Test of CPT and Lorentz Invariance from Muonium Spectroscopy

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Abstract. Following a suggestion of Kostelecký *et al.* we have evaluated a test of CPT and Lorentz invariance from the microwave spectrosopy of muonium. Precise measurements have been reported for the transition frequencies ν_{12} and ν_{34} for ground state muonium in a magnetic field H of 1.7 T, both of which involve principally muon spin flip. These frequencies depend on both the hyperfine interaction and Zeeman effect. Hamiltonian terms beyond the standard model which violate CPT and Lorentz invariance would contribute shifts $\delta\nu_{12}$ and $\delta\nu_{34}$. The nonstandard theory indicates that ν_{12} and ν_{34} should oscillate with the earth's sidereal frequency and that $\delta\nu_{12}$ and $\delta\nu_{34}$ would be anticorrelated. We find no time dependence in $\nu_{12} - \nu_{34}$ at the level of 20 Hz, which is used to set an upper limit on the size of CPT and Lorentz violating parameters.

Introduction

Much current theoretical work is devoted to finding a more fundamental and general underlying theory from which the standard model of particle physics could be deduced as the low energy limit. String theory and the inclusion of the gravitational interaction are central viewpoints, and in such theories particles may have structure and the CPT theorem may be violated.

Some years ago Kostelecký and coworkers [1,2,3] developed an extension of the standard model based on spontaneous breaking of CPT and Lorentz symmetry in an underlying theory. The analysis is done in the context of conventional relativistic quantum mechanics and quantum field theory in four dimensions retaining the usual gauge structure and renormalizability. The goal is to develop within an effective theory approach a plausible Lorentz and CPT violating extension of the standard model that provides a theoretical basis for establishing quantitative bounds on CPT invariance. Any CPT and Lorentz violation of the standard model must be highly suppressed to remain compatible with established experimental bounds. An appropriate dimensionless suppression factor is the ratio of the low-energy (standard model) scale to the Planck scale of the underlying theory, or $m_\mu/M_P \sim 10^{-21}$.

As an extension of QED considering only photons, electrons (±), and muons (±), the following CPT violating Lorentz-violating terms are added to the standard QED Lagrangian

$$\mathcal{L}_e^{\rm CPT} = -a_\nu^e \bar{\psi}_e \gamma^\nu \psi_e - b_\nu^e \bar{\psi}_e \gamma_5 \gamma^\nu \psi_e, \quad (1)$$

$$\mathcal{L}_{\mu}^{\mathrm{CPT}} = -a_{\nu}^{\mu}\bar{\psi}_{\mu}\gamma^{\nu}\psi_{\mu} - b_{\nu}^{\mu}\bar{\psi}_{\mu}\gamma_{5}\gamma^{\nu}\psi_{\mu}, \tag{2}$$

$$\mathcal{L}_{\gamma}^{\mathrm{CPT}} = \frac{1}{2}\left(k_{AF}\right)^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu}. \tag{3}$$

The notation is defined in ref.[2], where other, CPT even Lorentz-violating Lagrangian terms are also given. High precision experiments on muonium (M) can measure or set limits on the parameters of these symmetry violating terms, which are sensitive at the Planck scale level [4].

In this paper we consider the energy level diagram of the ground state of muonium in a magnetic field (Fig. 1). The transition frequencies ν_{12} and ν_{34} have been measured [5] with high precision and used to determine the hyperfine structure interval $\Delta\nu$ and the ratio μ_μ/μ_p of the muon magnetic moment to the proton magnetic moment. As applied to muonium the theory uses a modified

