

# Contents

- 1 Introduction . . . . . 1**
  - 1.1 Random Matrices and Vectors . . . . . 4
  - 1.2 Multivariate Normal Distributions . . . . . 7
  - 1.3 Distributions of Quadratic Forms . . . . . 11
    - 1.3.1 Results for General Covariance Matrices . . . . . 14
  - 1.4 Generalized Linear Models . . . . . 16
  - 1.5 Additional Exercises . . . . . 18
  - References . . . . . 20
- 2 Estimation . . . . . 21**
  - 2.1 Identifiability and Estimability . . . . . 22
  - 2.2 Estimation: Least Squares . . . . . 28
  - 2.3 Estimation: Best Linear Unbiased . . . . . 33
  - 2.4 Estimation: Maximum Likelihood . . . . . 34
  - 2.5 Estimation: Minimum Variance Unbiased . . . . . 35
  - 2.6 Sampling Distributions of Estimates . . . . . 37
  - 2.7 Generalized Least Squares . . . . . 38
  - 2.8 Normal Equations . . . . . 43
  - 2.9 Variance-Bias Tradeoff . . . . . 44
    - 2.9.1 Estimable Functions . . . . . 47
  - 2.10 Bayesian Estimation . . . . . 48
    - 2.10.1 Distribution Theory . . . . . 52
  - 2.11 Additional Exercises . . . . . 57
  - References . . . . . 59
- 3 Testing . . . . . 61**
  - 3.1 More About Models . . . . . 61
  - 3.2 Testing Models . . . . . 64
    - 3.2.1 Small Test Statistics . . . . . 71
    - 3.2.2 A Generalized Test Procedure . . . . . 72

3.3	Testing Linear Parametric Functions . . . . .	74
3.3.1	A Generalized Test Procedure . . . . .	83
3.3.2	Testing an Unusual Class of Hypotheses . . . . .	86
3.4	Discussion . . . . .	88
3.5	Testing Single Degrees of Freedom in a Given Subspace . . . . .	89
3.6	Breaking a Sum of Squares into Independent Components . . . . .	90
3.6.1	General Theory . . . . .	91
3.6.2	Two-Way ANOVA . . . . .	95
3.7	Confidence Regions . . . . .	97
3.8	Tests for Generalized Least Squares Models . . . . .	98
3.8.1	Conditions for Simpler Procedures . . . . .	101
3.9	Additional Exercises . . . . .	103
	References . . . . .	105
<b>4</b>	<b>One-Way ANOVA . . . . .</b>	<b>107</b>
4.1	Analysis of Variance . . . . .	108
4.2	Estimating and Testing Contrasts . . . . .	116
4.3	Additional Exercises . . . . .	120
<b>5</b>	<b>Multiple Comparison Techniques . . . . .</b>	<b>123</b>
5.1	Basic Ideas . . . . .	124
5.2	Scheffé's Method . . . . .	128
5.3	Least Significant Difference Method . . . . .	133
5.4	Bonferroni Method . . . . .	135
5.5	Tukey's Method . . . . .	135
5.6	Multiple Range Tests: Newman–Keuls and Duncan . . . . .	137
5.7	Summary . . . . .	139
5.7.1	Fisher Versus Neyman–Pearson . . . . .	142
5.8	Additional Exercises . . . . .	142
	References . . . . .	143
<b>6</b>	<b>Regression Analysis . . . . .</b>	<b>145</b>
6.1	Simple Linear Regression . . . . .	146
6.2	Multiple Regression . . . . .	148
6.2.1	Partitioned Model . . . . .	151
6.2.2	Nonparametric Regression and Generalized Additive Models . . . . .	153
6.3	General Prediction Theory . . . . .	155
6.3.1	Discussion . . . . .	155
6.3.2	General Prediction . . . . .	156
6.3.3	Best Prediction . . . . .	156
6.3.4	Best Linear Prediction . . . . .	160
6.3.5	Inner Products and Orthogonal Projections in General Spaces . . . . .	164
6.4	Multiple Correlation . . . . .	165

