Radiative Decay of Coupled States in an External dc Field

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Abstract. This paper examines two theoretical aspects of the interference of atomic states in hydrogen which comes from the application of an external electric field F to the $2s$ metastable state. The radiative corrections to the Bethe-Lamb formula and anisotropy contribution to the angular distribution, which arises from interference between electric-field-induced E1-radiation and forbidden M1-radiation, are analysed.

1 Theory and results

The interaction of hydrogen atoms with time-dependent external fields is a fundamental task of atomic physics with numerous applications. In particular, an atomic interferometer has been used in order to determine the Lamb shift Δ_L(2s-2p) in a hydrogen atom [1,2]. An optimal procedure for analysing the experimental data was adopted after an examination of several possibilities. In this procedure, two measurements had to be carried out during the detection of the flux density of $2p$ atoms emerging from the electric field F. One measurement was to be carried out with this field directed parallel to the velocity of the atoms, and the second with this field reversed. A main condition for determining whether a two-electrode interferometer was operating correctly was that the yield of $2p$ atoms be independent of the sign of the field F under the condition that the hydrogen atoms entered this field in a pure $2s$ state. In other words, the interference curves describing the yield of $2p$ atoms as a function of the field F should coincide even this field is reversed.

As it turned out, however, this independence was observed only at certain strictly determined values of the experimental parameters [3,4]. In general case, the reversal of F could cause a substantial discrepancy between experimental curves [3]. What was the reason for this? Our recent experimental results presented in [5] were interpreted as evidence of hydrogen $2s$ and $2p$ atomic states coherent mixing caused by $2s$ metastable hydrogen atoms passing through a slit cut in a metal plate. An astonishing feature of the zero-electric-field coherent mixing of atomic states is a rather high mixing amplitude observed at an enormous distance "atom-metal surface" on the atomic scale ($\simeq 100$ μm).

The purpose of this paper is to examine a similar coherent state mixing effect of the hydrogen atom in an applied electric field within the framework of atomic interferometer geometry for the Lamb shift measurement. The special attention is given to the several asymmetries in the angular distribution of the emitted radiation depending on the sign of the field F.

These results can be expected to play a role in the analysis of experiments, presented in [3,4,5].

When a hydrogen atom initially in the state $2s_{1/2}$ passes non-adiabatically into a region of atomic interferometer field strength F [1], the perturbed state vector to lowest order is

$$|\Psi(t)\rangle = \sum_n b_n(t)e^{-i\tilde{\varepsilon}_n t}|n\rangle \\ \simeq b_{1s}(t)e^{-i\tilde{\varepsilon}_{1s}t}|1s\rangle + b_{2s}(t)e^{-i\tilde{\varepsilon}_{2s}t}|2s\rangle + b_{2p}(t)e^{-i\tilde{\varepsilon}_{2p}t}|2p\rangle, \tag{1}$$

where $\tilde{\varepsilon}_n = \varepsilon_n - \frac{i}{2}\gamma_n$, γ_n is the decay width. The total Hamiltonian of this system is

$$H = H_0 + H_{int}. \tag{2}$$

H_0 is the unperturbed Hamiltonian and H_{int} is the interaction Hamiltonian

$$H_{int} = Ae^{i\omega t} - FD = Ae^{i\omega t} + V. \tag{3}$$

Here $Ae^{i\omega t}$ represents the interaction of the hydrogen atom with the radiation field, D is the z-component of the electric dipole moment of the atom.

The equations for the time dependence of the amplitudes $b_n(t)$ are

$$i\hbar\dot{b}_{2s} = b_{2p}(t)\langle 2s|V|2p\rangle e^{i(\tilde{\varepsilon}_{2s}-\tilde{\varepsilon}_{2p})t} + b_{1s}(t)e^{i(\tilde{\varepsilon}_{2s}-\tilde{\varepsilon}_{1s})t-i\omega t}\langle 1s|A|2s\rangle^* + i\delta(t), \\ i\hbar\dot{b}_{2p} = b_{1s}(t)\langle 1s|A|2p\rangle^* e^{i(\tilde{\varepsilon}_{2p}-\tilde{\varepsilon}_{1s})t-i\omega t} + b_{2s}(t)e^{i(\tilde{\varepsilon}_{2p}-\tilde{\varepsilon}_{2s})t}\langle 2s|A|2p\rangle, \\ i\hbar\dot{b}_{1s} = b_{2s}(t)\langle 1s|A|2s\rangle e^{i(\varepsilon_{1s}-\tilde{\varepsilon}_{2s})t+i\omega t} + b_{2p}(t)e^{i(\varepsilon_{1s}-\tilde{\varepsilon}_{2p})t+i\omega t}\langle 1s|A|2p\rangle, \tag{4}$$

