Spectroscopy of the Muonium Atom

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Abstract. Muonium is a hydrogen-like system which in many respects may be viewed as an ideal atom. Due to the close confinement of the bound state of the two 'point-like' leptons it can serve as a test object for Quantum Electrodynamics. The nature of the muon as a heavy copy of the electron can be verified. Furthermore, searches for additional, yet unknown interactions between leptons can be carried out. Recently completed experimental projects cover the ground state hyperfine structure, the 1s-2s energy interval, a search for spontaneous conversion of muonium into antimuonium and a test of CPT and Lorentz invariance. Precision experiments allow the extraction of accurate values for the electromagnetic fine structure constant, the muon magnetic moment and the muon mass. Most stringent limits on speculative models beyond the standard theory have been set.

1 Introduction

From electron-positron scattering at the highest achievable energies we can infer that leptons have dimensions of less than 10^{-18}m [1]. These particles may therefore be regarded as 'point-like' objects. The muonium atom (M=μ^+e^-) is the hydrogen-like bound state of leptons from two different particle generations, an antimuon(μ^+) and an electron (e^-) [2,3].

The dominant interaction within the muonium atom is electromagnetic. This can be treated most accurately within the framework of bound state Quantum Electrodynamics (QED). There are also contributions from weak interaction which arise from Z^0-boson exchange and from strong interaction due to vacuum polarization loops with hadronic content. Standard theory, which encompasses all these forces, allows to calculate the level energies of muonium to the required level of accuracy for all modern precision experiments[1].

In contrast to natural atoms and ions as well as to such artificial atomic systems, which involve hadronic constituents, muonium has the major advantage that there are no complications which would originate from the finite size and the internal structure of any of the charged particles within the atom. In natural hydrogen, for example, the interpretation of measurements of the ground state hyperfine structure (hfs) splitting is limited at the ppm level due to the not well known proton polarizability and the proton magnetic although the hfs frequency

[1] A detailed review of the theory of hydrogenic systems can be found in reference [4].

has been obtained more than six orders of magnitude better. Modern investigations of the hydrogen 1s-2s frequency interval are plagued by the proton mean square charge radius [5].

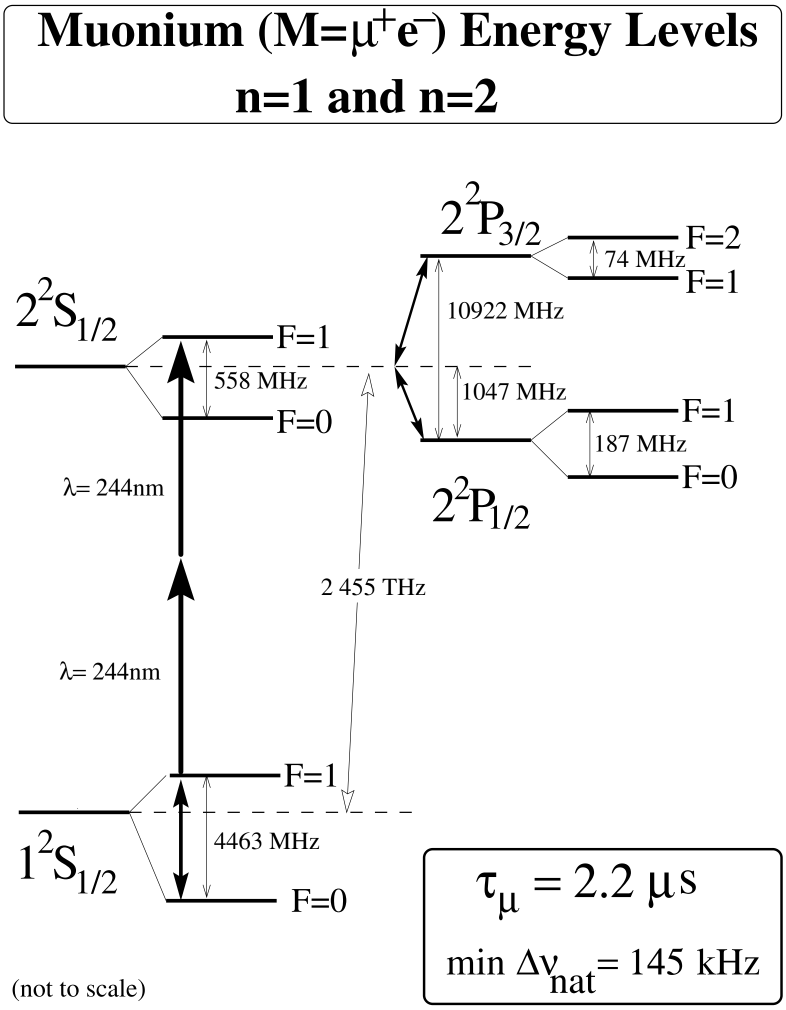

Fig. 1. Muonium energy levels for states with principal quantum numbers $n = 1$ and $n = 2$. The indicated transitions could be induced to date using modern techniques of microwave or laser spectroscopy. High accuracy has been achieved for the indicated transitions which involve the ground state. The atoms can be produced very efficiently only in the 1s state

Precision experiments in muonium provide sensitive tests for the standard theory, in particular of the most advanced field theory, QED. The nature of the muon as a heavy 'point-like' lepton (i.e. the electron-muon-tauon universality), which is fundamentally assumed in the standard theory, is always tested in a precision measurement. It should be stressed that the lack of an underlying theory, which can explain any of the lepton masses, the muon properties will remain solely astounding experimental facts with a possibility for surprise in every new precision experiment.

Because of our ability to describe muonium to very high precision on the ground of present solid knowledge, accurate values for fundamental constants like the muon mass m_μ, its magnetic moment μ_μ and anomaly a_μ and the electromagnetic fine structure constant α can be obtained in precision experiments [6]. Furthermore, searches for new and yet unknown forces in nature can be carried out. Those may show up as small deviations from the predicted standard model behaviour. With precision experiments parameters of speculative theories have been already severely restricted. Of particular interest are here models which have been invented as trials to expand the standard model in order to gain deeper insight into some of its not well understood features, like the masses of the fundamental fermions, the origin of parity violation in weak interactions, CP violation and many more. The verification of the smallness of the potential influence on muonium level energies, which would arise from any such new interaction, is very important for the reliability of extracted fundamental constants and their quoted accuracy.

2 Muonium Formation

Intense positive muon beams can be provided today with rates up to several MHz at different accelerator facilities worldwide. The μ^+ are born in positive pion (π^+) decays which themselves are generated by exposing a target consisting of a material with a low nuclear charge Z (typically carbon) to an intense beam of protons at 0.5 to 1 GeV energy. The recent experiments on muonium used 'surface' respectively 'subsurface' muon beams [7], i.e. muons which are born in pion decays at rest in the proximity of the production target surface or in deeper layers of the material. Such beams have momenta up to 29 MeV/c which corresponds to about 4 MeV energy. Parity violation in the weak decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ causes the μ^+-beam to be polarized.

Without exception, all high precision experiments, which could be performed in muonium up to date, have involved the 1s ground state (see Fig.3). The atoms can be produced in sufficient quantities only with principal quantum number $n = 1$ [3].

The most efficient mechanism to obtain M is e^- capture after stopping μ^+ in a suitable noble gas. This technique was employed already in the discovery experiment of the muonium atom through its characteristic muon spin rotation in a magnetic field by V.W. Hughes and collaborators in 1960. There the atoms were formed in Ar gas [8], where efficiencies of 65(5)% are possible. In the most recent precision measurements of $\Delta\nu_{HFS}$ and the muon magnetic moment μ_μ

at the Los Alamos Meson Physics Facility (LAMPF) in Los Alamos, USA [9] yields of 80(10)% were achieved for Kr gas targets [2] at atmospheric density.

The μ^+ moderation processes involve dominantly electro static interaction and there is no muon depolarization in strong axial magnetic fields ($B >>$ 0.16 T) [10]. At low fields the $m_F = 0$ atomic states are populated to 50 % which means a corresponding reduction of the polarization due to the muon spin oscillation at the hyperfine frequency $\Delta\nu_{HFS}$.