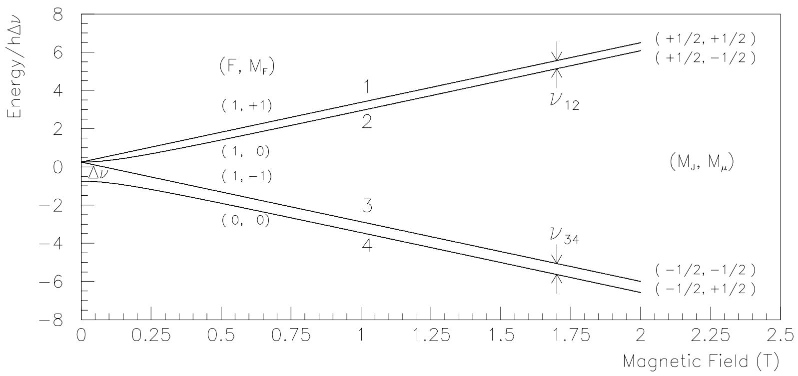

Fig. 1. Breit-Rabi levels

Dirac equation. Leading-order Lorentz violating energy shifts $\delta\nu_{12}$ and $\delta\nu_{34}$ can be obtained from a Hamiltonian using perturbation theory and relativistic two-fermion techniques. For our observed transitions at the strong magnetic field of 1.7 T, dominantly only muon spin flip occurs so the energy shifts are characterized by the muon parameters alone of the extended theory. The results of this approach are [4]:

$$\delta\nu_{12} \;\approx\; -\,\delta\nu_{34} \;\approx\; \tilde{b}_3^{\mu}/\pi, \tag{4}$$

where $\tilde{b}_3^{\mu} \equiv b_3^{\mu} + d_{30}^{\mu}m_{\mu} + H_{12}^{\mu}$. These laboratory-frame parameters all violate Lorentz invariance, and in addition, b_3^{μ} is CPT odd, while d_{30}^{μ} and H_{12}^{μ} are CPT even.

Predictions of the values of ν_{12} and ν_{34} from standard theory - dominantly the QED terms - requires values for many atomic constants including m_μ, μ_μ, and $\Delta\nu$ as well as the calculation of higher order QED radiative corrections. The relevant constants and calculations are not known to as high accuracy as the experimental determinations of ν_{12} and ν_{34}. Indeed several of these constants - $\Delta\nu$ and μ_μ/μ_p - are obtained from the muonium experiment. Hence accurate predictions for ν_{12} and ν_{34} can not be obtained by this approach, and the sensitivity to the non-standard model energy shifts $\delta\nu_{ij}$ (ij = 12 or 34) is small.

However, the theory with CPT and Lorentz violation involves spatial components in a celestial frame of reference, and since the laboratory rotates with the earth, these spatial components vary with time, and consequently the experimentally observed ν_{12} and ν_{34} may oscillate about a mean value at the earth's sidereal frequency $\Omega = 2\pi/23$ hr 56 m with amplitudes $\delta\nu_{12}$ and $\delta\nu_{34}$. No such signal would be obtained from the standard model. In the non-rotating celestial frame of reference with equatorial axes $\left\{\hat{X},\hat{Y},\hat{Z}\right\}$ where $\hat{Z}$ is oriented along the earth's rotational North Pole, an experimental constraint on $\delta\nu_{12}$ implies [4]

$$\frac{1}{\pi}|\sin\chi|\sqrt{\left(\tilde{b}_X^\mu\right)^2+\left(\tilde{b}_Y^\mu\right)^2}\leq\delta\nu_{12} \tag{5}$$

in which $\chi\sim 90°$ is the angle between $\hat{Z}$ and the quantization axis defined by the laboratory magnetic field at Los Alamos where the muonium experiment was done. The transformation from the lab frame quantity $\tilde{b}_3^\mu$ to the celestial frame quantities $\tilde{b}_J^\mu$ (where $J = X, Y, Z$) is given by

$$\tilde{b}_3^\mu \;=\; \tilde{b}_Z^\mu\cos\chi \;+\; \left(\tilde{b}_X^\mu\cos\Omega t \;+\; \tilde{b}_Y^\mu\sin\Omega t\right)\sin\chi .$$

