1.5.5.3.2.4 Arrott plots for two magnetic subsystems

The Arrott plots for a pure ferromagnetic system for which the magnetism originates from only one magnetic sublattice in the compound has been considered in the previous section. Now an arrangement is considered which is made up of two magnetic subsystems. The simplest configuration one may consider is a system made up of two different uncoupled magnetic phases. Thus the free energy takes the form

$$F = \frac{1}{2}\tilde{A}\,M^2 + \frac{1}{4}C\,M^4 - \boldsymbol{B}_0 \cdot \boldsymbol{M} + \frac{1}{2}\tilde{a}m^2 + \frac{1}{4}cm^4 - \boldsymbol{B}_0 \cdot \boldsymbol{m} \tag{45}$$

Here $\boldsymbol{M}$ and $\boldsymbol{m}$ are the magnetic moments on each of the magnetic subsystems. Minimisation of the free energy with respect to both $\boldsymbol{M}$ and $\boldsymbol{m}$ results in

$$\begin{aligned} \frac{\partial F}{\partial M} &= 0 \qquad \Rightarrow \quad \tilde{A}M + C\,M^3 - B_0 = 0 \\ \frac{\partial F}{\partial m} &= 0 \qquad \Rightarrow \quad \tilde{a}m + cm^3 - B_0 = 0 \end{aligned} \tag{46}$$

Each individual contribution on its own of either $\boldsymbol{M}$ or $\boldsymbol{m}$ is characterised by straight line Arrott plots. However, experimentally it is the total magnetic moment $\boldsymbol{M}_{\text{tot}} = p_{\text{M}}\boldsymbol{M} + p_{\text{m}}\boldsymbol{m}$ which is measured, with $p_{\text{M}}(p_{\text{m}})$ being the abundance of the phase characterised by the magnetic moment $\boldsymbol{M}$ ($\boldsymbol{m}$). Using the experimentally measured magnetic moment (which amounts to $\boldsymbol{M}_{\text{tot}}$) the Arrott plot will in general not be characterised by straight lines. This is illustrated with an example in Fig. 65, where the Arrott plots are shown for a ferromagnetic impurity phase ($p_{\text{M}} << 1$) within a paramagnetic matrix($p_{\text{m}} \approx 1$).

In the above discussion two magnetic subsystems have been considered. This situation may also occur in single phased samples with a more complicated crystallographic structure. If the magnetic atoms occupy crystallographically different lattice sites, the (ferro) magnetic moments of each sublattice are described by a free energy of the form as given in eq. (22). The total free energy takes a form which is a straightforward generalisation of eq. (45) to the appropriate number of subsystems. However, for subsystems in close contact with one another coupling terms can usually not be neglected.

Restricting the discussion to the ferromagnetic component only and using a two subsystem example to illustrate the magnetic behaviour, the lowest order coupling term takes the form

$$\gamma \boldsymbol{M} \cdot \boldsymbol{m} \tag{47}$$

Thus the free energy is given by

$$F = \frac{1}{2}\tilde{A}\,M^2 + \frac{1}{4}C\,M^4 - \boldsymbol{B}_0 \cdot \boldsymbol{M} + \frac{1}{2}\tilde{a}\,m^2 + \frac{1}{4}c\,m^4 - \boldsymbol{B}_0 \cdot \boldsymbol{m} + \gamma \boldsymbol{M} \cdot \boldsymbol{m} \tag{48}$$

Minimising the free energy with respect to the size of the magnetic moments (and leaving the question of the correct signs to be sorted out below) results in

$$\begin{aligned} \frac{\partial F}{\partial M} &= 0 \qquad \Rightarrow \quad \tilde{A}M + C\,M^3 - B_0 + \gamma m = 0 \\ \frac{\partial F}{\partial m} &= 0 \qquad \Rightarrow \quad \tilde{a}m + cm^3 - B_0 + \gamma M = 0 \end{aligned} \tag{49}$$

Depending on the sign of the coupling constant γ, a ferromagnetic coupling of $\boldsymbol{M}$ and $\boldsymbol{m}$ is favoured for $\gamma < 0$ while an antiparallel alignment will result for $\gamma > 0$.

The influence of the terms γM and γm on the magnetic moments m and M, respectively, is equivalent to a magnetic field, and the coupling terms proportional to γ in eq. (49) have thus to be understood and interpreted in such a manner. It is the molecular field (Weiss field) of one sublattice acting on the sites of the other sublattice. This source of an internal magnetic field is additional to the one of eq. (11) which originates from the coupling of the magnetic moments within the same magnetic sublattice.

