A lifetime in probability

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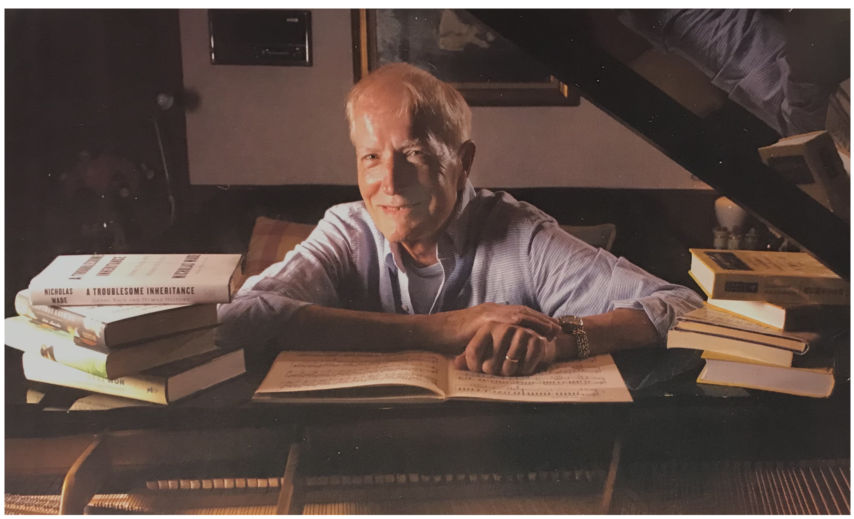
This interview was organized by the publishing editor of the *Probability Theory and Stochastic Modelling* series (`https://www.springer.com/series/13205`) where the third edition of Olav Kallenbergs *Foundations of Modern Probability* was recently published. In the following, Olav Kallenbergs books are quoted as follows:

(K05) *Probabilistic symmetries and invariance principles*, Springer 2005

(K17) *Random measures, theory and applications*, Springer 2017

(K21) *Foundations of modern probabillity*, 3rd ed., Springer 2021



Olav at the piano with books, 2014.

Dear Olav, thank you for taking your time for this interview. Most people in probability know your book "*Foundations of Modern Probability*" and would surely love to know a little about your life and career. Let's start with the beginning. What is your family background?

My father came from a simple servant family that had remained poor for generations. The curse of perpetual poverty was magically broken when my grandfather *Otto Kallenberg* got employed as a trusted servant to the Swedish king, moving into an apartment across from the royal castle. With the improved economy, the family could afford to send their youngest son, my father, to a Latin school, where he became the first member of the family to graduate from high school. To support any higher studies was out of the question, which is why he enrolled in the military academy where the education was free, thus becoming an officer in the Swedish army reserve.

My mother came from a prominent Norwegian family of doctors, lawyers, and public servants, whose ancestry can be traced back at least 17 generations. The brothers of my maternal grandfather had extraordinary careers, one becoming a prominent lawyer, another a famous architect, a third a successful business man, and a fourth an outstanding scientist. Though my grandfather was said to be the smartest of the lot, he opted for a simple life as *lensmann* in a rural island community north of the polar circle. My grandmother was sickly, staying for long periods at a sanatorium far from home, and so my mother was raised by her three older sisters. Since only the most elementary education was available in the local community, the children had to be sent for schooling to the nearest town, staying with friends and relatives. My mother must have been doing well in school, since she was the only daughter deemed worthy of a higher education, thus becoming the first girl of the family to graduate from high school.

The Early Years

You were born right after the outbreak of WWII. How did the war affect you and your family?

Though Sweden managed to stay neutral throughout the war, these were still very challenging times, with shortages of basic groceries in the stores and much of the working population tied up in the army. My mother's life was very lonely and difficult, as her entire family was living in Norway within occupied territory. When I was two years old, my grandfather was killed when traveling on board a steamship, also carrying some German soldiers, that was torpedoed and sank outside the Norwegian coast. My father was staying with his company at an undisclosed location along the Norwegian border, ready to fend off the Germans if they would decide to invade. I hardly knew my father in those days, and even after the war he was managing a refugee camp far away from home.

What about your early childhood?

