9 Isolated rings

9.1 Impurities

For an introduction into the consequences of a change in the impurity potential see Section 11.1.4 on page 285.

Random modifications of AB oscillations on a time scale of 10 to 40 h have been observed by Mailly et al [93M1, 94M1]. These fluctuations have been associated with slow relaxation processes of the impurities in the semiconductor, inducing changes in the scattering potential or in the Fermi level.

9.2 Interactions

The theoretically predicted value for the typical ensemble averaged persistent current (see Section 9.3.1 on page 251) was 1.5 orders of magnitude smaller than the PC found in an experiment on metallic rings [90L]. This discrepancy motivated attempts to incorporate the Coulomb interaction between the electrons into the theory. Using a Hartree approximation in early works to describe interacting electrons in a random impurity potential yielded too small currents. In the literature, mainly three different ways of including the Coulomb interaction exist. First, the combined effect of disorder and interaction is studied perturbatively by diagram techniques. It is claimed that the Coulomb interaction strongly enhances the PC. Second, numerical studies on discrete (finite) 1D rings with flux–dependent hopping matrix elements and diagonal disorder are performed. For half filling, the PC goes to zero with increasing strength of the interaction. For a fixed number of particles, a fixed system length and the number of lattice sites tending to infinity, the Coulomb interaction seems to counteract the suppression of the PC due to the impurity potential. Third, 1D continuum models with flux–dependent boundary conditions, a random impurity potential and a translationally invariant Coulomb interaction are studied by a combination of analytical and numerical techniques. A qualitative argument implies that the Coulomb interaction strongly counteracts the suppression of the PC due to random scatterers (see for example [93W2, 94K3, 95E2] and references therein).

As the carrier density in the GaAlAs/GaAs system examined by Mailly et al [93M1, 94M1] was very low, electron–electron interactions were more important than in metals. The good agreement between the results found in [93M1, 94M1] and theoretical predictions seemed to rule out the possibility that the electron–electron interactions change the value of the PC significantly.

9.3 Magnetic field

9.3.1 Persistent current

Consider an ideal 1D ring (no impurity scattering, no electron–electron interaction) of circumference L with a magnetic flux ϕ through its hole. Using a gauge for which the Hamiltonian of the system is independent of the vector potential $\boldsymbol{A}$, the wave function Ψ obeys the boundary condition $\Psi(x+L) = \Psi(x)e^{i2\pi\phi/\phi_0}$, where $\phi_0 = h/e$ $(c \equiv 1)$ is the magnetic flux quantum. This condition implies that the fluxes ϕ and $\phi + n\phi_0$ are indistinguishable and hence all physical properties of the system are periodic in ϕ with period ϕ_0. An equilibrium current, $I(\phi) = -\partial J/\partial\phi$, circulates around the ring whenever the appropriate thermodynamic potential J depends on flux, generating a magnetic moment $M(\phi) = \pi(L/2\pi)^2 I(\phi)$ perpendicular to the plane of the ring. (The quantity J represents the energy E $(T = 0)$ or the free energy F $(T \neq 0)$ in canonical ensembles, and $F - \mu N$ in grand canonical ones.) The equilibrium current is a consequence of the sensitivity of the eigenstates to twists in the boundary conditions. It does not dissipate and is therefore referred to as *persistent current* (PC) (see for example [88C4, 91B2, 91B3, 91I3, 91R3, 95E2, 97I1] and references therein).

For a given energy level, the contribution to the PC is proportional to the corresponding velocity of the electrons, $I_n(\phi) = -\partial E_n/\partial\phi = ev_n/L$, the sign is alternating for consecutive levels. Summing over all levels, strong cancellation takes place and the order of magnitude of the total current is determined by the Fermi velocity, $I_0 = ev_{\mathrm{F}}/L$. For an ideal ring of finite cross-section A one has to take the number of channels M into account which is of the order $M \approx k_{\mathrm{F}}^2 A$. The total current is $\sqrt{M}$ times the one–channel current (see for example [91B2, 91I3, 91R3, 97I1] and references therein).

At *finite temperature*, electrons are excited to higher levels, leading to additional cancellations in the sum of the single–level currents. The current is averaged over energy levels in the interval given by the thermal energy, $k_{\mathrm{B}}T$. The PC decreases exponentially with T with a linear temperature dependence in the exponent for a clean ring. The characteristic temperature T^* at which the PC begins to decrease is $k_{\mathrm{B}}T^* = \Delta_1 = hv_{\mathrm{F}}/L$, where Δ_1 is the level spacing of a 1D ring or the level spacing of one channel in case of a multi–channel ring. Disorder changes the characteristic temperature to the Thouless energy, $k_{\mathrm{B}}T^* = E_{\mathrm{c}} = \hbar D/L^2$, with the diffusion constant D. The exponential decrease is then governed by a $T^{1/2}$ dependence in the exponent (see for example [91B2, 91R3, 97I1] and references therein).

The PC contains *higher harmonics* oscillating with periods ϕ_0/k (k an integer). In the case of odd k, the corresponding currents have different signs for an even or odd number of electrons in the ring. The even harmonics are positive in both cases. Due to the random sign of the fundamental mode of the PC, the ϕ_0 periodic part vanishes when averaging over an ensemble of rings. The even harmonics yield a contribution proportional to the number of rings if the average is performed with a fixed electron number for each ring. When averaging with a fixed chemical potential, the even harmonics also vanish (see for example [91B2, 91B3, 91I3, 91R3, 97I1] and references therein).