For a negative coupling constant γ a ferromagnetic coupling of the two subsystems is favoured, and the effect of an external magnetic field is to simply enhance the internal magnetic field. For $\gamma > 0$ a more interesting situation arises. Here both $\boldsymbol{M}$ and $\boldsymbol{m}$ are coupled antiparallel with respect to one another, as this minimises the free energy due to the term $\gamma \boldsymbol{M} \cdot \boldsymbol{m}$. The sizes of the magnetic moments are in general such that $|\boldsymbol{M}| \neq |\boldsymbol{m}|$, as there is no symmetry element which links the modulus of $\boldsymbol{M}$ with $|\boldsymbol{m}|$. The difference $|\boldsymbol{M}| - |\boldsymbol{m}|$ is usually non zero. Thus when an external magnetic field is applied and $\boldsymbol{M}$ and $\boldsymbol{m}$ are non zero and antiparallel the moments will orient in such a way as to have the net magnetic moment $\boldsymbol{M} - \boldsymbol{m}$ oriented parallel to the external magnetic field direction. Taking $|\boldsymbol{M}| > |\boldsymbol{m}|$ and $\gamma > 0$ eq. (49), corrected for the appropriate signs, now reads

$$\begin{aligned} AM + CM^3 - B_0 - \gamma m &= 0 \\ am + cm^3 + B_0 - \gamma M &= 0 \end{aligned} \tag{50}$$

The set of coupled equations in (48) may be solved numerically and the Arrott plots constructed for $\boldsymbol{M}_{\text{tot}} = \boldsymbol{M} - \boldsymbol{m}$. The result of a model calculation is shown in Fig. 66. For this calculation the two subsystems are taken to be paramagnetic and characterised by a weakly temperature dependent Pauli paramagnetic susceptibility resulting in

$$\begin{aligned} a(T) &= a_0 + a_1 T^2 \\ A(T) &= A_0 + A_1 T^2 \end{aligned} \tag{51}$$

Here a_0, a_1, A_0 and A_1 are appropriate constants. It is to be noted that each subsystem has a positive $A(a)$-coefficient and, without the application of an external magnetic field, the free energy is minimised by a zero magnetic moment. However, due to the coupling term proportional to γ the internal field may create an instability towards a magnetic polarisation of the subsystems. While the free energy will increase when magnetic moments are created on each of the subsystems, an overall reduction of the free energy is brought about by the coupling term $\gamma \boldsymbol{M} \cdot \boldsymbol{m}$. For a positive γ value of sufficient size the reduction of the free energy (Fig. 66d) due to the coupling term more than compensates the free energy increase for the creation of the magnetic moments. Thus a magnetic state will result with a net ferromagnetic moment. Using the Arrott plots as shown in Fig. 67 this situation is indicated for strong coupling in Fig. 66b in the low external field region (I) and for low temperatures. This region is determined by the ratio of the external and the internal magnetic field. In region I the internal magnetic field is much stronger than the externally applied magnetic field.

For large external fields the alignment of moments is driven from a ferrimagnetic arrangement to a ferromagnetic one. This can be seen from the (almost straight) lines of the Arrott plots in Fig. 67 for high fields (III) where the high field region is characterised by a parallel alignment of the magnetic moments. The regions of dominating internal and external magnetic fields are separated by a transition region (II in Fig. 67b) which displays a complex magnetic behaviour and indicates strong competition between a ferrimagnetic and a ferromagnetic arrangement of magnetic moments.

It is noted that for a Pauli paramagnet, and due to its weak temperature dependence, the physically interesting temperature region lies between T_1 and T_2 in Fig. 67. Within this temperature interval the size of the ferromagnetic moment in the low external field region is essentially independent of the external magnetic field strength. This is due to the fact that this region is governed by the strength of the internal magnetic field. The external field is much smaller than the internal one, and the external field is thus not able to significantly affect the size of the net ferromagnetic moment.