The other boys in our neighborhood were only interested in running around with sticks and playing war games. Since this didn't interest me, I spent my days playing alone in my mother's kitchen. My sister and I had very few toys, and so if we wanted some we had to make them ourselves. In this way, I soon become an expert on building all kinds of railroads and bridges, and I even used an old alarm clock to make an engine for the train. Like all other Swedish kids, I started school the year I turned seven. (There was no kindergarten in those days.) Since I already knew how to read and write plus some elementary arithmetic, the first few years in school were extremely boring, the only relief being some simple Bible stories. Though my parents had only high school diplomas, they always stressed the importance of a good education.

Do you remember your first exposure to mathematics?

Though the simple algebra and word problems we learned in school were again extremely boring, the advantage with math was that there was nothing to memorize, and you could easily figure things out if you were only smart enough. Math got more interesting when we started with Euclidean geometry, which was a subject I truly loved. Oddly enough, our textbook was more or less a literal translation of Euclid's original, with all the nonsense faithfully reproduced, such as "a line is that which has no width." But there were so many beautiful propositions, with short but clever logical proofs.

Somehow I always got the top grade A in math, all through school, which was very unusual, since an A was hardly given except in the graduating class. From my teachers I got some contradictory advice. One thought that if you were good at math, you should focus on that, and all other subjects would take care of themselves. The opposite advice came from another teacher, who kept reminding me of the danger of a narrow specialization, becoming a *fackidiot*. By the end of high school we started with some very elementary calculus, which again got quite interesting. Every year there was a math competition for Swedish high school students, where I won the first prize. The outcome was even publicized in a Scandinavian math journal.

After graduation from high school plus nine months of military service, I had five months free. Then I took a simple job at a local insurance company, spending my spare time to study Courant's calculus books, where I read every section and did all the exercises in both volumes. This turned out to be a great investment, since most undergraduate courses were calculus based and became very easy.

Studies at KTH in Stockholm

For your undergraduate studies, you enrolled at the Royal Technical University (KTH) in Stockholm. How did that come about?

This was a special challenge, since the division of Technical Physics (F) at KTH where I was admitted had the highest entrance requirements in the country, but also the most demanding courses. We were only 30 students in the F division, and because of the high entrance demands, all my classmates were extremely bright with broad interests. In fact, of us 30 students, at least five or six eventually became university professors in our own right.

The teaching was divided into lectures by the main professor and problem solving sessions held by teaching assistants, and we were advised that the latter were absolutely necessary to attend, whereas the former we could safely skip. With my usual stubbornness I did the exact opposite.

The grading scale at KTH consisted of the numerals 1–7, where 3 was required for passing. However, we could only get up to 5 on the regular course, and if we wanted a higher grade, we had to take an extra reading course with an oral exam for the professor, not too hard for a 6 but very challenging for grade 7. In this way, I took reading courses in linear and abstract algebra, functional and harmonic analysis, complex variables, probability, etc. This is how I got a good foundation in graduate level mathematics. My assignment in probability was to read a part of Feller's classical book, arguably the best math book ever written. The only problem was with the numerous little errors, and part of the challenge was to "fix" the proofs. I remember how the professor *Carl-Gustav Esseen* complimented me after the exam, addressing me formally in third person: "This the candidate has done thoroughly!" Already from my third year in college I became a teaching assistant, first in physics, then in numerical analysis, and finally in calculus.

At what stage did you get seriously interested in mathematics?

Already the first calculus lectures, held by professor *Hans Rådström* in the huge KTH auditorium, were fascinating, as he presented from memory complete proofs of all results, maybe up to the level of the Heine–Borel theorem. This was when I truly fell in love with mathematics. But then, after reading the famous book of *E.T. Bell*, I thought that all great mathematicians had been child prodigies. For example, Galois had revolutionized algebra before dying in a duel at age 21. Hardy said in his *A Mathematician's Apology* that "mathematics is a young man's game." Though I would have loved to become a mathematician, it was only too bad that I was simply too old. I was only 19.

Before graduation I needed to do a special project, and I asked professor Esseen if he had a problem for me. As an expert on harmonic analysis, he had once written a famous dissertation, including the celebrated *Berry–Esseen* bound. The theses also contained an open problem involving probability measures on the line and their characteristic functions, and he proposed that I make some computer simulations that might suggest the answer. Using computers in those days was very complicated, using punching cards for the input and then studying carefully the similarly encoded output. Thus, I decided to see what I could do by some pure reasoning. After a few days I had solved Esseen's problem, and I wrote a short paper with my proof, which I handed over to Esseen. I was then ready to graduate as a civil engineer, and I was selected as one of the two top students in my division.

