters	Value
Decoding algorithm	Successive Cancellation
Channel characteristic	Rayleigh fading
Polar codes length	32
Polar codes rate (r = k/N)	0.5
fft size	64
Modulation	BPSK
Number of transmitting and receiving antennas of Polar-STBC	$(N_t = 2 \text{ and } N_r = 2)$
Number of transmitting and receiving antennas of STBC only	$N_t = 2$ and $(N_r = 2, N_r = 3)$

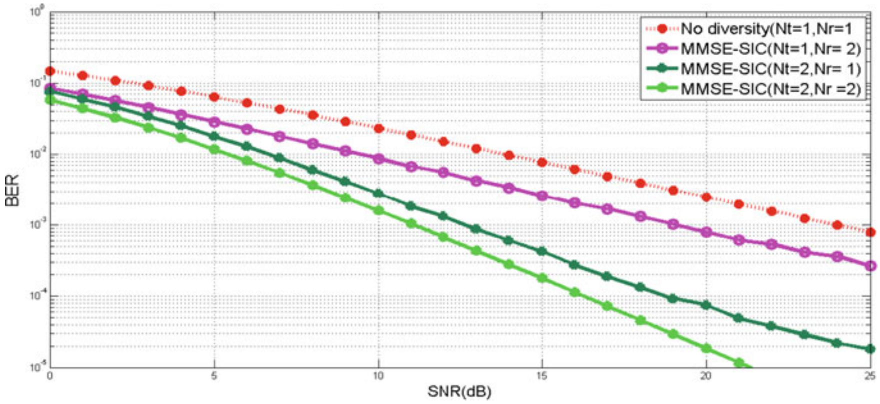

Fig. 2 Comparison between the different diversity schemes

5 Polar Successive Cancellation Decoding

In essence, the SC algorithm is an efficient method of calculation $N = 2^n$ Probability pairs, corresponding to n recursively defined channels. The SC decoding algorithm is wholly described in [5, 12]. We use this algorithm for computing the code word in this paper.

The parameters simulation are placed on the Table 1.

6 Simulation Results 1

In this section, we present the BER results for the proposed MMSE-SIC receiver using Polar-STBC in Fig. 2.

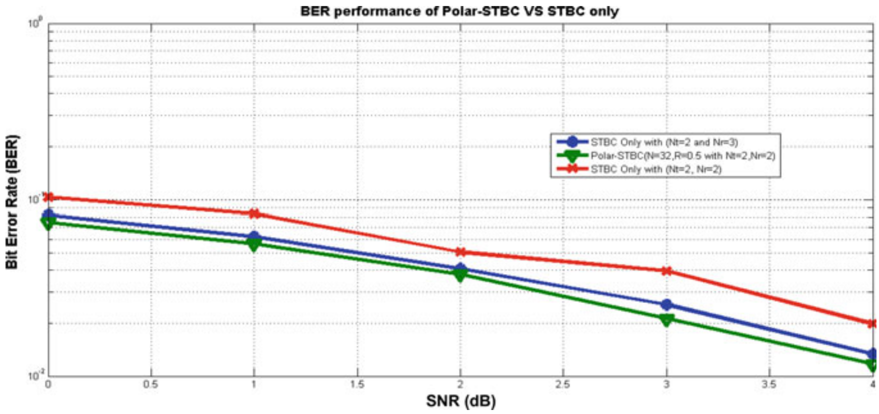

Fig. 3 BER performance between small polar-STBC and STBC only

For instance, at an SNR = 20 dB, the SISO system (1Tx, 1Rx) gives a BER = 5×10^{-3} while the SIMO (1Tx, 2Rx) gives a BER = $2 \times 5\ 10^{-3}$, the MISO system (2Tx, 1Rx) gives a BER = 8×10^{-5} while the MIMO (2Tx, 2Rx) gives BER = 3×10^{-6}. Further at 10^{-5} we noted an improvement over the SISO (1Tx, 1Rx), SIMO and MISO respectively.

7 Simulation Results 2

In Fig. 3, BER performances is analysis at 6×10^{-2} for Polar-STBC using $N_t = 2$ and $N_r = 2$ versus STBC only using $N_t = 2$ and $N_r = 2$ and $N_t = 2$ and $N_r = 3$.

We noted a big improvement when the BER is 6×10^{-2}, the SNR for Polar-STBC 2 * 2 MIMO antennas is about 1 dB improvement over the STBC only using $N_t = 2$ and $N_r = 2$ and better than STBC only using $N_t = 2$ and $N_r = 3$ around 0.3 dB improvement.

At the other hand we introduces the frame error rate performance versus SNR per receiver antenna (e.g. SNR = $2 * E_s/N_0$) in Fig. 4.

Its shown that the FER at 6×10^{-2}, a slight improvement of Polar-STBC using $N_t = 2$ and $N_r = 2$ versus OSTBC only using $N_t = 2$ and $N_r = 2$ about 1.7 dB, but also outperform STBC only using $N_t = 2$ and $N_r = 3$ MIMO around 0.4 dB. These results illustrate that the STBC used in MIMO provide transmit diversity communication over fading channel, but also the coding gain is improved by using polar channel coding.

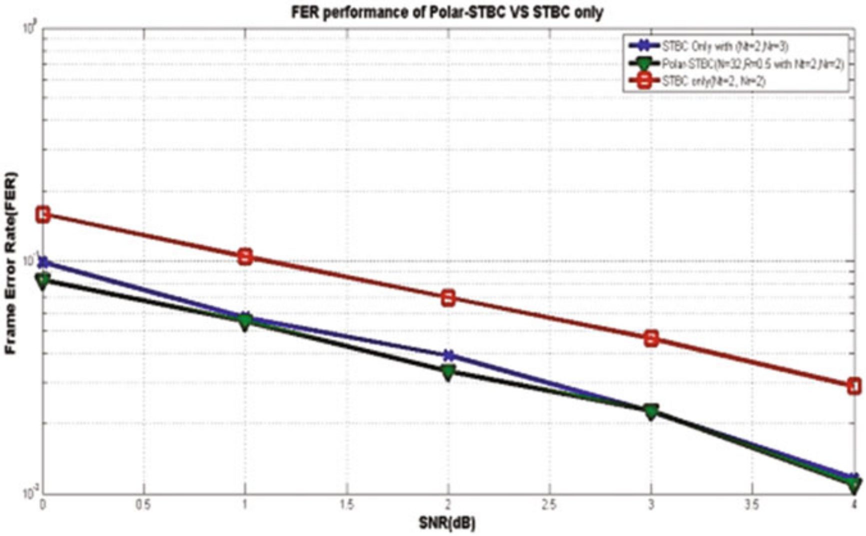

Fig. 4 FER performance between small polar-STBC and STBC only

8 Conclusion

In this paper, after concatenation of small polar codes and STBC codes in order to obtain maximum diversity gain and coding gain, MMSE-SIC receiver is used at the output to further exploit the soft values. In addition, applying successive cancellation decoding the BER is further reduced. The results obtained show that the Polar-STBC has better performance than the STBC even with more antennas. This proposed scheme inherits the advantages of Polar Codes that have both low encoding and decoding complexity but also the advantage of good receiver MMSE-SIC and very good decoding SCD. Indeed, this task is particularly challenging. This proposition open many perspectives such as architecture implementation for small Polar-STBC codes.

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