**List of known updates: January 2021**

The following updates to problems have been gathered since the publication of this book. Eleanor Lingham would like to thank a reviewer for sharing many of these:

**Problem 4.5:** (Sendov's conjecture) Terence Tao [25] has published a result on arxiv showing that the conjecture holds for polynomials of sufficiently large degree.

**Problem 5.20:** (Plessner's theorem) Stephen Gardiner and Myrto Manolaki [16] show that the second statement can be replaced by a much stronger assertion.

**Problem 6.61:** (Conformal sewing of rectifiable curves) Answered negatively by Christopher Bishop [9].

**Problem 6.93:** ($L^p$ version of the Hayman-Wu theorem) Fernández, Heinonen and Martio [14] show this is true for some $p > 1$ and an example of Baernstein [7] shows it fails for some $p < 2$. See also Exercise VII.7 of [17].

**Problem 6.114:** (Hayman-Wu theorem for regular curves) This is proven by Christopher Bishop and Peter Jones [11]. Quasiconformal analogues are considered in [3] (fails in $n = 2$) and [19] (true in higher dimensions).

**Problem 6.115:** (Generalised F. and M. Riesz theorem) This is also proven by Christopher Bishop and Peter Jones [11]. See also [12]. Higher dimensional analogues are considered in [6, 5]. These papers and other recent results by various subsets of the authors are landmarks in geometric measure theory and PDE.

**Problem 7.55:** (Absolute continuity quasiconformal inverses) Dimitrios Ntalampekos and Matthew Romney [24] construct a quasiconformal mapping of $\mathbb{R}^3$ to itself and subset of $\mathbb{R}^2$ of positive area that is mapped to zero area.

**Problem 7.56:** (Conformal weldings) As noted in the update, Oikawa and Huber have shown that not every circle homeomorphism is a conformal welding. A particularly simple example is given in [10, Remark 1], and for sufficient conditions beyond quasisymmetry see [21, 13, 10].

**Problem 7.83(a):** (Conjugations between Fuchsian groups) George Mostow [23] proved any circle homeomorphism between finitely generated Fuchsian groups of the first kind is either Möbius or singular. See also [20, 1, 27, 8] for simplifications of the proof and strengthenings of the result.

**Problem 7.83(b):** (Conjugations between Fuchsian groups) Katsuhiko Matsuzaki [22] shows that for finitely generated Fuchsian groups of the first kind, the Teichmüller space is not dense in the space of Schwarzian derivatives of univalent maps, building on early examples (without a group action) in [18, 15, 2, 26, 4].

If you have any more updates on problems to share, please send the details and citations to Eleanor Lingham at: `haymanslist@gmail.com`.

## Bibliography


[1] S. Agard. A geometric proof of Mostow's rigidity theorem for groups of divergence type. *Acta Math.*, 151(3-4):231–252, 1983.

[2] K. Astala. Self-similar zippers. In *Holomorphic functions and moduli, Vol. I (Berkeley, CA, 1986)*, volume 10 of *Math. Sci. Res. Inst. Publ.*, pages 61–73. Springer, New York, 1988.

[3] K. Astala, J.L. Fernández, and S. Rohde. Quasilines and the Hayman-Wu theorem. *Indiana Univ. Math. J.*, 42(4):1077–1100, 1993.

[4] K. Astala and F.W. Gehring. Crickets, zippers and the Bers universal Teichmüller space. *Proc. Amer. Math. Soc.*, 110(3):675–687, 1990.

[5] J. Azzam, S. Hofmann, J.M. Martell, S. Mayboroda, M. Mourgoglou, X. Tolsa, and A. Volberg. Harmonic measure is rectifiable if it is absolutely continuous with respect to the co-dimension-one Hausdorff measure. *C. R. Math. Acad. Sci. Paris*, 354(4):351–355, 2016.

[6] J. Azzam, S. Hofmann, J.M. Martell, S. Mayboroda, M. Mourgoglou, X. Tolsa, and A. Volberg. Rectifiability of harmonic measure. *Geom. Funct. Anal.*, 26(3):703–728, 2016.

[7] A. Baernstein, II. A counterexample concerning integrability of derivatives of conformal mappings. *J. Analyse Math.*, 53:253–268, 1989.

[8] C. Bishop and T. Steger. Representation theoretic rigidity in psl (2,r). *Acta Math.*, 170(1):121–149, 1993.

[9] C.J. Bishop. Conformal welding of rectifiable curves. *Math. Scand.*, 67(1):61–72, 1990.

[10] C.J. Bishop. Conformal welding and Koebe's theorem. *Ann. of Math. (2)*, 166(3):613–656, 2007.

[11] C.J. Bishop and P.W. Jones. Harmonic measure and arclength. *Ann. of Math. (2)*, 132(3):511–547, 1990.

[12] C.J. Bishop and P.W. Jones. Harmonic measure, $L^2$ estimates and the Schwarzian derivative. *J. Anal. Math.*, 62:77–113, 1994.

[13] G. David. Solutions de l'équation de Beltrami. *Séminaire Équations aux dérivées partielles (Polytechnique) dit aussi "Séminaire Goulaouic-Schwartz"*, 1986-1987. talk:8.

[14] J.L. Fernández, J. Heinonen, and O. Martio. Quasilines and conformal mappings. *J. Anal. Math.*, 52:117–132, 1981.

[15] B.B. Flinn. Jordan domains and the universal Teichmüller space. *Trans. Amer. Math. Soc.*, 282(2):603–610, 1984.

[16] S.J. Gardiner and M. Manolaki. A strong form of Plessner's theorem. *Advances in Mathematics*, 376:107489, 2021.

[17] J.B. Garnett and D.E. Marshall. *Harmonic measure*, volume 2 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2005.

[18] F.W. Gehring. Spirals and the universal Teichmüller space. *Acta Math.*, 141(1-2):99–113, 1978.

[19] J. Heinonen. The diameter conjecture for quasiconformal maps is true in space. *Proc. Amer. Math. Soc.*, 123(6):1709–1718, 1995.

[20] T. Kuusalo. Boundary mappings of geometric isomorphisms of Fuchsian groups. *Ann. Acad. Sci. Fenn. Ser. A. I.*, (545):7, 1973.

[21] O. Lehto. Homeomorphisms with a given dilatation. In *Proceedings of the Fifteenth Scandinavian Congress (Oslo, 1968)*, Lecture Notes in Mathematics, Vol. 118, pages 58–73. Springer, Berlin, 1970.

[22] K. Matsuzaki. Simply connected invariant domains of Kleinian groups not in the closures of Teichmüller spaces. *Complex Variables Theory Appl.*, 22(1-2):93–100, 1993.

[23] G.D. Mostow. *Strong rigidity of locally symmetric spaces*. Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo, 1973. Annals of Mathematics Studies, No. 78.

[24] D. Ntalampekos and M. Romney. On the inverse absolute continuity of quasiconformal mappings on hypersurfaces, 2018.

[25] T. Tao. Sendov's conjecture for sufficiently high degree polynomials, 2020.

[26] W. Thurston. Zippers and univalent functions. In *The Bieberbach conjecture (West Lafayette, Ind., 1985)*, Math. Surveys Monogr., 21, pages 185–197. Amer. Math. Soc., Providence, RI, 1986.

[27] P. Tukia. A rigidity theorem for Möbius groups. *Invent. Math.*, 97(2):405–431, 1989.

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