How did you decide to pursue graduate studies in probability?

Well, after my graduation as a civil engineer, it was natural to proceed with graduate studies, since at this point I truly loved all theoretical subjects and especially mathematics. But then I thought, fairly or not, that if I would choose a subject in classical math, I would end up doing some technical calculations and estimates in a very specialized area. Probability theory seemed to be so totally different. Here was an emerging field of modern mathematics, where all the standard tools and techniques of classical math came into play, including real and complex, functional, and harmonic analysis, occasionally even abstract algebra and topology. So, I decided to go into probability theory. A contributing factor was that Carl-Gustav Esseen was known to be such a famous man with a world-wide reputation. What I didn't know was that, after writing his famous dissertation in his youth, he had published very little.

A complication following me all through my career was that, for mostly historical reasons, probability was classified in Sweden and elsewhere as part of mathematical statistics, thus belonging to a separate department. In fact, up to around 1930, probability and statistics were more or less the same subject, whereas today they are light-years apart.[1] The effect was that probability theory, though firmly established as an integral part of real analysis, belonged to the separate department of mathematical statistics. Thus, I became a GTA in Esseen's department, responsible for teaching two statistics courses per semester plus supervising the examination and grading of hundreds of undergraduate students.

Since we had no graduate courses in those days and very few seminars, I was only handed an extensive list of course literature, with associated oral exams for the professor. I remember especially how I spent a whole summer studying for the last and most challenging exam, where I was responsible for knowing every theorem and proof in Loève's book plus half of the book of Doob, totalling some 1,000 pages. Since studying at home or in my office soon became too boring, I chose instead to study in different parks around Stockholm, choosing a new bench for each proof.

I also had to produce a master's thesis, and Esseen proposed that I do some work in the notoriously difficult area of convolution factorization of probability measures, leading into some hard problems in complex and harmonic analysis. Apart from the mathematical challenges, some of the major literature in the area was written in Russian, and I remember sitting with a dictionary, trying to make sense of some Russian papers.

[1] No offense intended. Many statisticians are my close friends, and I respect their work while they respect mine. It is just that we have nothing professionally in common.

After graduating with a masters degree from KTH, you decided to leave Stockholm?

Yes. In those days, a masters degree in Sweden would roughly correspond to a Ph.D. abroad, whereas a Swedish Ph.D. was approximately equivalent to a German habilitation. After getting my masters degree from KTH, I could have stayed on to pursue a Ph.D. under Esseen. However, for several reasons I felt that this would have been pretty pointless, and it was time to move on. By then we had no graduate courses, seminars, or supervision of any kind, and I would essentially be left alone, identifying and solving my own problems. While the undergraduate studies at KTH had been extremely stimulating, my graduate studies were thoroughly disappointing, and I wanted to get back to a full and interesting life.

Based on the strength of my masters thesis, I got a permanent position as a lecturer at the University of Lund, naturally again in the department of mathematical statistics. Though the job was well paid, the working load was enormous, about 14 hours a week of elementary teaching, plus the supervision of teaching and grading of hundreds of undergraduate students. Since we were now isolated from the math department, there was very little of intellectual stimulation, and I felt like being degraded to the rank of a department janitor.

After a couple of years in Lund, I started looking around for some other employment, and I got hired by a consultant company in Gothenburg, where I was assigned some challenging queuing problems, with the aim of maximizing the profitability of the client businesses. My working load was again enormous, since every hour of work was charged to the client companies, and all travels across Sweden had to occur on my free time. In addition to a full working load during the week, we often had meetings on Saturdays to discuss business policy, held in a small, smoke-filled room without ventilation, and I would come home in the evening exhausted and with bad headache. After one year on the job I gave up again, this time applying for a temporary position as a lecturer at Chalmers University in Gothenburg.

Chalmers University in Gothenburg

Tell me about your early years at Chalmers.