Muonium atoms travelling freely in vacuo can be obtained by stopping μ^+ close to the surface of a SiO_2 powder target The atoms are formed through e^- capture and some percent of them diffuse through the target surface into the surrounding vacuum[2] [12]. There the atoms have a thermal Maxwell-Boltzmann velocity distribution with an average velocity of 0.74(1) mm/μs for 300 K target temperature. The development of this production technique was an essential prerequisite for Doppler-free two-photon laser spectroscopy of the $1^2S_{1/2}$-$2^2S_{1/2}$ frequency interval $\Delta\nu_{1s2s}$ which were carried out in a pioneering approach at the KEK facility in Tsukuba, Japan [13], and in a precision measurement at the Rutherford Appleton Laboratory (RAL) in Chilton, United Kingdom [14]. Such measurements aim for an accurate value for m_μ and a test of the muon-electron charge ratio. Thermal muonium in vacuo was also the key to a sensitive search for a conversion of muonium into its anti-atom ($\overline{\mathrm{M}}$) at the Paul Scherrer Institut (PSI) in Villigen, Switzerland [15,16]. Further, a first unambiguous demonstration of hyperfine transitions in vacuo could be made [17].

Metastable muonium atoms in the 2s state have been produced with a beam foil technique at LAMPF and at the Tri University Meson Physics Facility (TRIUMF) at Vancouver, Canada. Only moderate numbers of atoms could be obtained. The velocity resonance nature of the electron transfer reaction results in a muonium beam at keV energies. Very difficult and challenging experiments using electromagnetic transitions in excited states, particularly the $2^2S_{1/2}$–$2^2P_{1/2}$ classical Lamb shift and $2^2S_{1/2}$-$2^2P_{3/2}$ splitting could be induced with microwaves. However, the achieved experimental accuracy at the 1.5 % level [18,19,20], does not represent a severe test of theory yet.

3 Ground State Hyperfine Structure

The by far largest part of the hfs splitting $\Delta\nu_{HFS}$ in the muonium 1s state is given by the well known Fermi energy[21], which arises from the interaction of the muon and electron magnetic moments. Including all contributions it is given by

$$\Delta\nu_{HFS} = \frac{16}{3}(Z\alpha)^2 R_\infty \frac{\mu_\mu}{\mu_B}\left[1+\frac{m_e}{m_\mu}\right]^{-3}(1+\varepsilon_{rad}+\varepsilon_{rec}+\varepsilon_{rad-rec}) \qquad (1)$$

[2] Another powerful technique uses hot metal foils as converters, where tungsten and rhenium give best results. This method was the basis for a successful production of slow muons by resonant two step laser ionization of M atoms at KEK in Tsukuba, Japan [11]. This work is at present continued at RAL to provide an intense beam of slow muons for condensed matter, particularly surface science.

$$+\Delta\nu_{strong} + \Delta\nu_{weak} + \Delta\nu_{exotic} \ ,$$

with μ_B the Bohr magneton, m_e the muon mass and $R_\infty = \alpha^2 \cdot m_e c^2/2 \cdot h$ the Rydberg constant, where c is the speed of light and h the Planck constant. QED corrections for radiative effects ε_{rad} are of order α due to the lepton magnetic anomalies, recoil contributions ε_{rec} are of order $\alpha m_e/m_\mu$ and combined radiative and recoil terms $\varepsilon_{rad-rec}$ start at $\alpha^2 m_e/m_\mu$. The strong interaction adds $\Delta\nu_{strong} = 250$ Hz and weak interaction through parity conserving axial-axial vector currents yields $\Delta\nu_{weak} = -65$ Hz [4]. The sign of this effect is opposite to the sign of the effect in hydrogen, the μ^+ is an anti-particle in contrast to the proton. Among the possible exotic interactions which could contribute to $\Delta\nu_{HFS}$ is the conversion of muonium (M) to antimuonium ($\overline{\mathrm{M}}$). Although this process may appear in the environment of atomic physics as a somewhat remote possibility, it must be mentioned that it is a full analogy for leptons to the well known K^0-$\overline{\mathrm{K}^0}$ oscillations. If the process exists, it would cause a splitting of hyperfine levels in nS states up to [15]

$$\Delta\nu_{exotic} = \Delta\nu_{\mathrm{M}-\overline{\mathrm{M}}}(nS) = \langle M|H_{M\overline{M}}|\overline{M}\rangle = \frac{519}{n^3} \cdot (G_{M\overline{M}}/G_F)\sqrt{S_B} \ \mathrm{Hz}, \quad (2)$$

where $H_{M\overline{M}}$ stands for the interaction Hamiltonian, $G_{M\overline{M}}$ represents the coupling constant of the process, G_F is the Fermi weak interaction constant and $S_B \leq 1$ reflects the magnetic field dependence of the exotic coupling. Therefore it could influence the interpretation of precision measurements and affect the validity of extracted Fundamental constants, unless proven to be small. From a very recent direct search (see chapter 6) at the Paul Scherrer Institute (PSI) in Villigen, Switzerland, an upper limit of $\Delta\nu_{\mathrm{M}-\overline{\mathrm{M}}}(1S) \leq 1.5 \ \mathrm{Hz}/\sqrt{S_B}$ can be concluded for an expected line splitting [3,15].

In the latest experiment at LAMPF [9] a muon beam was stopped in a Kr gas target at typically atmospheric density inside of a microwave cavity. This device was centered in a Magnetic Resonance Imaging magnet at $\boldsymbol{B}$ =1.7 Tesla field with ppm homogeneity. Microwave transitions between the two energetically highest respectively two lowest Zeeman sublevels at the frequencies ν_{12} and ν_{34} (Fig.3) involve a muon spin flip. They were detected through a change in the spatial distribution of e^+ from the decays $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$, because due to parity violation in this weak interaction process the e^+ are preferentially emitted in the direction of the μ^+ spin. As a consequence of the Breit-Rabi equation, which describes the behaviour of the levels in a magnetic field $\boldsymbol{B}$, the sum of these frequencies for the same value of $\boldsymbol{B}$ equals the splitting in zero field $\Delta\nu_{HFS}$ and their difference yields in a known field μ_μ [2]. The experiment utilized the technique of "old muonium", which allowed to reduce the linewidth of the signals (Fig.3) below one half of the natural linewidth

$$\delta\nu_{nat} = (\pi \cdot \tau_\mu)^{-1} = 145 \ \mathrm{kHz}, \quad (3)$$

where $\tau_\mu = 2.2 \ \mu$s is the muon lifetime. For this purpose an essentially continuous muon beam was chopped by an electrostatic kicking device into 4 μs long pulses

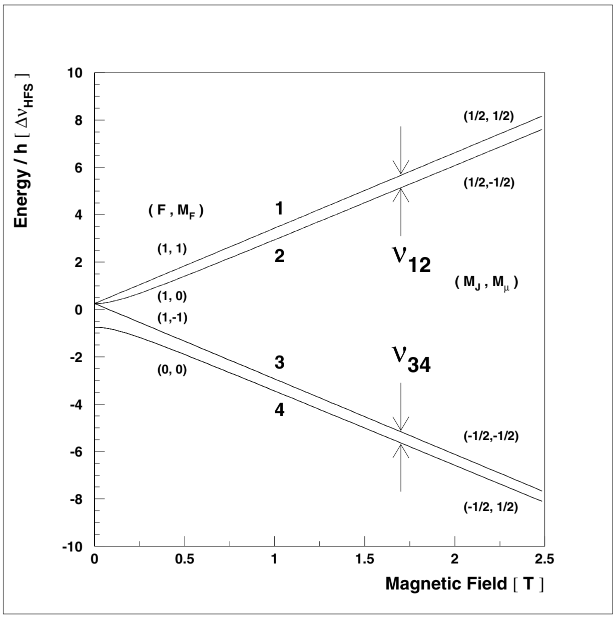

Fig. 2. Ground state Zeeman levels in an external magnetic field. The sum of the frequencies of the indicated transitions $\Delta\nu_{12}$ and $\Delta\nu_{34}$ at the same magnetic field equals the zero field splitting $\Delta\nu_{HFS}$ and their difference allows to determine the muon magnetic moment

with 14 μs separation. Only atoms which were interacting coherently with the microwave field for periods longer than several muon lifetimes were detected [22].

The results are mainly statistics limited and improve the knowledge of both $\Delta\nu_{HFS}$ and μ_μ by a factor of three [9] over previous measurements [23]. The zero field splitting is determined to

$$\Delta\nu_{HFS}(\text{expt.}) = \nu_{12} + \nu_{34} = 4\,463\,302\,765(53)\ \text{Hz} \quad (12\ \text{ppb})\ . \tag{4}$$

This value agrees well with the theoretical prediction of [24]

$$\Delta\nu_{HFS}(\text{theory}) = 4\,463\,302\,563(510)(34)(\leq 100)\ \text{Hz} \quad (120\ \text{ppb})\ . \tag{5}$$

Here the first quoted uncertainty is due to the accuracy to which the muon-electron mass ratio m_μ/m_e is known, the second error is from the knowledge of α as obtained from electron $g-2$ measurements [25], and the third value corresponds to estimates of uncalculated higher order terms.

