

Errata for *Spectral Theory*

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Please email any additional corrections to dborthw@emory.edu.

Chapter 4

p. 85: Second resolvent identity

This formula should read,

$$(S - z)^{-1} - (T - z)^{-1} = (S - z)^{-1}(T - S)(T - z)^{-1},$$

both here and in Exercise 4.1

p. 96: Exercise 4.1

The second resolvent identity should read

$$(S - z)^{-1} - (T - z)^{-1} = (S - z)^{-1}(T - S)(T - z)^{-1}.$$

p. 116: Proof of Thm 5.11

Missing brackets in the second paragraph. The function $Q^{-1}\phi$ has support on $\{\alpha = \lambda\}$.

p. 133: First equation

The γ is erroneous. The first equation should read

$$|\langle f, v \rangle| \leq \|f\| \|v\| \leq \|f\| \|v\|_{H^1},$$

p. 136: Proof of Thm 6.8

The first sentence of the second paragraph is mixed up. This paragraph should read as follows:

If $u \in \mathcal{D}(-\Delta_D)$, then by (6.13) we have

$$(1) \quad \|u\|_{H^1}^2 = \langle u, (-\Delta + 1)u \rangle.$$

By Cauchy-Schwarz and the fact that $\|u\| \leq \|u\|_{H^1}$, this implies that

$$\|u\|_{H^1} \leq \|(-\Delta + 1)u\|.$$

This shows that $(-\Delta_D + 1)^{-1}$ is bounded as a map $L^2(\Omega) \rightarrow H_0^1(\Omega)$. Therefore $(-\Delta_D + 1)^{-1}$ is compact as a map $L^2(\Omega) \rightarrow L^2(\Omega)$ by Theorem 6.9.

p. 144: Corollary 6.16

The assumption on ψ should read $\psi \in H^1(\Omega)$.

p. 151: Equation (6.48)

The σ in the brackets should be $\sigma(-\Delta_D)$.

p. 151: Equation (6.49)

These inequalities are backwards. The equation should read:

$$N_{\mathcal{R}_1}(t) \leq N_{\Omega}(t) \leq N_{\mathcal{R}_2}(t).$$

p. 172: Proof of Thm 6.34

The definition of $\psi_1^\pm$ should read

$$\psi_1^\pm(x) := \max\{\pm\psi_1(x), 0\}.$$

p. 195: Proof of Thm 7.7

In last part of the proof, B was mistakenly assumed to be closed. Here is a clean version of the final two paragraphs:

Now assume that A is merely essential self-adjoint. If $u \in \mathcal{D}(\overline{A})$, then there exists a sequence $u_n \rightarrow u$ with $u_n \in \mathcal{D}(A)$, such that Au_n converges to $\overline{A}u$. By the assumption (7.24), the sequence Bu_n also converges, so that $u \in \mathcal{D}(\overline{B})$ (since B is closed). By continuity, we can extend (7.24) to

$$(2) \quad \|\overline{B}u\| \leq \alpha\|\overline{A}u\| + \beta\|u\|$$

for all $u \in \mathcal{D}(\overline{A})$. By the first part of the proof, this implies that $\overline{A} + \overline{B}$ is self-adjoint on the domain $\mathcal{D}(\overline{A})$.

It remains to check that $\overline{A + B} = \overline{A} + \overline{B}$. Since $\overline{A} + \overline{B}$ is a closed extension of $A + B$, we have $\overline{A + B} \subset \overline{A} + \overline{B}$. On the other hand, the assumption (7.24) gives

$$\|(A + B)u\| \leq (\alpha + 1)\|Au\| + \beta\|u\|.$$

For $u \in \mathcal{D}(\overline{A})$ this implies that $u \in \mathcal{D}(\overline{A + B})$ and that $(\overline{A + B})u = (\overline{A} + \overline{B})u$. In other words,

$$\overline{A + B} \subset \overline{A} + \overline{B}.$$

We conclude that $\overline{A + B}$ is self-adjoint on $\mathcal{D}(A)$.

Spectral Theory

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