My initial teaching load was again enormous, but already after one semester I got a special scholarship with a much reduced teaching load. Besides, this was an exciting environment with many outstanding graduate students. The main professor in the department, *Harald Bergström*, was close to retirement but still very active, though only in the classical area of the central limit theorem, where the most general case had already been settled more than 30 years earlier. There was also the young, dynamic, and extremely clever postdoc *Peter Jagers*, who had managed to break away from the perpetual work on the classical limit theorems, soon to become a leading expert on branching processes. He was the one who had attracted all those brilliantly gifted graduate students, including *Torgny Lindvall*, *Olle Nerman*, and many others. (*Hermann Thorisson* and *Tommy Norberg* joined us later.)

When I came to Chalmers in the fall of 1970, Peter Jagers had just left for a sabbatical at Stanford University. However, in October 1971 he was back, and he brought some lecture notes on random measures and point processes that he had written during his absence. We started right away with a seminar on the subject, and very soon I made some interesting discoveries. Already by the next April I had so many new and interesting results that I decided to submit my paper as a doctoral dissertation, which I defended publicly in May 1972. This was also the year when my father died from incurable cancer, and I took the train to see him in Stockholm every second weekend.

My paper was long, technical, and filled with lots of new and (as I thought) exciting results, and I sent it to a leading probability journal, where it soon got rejected. However, I also sent a copy to *Klaus Matthes* in East-Berlin, the undisputed leader of the emerging field of general point processes, and what a difference! More or less in return mail, he offered to publish my thesis in their monograph series, and he invited me for a visit. He even send me a Xerox copy of the entire manuscript of their famous point process book (German edition), which was just about to get published. I also sent my thesis to *Ross Leadbetter* in Chapel Hill, who invited me right away to spend a year as a visitor in their famous department.

From this time on, you visited many different countries?

Yes, I did spend a year in Chapel Hill, and for many years I traveled extensively in both eastern and western Europe. To East-Germany (DDR) I was invited for visits more or less every year, until I left for the US. Matthes was the powerful director of the celebrated *Karl Weierstrass Institute* of the *Academy of Sciences* in DDR, and a dynamic personality in his own right, inspiring a whole generation of graduate students in especially Berlin and Jena. This was during communist times, and I had endless political discussions with everybody, except with Matthes himself, who was also a communist party boss. On my visits I was always treated as a guest of honor. I recall an evening reunion with dozens of participants, where somebody proposed a guessing game: Someone would think of a famous mathematician, and the others would ask questions, until they could guess who the person was. The right answer turned out to be *Herr Dr. Kallenberg*, and I was blushing. Suddenly I was a famous man, at least in the DDR.

Already at this time I had very broad mathematical interests and was working on projects all across probability theory. During my visit to Chapel Hill, I went to the library every day to scan the incoming math journals for papers in probability theory. I was especially proud of a long paper I wrote on stochastic integration (now obsolete), which became my favorite work to present at seminars and conferences. Already at this time, I was sometimes criticized for not speaking about point processes, which was supposed to be "my field." During my stay there was also a prominent Russian visitor, whom I joined for lunch every day at a simple hamburger restaurant. When I met him again years later, all he wanted to talk about was hamburgers.

Your great breakthrough came with your solution of the Rollo Davidson problem?

Yes. Rollo Davidson was a math prodigy at Cambridge who died tragically in a climbing accident at age 25, leaving behind a stack of unpublished manuscripts, plus his "big problem" about stationary line processes in the plane, where some evidence seemed to suggest that, under suitable regularity conditions, such processes would have to be of Cox type (mixtures of Poisson). Davidson was a pioneer in the emerging field of stochastic geometry, and I got fascinated by those line and flat processes, especially in their relation to certain infinite particle systems. For a couple of years, this general area had been one of my main interests, and after writing a couple of papers on the subject, it suddenly occurred to me that the Davidson conjecture was actually false, and I could give a simple counterexample. This happened to be during the Christmas break, and I remember spending about a week to write a short paper, explaining my argument. Though I didn't think it was of much interest, I submitted my note to a journal, and I also sent copies to a few colleagues interested in the area.