Analysis of Muonium Spectroscopy Data

The accurate measurements of ν_{12} and ν_{34} were done in a microwave magnetic resonance experiment [5]. Resonance lines were observed by varying the magnetic field with fixed microwave frequency and by varying the microwave frequency with fixed magnetic field. A line narrowing technique was used involving observation of a transition signal only from M atoms which have lived considerably longer than $\tau_\mu \sim 2.2\ \mu$s (Fig. 2). The values reported for ν_{12} and ν_{34} at a magnetic field strength corresponding to a free proton precession frequency of 72.320 000 MHz were

$$\nu_{12}(\text{exp}) = 1\ 897\ 539\ 800\ (35)\ \text{Hz}\ (18\ \text{ppb}), \tag{6}$$
$$\nu_{34}(\text{exp}) = 2\ 565\ 762\ 965\ (43)\ \text{Hz}\ (17\ \text{ppb}) \tag{7}$$

in which one standard deviation errors include both statistical and systematic errors. To search for a time dependence of ν_{12} and ν_{34}, we have employed the following algorithm. Data from each resonance line run (each lasting about half an hour) are fit at the measured magnetic field strength and Kr pressure to determine provisional line centers for ν_{12} and ν_{34}. These line centers were then transformed to their values in a magnetic field strength corresponding to a free proton precession frequency of 72.320 000 MHz. The data were taken at Kr

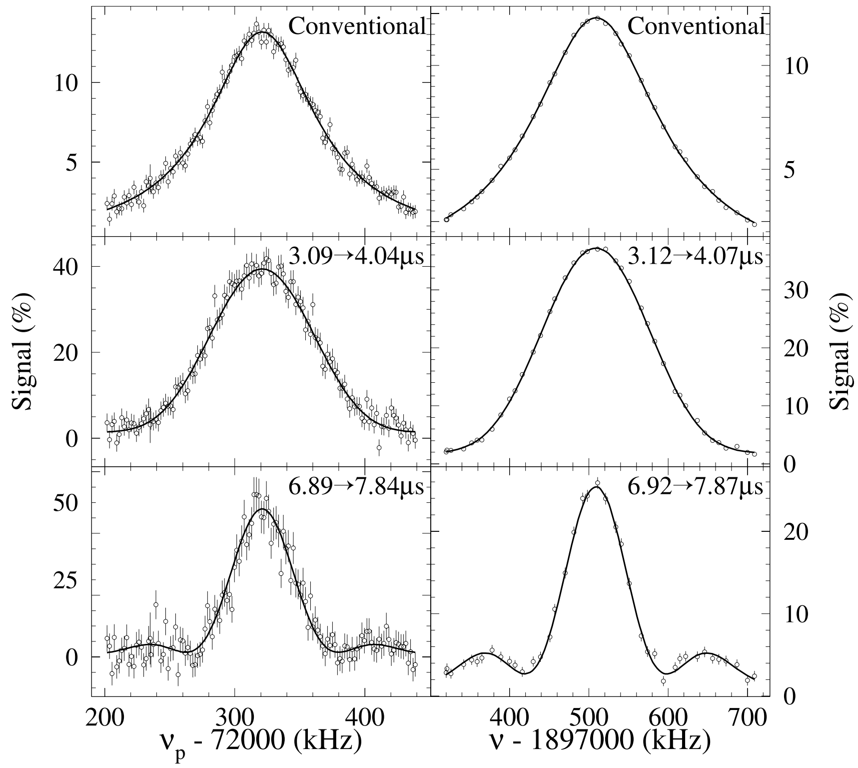

Fig. 2. Resonance lines

pressures of 0.8 and 1.5 atm, so the line centers were corrected for a small quadratic pressure shift, and then were extrapolated linearly to their values at zero pressure, using a pressure shift coefficient determined from the data. Line centers for ν_{12} and ν_{34} obtained in this way were then grouped as a function of sidereal time, where time zero has been set as the time in 1995 when we obtained our first data.