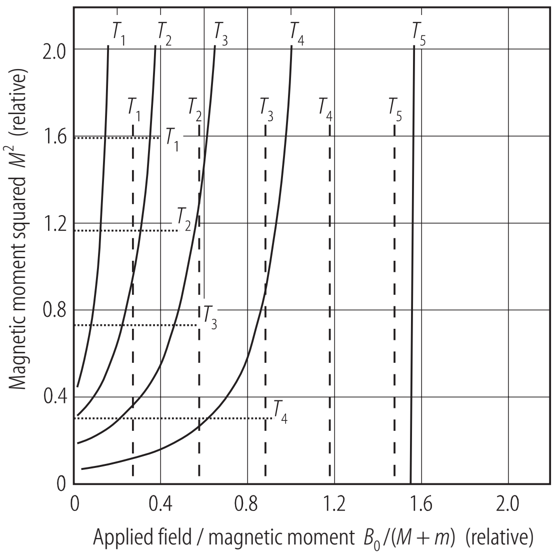

Fig. 65. Arrott plots of a ferromagnetic phase (dotted line) and a paramagnetic phase (dashed line). The Arrott plots for a sample which consists of a mixture of 5 % of the ferromagnetic phase and 95 % of the paramagnetic phase is shown by the full line.

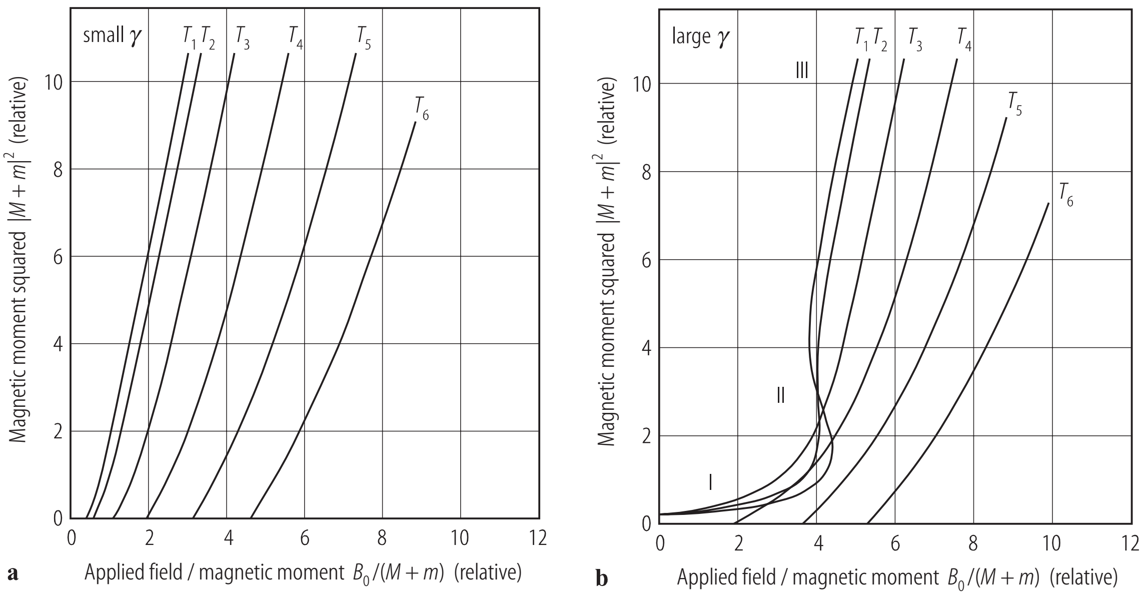

Fig. 67. Arrott plots for two coupled sublattices. (**a**) corresponds to weak coupling (i.e. γ small) and (**b**) depicts the situation for larger γ values. Regions I to III in (**b**) indicate the regimes for which the internal field (region I) or the applied field (region III) dominate. In region II, both fields are of comparable magnitude.