The reaction was astounding. *Klaus Krickeberg* called from Paris to invite me to spend a year at the *Sorbonne*, and from Berkeley I was invited to spend a year in their department, in a letter written by the famous statistician *Erich Lehmann*, saying: "I hope you can come—Kolmogorov will also be there." From Cambridge I was notified that I had been awarded the prestigious *Rollo Davidson Prize*. When I came to Cambridge, few people cared about my regular colloquium talk about some recent work, but an extra seminar was arranged to discuss my solution to the Davidson problem, where the auditorium was filled to the last seat. It was a little embarrassing, since I didn't even care to bring a copy of my Rollo Davidson paper. For personal reasons I couldn't go to either Paris or Berkeley, but a few years later I did spend a wonderful year in Vancouver, where I could go downhill skiing three times a week all through the long winter season.

To explain my attitude, you need to understand how mathematicians work. When faced with a problem we make some guesses, and then we try to determine whether those guesses are true or false. If true, then we have a theorem and we are ready to move on to the next step; if false, we need to modify the conjecture or perhaps discard it altogether. In the case of the Davidson problem, the conjecture was supported by very little evidence, and it turned out to be false. Not a big deal, it happens all the time and is simply part of a mathematician's life.

Left: opening lecture at SPA 2018.
Right: lecture at the Kallenberg workshop, Mittag-Leffler Institute 2013.

Then you stayed at Chalmers for many years?

Yes, that was my good luck. The years I spent in Gothenburg on various postdoc positions were among the best of my life. Already during my first year at Chalmers, Harald Bergström suggested that I give a course on weak convergence based on Billingsley's recent book. Thus, curiously, the first graduate course I experienced during my studies was one that I taught myself. During the following years, I was allowed to teach only graduate courses in probability, and I chose a new topic for each semester, including Markov processes, martingales, stochastic calculus, ergodic theory, etc. One day I was approached by some colleagues in the math department, who suggested that I teach a course on "Probability theory for mathematicians."[2] This is how I started teaching courses to a general mathematical audience, which became very popular among faculty and graduate students alike. They may be regarded as the starting point of my "Foundations" project. Already at this time, I had been working in practically every area of modern probability, and I regarded every one of them as "my field."

My appointments at Chalmers were only temporary, and I was always nervous about the renewal of my scholarships. During my last year I got a personal research position funded by the Swedish national research counsel[3], again with a very modest teaching load. The only caveat was that, whenever a chair of full professor was advertised in Sweden, I was obliged to apply, and presumably to accept the position if I got selected. Since there were very few such "chairs" in Sweden (in my case only those in mathematical statistics were relevant), in principle I might have to wait for years until

[2] Only a probabilist will understand what is so funny about the title.
[3] counterpart of the NSF in the US

somebody would retire or die. Now it so happened that, already after one year, three such positions were advertised at the same time, one at each of the universities in Uppsala, Lund, and Stockholm. Thus, I had to apply to all three, and since there was a common hiring committee, I was selected for all of them. Then I could just choose where to go, and naturally I chose Uppsala, the oldest and most venerable among Swedish universities. In this way, I was appointed to the Uppsala chair, in a formal letter signed by the Swedish king, and at the same time I automatically lost my job at Chalmers.

All through my life there had been a crisis on the Swedish house market, and at this time it was worse than ever. I owned a simple row house outside Gothenburg, and because of various government regulations, it had suddenly become impossible to sell a house without a substantial loss. In the newspaper I read about people who had to take out huge bank loans to afford selling their houses. In Uppsala the situation was very much the opposite, where it was virtually impossible to get even the simplest apartment without a huge down-payment. The situation was compounded by my special family situation. I was newly married with a little baby child, but my wife was Korean and was not eligible for the social benefits that everybody else in Sweden took for granted.

Moving to America

How did it happen that you decided to move to the US?

Well, I was in a desperate financial situation, was stuck with a house that I couldn't sell, and was unable to find even the simplest affordable apartment in the place of my new employment. In my desperation, I wrote to my friend *Ross Leadbetter* in Chapel Hill, asking for his advice. More or less in return mail, he offered me to spend the next academic year at the *Center for Stochastic Processes*, where he was one of the managers, the others being *Stamatis Cambanis* and *Gopi Kallianpur*. This is how I decided right away to go to Chapel Hill.