Non-zero values for $\delta\nu_{12}$ and $\delta\nu_{34}$ could arise from systematic effects which lead to variations in the parameters affecting the line centers - particularly variations having a period of $\approx$ 24 hr. Principal concerns are possible day-night variations of the magnetic field strength, and of the density and temperature of the Kr gas.

Day-night variations of several °C in the temperature of the experimental hall lead to oscillations in the magnetic field strength of the persistent-mode superconducting solenoid of ≤ 1 ppm. Changes of 0.05 ppm in the field strength were easily resolved, and the oscillation's effects on the line centers were accounted for in extracting the line centers. Temperature changes also affected the diamagnetic shielding constant of the water in the NMR probes used to monitor the field (the probes were not temperature-stabilized, but were in good thermal contact with the microwave cavity which was temperature-stabilized to 0.1 °C). The maximum conceivable 2 °C day-night changes in water temperature would change the NMR frequencies by 0.02 ppm, leading to errors in the line centers of about 2.5 Hz; of opposite sign for ν_{12} and ν_{34}. This potential effect is well below the statistical sensitivity for sidereal variations of 12 to 15 Hz.

The effect on these data of the variation of Kr pressure with time has been evaluated. The front end window to the Kr stopping target was of 3 mil mylar, and flexed with day-night variations of the external atmospheric pressure. This induced fractional day-night oscillations in the Kr gas target pressure which were measured to be about 2.5×10^{-4} (see Fig. 3). Through pressure shift coefficients of about -16.5 kHz/atm for ν_{12} and -19.5 kHz/atm for ν_{34}, the resulting shifts in the line centers (typically 7.5 Hz in ν_{34} and 6 Hz in ν_{12}) were automatically accounted for in performing the linear extrapolation to zero pressure, and should not contribute any significant time variation to ν_{ij}. The pressure shift coefficients

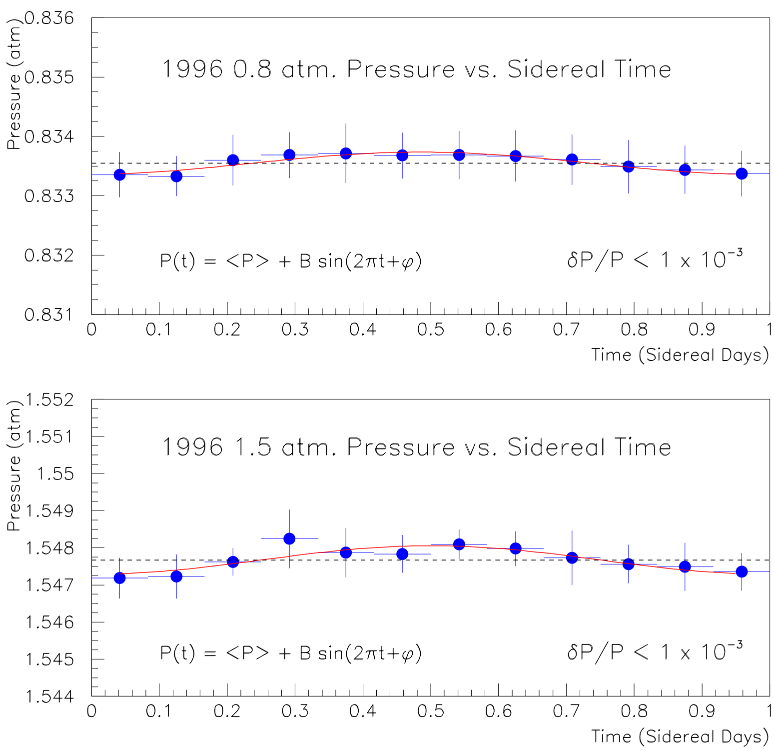

Fig. 3. Kr gas pressure versus time

depend on the average velocities of the atoms, and so are functions of temperature. The fractional changes in the transition frequencies with temperature (measured in hydrogen and its isotopes[6]) are roughly 1×10^{-11} °C^{-1}Torr^{-1}. Given the temperature stability of the Kr gas of about 0.1 °C, temperature dependent errors in the extrapolation of the line centers to their vacuum values would be limited to a few Hz, well below the statistical sensitivity of our test.

