Questions from Colleagues and Friends

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More detailed answers to some of the following questions can be found in the interview and in Olav's autobiographical and mathematical Notes.

My name is Richard Kruel (R.K.) and I am the publishing editor for Springers *Probability Theory and Stochastic Modelling* series. As we finalized Olav Kallenbergs third edition of his *Foundations of Modern Probability* (FMP), I thought it would be interesting to organize an interview with the author to get to know more about him and his work. I wrote to some of his collaborators and friends to help me prepare the interview. I particularly wanted to thank Robert Adler, David Aldous, Amarjit Budhiraja, Kamesh Casukhela, Steven Evans, Anders Grimvall, Olle Häggström, Sture Holm, Martin Jacobsen, Peter Jagers, Svante Janson, Takis Konstantopoulos, Klaus Krickeberg, Andreas Kyprianou, Günter Last, Ross Leadbetter, Ming Liao, Torgny Lindvall, Ilya Molchanov, Lisa Peterson, Holger Rootzén, Hermann Thorisson, Anton Wakolbinger, Martina Zähle and Hans Zessin and for their help.

Here are some of their questions and Olav's initial answers. Other questions implicitely found their way into our interview that you can find here. The interview also prompted Olav to elaborate a bit more on his life and work in two separate files that you can find on the same webpage.

In the following, Olav Kallenbergs books are quoted as follows:

(K05) *Probabilistic symmetries and invariance principles*, Springer 2005

(K17) *Random measures, theory and applications*, Springer 2017

(K21) *Foundations of modern probabillity*, 3rd ed., Springer 2021

Questions from Andreas Kyprianou, University of Bath, UK

A.K. What was the motivation behind writing your *Foundations of Modern Probability* in the first place?

O.K. I started writing lecture notes for my class. Then it got fun when I discovered that practically every proof given in the standard literature could be simplified, and I also wanted a different emphasis, more on understanding and overview, less on computations. Finally, it was when I became Editor-in-Chief of PTRF and I had so little time, so I wanted to do something easy.

A.K. Are you aware of how important your book has become ? It is often seen as a modern classic in probability.

O.K. I am happy to hear from people who like my book, since it strengthens my belief in my primary mission, trying to keep the subject together.

A.K. If you had more time and energy, what else would you include in your book?

O.K. I regret I didn't have time, energy, or space to include the infinite particle systems, as presented in the excellent books by Liggett. There is so much else. I was looking for a while for a simple, noncomputational proof of the semicircle law, which is such a beautiful theorem. Then SPDEs is such a huge and interesting subject, which I should have done. Finally, I wish I could have gone further into Malliavin calculus, trying to avoid the technical complications.

A.K. What is your perception of how far probability has evolved in the last 50 years?

O.K. I always claimed that the period 1900-80 was the golden age of probability. Now I am not seeing much of new revolutionary ideas, but probably this just reveals my ignorance. So many clever people are still around, mostly concerned with technical extensions or applications to physics and other areas. Much of this is very impressive, but it is not "foundations" anymore. Whether there is anything else that remains to be discovered, we never know. Already after Euclid or after Newton, people said that all the important stuff was done, and what remains is to fill in some details and solve a few outstanding open problems.

A.K. Given the breadth of topics in probability theory, how did you choose your writing style, in particular to accommodate generic consumption?

O.K. It is all based on my own taste and feeling of what is most important. I always prefer the simple basic ideas that give important insight, avoiding computational stuff that tells us less.

Questions from David Aldous, University of California, Berkeley, USA

D.A. You are unusual nowadays in mostly working alone. Can you tell us a bit more about your working process ? Could you comment on Terry Tao's "three stages of learning to do mathematics"?

O.K. I actually love collaboration with others. The main reason for my solitary style is that I have always been attracted by problems that nobody else seemed to care about, simply because they were not "mainstream" or fashionable, or were not "in my field."

Tao's comments are very interesting; it is fascinating to see how such a great and famous mathematician is thinking and working. But then I have to admit that my approach is the exact opposite, in that I would bypass step 1-2 and go directly to 3. To me intuition always has to come first, and it always comes when I am not "working," such as when I am practicing the piano or taking a walk. When I have a hard problem, I may wake up in the middle of the night and see how to solve my problem. Then I will get up and scribble some notes, before going back to sleep. Sometimes it may take days, perhaps even weeks, until I have a complete technical proof. To be honest I am not very good at computation, and to this day I never managed to memorize the multiplication table or the quadratic formula.

Questions from Peter Jagers, Chalmers University of Technology, Sweden

P.J. I would be interested in hearing about your move from Sweden to Alabama, differences and similarities socially, scientifically and, culturally. I am also interested in your opinion of people you met and were stimulated by. I know the German school of random measures and point processes was important to you. What was your relationship to Klaus Matthes?