The measurements give a muon magnetic moment of

$$\mu_\mu/\mu_p = 3.183\,345\,13(39) \ \ (120\ \text{ppb}) \tag{6}$$

which translates into a muon/electron mass ratio of

$$m_\mu/m_e = 206.768\,277(24) \ \ (120\ \text{ppb})\ . \tag{7}$$

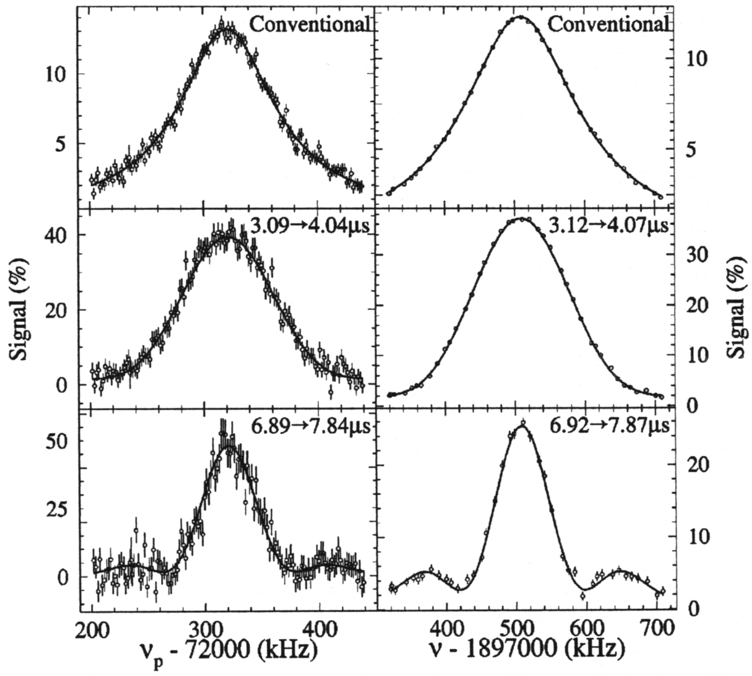

Fig. 3. Samples of conventional and 'old muonium' resonances at the frequency ν_{12} (See Fig. 2). The narrow 'old' signals have also a higher amplitude and a characteristic line shape [22]. The lines in the left column were recorded by sweeping the magnetic field, which was measured in units of the proton NMR frequency (ν_P). The lines on the right were obtained using microwave frequency (ν) scans

A highly accurate value for the μ can also be obtained from the zero field splitting $\Delta\nu_{HFS}$. Using the fine structure constant α derived from the magnetic anomaly of the electron, one finds

$$m_\mu/m_e = 206.768\,267\,0(55) \quad (27 \text{ ppb}) \ . \tag{8}$$

This value depends strongly on the correctness of the theory both for the muonium hyperfine structure and the electron magnetic anomaly. Alternatively, extracting a value for α from $\Delta\nu_{HFS}$ instead represents a most valuable stringent consistency test for different branches of physics, which each allow to obtain a precise value of α (see Fig.3).

The hyperfine splitting is proportional to $\alpha^2 R_\infty$, with the very precisely known Rydberg constant R_∞. Comparing experiment and theory yields [9]

$$\alpha_2^{-1} = 137.035\,996\,3(80) \quad (58 \text{ ppb}) \ . \tag{9}$$

If R_∞ is decomposed into even more fundamental constants, one finds $\Delta\nu_{HFS}$ to be proportional to $\alpha^4 m_e/h$. Using the value h/m_e as determined in measure-

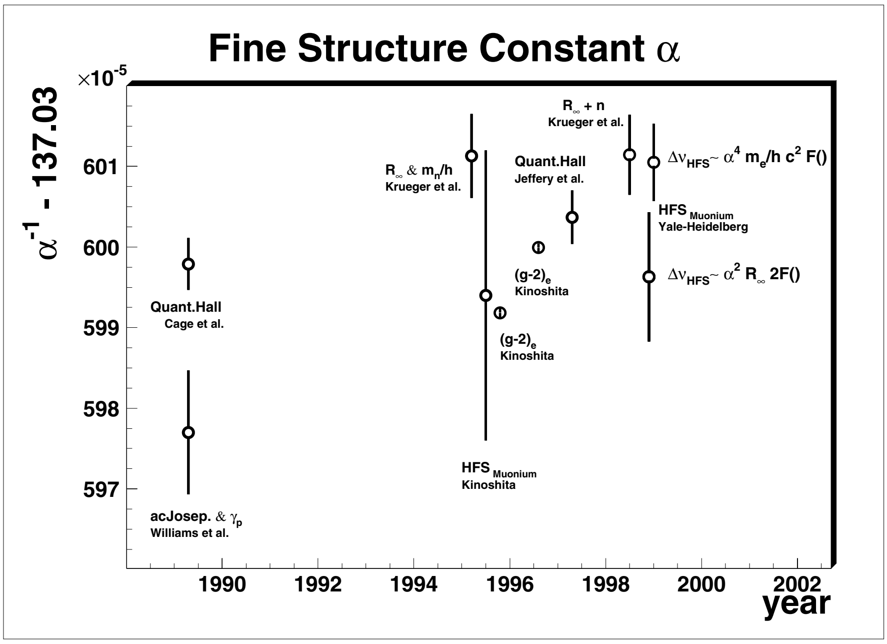

Fig. 4. The fine structure constant α has been determined with various methods [25-28]. most precise is the determination from the magnetic anomaly of the electron. The muonium atom offers two different routes which uses independent sets of fundamental constants. The disagreement (the error bars are mostly statistical) seem to indicate that the value h/m_e from neutron de Broglie wavelength measurements may be quoted with too high accuracy

ments of the neutron de Broglie wavelength [28] gives

$$\alpha_4^{-1} = 137.036\,004\,7(48) \ \ (35\text{ ppb}) \ . \qquad (10)$$

In the near future a small improvement in α_4^{-1} can be expected from ongoing determinations of h/m_e in measurements of the photon recoil in Cs atom spectroscopy and a Cs atomic mass measurement [29]. The present limitation for accuracy of α_4^{-1} arises mainly from the muon mass uncertainty. Therefore any better determination of the muon mass, e.g. through a precise measurement of the reduced mass shift in $\Delta\nu_{1s2s}$, will result in an improvement of α_4^{-1}.

At present the good agreement within two standard deviations between the fine structure constant determined from muonium hyperfine structure and the one from the electron magnetic anomaly is generally considered the best test of internal consistency of QED, as one case involves bound state QED and the other one QED of free particles.

Among the mostly fundamentally assumed symmetries in nature are the Lorentz invariance and the validity of the CPT theorem which demands an invariance of nature under simultaneous charge conjugation (C), parity operation

(P) and time reversal (T). It is particularly difficult to compare tests which were made in different systems, i.e. often quoted upper limits on relative changes in particle properties are only little justified. Small numbers per se may not be too important. A comparison on the basis of the strength of the potential interaction could give much more information; however, in the absence of a positive signal and of a clear theory this is hardly possible. In a recent approach by by Bluhm et al. [30] it was suggested to compare different systems on a common basis, i.e. through the energies of the involved states. Within a generic extension of the standard model [30] diurnal variations of the ratio [31]

$$(\Delta\nu_{12} - \Delta\nu_{34})/(\Delta\nu_{12} + \Delta\nu_{34}). \tag{11}$$

were suggested for muonium. A reanalysis of the data recorded at LAMPF [9] could show that in the course of a day the changes in any frequency are less than about 15 Hz [32] and are therefore of no concern for the above mentioned interpretation of the experiment and in particular for the obtained fundamental constants. With significantly increased muon flux and a pulsed time structure with narrow μ^+ bunches of width $\delta T \leq \tau_\mu$, separated by $T \approx 10 \cdot \tau_\mu$, an extinction ratio $\varepsilon \ll \exp\big(-T/\tau_\mu\big)$ and straight forward improvements in the setup a gain of one order of magnitude for $\Delta\nu_{HFS}$, μ_μ/μ_p, α_2^{-1} and α_4^{-1} should be attainable with the same 'old muonium' technique.