The last term in the first equation (4) provides the initial non-adiabatical condition at $t = 0$ for $2s$-metastable state of hydrogen atom. The corresponding terms describing the $2p$ to $2s$ spontaneous transitions and the $1s$ Stark effect have been neglected as small.

The simplest way of solving these equations is to Fourier transform the amplitudes b_n:

$$b_n(t) = (-1/2\pi i)\int_{-\infty}^{+\infty} d\,E\,G_n(E)e^{i(\tilde{\varepsilon}_n - E)}\ , \\ i\delta(t) = (-1/2\pi i)\int_{-\infty}^{+\infty} d\,E\,e^{i(\tilde{\varepsilon}_{2s} - E)}\ . \tag{5}$$

Inserting these expressions into (4), we obtain the equations for the new amplitudes $G_n(E)$:

$$G_{2s}(E)(E - \tilde{\varepsilon}_{2s}) = G_{2p}(E)\langle 2s|V|2p\rangle + G_{1s}(E)\langle 1s|A|2s\rangle^* + 1, \\ G_{2p}(E)(E - \tilde{\varepsilon}_{2p}) = G_{1s}(E)\langle 1s|V|2p\rangle^* + G_{2s}(E)\langle 2p|V|2s\rangle, \\ G_{1s}(E)(E - \varepsilon_{1s} - \omega) = G_{2s}(E)\langle 1s|A|2s\rangle + G_{2p}(E)\langle 1s|A|2p\rangle. \tag{6}$$

The solutions for the amplitudes $G_b(E)$ are determined analytically. In order to get the amplitudes $b_n(t)$, we have used these solutions and have applied the theory of residues by countour integration in (5). For example, the resulting expressions for $b_{2s}(t)$ and $b_{2p}(t)$ are:

$$\begin{aligned} b_{2s}(t) &= \frac{e^{i(\tilde{\varepsilon}_{2s}-\tilde{\varepsilon}_{2p})t}}{(\omega_1-\omega_2)}\left[\omega_1 e^{\omega_1 t}-\omega_2 e^{\omega_2 t}\right], \\ b_{2p}(t) &= -i\frac{\langle 2s|V|2p\rangle}{(\omega_1-\omega_2)}\left[e^{\omega_1 t}-e^{\omega_2 t}\right], \end{aligned} \tag{7}$$

where $\omega_{1,2}$ are the roots of the secular equation

$$\omega_{1,2} = -\frac{i(\tilde{\varepsilon}_{2s}+\tilde{\varepsilon}_{2p})}{2} \pm \sqrt{\left|\frac{i(\tilde{\varepsilon}_{2s}-\tilde{\varepsilon}_{2p})}{2}\right|^2 - |\langle 2s|V|2p\rangle|^2}. \tag{8}$$

The probability of the state $|n\rangle$ is proportional to $|b_n(t)|^2$. For example, with this definition the probability of the $2p$-state may be written as

$$|b_n(t)|^2 = \frac{2}{\hbar^2}\frac{|\langle 2s|V|2p\rangle|^2 e^{t(c_1+c_2)}}{[(c_1-c_2)^2+(d_1-d_2)^2]}\left\{\cosh[t(c_1-c_2)]-\cos[t(d_1-d_2)]\right\}. \tag{9}$$

Here $c_n = \mathrm{Re}\,\{\omega_n\}$, $c_n = \mathrm{Im}\,\{\omega_n\}$. The time dependence of the probability of the $2p$-state, which given by formula (9), corresponds directly to the experimental interference curve presented in [3] at condition $t = 2.5\times10^{-9}$ s, and F in the range $0 \div 300$ V/cm.