6.4.1	Squared Predictive Correlation . . . . .	168
6.5	Partial Correlation Coefficients . . . . .	170
6.6	Best Linear Unbiased Prediction . . . . .	172
6.7	Testing Lack of Fit . . . . .	177
6.7.1	The Traditional Test . . . . .	178
6.7.2	Near Replicate Lack of Fit Tests . . . . .	180
6.7.3	Partitioning Methods . . . . .	182
6.7.4	Nonparametric Methods . . . . .	185
6.8	Polynomial Regression and One-Way ANOVA . . . . .	186
6.9	Additional Exercises . . . . .	191
	References . . . . .	194
<b>7</b>	<b>Multifactor Analysis of Variance . . . . .</b>	<b>197</b>
7.1	Balanced Two-Way ANOVA Without Interaction . . . . .	197
7.1.1	Contrasts . . . . .	203
7.2	Balanced Two-Way ANOVA with Interaction . . . . .	204
7.2.1	Interaction Contrasts . . . . .	207
7.3	Polynomial Regression and the Balanced Two-Way ANOVA . . . . .	214
7.4	Two-Way ANOVA with Proportional Numbers . . . . .	217
7.5	Two-Way ANOVA with Unequal Numbers: General Case . . . . .	219
7.5.1	Without Interaction . . . . .	219
7.5.2	Interaction . . . . .	222
7.5.3	Characterizing the Interaction Space . . . . .	228
7.6	Three or More Way Analyses . . . . .	230
7.6.1	Balanced Analyses . . . . .	230
7.6.2	Unbalanced Analyses . . . . .	232
7.7	Additional Exercises . . . . .	238
	References . . . . .	240
<b>8</b>	<b>Experimental Design Models . . . . .</b>	<b>241</b>
8.1	Completely Randomized Designs . . . . .	242
8.2	Randomized Complete Block Designs: Usual Theory . . . . .	242
8.3	Latin Square Designs . . . . .	243
8.4	Factorial Treatment Structures . . . . .	247
8.5	More on Factorial Treatment Structures . . . . .	250
8.6	Additional Exercises . . . . .	253
	References . . . . .	253
<b>9</b>	<b>Analysis of Covariance . . . . .</b>	<b>255</b>
9.1	Estimation of Fixed Effects . . . . .	256
9.1.1	Generalized Least Squares . . . . .	260
9.2	Estimation of Error and Tests of Hypotheses . . . . .	261
9.3	Another Adjusted Model and Missing Data . . . . .	264
9.4	Balanced Incomplete Block Designs . . . . .	267

9.5	Testing a Nonlinear Full Model . . . . .	276
9.6	Additional Exercises . . . . .	277
	References . . . . .	279
<b>10</b>	<b>General Gauss–Markov Models . . . . .</b>	<b>281</b>
10.1	BLUEs with an Arbitrary Covariance Matrix . . . . .	282
10.2	Geometric Aspects of Estimation . . . . .	288
10.3	Hypothesis Testing . . . . .	291
10.4	Least Squares Consistent Estimation . . . . .	296
10.5	Perfect Estimation and More . . . . .	304
	References . . . . .	311
<b>11</b>	<b>Split Plot Models . . . . .</b>	<b>313</b>
11.1	A Cluster Sampling Model . . . . .	314
11.2	Generalized Split Plot Models . . . . .	318
11.2.1	Estimation and Testing of Estimable Functions . . . . .	322
11.2.2	Testing Models . . . . .	326
11.2.3	Unbalanced Subplots . . . . .	328
11.3	The Split Plot Design . . . . .	328
11.4	Identifying the Appropriate Error . . . . .	332
11.4.1	Subsampling . . . . .	332
11.4.2	Two-Way ANOVA with Interaction . . . . .	335
11.5	Exercise: An Unusual Split Plot Analysis . . . . .	337
	References . . . . .	338
<b>12</b>	<b>Model Diagnostics . . . . .</b>	<b>341</b>
12.1	Leverage . . . . .	344
12.1.1	Mahalanobis Distances . . . . .	345
12.1.2	Diagonal Elements of the Projection Operator . . . . .	347
12.1.3	Examples . . . . .	348
12.2	Checking Normality . . . . .	354
12.2.1	Other Applications for Normal Plots . . . . .	360
12.3	Checking Independence . . . . .	362
12.3.1	Serial Correlation . . . . .	363
12.4	Heteroscedasticity and Lack of Fit . . . . .	369
12.4.1	Heteroscedasticity . . . . .	369
12.4.2	Huber–White (Robust) Sandwich Estimator . . . . .	374
12.4.3	Lack of Fit . . . . .	377
12.4.4	Residual Plots . . . . .	380
12.5	Updating Formulae and Predicted Residuals . . . . .	380
12.6	Outliers and Influential Observations . . . . .	384
12.7	Transformations . . . . .	388
	References . . . . .	390