4 1s-2s Energy Interval

Doppler-free excitation of the 1s-2s transition had been achieved in the past at KEK [13] and at RAL [33,34]. The transitions were induced by counter-propagating intense pulsed UV light at 244 nm wavelength which was generated by frequency doubling of blue laser radiation. The accuracy of these experiments was limited by properties of the employed laser systems, which allowed little control over an ac Stark effect and laser frequency chirping. The latter was caused by rapid changes of the index of refraction in the dye solutions of laser amplifier stages for the blue light. The experiments were only possible at pulsed accelerators sites, because with the presently available muon beam fluxes and muonium production yields reasonable transition rates can only be expected for high excitation probabilities. This requires pulsed lasers.

A new measurement, which was tuned for precision, has been performed recently at the worlds brightest pulsed surface muon source at RAL [7,14]. The $1^2S_{1/2}(F{=}1) \rightarrow 2^2S_{1/2}(F{=}1)$ transition was induced when muonium atoms, which had emerged from a SiO_2 target, interacted about 8 mm above the target surface with the light field of two counter-propagating laser beams of wavelength 244 nm. The two-photon excitation was detected by photo-ionization of the 2s state in the same light field. The released muons were identified in a mass spectrometer which selected particles by a combination of electric and magnetic fields as well as their time of flight (Fig. 4). The positrons from muon decay were recorded as part of an event signature. The muon count rate as a function of laser frequency represents the experimental signal.

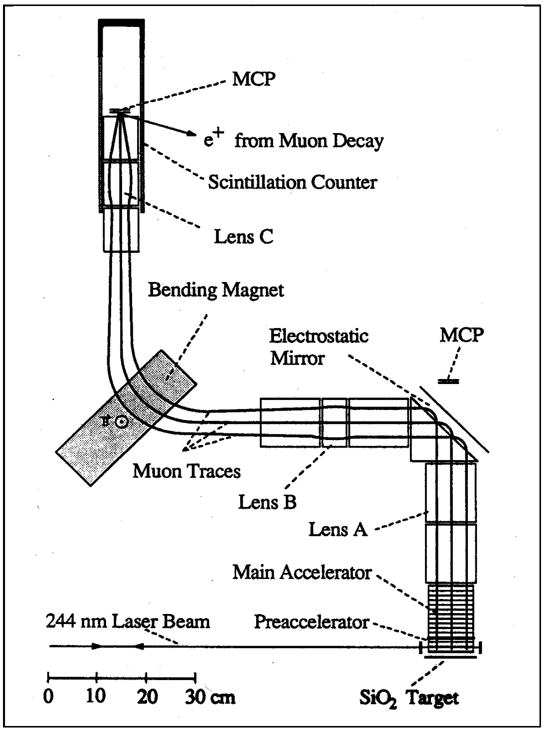

Fig. 5. Detection scheme for the muonium 1s-2s-photo-ionization transition. The muon released in the process is accelerated to typically 2 keV in a two stage device. The particle is identified in a mass spectrometer consisting of a 1.66 m time of flight path with an electrostatic deflector and a bending magnet. The positron from muon decay is observed as part of the signature

The necessary high power UV laser light was generated by frequency tripling the output of an alexandrite ring laser amplifier in crystals of LBO and BBO. The alexandrite laser was seeded with light from a continuous wave Ar ion laser pumped Ti:sapphire laser at 732 nm (Fig. 4) [35]. Fluctuations of the optical phase during the laser pulse (laser frequency chirp) were compensated with two electro-optic devices in the resonator of the ring amplifier to give a swing of the laser lights frequency chirping of less than about 5 MHz. The fundamental optical frequency was calibrated by frequency modulation saturation spectroscopy of the a_{15} hyperfine component of the 5-13 R(26) line in thermally excited $^{127}I_2$ vapour which lies about 700 MHz lower than 1/6 of the M transition frequency. It has been calibrated to 0.4 MHz at the Institute of Laser Physics in Novosibirsk, Russia, and at the National Physical Laboratory in Teddington, United Kingdom [36]. The cw light was frequency up-shifted by passing through two acousto-optic modulators (AOM's). For the muonium measurements 25 preselected values of the AOM frequency were chosen. Every minute one of them was randomly chosen.

Among the many further experimental details we would like to mention that in total 3 million laser shots were fired with a chirp swing below 15 MHz. Altogether 99 events were found (Fig. 4). To obtain the theoretical line shape, we calculated numerically for a randomly selected sample of 20% of all the laser

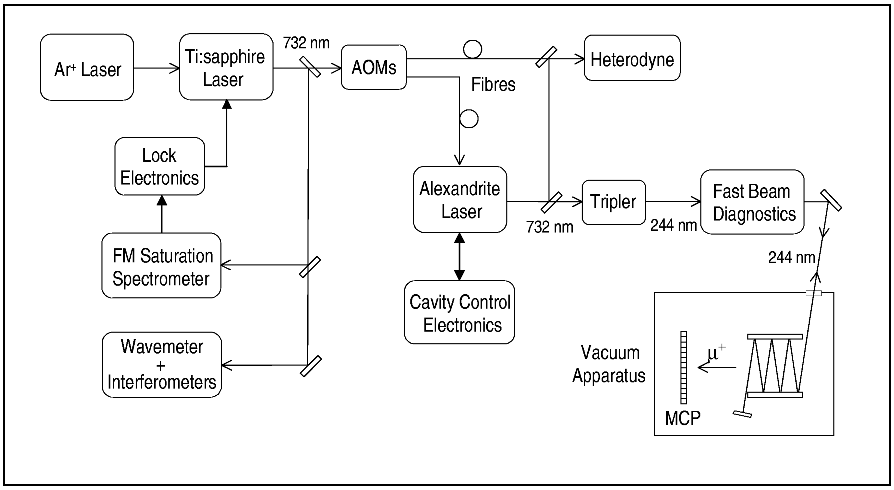

Fig. 6. Laser system in the muonium 1s-2s experiment. A cw laser at 732 nm is locked to a molecular I_2 resonance. Its light is amplified in an alexandrite ring amplifier and then frequency tripled. The light frequency is scanned using acousto-optic modulators

pulses the probability for a resonant ionisation event using a line shape theory which is based on a density matrix model. This allows the inclusion in each case of the recorded time dependent phase shift, the intensity and the beam cross section for the laser light; details are given in reference [37,38]. We verified that the remaining 80% and all the pulses which produced an event had the same average distributions for chirp, chirp swing, intensity and spatial profiles. The experimental setup has been tested and the novel analysis procedures were verified using measurements of the two hyperfine components of the 1s-2s transition in deuterium [14].

The muonium experiment gives as a result

$$\Delta\nu_{1s2s}(\text{expt.}) = 2\,455\,528\,941.0(9.8)\ \text{MHz}\ . \tag{12}$$

This constitutes a significant improvement in accuracy compared to the earliest efforts (Fig. 4) [13], where the errors were totally dominated by systematics and their treatment proper[39]. The new result is in good agreement with the theory value [40]

$$\Delta\nu_{1s2s}(\text{theory}) = 2\,455\,528\,935.4(1.4)\ \text{MHz}\ . \tag{13}$$

From these figures the muon/electron mass ratio is found to be

$$m_{\mu^+}/m_{e^-} = 206.768\,38(17). \tag{14}$$

In an alternate interpretation the muon/electron charge ratio can be extracted. For hydrogen-like systems the leading order for the gross structure energy is proportional to $(Z^2\alpha)\alpha/n^2$ where Z is the nuclear charge in units of the