O.K. The differences between Sweden and the US are huge. The universities in the US form a hierarchy, with the ivy league universities and Berkeley, UCLA, etc on the top. All parents want their children into a university as high up on the ladder as possible, which leads to a negative selection for the less famous universities. Auburn University is one of the top universities in Alabama, but can't compare with the famous schools in the US. As for math, most American kids want to become doctors, lawyers, or engineers, and if they fail to get into any of those professions, there is only math left, which leads to a negative selection into math too. Most undergraduate students who have to take a math class have already decided from day one that they hate the subject. Then there are exceptions, and in every class there are always a few very bright kids. The tempo of math teaching is much higher, and the students are used to have tests and quizzes more or less every week. With respect to graduate studies, we are getting lots of oriental students who are usually brilliant and very ambitious. Nowadays, most of the faculty hired are oriental, and native-born professors are getting rare.

A huge cultural difference is that English is a world language, and in a good bookstore you can find hundreds of excellent books on every subject. Even the top newspapers are fantastic. Politically, the US is very conservative, especially here down in the South. In Sweden I had a constant problem with housing caused by annoying government regulations, whereas in the US life is so easy, there are affordable apartments everywhere. Even a moderately liberal person like me would probably be regarded as far-left here in the US. Then finally, the US is a very religious country, where a large proportion of the population are going regularly to church and even attend bible studies. By a recent survey, some 40 % of the American people are creationists and believe that the world was created by God some 6,000 years ago. Even Newton would have agreed, but that was before Darwin.

Matthes was always very kind and helpful, and he was such a great communicator and a brilliant mathematician. But he resisted any attempt to a more relaxed and informal relationship.

Question from Ilya Molchanov, Universität Bern, Switzerland

I.M. Do you foresee that future generations of probabilists will continue your work and in which directions ?

O.K. If you mean my research, I can only hope so. If you mean my work on FMP, then an unqualified yes. I believe that it will be generally recognized, sooner or later, that we must make every effort to keep not only probability but all of math together. If we fail, then the subject will eventually disintegrate into thousands of little subfields, and we will lose our overview.

Questions from Martina Zähle, Universität Jena, Germany

M.Z. How do you stay motivated to work on your lenghty book projects? You have worked in multiple areas of probability. How important are is linking different areas together in your approach to research ?

O.K. Writing FMP I never regarded as a great effort, it was only something I enjoyed. Since I had been working myself in most of the covered areas, I already knew most of the covered material. To me it is absolutely crucial for the research to have a broad knowledge of probability and indeed of all of mathematics. That may help you to recognize a martingale or a Hilbert space when you need it.

Question from Hans Zessin, Universität Bielefeld, Germany

H.Z. Can you tell us a bit about your work in point processes ?

O.K. Point processes are just one of my many interests, but it was exciting to experience the great interest in the area in those days.

Questions from Anton Wakolbinger, Universität Frankfurt, Germany

A.W. At a workshop at the Mittag-Leffler Institute in 2013 you gave a beautiful talk on "Some failures, problems and conjectures from a lifetime in probability". Eight years later, which of these in your view is (still) the most intriguing one, and in which of them did you see signs of progress in these years?

Your FMP book has become one of the most remarkable classics in its genre, combining (and reconciling) features of an encyclopedia, a monograph, and a (very advanced and comprehensive) textbook. If you were to set out for a fourth edition right now, which chapters would you pay the most substantial attention? What would your answer be for a second edition of *Probabilistic Symmetries and Invariance Principles* ?

O.K. I haven't thought of those six problems since I gave my talk, and to my knowledge nobody else did either. So, they seem to be still open. Regarding you second and third question, it is a little early to think about what I would like to change in a hypothetical fourth edition, since I have just finished the third one. Please ask me again some ten years from now, if I am still alive.

You are saying that my FMP is "very advanced", which I hope it is not. Apart from some topics at the end of each chapter, I think that the level of difficulty is comparable to that of other graduate level textbooks in probability. I used it many times in my own graduate classes, and I never had any such problems.

Questions from Günter Last, Karlsruhe Institue of Technology, Germany

G.L. Can you tell us a bit about your opinion about the relationship between the natural sciences and mathematics ? How much progress in mathematics has been stimulated by so called applied problems ? The answer might depend on the field of mathematics we're talking about. In my opinion a lot of ideas in probability come from statistical mechanics. One a more personal note, I wonder where you get your energy for all your ambitious projects.

O.K. I believe (like Galileo?) that many areas of the sciences can only be understood through mathematics. I also agree (with Einstein) that the most remarkable thing about our world is that it is comprehensible (through mathematics). In my opinion, the greatest challenge of science is to make sense of quantum mechanics. I believe that the Copenhagen interpretation of quantum mechanics is wrong and must be discarded. I also believe that the arrow of time is real and not just an illusion caused by the second law of thermodynamics. What areas of mathematics are needed for a full understanding is impossible to tell. It may turn out to be some new math that is not discovered yet. I also believe that all basic propositions in mathematics exist "out there" independent of our discovery. In that sense, math itself is part of natural science.

Regarding my "energy," I am very lazy by nature and am working very little. All my inspiration ultimately comes from my passion for music and art.

Question from Steven Evans, University of California, Berkeley, USA

S.E. I would want to know what would be in the imagined "Volume 2" of "Foundations of Modern Probability". Some topics that come to mind are random matrices, the stochastic Loewner equation, and the Gaussian free field. Do you think that any of these objects (or perhaps some others) have become fundamental enough to be included?