<b>13</b>	<b>Collinearity and Alternative Estimates</b>	393
13.1	Defining Collinearity	394
13.2	Tolerance and Variance Inflation Factors	398
13.3	Regression in Canonical Form and on Principal Components	401
13.3.1	Regression in Canonical Form	401
13.3.2	Principal Component Regression	404
13.3.3	Generalized Inverse Regression	404
13.4	Classical Ridge Regression	405
13.4.1	Ridge Applied to Principal Components	408
13.5	More on Mean Squared Error	410
13.6	Robust Estimation and Alternative Distance Measures	410
13.7	Orthogonal Regression	413
	References	416
<b>14</b>	<b>Variable Selection</b>	419
14.1	All Possible Regressions and Best Subset Regression	421
14.1.1	$R^2$	421
14.1.2	Adjusted $R^2$	422
14.1.3	Mallows's $C_p$	423
14.1.4	Information Criteria: AIC, BIC	425
14.1.5	Cost Complexity Pruning	427
14.2	Stepwise Regression	428
14.2.1	Traditional Forward Selection	428
14.2.2	Backward Elimination	431
14.2.3	Other Methods	433
14.3	Discussion of Traditional Variable Selection Techniques	433
14.3.1	$R^2$	434
14.3.2	Influential Observations	435
14.3.3	Exploratory Data Analysis	435
14.3.4	Multiplicities	436
14.3.5	Predictive Models	436
14.3.6	Overfitting	437
14.4	Modern Forward Selection: Boosting, Bagging, and Random Forests	437
14.4.1	Boosting	438
14.4.2	Bagging	441
14.4.3	Random Forests	445
	References	446

<b>Appendix A: Vector Spaces</b> .....	447
<b>Appendix B: Matrix Results</b> .....	457
B.1 Basic Ideas .....	457
B.2 Eigenvalues and Related Results .....	459
B.3 Projections .....	463
B.4 Miscellaneous Results .....	471
B.5 Properties of Kronecker Products and Vec Operators .....	473
B.6 Tensors .....	474
B.7 Exercises .....	476
<b>Appendix C: Some Univariate Distributions</b> .....	481
<b>Appendix D: Multivariate Distributions</b> .....	485
D.1 Identifiability .....	489
<b>Appendix E: Inference for One Parameter</b> .....	491
E.1 Testing .....	493
E.2 $P$ Values .....	495
E.3 Confidence Intervals .....	496
E.4 Final Comments on Significance Testing .....	497
<b>Appendix F: Significantly Insignificant Tests</b> .....	499
F.1 Lack of Fit and Small $F$ Statistics .....	501
F.2 The Effect of Correlation and Heteroscedasticity on $F$ Statistics ...	503
<b>Appendix G: Randomization Theory Models</b> .....	509
G.1 Simple Random Sampling .....	509
G.2 Completely Randomized Designs .....	511
G.3 Randomized Complete Block Designs .....	513
<b>References</b> .....	516
<b>Author Index</b> .....	517
<b>Subject Index</b> .....	521

Plane Answers to Complex Questions

The Theory of Linear Models

Christensen, R.

2020, XXII, 529 p. 33 illus., Hardcover

ISBN: 978-3-030-32096-6