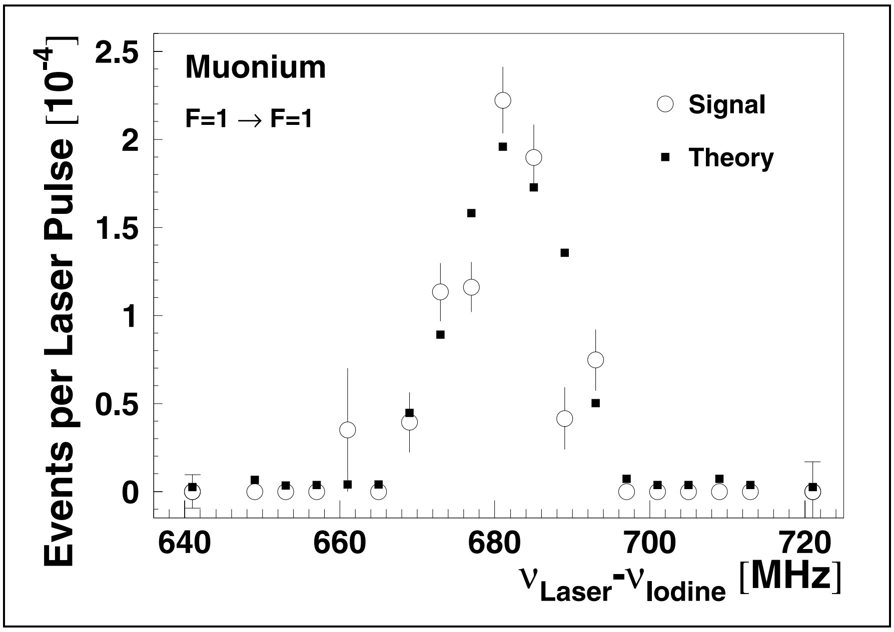

Fig. 7. Muonium 1s-2s signal. The frequency corresponds to the offset of the Ti:sapphire laser from a molecular iodine reference line. The open circles are the observed signal, the solid squares represent the theoretical expectation based on measured laser beam parameters and a line shape model [38]

electron charge. By comparing $\Delta\nu_{1s2s}$(expt.) and $\Delta\nu_{1s2s}$(theory) we found for the charge ratio

$$Z = q_{\mu^+}/q_{e^-} = -1 - 1.1(2.1) \cdot 10^{-9} \ . \tag{15}$$

This is the best verification of charge equality in the first two generations of particles. We note that the existence of one single fundamental quantized unit of charge is solely an empirical observation and no associated underlying symmetry has yet been revealed. Gauge invariance, unfortunately, assures charge quantization only within one generation of particles.

Major progress in the laser spectroscopy of muonium can be expected from a cw laser experiment, where the light frequency accuracy is not expected to present any problem in the foreseeable future. Prior to such a project two major technological advances will be needed: (i) optical coatings in the 244 nm region which will allow to set up an enhancement cavity with kW circulating power and (ii) a pulsed muon facility with some two orders of magnitude increased flux over present beams with pulse widths below τ_μ and pulse separations of at least several τ_μ. Most important is in this context the overall number of muonium atoms in the laser beam. A pulsed beam time structure is of advantage primarily for background suppression.

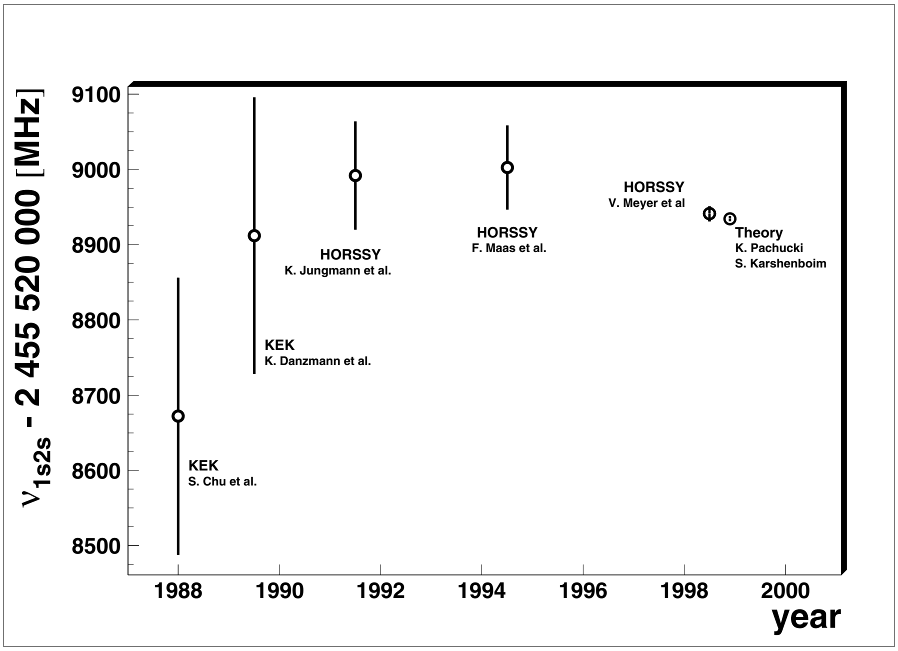

Fig. 8. Evolution of muonium 1s-2s measurements. The two results displayed for the KEK accelerator facility refer to one single measurement by a Japanese - American collaboration [13] and its reanalysis [39]. The newer measurements were made by the **H**eidelberg - **O**xford - **R**utherford - **S**ussex - **S**iberia - **Y**ale collaboration and have now reached a level of accuracy comparable to theory, where the limitation arises primarily from the muon mass

5 Connection to a New Measurement of the Muon Magnetic Anomaly

The muon magnetic anomaly is given, like in case of the electron, mostly by photon and electron-positron fields. However, the effects of heavier particles, which are introduced through vacuum polarization loops, is enhanced by the square of the mass ratio $m_\mu/m_e \approx 4 \cdot 10^4$. The contributions from strong interaction amount to 58 ppm. They can be determined from a dispersion relation with the input from experimental data on e^+-e^- annihilation into hadrons and from hadronic τ-decays. The weak interaction adds 1.3 ppm through loops with $W^\pm$ and Z^0 bosons and such with additional photons. At present standard theory yields a_μ to 0.66 ppm [25]. Contributions from physics beyond the standard model may be as large as a few ppm. Such could arise from, e.g., supersymmetry, compositeness of fundamental fermions and bosons, CPT violation and many others. Of urgent actuality is the possibility to restrict for minimal supersymmetric models the value of $\tan\beta$, which is the ratio of the vacuum expectation value of the involved two Higgs fields.

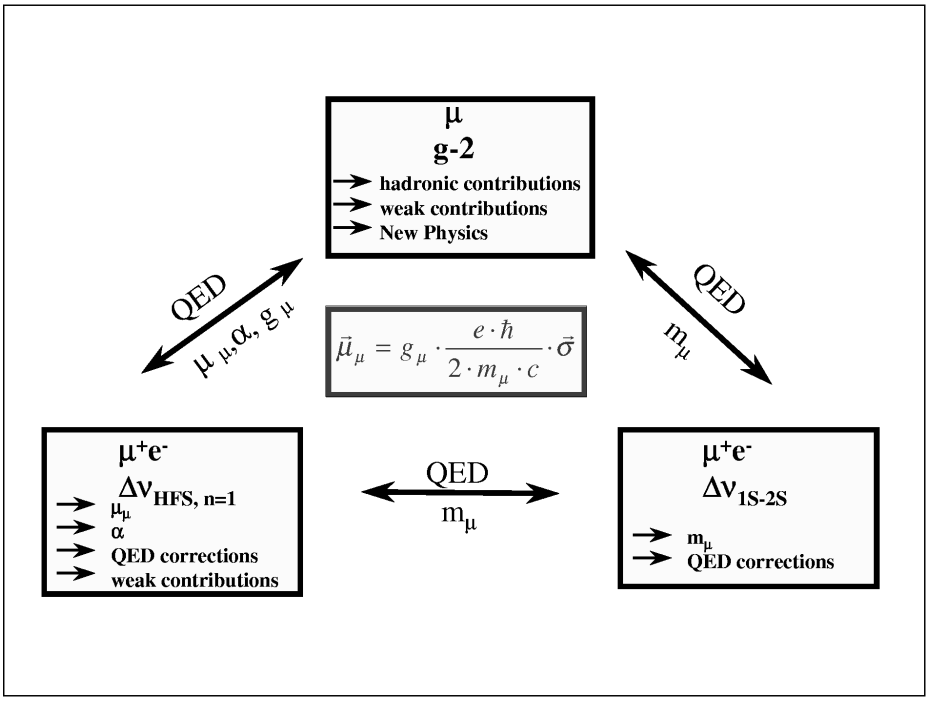

Fig. 9. The spectroscopic experiments on the hyperfine structure of muonium and the 1s-2s energy interval are closely related to a precise measurement of the muon muon magnetic anomaly. The measurements put a stringent test on the internal consistency of the theory of electroweak interaction and on the set of the involved fundamental constants

The spectroscopic experiments in muonium, in particularly on the hyperfine and the 1s-2s intervals, are closely inter-related with the determination of the muon magnetic anomaly a_μ through the fundamental relation $\mu_\mu = (1 + a_\mu) \cdot e\hbar/(2m_\mu c)$. The results from all experiments establish a self consistency requirement for QED and electroweak theory and the set of fundamental constants involved (Fig. 5) The constants α, m_μ, μ_μ are the most stringently tested important parameters. Although, in principle, the muon-electron system could provide the relevant electroweak constants, the Fermi coupling constant G_F and Weinberg angle $\sin^2\theta_W$, the use of more accurate values from independent measurements may be chosen for higher sensitivity to new physics.