Other potential concerns involve the two frequency references used in the experiment - the proton precession frequency forming the basis of the magnetic field determination, and the Loran-C 10 MHz frequency reference used for the NMR and microwave frequency synthesizers. The Loran-C standard is based on hyperfine transitions in Cs with m_F=0, and so is insensitive to any preferred spatial orientation, and would not introduce a signature for Lorentz violation into the spectroscopic measurements. Bounds on clock comparisons of ^{199}Hg and ^{133}Cs [7,8] place crude limits on the Lorentz violating energy shifts in the precession frequency of a proton of 10^{-27} GeV, which imply the NMR measurements are free of shifts well below the Hz level.

Results

All the data obtained in 1995 and 1996 are plotted as a function of time measured as a function of a sidereal day in Fig. 4, where twelve points at $\approx$ 2 hr. intervals are plotted, and the vertical scale is in Hz. The data for ν_{12} and ν_{34} were fit by the functions

$$\nu_{ij}(t) = \langle \nu_{ij} \rangle + \delta\nu_{ij} \sin\left(2\pi t + \phi_{ij}\right)$$

where t is the time in sidereal days, $\langle \nu_{ij} \rangle$ is a constant, and $\delta\nu_{ij}$ is the amplitude of the possible time variation. The values for $\delta\nu_{12}$ and $\delta\nu_{34}$ are consistent with zero within the errors of 12 to 15 Hz.

As stated above, the theory being tested requires $\delta\nu_{12} \approx -\delta\nu_{34}$. A plot of $\nu_{12} - \nu_{34}$ is shown in Figure 5, where a common phase is assumed between ν_{12} and ν_{34}. The data exhibit no variation with time within $\pm$ 20 Hz, which corresponds to $\tilde{b}_3^\mu \leq 2 \times 10^{-23}$ GeV at the one sigma level (see Eqn. 4). The figure of merit of these results as a test of CPT violation is taken as $2\pi \mid \delta\nu_{12} \mid / m_\mu < 5 \times 10^{-22}$ with similar values arriving from $\delta\nu_{34}$ and $\delta(\nu_{34} - \nu_{12})$. The viewpoint justifying this choice of figure of merit is that the standard model takes particle masses as input parameters. However, once given the muon mass the values of all other observables for muonium are calculable. A non-zero value for $\delta\nu_{12}$ would violate the standard model. Hence the figure of merit chosen is a reasonable one.

Other Tests of CPT

The result from muonium can be compared with some other tests of CPT violation based on this theory (Table 1). Another type of experiment listed in Table 1 is the search for a time variation during a sidereal day of a nuclear magnetic resonance signal[9]. The first such experiment was done on ^{7}Li in which the