In the presence of *impurities*, the elastic mean free path l characterizes the amount of disorder and ξ is the localization length. One distinguishes three different regimes: ballistic motion ($L < l$), diffusive motion ($l < L < \xi$), and a localized regime ($\xi < L$). In the ballistic regime, the typical current, $I_{\mathrm{typ}} = \sqrt{\langle I^2\rangle}$, (averaged over different realizations of disorder) is of the order of $\sqrt{M}I_0$. In the diffusive regime, it is proportional to l and independent of M, $I_{\mathrm{typ}} = ev_{\mathrm{D}}/L = ev_{\mathrm{F}}l/L^2$, where v_{D} is the diffusion velocity of the electrons. The typical single–level current near E_F depends on M, $\langle I_n^2\rangle^{1/2} \propto \pm I_0 M^{-1/2}(l/L)^{1/2}$. The average current periodic in $\phi_0/2$ calculated for a fixed number of electrons is proportional to the second–harmonic typical current times $\sqrt{Ml/L}$ and thus also depends on M, contrary to the typical current. In the localized regime, the wave functions overlap only via exponentially small tails. The typical current decreases inversely with M and exponentially with L, $I_{\mathrm{typ}} = (I_0/M)e^{-L/\xi}$. Current amplitudes of higher harmonics are reduced even faster with the system length (see for example [91B3, 91R3, 97I1] and references therein).

The PC is a quantum–coherence effect. When *inelastic scattering* is present in the ring, the PC decreases exponentially with the phase coherence length l_φ, $I = I_0 e^{-L/2l_\varphi}$. Higher harmonics of the PC are cut off faster by dephasing. Even though the temperature dependence is also an exponential law, it is fundamentally different from the exponential decay due to dephasing. A finite temperature leads to energy–averaging by a superposition of different interference patterns, dephasing causes a trace in the environment (see for example [91B2, 91I3, 97I1] and references therein).

Mailly et al [93M1, 94M1] found AB oscillations and a PC in a mesoscopic GaAlAs/GaAs ring. The elastic mean free path was $l = 11 + \mu$m, the phase coherence length was $25\,\mu$m. Via etching, the 2DEG was depleted to form a ring with an internal diameter of $2\,\mu$m and an external diameter of $3.4\,\mu$m. The actual width of the loop was $w_{\mathrm{eff}} = 0.16\,\mu$m. The mean resistance of the ring was $1\,\mathrm{k}\Omega$. Two Schottky gates were fabricated, the first one on the two outgoing wires to make insulation of the ring from the measuring wires possible. The second one was placed on one branch of the ring and allowed suppression of all interference effects. A SQUID was fabricated on the same chip as the sample. Typical results for signal and noise at $T = 15\,\mathrm{mK}$ are shown in Fig. 244 for the resistance and in Fig. 245 for the magnetic response. The vertical scale is the square root of the power spectrum which is directly expressed in Ω in Fig. 244 and in nA in Fig. 245. The resistance signal shows AB oscillations with periods h/e and $h/2e$. In the magnetization signal, an h/e frequency component due to a PC has been observed in most of the measurements, while such a component has never been present in the noise spectrum. Mailly et al found a typical persistent current amplitude of $4 \pm 2\,\mathrm{nA}$, comparable to a theoretical value $I_0 = ev_\mathrm{F}/L = 5\,\mathrm{nA}$.

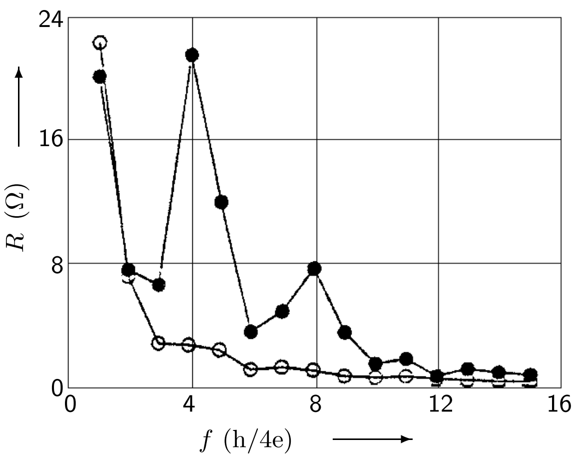

Fig. 244: Square root of power spectrum of the resistance fluctuations vs. frequency f of the ring [93M1]. Open circles correspond to experimental noise, solid circles correspond to the experimental signal.

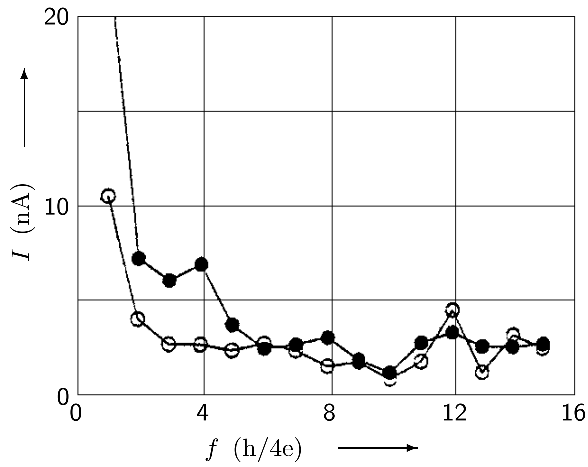

Fig. 245: Square root of power spectrum of the magnetization of the ring [93M1]. The values are converted into the equivalent current in the ring. Open circles correspond to experimental noise, solid circles correspond to the experimental signal.

9.4 References for Section 9

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