A new determination of a_μ [41] is presently carried out in a superferric magnetic storage ring at the Brookhaven National Laboratory (BNL) in Upton, USA.

The experiment uses a g–2 technique in which the difference $\omega_a = \omega_s - \omega_c$ of the spin precession and the cyclotron frequencies (ω_s, ω_c) is measured. This project aims for a final precision of 0.35 ppm. In order to be able to reach this goal, it is essential to have m_μ (respectively μ_μ/μ_p) available with an accuracy at the 0.1 ppm level, because this quantity is needed for extracting the experimental result

$$a_\mu = \frac{g-2}{2} = \frac{\omega_a m_\mu c}{eB} \qquad (16)$$

with the precisely measured magnetic field $\boldsymbol{B}$. At this stage, these relevant muon parameters can only be provided by muonium spectroscopy to sufficient and reliable accuracy.

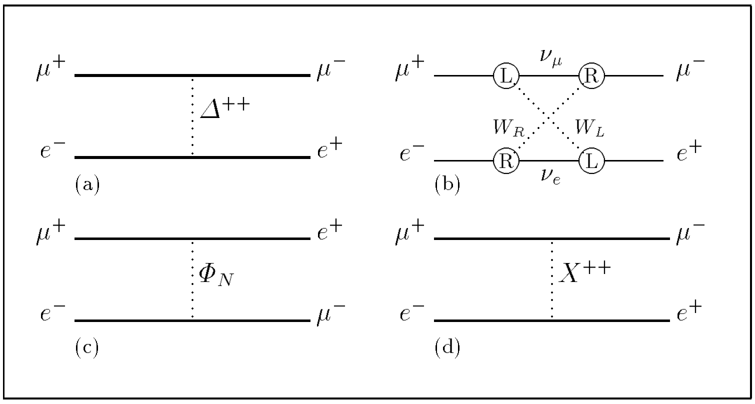

Fig. 10. Muonium-antimuonium conversion in theories beyond the standard model. The interaction could be mediated by (a) a doubly charged Higgs boson Δ^{++} [52,53], (b) heavy Majorana neutrinos [52], (c) a neutral scalar Φ_N [54], e.g. a supersymmetric τ-sneutrino $\tilde{\nu}_\tau$ [55,56], or (d) a bileptonic gauge boson X^{++} [57]

The BNL experiment is planed for both μ^+ and μ^- as a test of CPT invariance. This is of particular interest in view of CPT violating models [30] (see chapter 3). For measurements of magnetic anomalies a comparison of measurements is suggested through the energies of particles with spin down and of anti-particles with spin up in an external magnetic field. The nature of $g-2$ experiments is such that they provide a figure of merit $r = |a^- - a^+| \cdot \hbar\omega_c/m \cdot c^2$ [42] for such a CPT test, where a^- and a^+ are the positive and negative particles magnetic anomalies and m is the particle mass. For the past electron and positron measurements one has $r_e = 1.2 \cdot 10^{-21}$ [42]. This may be viewed as a much tighter CPT test than in the case of the neutral kaon system, were the mass differences between K^0 and $\overline{K}^0$ yield $r_K = 4 \cdot 10^{-19}$ [43]. An even more stringent CPT test arises already from the past muon magnetic anomaly measurements [44] where $r_\mu = 3.5 \cdot 10^{-24}$. This may therefore already be regarded as the presently best known CPT test based on system energies [45]. With im-

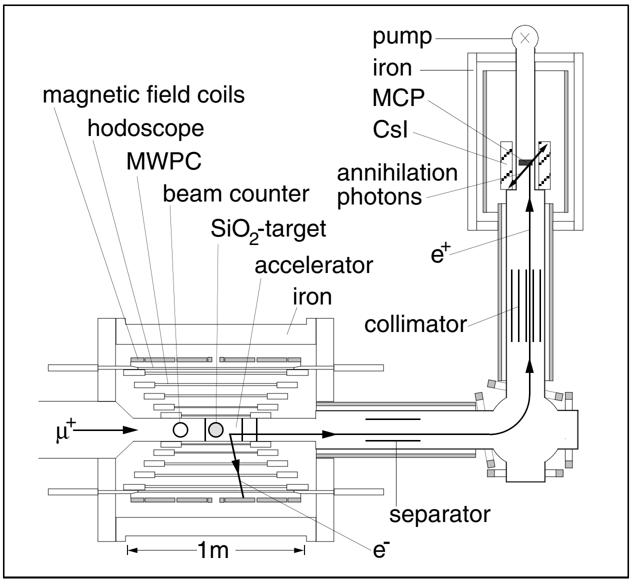

Fig. 11. Top view of the MACS (Muonium - Antimuonium - Conversion - Spectrometer) apparatus at PSI to search for $\mathrm{M}-\overline{\mathrm{M}}$ - conversion [59]

provement expected in the BNL $g-2$ experiment one can look forward to a 20 times more precise test of this fundamental symmetry.

6 Muonium-Antimuonium Conversion

Beyond atomic spectroscopy muonium renders the possibility to search directly and sensitively for yet unknown interactions between the two charged leptons from two different generations. Among the mysteries observed for leptons are the apparently conserved lepton numbers. As a matter of fact, several distinctively different lepton number conservation schemes appear to hold, some of which are additive and some are multiplicative, parity-like. Some of them distinguish between lepton families and others don't [46,47,48,49,50]. No local gauge invariance has been revealed yet which would be associated with any of these empirically established laws. Since there is common believe [51] that any discrete conserved quantity is connected to a local gauge invariance, a breakdown of lepton number conservation is widely expected, particularly in the framework of many speculative models.

A potential M-$\overline{\mathrm{M}}$-conversion would violate additive lepton family number conservation and is discussed in many of the speculative theoretical approaches (see Fig. 10). It would be a full analogy in the sector of leptons to K^0-$\overline{\mathrm{K}}^0$ oscillations, which are well known and established in the quark sector of the standard model.

A dedicated search experiment was performed at PSI [15,59]. The setup (Fig. 11) was designed to employ a powerful signature, which had been developed in a predecessor experiment at LAMPF. The coincident identification of both charged particles released in the anti-atom's decay was required. [58,59]. Muonium atoms in vacuo with thermal velocities were produced from a SiO_2 powder target. They were observed for antimuonium decays. Energetic electrons from the decay of the μ^- in the antiatom were traced in a magnetic spectrometer at 0.1 T magnetic field. This instrument consisted of five concentric multiwire proportional chambers and a 64 fold segmented hodoscope. The positron in the atomic shell of the antiatom is expected to be left behind after the muon decay. This positron has 13.5 eV average kinetic energy, the systems Rydberg energy [60,61]. It could be accelerated to 10 keV in a two stage electrostatic device and guided in a magnetic transport system onto a position sensitive microchannel plate detector (MCP). Annihilation radiation into two γ rays of 511 keV could be observed in a 12 fold segmented pure CsI calorimeter around the MCP. For normalization purposes the muonium production was monitored regularly every 5 hours by reversing all electromagnetic fields in the apparatus.

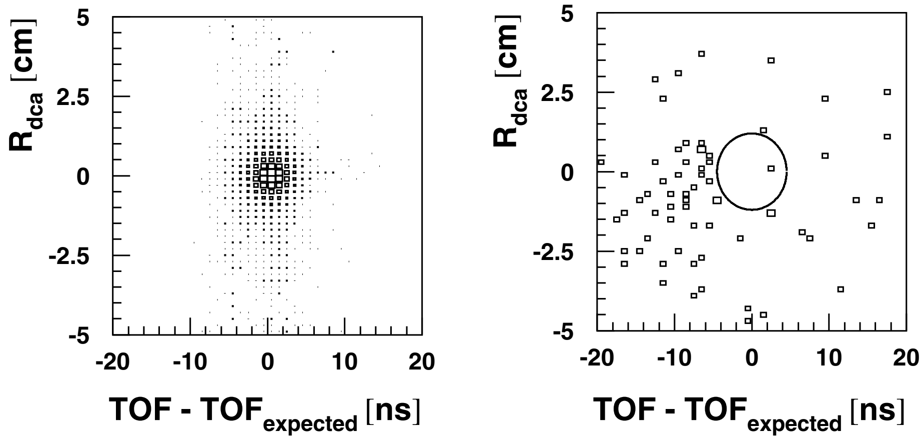

Fig. 12. Time of flight (TOF) and vertex quality for a muonium measurement (left) and the same for all data of the final 4 month search for antimuonium (right). One event falls into the indicated 3 standard deviations area

