2. REVIEW QUESTIONS

1. Integration of scalar fields follows the rules of calculus: These are not affected by the rules of vector calculus *T/F*. Explain

2. What is a line integral? How does it differ from a regular scalar integral?

3. What do line integrals represent? Give examples.

4. Define the idea of circulation. How does it relate to line integrals?

5. Is a line integral always independent of path? Explain.

6. What do surface integrals represent? Give examples.

7. What does a closed surface integral represent? Give examples.

8. What does an open surface integral represent? Give examples.

9. What does a volume integral represent? Give examples.

10. A volume integral can be:

(a) A vector quantity only.

(b) A scalar quantity only.

(c) A vector or scalar quantity.

11. Define the gradient. What is the physical meaning of the gradient? Give examples.

12. The gradient operates on a scalar function and results in a vector function *T/F*.

13. The gradient is perpendicular to surfaces of constant scalar quantities. Show how this property can be used to:

(a) Find surfaces of constant scalar values.

(b) Find the normal unit vector to a surface at a point.

14. The symbolic operation indicated by the gradient is identical in all systems of coordinates *T/F*.

15. The del operator is identical in all systems of coordinates *T/F*.

16. The gradient of a scalar function is different in different systems of coordinates *T/F*.

17. List the important uses of the gradient.

18. Derive the divergence of a vector field. What does it signify?

19. The notation $\nabla \cdot \mathbf{A}$ represents a scalar product *T/F*.

20. State the divergence theorem. Why is it an important theorem?

21. What is the physical meaning of the divergence theorem?

22. The divergence (mark correct answer):

(a) Acts on a vector field and results in a vector field.

(b) Acts on a vector field and results in a scalar field.

(c) Acts on a scalar field or a vector field.

23. Discuss the divergence theorem and the link between surface and volume quantities it uses.

24. Define circulation of a vector field.

25. Define the curl of a vector field. How do you understand this definition? What does it imply?

26. The curl operates on a vector function and results in a vector function *T/F*.

27. The symbolic operation indicated by the curl is identical in all systems of coordinates *T/F*.

28. The curl of a vector function is the same in different systems of coordinates *T/F*.

29. List the important uses of the curl.

30. The curl of a vector field indicates (mark correct answer):

(a) That the vector field is circular.

(b) That the vector field varies in space.

(c) That the vector field is constant in space.

(d) None of the above

31. State Stokes' theorem. What is the physical meaning of Stokes' theorem?

32. Discuss Stokes' theorem and the link between surface and line quantities it uses.

33. Indicate which of the following expressions are correct. If an expression is not correct, indicate the correction needed to make true the right-hand side only:

(a) $\int_s (\nabla \times \mathbf{A}) \cdot \hat{\mathbf{n}}\, ds = \oint_C \mathbf{A} \cdot d\mathbf{l}$

(b) $|\nabla \cdot \mathbf{D}| = \left(\frac{\partial D_x}{\partial x}\right)^2 + \left(\frac{\partial D_y}{\partial y}\right)^2 + \left(\frac{\partial D_z}{\partial z}\right)^2$

(c) $|\nabla \cdot \nabla \phi| = 0$

(d) $|\nabla \times \nabla \phi| = 0$

(e) $(\nabla \times \mathbf{A})_x = \frac{1}{dy\, dz} \oint_s \mathbf{A} \cdot \hat{\mathbf{n}}\, ds$

34. What is a conservative field?

35. What is the necessary condition for a vector field to be conservative?

36. If a vector field is defined as the gradient of a scalar, the vector field is nonconservative *T/F*.

37. What does "conservative" in "conservative field" stand for?

38. State the Helmholtz theorem.

39. What is the main consequence of the Helmholtz theorem in terms of the unique definition of vector fields?

40. What is a solenoidal, irrotational field? Give examples.

41. What is a nonsolenoidal, irrotational field? Give examples.

42. How many types of vector fields does the Helmholtz theorem allow? What are they?