O.K. My criteria for inclusion were not only whether the subjects were "foundational," but also whether it would be possible to do them justice in a short chapter of some 20-25 pages. Regarding the three topics mentioned by Steve, I may comment as follows:

1. Random matrices: In the new edition of FMP, I do have a chapter on random arrays (#28). I did spend some effort to look for a short, non-computational proof of the semi-circle law, but I couldn't find any (partly due to a lack of time). The area has been very fashionable for a very long time, but all extensions I have seen are extremely technical and physically irrelevant, as explained in my historical notes to that chapter.

2. The Loewner evolution would probably have been an ideal subject for an additional chapter.

3. I don't know enough about the Gaussian free field to have any opinion.

Other subjects I would have loved to include in a hypothetical fourth edition might be Liggett-type particle systems, SPDE's and super-processes.

Question from Hermann Thorisson, University of Iceland

H.T. Modern probability rests mainly on measure theory and to some extent on topology. Have the foundations of modern probability now reached the same level of maturity as those fellow 20th century fields? Or is probability still a frontier field?

O.K. The moment when probability theory attained full maturity can be dated quite precisely to 1933 (88 years ago), when Kolmogorov gave the modern definition of conditional expectations and proved the existence of random processes with given finite-dimensional distributions. Since then, probability has been an integral part of especially real and functional analysis, with close ties to ordinary and partial differential equations. Work in probability theory also relies routinely on topology and occasionally even on abstract algebra and differential geometry. Close ties with potential theory were established in the 1950's through the work of Doob, Hunt, and many others, and many books have since been written about the profound (two-way!) interaction between analysis and probability. The 20th century was no doubt the golden age of modern probability. — Unfortunately, the role of probability theory as an integral part of mathematical analysis is totally unknown by the general public, and surprisingly even by many fellow mathematicians.

Questions from Takis Konstantopoulos, University of Liverpool, UK

T.K. In the old days it was believed that you need measure theory to develop probability theory, but not anymore. How do you think that this will impact the future?

O.K. I am very much concerned about the future of mathematics, where the amount of knowledge is growing exponentially at a high rate, whereas the teaching at most colleges in at least the US has got stuck in a pattern from a hundred years ago. Most of my former colleagues still think of mathematics as divided into algebra, topology, and analysis, and all incoming students in my department are recommended to take year-long courses in those three areas. If they ever come to the point of studying probability, they have already used up most of their required course hours, and it is time to get started on a research topic. The solution is to start with probability already on day one, and make sure to study all the basic areas of the subject, to get as broad general background as possible. Most of the required facts from measure theory, functional analysis, etc., the students will pick up on the way.

T.K. Nowadays, young probabilists have to specialize immediately without any chance to grasp the subject "holistically". What is to be done with this over-specialization?

O.K. My answer is the same as before. On the question of what is most important in math, to be broad or deep, my answer is that you need to be both.

T.K. I once heard you say that whereas we used to think of Brownian motion as the fundamental stochastic process, we must now shift our attention to the Dawson-Watanabe process. Can you elaborate on that?

O.K. This is a misunderstanding: I still believe in the fundamental importance of Brownian motion. What I have said many times is that the

Dawson-Watanabe super-process may be the single most interesting (!) object in probability theory. Several book-length surveys have been published by different authors, and they are all totally different, focusing on different aspects of the theory. Alison Etheridge wrote a wonderful introduction, but a more detailed and comprehensive coverage would require a thousand pages.

T.K. As someone who's been an expert in probabilistic symmetries, I'd love to hear your view on how we can use symmetry in teaching and research.

O.K. Aspects of symmetry and invariance are fundamental in most areas of modern mathematics, but in probability they have been somewhat downplayed or neglected. I think that they need to be brought up in forefront as fundamental notions of probability, beside the properties of dependence. My own contributions are a whole book on symmetries, plus two new chapters in FMP-3.

R.K. Dear Olav, after reading the questions you got from your friends and collaborators, are there any other topics that you thought you would be asked about?

O.K. I thought I would get a lot of questions about my research in Mathematics. If the roles were reversed, I might ask myself:

1. What is the most difficult thing you ever did in math?
2. Of all your mathematical discoveries, which one do you regard as the most surprising?

R.K. I am curious to know the answers to these questions

O.K. To characterize the multi-variate symmetries was doubtless the hardest thing I have ever done, taking me some 10-15 years from my first conjectures to the complete proofs, requiring about 100 pages of tight reasoning covered in Chapters 7 and 8 of my *Probabilistic Symmetries and Invariance Principles* book.

The characterization of strong stationarity [1] may be the result that surprised me the most. For the context, recall that the three basic dependence structures in probability theory are stationarity, Markov processes, and martingales. Of those three, the Markov property extends to a strong version in terms of optional (stopping) times, and similarly for the martingale property via the optional sampling theorem. But the extension to optional times fails for stationarity, and in fact invariance in distribution under optional shifts is equivalent to the de Finetti property of conditional i.i.d. sequences.

[1] K05, Sec. 2.1; K21 Ch. 27.

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