The relevant measurements were performed during in total 6 month distributed over 4 years during which $5.7 \cdot 10^{10}$ muonium atoms were in the interaction region. Out of those, one event fell within a 99% confidence interval of all relevant distributions (Fig. 12). The expected background due to accidental coincidences was 1.7(2) events. Thus an upper limit on the conversion probability of

$$P_{M\overline{M}} \leq 8.2 \cdot 10^{-11}/S_B \quad (90\% \text{ C.L.}) \tag{17}$$

was found, where S_B accounts for the interaction type dependent suppression of the conversion in the magnetic field of the detector due to the removal of degeneracy between corresponding levels in muonium and $\overline{M}$. The reduction is strongest for (V±A)×(V±A), where S_B=0.35 [62,63]. This yields for the traditionally quoted upper limit on the coupling constant in an effective four fermion interaction

$$G_{M\overline{M}} \leq 3.0 \cdot 10^{-3} G_F \quad (90\%\ \mathrm{C.L.}) \quad . \tag{18}$$

This new result, which exceeds bounds from previous experiments [58,64] by a factor of 2500 and the one from an early stage of the experiment [59] by 35, has some impact on speculative models. A certain Z_8 model is ruled out with more than 4 generations of particles where masses could be generated radiatively with heavy lepton seeding [66]. A new lower limit of

$$m_{X^{\pm\pm}} \geq 2.6\ \mathrm{TeV/c}^2 \cdot g_{3l} \quad (95\%\mathrm{C.L.}) \tag{19}$$

on the masses of flavour diagonal bileptonic gauge bosons in GUT models is extracted which lies well beyond the value derived from direct searches, measurements of the muon magnetic anomaly or high energy Bhabha scattering [57,65]. Here g_{3l} is of order 1 and depends on the details of the underlying symmetry. For 331 models this translates into $m_{X^{\pm\pm}} \geq 850$ GeV/c^2 which disfavours their minimal Higgs version in which an upper bound of 600 GeV/c^2 has been extracted from an analysis of electroweak parameters [67,68]. The 331 models may still be viable in some extended form involving a Higgs octet [69]. In the framework of R-parity violating supersymmetry [56,55] the bound on the coupling parameters could be lowered by a factor of 15 to $\mid \lambda_{132}\lambda^*_{231} \mid \leq 3 \cdot 10^{-4}$ for assumed superpartner masses of 100 GeV/c^2. Further the achieved level of sensitivity allows to narrow slightly the interval of allowed heavy muon neutrino masses in minimal left-right symmetry [53] (where a lower bound on $G_{M\overline{M}}$ exists, if muon neutrinos are heavier than 35 keV) to ≈ 40 keV/c^2 up to the present experimental bound at 170 keV/c^2. In minimal left right symmetric models, in which $M\overline{M}$ conversion is allowed, the process is intimately connected to the lepton family number violating muon decay $\mu^+ \rightarrow e^+ + \nu_\mu + \overline{\nu}_e$. With the limit achieved in this experiment this decay is not an option for explaining the excess neutrino counts in the LSND neutrino experiment at Los Alamos [70,71].

A future $M - \overline{M}$ experiment could take particularly advantage of high intensity pulsed beams, with pulses short compared to τ_μ and separated by several τ_μ. In contrast to other lepton number violating muon decays, the M-$\overline{M}$-conversion through its nature as particle - antiparticle oscillation, has a time evolution analogous to usual two level systems, like they can be found in many cases in atomic physics. Hence for coupling constants of the demonstrated smallness the probability for finding $\overline{M}$ in the ensemble increases quadratically in time. This gives the signal an advantage growing in time over major background, which can be assumed to decay exponentially [16]. Particularly for double or triple coincidences in the event signature this could be advantageous.

7 Long Term Future Possibilities

It appears that the availability of muons limits the ability to perform more accurate spectroscopy and to find very rare processes like M-$\overline{M}$-conversion. All the mentioned experiments on muons and muonium are limited by statistics. Systematic errors could even be reduced much further by straightforward means, if required. Therefore any measure to boost the particle fluxes of existing muon beam lines will be a very important step forward.

For significant improvements, we need significantly more intense accelerators, such as they are presently discussed at various places. In the intermediate future a possible European Spallation Source (ESS) or the Japanese Hadron Facility (JHF) are important options. Also at the Oak Ridge neutron spallation source could accommodate intense muon beams. A new planned intense proton machine at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, Germany, could be utilized to produce muon beams with optimized particle flux, time structure and phase space. The most promising facility would be, however, a muon collider [72] or a neutrino factory, the front end of which could provide muon rates 5-6 orders of magnitude higher than present beams (see Table 1). Very attractive are new muon beam designs, where relatively more muons (compared to present schemes) can collected at the production target and where new techniques like phase rotation will be employed. Among those the proposal of the PRISM beam for JHF is very appealing [73]. Such a scheme could be adapted to many of the other mentioned accelerators.

A major advantage of facilities with significantly increased muon flux would be the possibility to use novel experimental techniques which could not be exploited so far [74] like, e.g. the use of cw lasers for optical spectroscopy or an 'old muonium' approach for a new generation M-$\overline{\mathrm{M}}$ search. Further, a wider class of muonic atoms would be become accessible for precision spectroscopy [75] beyond the already started laser investigations of muonic hydrogen [76].

Table 1. Muon fluxes of some existing and future facilities. Rutherford Appleton Laboratory (RAL), Japanese Hadron Facility (JHF), European Spallation Source (ESS), Muon collider or neutrino factory (MC)

	RAL(μ^+)	PSI(μ^+)	PSI(μ^-)	JHF(μ^+)	ESS(μ^+)	MC (μ^+, μ^-)
Intensity [μ/s]	3×10^6	3×10^8	1×10^8	4.5×10^{11}	4.5×10^7	7.5×10^{13}
Momentum bite						
Δ p_μ/p[%]	10	10	10	10	10	5-10
Spot size						
[cm × cm]	1.2×2.0	3.3×2.0	3.3×2.0	1.5×2.0	1.5×2.0	few×few
Pulse structure	82 ns	50 MHz	50 MHz	300 ns	300 ns	50 ps
Repetition rate	50 Hz	continuous	continuous	50 Hz	50 Hz	15 Hz

8 Conclusions

Although the nature of the muon - the reason for its existence - still remains a mystery, both the theoretical and experimental work in basic muon physics, have contributed to an improved understanding of basic particle interactions and symmetries in physics. Particularly muonium spectroscopy has verified the behaviour of the muon as a point-like heavy lepton which differs only in its mass related parameters from the others. This fact is fundamentally assumed in every precision calculation within standard theory. In addition, the measurements provide accurate values of fundamental constants.

It is the interplay between particle physics and QED phenomena in the muonium atom which cause increasing understanding of fundamental forces and increasing reliability of extracted fundamental constants. None of both sides could reach significant results without the other. With the significant improvement expected for muon beam rates at various places we can look forward to further insights and maybe hints why there are particle generations.

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References

1. H.U. Martyn: in *Quantum Electrodynamics*, ed. T. Kinoshita, World Scientific, pp. 92–161 (1990); T. Kinoshita and W.J. Marciano, ibid. pp. 419–478
2. V.W. Hughes and G. zu Putlitz: *Quantum Electrodynamics*, ed. T. Kinoshita, World Scientific, pp. 822–904 (1990)
3. K. Jungmann: in *Muon Science*, eds. S.L. Lee, S.H. Kilcoyne and R. Cywinsky, Inst. of Physics Publ., pp. 405–461 (1999) and references therein
4. M.I. Eides, H. Grotch and V.A. Shelyuto: Phys. Rep., to be published (2000), hep-ph/000215 (2000)
5. F. Biraben, T.W. Hänsch et al.: *this book*, pp. 17–41
6. P. Mohr: *this edition*, pp. 145–156
7. G.H. Eaton an S.H. Kilcoyne: *Muon Science*, eds. S.L. Lee, S.H. Kilcoyne and R. Cywinsky, Inst. of Physics Publ., p. 11 (1999)
8. V.W. Hughes et al.: Phys. Rev. Lett. **5**, 63 (1960)
9. W. Liu et al.: Phys. Rev. Lett. **82**, 711 (1999)
10. V.W. Hughes and T. Kinoshita: in *Muon Physics*, ed. V.W. Hughes and C.S. Wu, Academic Press, pp. 11–199 (1977)
11. Y. Myake et al.: Hyperfine Interactions **106**, 237 (1997); A. Matsushita et al., Surf. Sci. **357-358**, 961 (1996); A.P. Mills et al.: Phys. Rev. Lett. **56**, 1464 (1986)
12. K. Woodle et al.: Z. Phys. D **9** 59 (1988); see also: A.C. Janissen et al.: Phys. Rev. A **42**, 161 (1990)
13. Steven Chu et al.: Phys. Rev. Lett. **60**, 101 (1988); see also: K. Danzmann et al.: Phys. Rev. A **39**, 6073 (1989)
14. V. Meyer et al., Phys. Rev. Lett. **84**, 1136 (2000)
15. L. Willmann et al., Phys. Rev. Lett. **82**, 49 (1999)