The line shape of the emitted radiation can be obtained from the solution for $b_{1s}(t)$ and is proportional to $|b_{1s}(+\infty)|^2$:

$$|b_{1s}(+\infty)|^2 = \frac{B}{C}, \tag{10}$$

where

$$\begin{aligned} B &= |\langle 1s|A|2s\rangle|^2\left\{[\omega-(\varepsilon_{1s}-\varepsilon_{2s})+\Delta_L]^2+\frac{1}{4}\gamma_{2p}^2\right\}+|\langle 1s|A|2p\rangle|^2|\langle 2s|V|2p\rangle|^2 \\ &+ 2\langle 2s|V|2p\rangle\left\{[\omega-(\varepsilon_{1s}-\varepsilon_{2s})+\Delta_L]\,\mathrm{Re}[\langle 1s|A|2s\rangle\langle 1s|A|2p\rangle^*]\right. \\ &\left. - \frac{1}{2}\gamma_{2p}\mathrm{Im}[\langle 1s|A|2s\rangle\langle 1s|A|2p\rangle^*]\right\}, \\ C &= [\omega-(\varepsilon_{1s}-\varepsilon_{2s})]^2[\omega-(\varepsilon_{1s}-\varepsilon_{2s})+\Delta_L]^2+\frac{1}{16}\gamma_{2p}^2\gamma_{2s}^2 \\ &+ |\langle 2s|V|2p\rangle|^4+\frac{1}{4}\gamma_{2s}^2[\hbar\omega-(\varepsilon_{1s}-\varepsilon_{2s})+\Delta_L]^2 \\ &- 2|\langle 2s|V|2p\rangle|^2\left\{[\omega-(\varepsilon_{1s}-\varepsilon_{2s})][\omega-(\varepsilon_{1s}-\varepsilon_{2s})+\Delta_L]-\frac{1}{4}\gamma_{2s}\gamma_{2p}\right\}. \end{aligned} \tag{11}$$

The sign electric field dependence in (11) may be defined by anisotropy parameter

$$R = \frac{I^+ - I^-}{I^+ + I^-}, \tag{12}$$

where I^+ and I^- are induced $\mathrm{L_y}$-α intensities emitted with the field directed parallel to the velocity of atomic beam and with a reversed field, respectively. Anisotropy parameter R is linearly proportional to a dipole matrix element $\langle 2s|V|2p\rangle$. Numerical estimation shows, that $R \approx -3 \times 10^{-6}$ for $F = 1\,\mathrm{V/cm}$ and $R \approx -1 \times 10^{-8}$ for $F = 300\,\mathrm{V/cm}$. These results are in good agreement with previous estimations [6,7].

The radiative correction in the lowest order to the emission line shape of metastable hydrogen atom can be written in the form of additional contribution to a numerator in (10)

$$|\langle 2s|V|2p\rangle|^2 \left[a_1\gamma_{2p} + a_2\Delta_L\right] . \tag{13}$$

Here a_1 corresponds to a dispersive correction to the Stark quenching line shape, a_2 represents a small vertex correction (of order α^3) to Stark Hamiltonian. Numerical values for these quantities are -1.85×10^{-9} and 5.13×10^{-8}, respectively. The details of calculations demand a separate examination and will be discussed in our forthcoming work.

References

1. V. G. Pal'chikov, Yu. L. Sokolov, V. P. Yakovlev: Metrologia **21**, 99 (1985)
2. V. G. Pal'chikov, Yu. L. Sokolov, V. P. Yakovlev: Physica Scripta **55**, 33 (1997)
3. Yu. L. Sokolov, V. P. Yakovlev, V. G. Pal'chikov, D. N. Lin: Nuovo Cimento D **14**, 183 (1992)
4. Yu. L. Sokolov, V. P. Yakovlev, V. G. Pal'chikov: Physica Scripta **49**, 86 (1994)
5. Yu. L. Sokolov: Physics-Uspekhi **42**, 481 (1999)
6. P. J. Mohr: Phys. Rev. Lett. **40**, 854 (1978)
7. G. W. F. Drake: Phys. Rev. Lett. **40**, 1705 (1978)