I had visited Chapel Hill before, but with the new *Center for Stochastic Processes*, they had created a marvelous environment, with up to a dozen visitors from all over the world constantly coming and going. Every week there was a seminar, always followed by an intense discussion. The three organisers were specialists in different areas and attracted visitors accordingly. Gopi Kallianpur was the one who influenced me the most, as he was following every word of the presentations and was leading the subsequent discussion with some penetrating comments and questions. When giving a talk with him in the audience, I felt like talking to him alone, as he was sitting in the first row and would sometimes interject some quick comments, always exactly to the point. I soon became very active too, and the initial comments often led to an intense discussion between the two of us. After this first year at the Center, I truly loved the place and came back for long visits every summer. Stamatis Cambanis told me that I was welcome to visit as often as I wished. Unfortunately, he died tragically from a cancer tumour when he was only 50 years old, and soon Kallianpur also got sick and died as well. Then of the organizing trio there was only Leadbetter left, and the exciting activity soon faded out.

My stay at the Center gave only a temporary relief to my Swedish problems, and for a permanent solution I needed to look for a job elsewhere. In the Scandinavian countries I could get any academic position I wanted, and in many European countries I had a very high reputation as well. Thus, I thought it would be easy to get a job in the US. Together with a young German visitor we applied to all open positions we could find in the ads. He got invited for interviews everywhere, but for me I just got a card saying; “Thanks for applying. We regret that we have no positions at this time.” I started getting desperate.

Then one day I got a phone call from Auburn University with an invitation to come over for a colloquium talk. Such invitations were not unusual, and I accepted right away. Since I had never heard of Auburn, I got a map of the US and found Auburn as a little dot far down in the south. When I came to Auburn to give my talk, I was told that: “By the way, we have an open position in probability, and you might be interested.” — “Well, I am actually looking for a job, so please tell me about it.” After a few days I got an offer from Auburn, and I calculated that with the offered salary, my living standards would improve by a factor five. When I asked around for advice, many thought that it was a good offer, but my Jewish friend *Robert Adler* insisted that: “You shouldn’t go to Auburn, you are too good for Auburn.” Well, I had no choice so I accepted the offer, and in the fall I moved with my young family down to Alabama. I thought that my stay at Auburn would be only temporary, but once you have a permanent position in the US, it is not so easy to move elsewhere. No great offers came my way, and I ended up spending more than 30 years at Auburn University.

Tell me about your teaching in America.

Well, the first few years at Auburn were wonderful. Apart from our basic graduate courses in probability theory, I got to teach graduate courses in some of my favorite areas, including real, complex, and functional analysis. We had some very good students in those days, and I supervised several of them for a Ph.D. Many students were interested in getting into statistics, and since nobody even among advising faculty knew the difference between probability and statistics, they came to us instead. This influx came to an end when eventually a separate statistics group was created in our department.

A similar problem arose in pure mathematics, when some colleagues in analysis started teaching probability courses under different names. After the first few years, I never again got a chance to teach the attractive analysis

With family, early 1990's.

courses, where we had a waiting list of interested faculty of some 10–15 years. Towards the end of my long teaching career, my favorite became an undergraduate course in the history of mathematics, which nobody else wanted to teach. Here as elsewhere, I found the existing textbooks useless, which forced me to design my own course.

Our probability group of originally three people was gradually enlarged to five, through the hiring of *Jurek Szulga* and *Ming Liao*, superb experts on functional analysis and differential geometry, respectively, and for many years we had a nice seminar where we alternated to speak. Soon an energetic dean reorganized the math departments, which led to an unfortunate split of the probability group. It was with mixed feelings that I eventually decided to retire from the university, thus allowing me to work full time on my research and book writing.

Throughout my career I have also served as an associate editor of numerous math journals, and for three years I was the editor-in-chief of *Probability Theory and Related Fields* (PTRF), which was among the most difficult things I have ever done. The problem was that, in order to keep uniform standards when the submission rate was very high and few papers could be accepted, I was forced to read practically every paper myself. Thus, during my entire stint as editor, I read about 1,000 papers, good or bad.

Apart from your "Foundations" book, you have published two massive research monographs in different areas. Tell me about your work on probabilistic symmetries.