16. L. Willmann and K. Jungmann: in *Atomic Physics Methods in Modern Research*, eds. K. Jungmann, J. Kowalski, I. Reinhard and F. Träger, Springer, pp. 43–56 (1997)
17. K. Jungmann et al.: Appl. Phys. B **60**, S159 (1995)
18. C.J. Oram et al.: Phys. Rev. Lett. **52**, 910 (1984)
19. A. Badertscher et al.: Phys. Rev. Lett. **52**, 914 (1984) and Phys. Rev. A **41**, 93 (1990)
20. S.H. Kettell: PhD thesis, Yale university (1990)
21. E. Fermi, Z. Phys. **60**, 320 (1930)
22. M.G. Boshier et al.: Phys. Rev. A **52**, 1948 (1995)
23. F.G. Mariam et al.: Phys. Rev. Lett. **49**, 993 (1982)
24. T, Kinoshita and M. Nio: *Frontier Tests of QED and the Physics of the Vacuum*, eds. E. Zavattini, D. Bakalov and C. Rizzo, Heron Press, pp. 151–167 (1998); T. Kinoshita: hep-ph/9808351 (1998); K. Pachucki: Phys. Rev. A **54**, 1994 (1996); S. G. Karshenboim: Z. Phys. D **36**, 11 (1996); S. A. Blundell, K.T. Cheng and J. Sapirstein: Phys. Rev. Lett. **78**, 4914 (1997); M. I. Eides, H. Grotch and V.A. Shelyuto: Phys. Rev. D **58**, 013008 (1998); V. Hund and H. Pilkuhn: J. Phys. B **33**, 1617 (2000)
25. V.W. Hughes and T. Kinoshita: Rev. Mod. Phys. **71**, S133 (1999); see also: T. Kinoshita: IEEE Trans. Instr. Meas. **44**, 498 (1996), IEEE Trans. Instr. Meas. **46**, 108 (1997) and Rep. Prog. Phys. **59**, 1459 (1996)
26. A.M. Jeffrey et al.: IEEE Trans. Instr. Meas. **46**, 264 (1997)
27. E.R. Williams et al.: IEEE Trans. Instr. Meas. **38**, 233 (1989)
28. E. Krüger, W. Nistler and W. Weirauch, IEEE Trans. Instr. Meas. **46**, 101 (1997) and Metrologia **32**, 117 (1995); W. Nistler, priv. com. (1998)
29. S. Chu, 17th International Conference on Atomic Physics, Florence, Italy (2000)
30. R. Bluhm, V.A. Kostelecky and N. Russel: Phys. Rev. D **57**, 3932 (1998)
31. R. Bluhm, V.A. Kostelecky, C.D. Lane: Phys. Rev. Lett. **84**, 1098 (2000)
32. for details see: V.W. Hughes et al.: *this edition*, pp. 397–406
33. K. Jungmann et al.: Z. Phys. D **21**, 241(1991)
34. F.E. Maas et al.: Phys. Lett. **187**, 247 (1994)
35. P. Bakule et al.: Appl. Phys. B **71**, 11 (2000)
36. S.L. Cornish et al.: J. Opt. Soc. Am. B **17**, 6 (2000)
37. V. Yakhontov and K. Jungmann: Z. Phys. D **38**, 141 (1996)
38. V. Yakhontov, R. Santra and K. Jungmann: J. Phys. B **32**, 1615 (1999)
39. A.P. Mills: Hyperfine Interactions **76**, 233 (1993)
40. K. Pachucki et al.: J. Phys. B **29**, 177 (1996); S. G. Karshenboim: Z. Phys. D **39**, 109 (1997) and Can. J. Phys. **77** , 241 (1999); K. Pachucki and S. G. Karshenboim: priv. com. (1999)
41. R.M. Carey et al.: Phys. Rev. Lett. **82** 1632 (1999)
42. H.G. Dehmelt et al.: Phys. Rev. Lett. **83**, 4694 (1999)
43. A. Angelopoulos et al.: Phys. Lett. B **471**, 332 (1999)
44. J. Baily et al.: Nucl. Phys. B **150**,1 (1979)
45. K. Jungmann: Hyperfine Interactions **127**, 189 (2000)
46. Y.B. Zeldovitch: Dan. SSR **86**, 505 (1952)
47. B. Pontecorvo: Sov. Phys. JETP **37**, 1751 (1959) and Sov. Phys. JETP **6**, 381 (1958)
48. N. Cabbibo and R. Gatto: Phys. Rev. Lett. **5**, 114 (1960); N. Cabbibo: Nuovo Cim. **19**, 612 (1961)
49. E.J. Konopinski and H.M. Mahmoud: Phys. Rev.**92**, 1045 (1953)

50. G. Feinberg and S. Weinberg: Phys. Rev. Lett. **6**, 381 (1961)
51. T.D. Lee and C.N. Yang: Phys. Rev. **98**,1501 (1956)
52. A. Halprin: Phys. Rev. Lett. **48**, 1313 (1982)
53. P. Herczeg and R.N. Mohapatra: Phys. Rev. Lett. **69**, 2475 (1992)
54. W.S. Hou and G.G. Wong: Phys. Rev. D **53** 1537 (1996)
55. A. Halprin and A. Massiero: Phys. Rev. D **48**, 2987 (1993)
56. R.N. Mohapatra: Z. Phys. C **56**, S117 (1992)
57. H. Fujii et al.: Phys. Rev. D **49**, 559 (1994)
58. B.E. Matthias et al.: Phys. Rev. Lett. **66**, 2716 (1991)
59. R. Abela et al.: Phys. Rev. Lett. **77**, 1951 (1996)
60. L. Chatterjee et al.: Phys. Rev. D **46**, 46 (1992)
61. A. Czarnecki, G.P. Lepage and W.J. Marciano: Phys. Rev. D **61**, 073001 (2000)
62. K. Horrikawa and K. Sasaki: Phys. Rev. D **53**, 560 (1996)
63. G.G. Wong and W.S. Hou: Phys. Lett. B **357**, 145 (1995)
64. V.A. Gordeev et al.: JETP Lett. **59**, 589 (1994)
65. F. Cuypers and S. Davidson: Eur. Phys. J. C **2**, 503 (1998)
66. G.G. Wong and W.S. Hou: Phys. Rev. D **50**, R2962 (1994)
67. P. Frampton: Phys. Rev. Lett. **69**, 1889 (1994); see also: hep-ph/97112821 (1997)
68. P. Frampton and S. Harada: hep-ph/9711448 (1997) and hep-ph/0002017 (2000)
69. P. Frampton: priv. comm. (1998); V. Pleitez: Phys. Rev. D **61**, 057903 (2000)
70. P. Herczeg: in *Beyond the Desert 1997*, ed. H.V. Klapdor-Kleingrothaus and H. Päs, Inst. of Physics Publishing, pp. 124–133 (1998);
71. C. Athanassopoulos et al.: Phys. Rev. C **54**, 2685 (1996); see also: nucl-ex/9709006
72. R.B. Palmer: in *Handbook of Accelerator Physics and Engineering*, eds. A. Wu Chao and M. Tigner, World Scientific, pp. 33–35 (1999)
73. Y. Kuno: in *Proceedings of the KUICR98 Workshop*, Iji, Kyoto, Japan (1998); see also: in *Proceedings of the HISMUS99 Workshop*, KEK, Tsukuba, Japan (1999), in print
74. D. Kawall et al.: *Proceedings of the Workshop at the First Muon Collider and the Front End of a Muon Collider*, eds. S. Geer and R. Raja, AIP, pp. 486–493 (1998)
75. M.G. Boshier et al.: Comm. At. Mol. Phys. **33**, 17 (1996); K. Jungmann: Z. Phys. C **56**, 59 (1992)
76. R. Pohl et al.: *this edition*, pp. 454–466