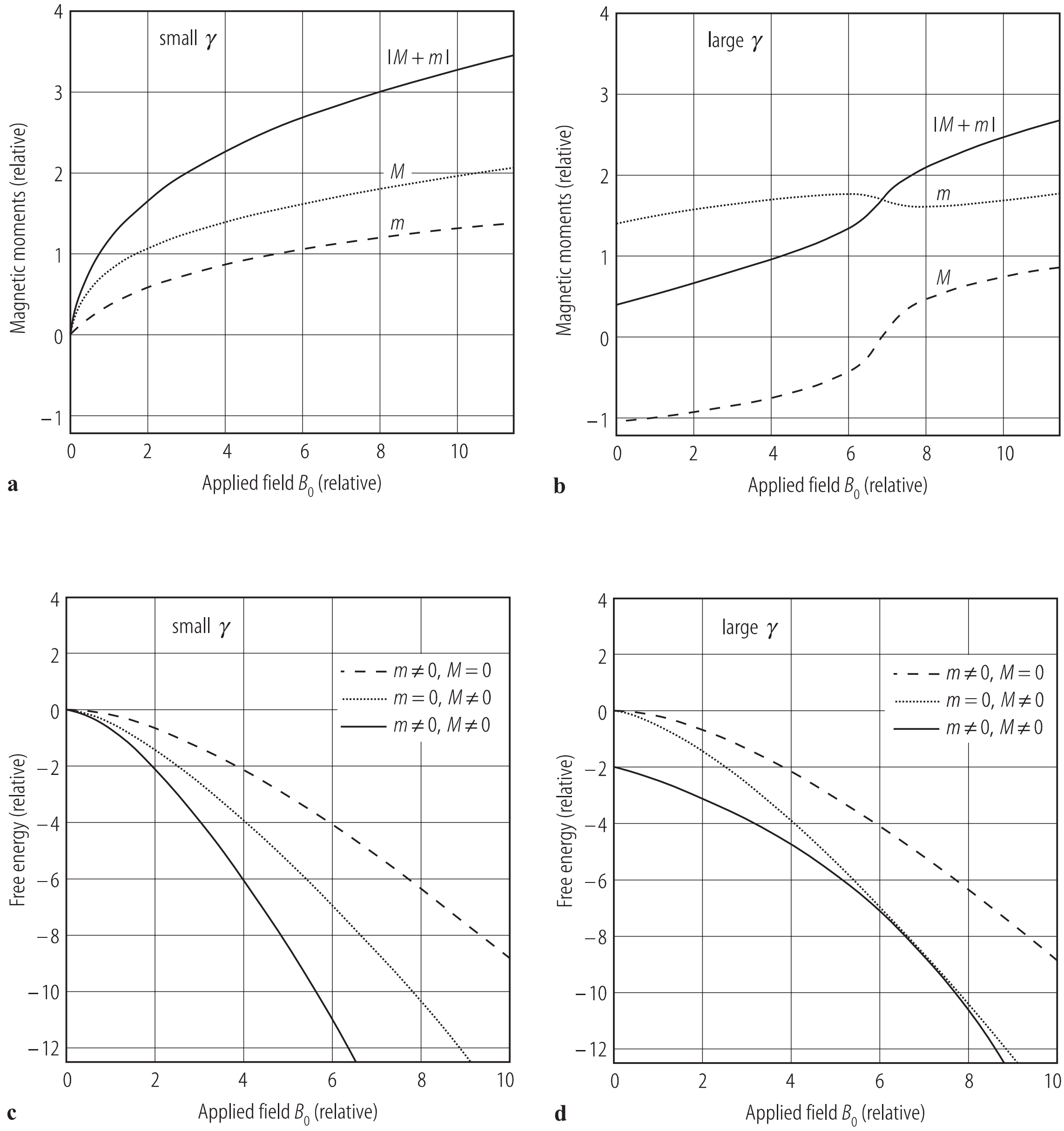

Fig. 66. Magnetic moments (M: dotted line, m: dashed line, $|M + m|$: full line) for small (**a**) and large (**b**) coupling constants γ. The corresponding free energies are shown in (**c**) and (**d**) for small and large γ values, respectively. The dashed (dotted) curves in (**c**) and (**d**) correspond to the free energy with $m \neq 0$, $M = 0$, ($M = 0$, $M \neq 0$) and the full line to $m \neq 0$, $M \neq 0$.

1.5.5.3.2.5 Ferromagnetic and / or antiferromagnetic order

Here the situation of more than one magnetic interaction present in the sample is considered. For a structure with only one magnetic sublattice and taking into account only one ferromagnetic moment $\boldsymbol{M}$ and an antiferromagnetic one $\boldsymbol{L}$, the free energy may be written in the form

$$F = \frac{1}{2}\tilde{A}\,M^2 + \frac{1}{4}C\,M^4 - \boldsymbol{B}_0 \cdot \boldsymbol{M} + \frac{1}{2}\,\tilde{a}\,L^2 + \frac{1}{4}c\,L^4 + \frac{1}{2}\,\gamma L^2 M^2 + \delta(\boldsymbol{L} \cdot \boldsymbol{M})^2 \qquad (52)$$