Well, this an area that has interested me ever since I wrote my dissertation. My symmetry book has over 500 pages, where at least some 300–400 pages represent my own research, so I can only give some examples. I think the *predictable sampling theorem* is especially interesting and important [4]. Here we consider any finite or infinite sequence $\xi = (\xi_1, \xi_2, \ldots)$ of exchange-

[4] K05, Ch. 4; K21, Ch. 27

able random variables, including the case of i.i.d. sequences. By the definition of exchangeability, the distribution of ξ is invariant under permutations of the elements, and the theorem says that the invariance remains true under any *predictable* permutation. In other words, ξ has the same distribution as the sequence $(\xi_{\tau_1}, \xi_{\tau_2}, \ldots)$ for any a.s. distinct, predictable times $\tau_1, \tau_2, \ldots$ taking values in the index set of ξ. This result is powerful enough to imply Lévy's third and most difficult arcsine law for Brownian motion. A related but weaker result was proved in *fluctuation theory* by *Sparre Andersen*, where Feller says that S-A's discovery "was a sensation greeted with incredulity, and the original proof was of an extraordinary intricacy and complexity." My result has also a much deeper continuous-time counterpart.

My hardest results in the area concern *contractable, exchangeable*, and *rotatable arrays* of random variables, some of which have important applications to random graphs and networks [5]. In the simplest case, we consider a two-dimensional random array $X = (X_{ij})$ on N^2, said to be *separately exchangeable* if its distribution is invariant under arbitrary permutations p and q in the two indices, so that the array (X_{p_i,q_j}) has the same distribution as X. Similarly, X is said to be *jointly exchangeable* if the array (X_{p_i,p_j}) has the same distribution as X for every single permutation p. The celebrated *Aldous–Hoover theorems* give characterizations of such arrays in terms of suitable *coding representations*, and an obvious challenge is to extend the latter to the contractable case of sub-sequence invariance. The surprising conclusion is that a sub-diagonal array of arbitrary dimension d is contractable iff it admits an exchangeable extension to the entire index set N^d. It has long been my principal open problem to find a direct proof that doesn't depend on the subtle coding representations.

The case of rotation invariance is again much more difficult. Here the natural setting is in terms of unitary operators on products of separable Hilbert spaces, where one obtains representations in terms of tensor products of *multiple Wiener–Itô integrals*. (These are the same integrals that play a crucial role in Malliavin calculus.) We can now go back and derive some general representations of contractable and exchangeable *random sheets* of arbitrary dimension.

What about your work on random measures?

Well, my monograph in that field has almost 700 pages, at least half of which represent my own research, so again I can only take some examples. One of my most important discoveries may be the existence of the *Gibbs kernel* of a particle system, which may be regarded as dual to the *Palm kernel* comprised of multivariate Palm distributions [6]. Here the theory is closely related to certain developments in statistical mechanics by *Dobrushin* and

[5] K05, Ch's 7–8; K21, Ch. 28

[6] K17, 8; K21, Ch. 31

others. Specializing to the case of point sets of cardinality one, one obtains the *Papangelou kernel*, which was originally introduced by Papangelou in the context of stochastic geometry to prove the Cox property of certain line processes. General Gibbs and Palm kernels can be used to describe the inner and outer conditioning in particle systems.

For a further example, consider an optional time τ with associated mark χ, and let $\hat{\xi}$ be the compensator of the random point mass $\xi = \delta_{\tau,\chi}$, in the sense of the Doob–Meyer decomposition. When the filtration is the one induced by ξ, we may express $\hat{\xi}$ by a simple formula in terms of the underlying distribution μ of ξ. For a general filtration $\mathcal{F}$, we can solve this equation for μ to obtain a predictable random measure ζ, also expressible as a *Doléans exponential* of $\hat{\xi}$. This so-called *discounted compensator* ζ has the most powerful mapping properties [7]. Thus, we can use the discounted compensator to map any set of random points as before into a set of independent random elements with specified distributions, in a similar way as the ordinary compensator $\hat{\xi}$ can be used to map any ql-continuous, simple point process into Poisson. In particular, this mapping property yields a simple proof of the predictable mapping theorem mentioned earlier. My proofs in this area are based on the subtle stochastic calculus for general semi-martingales.

Olav's Springer books, 2005, 2017 and 2021.

For a third example [8], let ξ be a random measure on a space S, and let η be a random element in T. Then the Palm measures of η with respect to ξ can be obtained by a disintegration of the form $\rho = \nu \otimes \mu$, where ρ is the *Campbell measure* of the pair (ξ, η), ν is a *supporting measure* of ξ, and μ is the desired *Palm kernel* from S to T. When ξ and η are jointly stationary under the action of a locally compact, second countable group G acting on both S and T, we may choose both ρ and ν to be G-invariant, and it becomes important to choose even μ to be G-invariant, in the sense that $\mu_{rs} = \theta_r \mu_s$ for all $r \in G$ and $s \in S$, where the θ_r are shift operators on the measure space $\mathcal{M}_T$. For certain purposes, it is also important to consider the more general case where ρ and ν are *jointly stationary random measures* on $S \times T$ and S,

[7] K17, Sec. 9.4; K21, Ch. 10
[8] K17, Sc's 7.5–6

respectively, in which case we need to choose the kernel μ to be stationary as well. Noting that the *Besicovitch covering theorem* remains valid on any Riemannian manifold, it is not too hard to give a proof when G is a *Lie group*. To extend the result to general locally compact groups G as above, we may then use the fact that G has an open subgroup H that is the *projective limit of Lie groups*. This is one of the hardest results I have proved in recent years.

You have even included some of your own results in the "*Foundations of Modern Probability*" book.

Well, that is something I wanted to avoid, unless the results can qualify as truly "foundational." I did include some easy versions of the results described before, and I also included my functional representation of solutions to a *stochastic differential equation*, improving the classical Yamada–Watanabe theorem [9]. While working on the new edition of my Foundations book, I made some interesting discoveries in stochastic differential geometry that I decided to include in the last chapter [10].

For the context, recall that already Itô and his followers developed a stochastic calculus on Riemennian manifolds. Around 1980, the famous mathematicians *Laurent Schwartz*[11] and *P.A. Meyer* made the startling discovery that much of the theory could be developed on a general differential manifold S, leading to a beautiful theory eloquently described in monographs by Émery. Here continuous[12] semi-martingales can be defined without any additional structure, but for the definition of martingales we need S to be endowed with a *connection* ∇. Then a semi-martingale X in S is said to be a *martingale* if $f(X) \overset{m}{=} \frac{1}{2}\nabla f[X]$ for every smooth function f on S, where $\nabla f[X]$ is the quadratic variation process associated with the bilinear form ∇f and $\overset{m}{=}$ denotes equality up to a martingale term. For a general semi-martingale X, we may then look for some *intrinsic local characteristics* of X, such that X is a martingale iff the associated drift rate vanishes.

To appreciate the encountered difficulties, note that X has no Doob–Meyer decomposition $M + A$, due to the absence of any linear structure on S. Still we can define intrinsically some drift and diffusion rates of X, which can be shown to have the desired projection properties, when S is embedded into a Euclidean space. This connects the abstract Schwartz–Meyer theory to the classical theory on Riemannian manifolds.

[9] K21, Ch. 32
[10] K21, Ch. 35
[11] creator of distribution theory
[12] henceforth always understood

You have sometimes been characterized as a renaissance man. Tell me about your interests outside of mathematics.

Well, apart from the family that always comes first, I am the most passionate music lover you have ever known, and I can't imagine a day without music. All my work is inspired by the music of Bach, Beethoven, Schubert, Brahms, ..., and the useful ideas are always coming when I am practicing the piano or listening to music. I used to have some piano students coming for lessons every Sunday morning, and at one point I even worked with a string quartet of little kids. We knew many professional musicians in neighboring cities, and a couple of times a year we arranged with music recitals in our living room attended by some 30-40 people, where my role was to organize the event and write some program notes about the music performed.

I am also a passionate art lover, and I am constantly reading books about especially cultural history and modern science. My wife used to tell me that I can buy as many books as I want, but no more book cases. As a result, the books keep piling up everywhere in our house, but what can I do, I can't live without them? I am also a passionate hiker and downhill skier. People often ask me whether I am missing Sweden. The answer is that I miss all of Europe so badly, and whenever I come to Paris, Rome, Venice, ..., even Stockholm, I am taking a deep breath and saying to myself: "Wow, now I am back home again!"

Is there anything else you wanted to say before we end our conversation ?

Yes ! Thank you for all your efforts with this interview, and thanks to all the people you have contacted for their excellent questions.

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