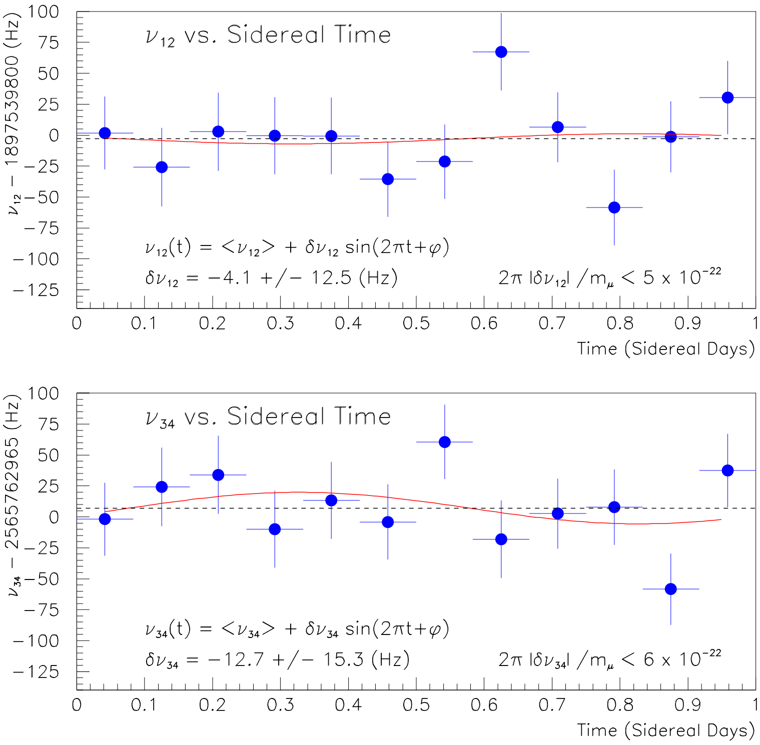

Fig. 4. ν_{12} and ν_{34} versus time

Table 1. Limits from other tests of CPT

Figure of Merit	Value	Reference
$\frac{m(K_0)-m(\bar{K}_0)}{m(K_0)}$	$< \ 10^{-18}$	[10]
$\left(\omega_a^{e^-} - \omega_a^{e^+}\right)/2m_e$	$1.2 \ \times 10^{-21}$	[11]
$\left(\omega_a^{\mu^-} - \omega_a^{\mu^+}\right)/m_\mu$	$2 \ \times 10^{-22}$	[4]
$\frac{2\pi\delta\nu_{12}}{m_\mu}$	5×10^{-22}	(This paper)
$\frac{\delta\nu_{\mathrm{NMR}}(^7\mathrm{Li})}{m_p}$	10^{-20}	[9]
$\frac{\delta\nu_{\mathrm{NMR}}(^9\mathrm{Be})}{m_p}$	3×10^{-23}	[8]

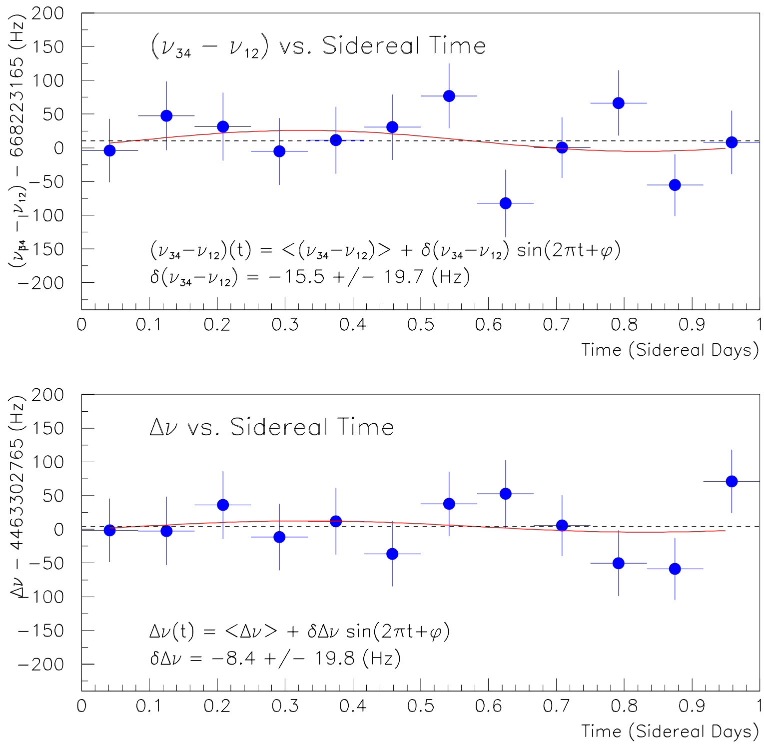

Fig. 5. $\nu_{34} - \nu_{12}$ versus time

NMR frequency of ^{7}Li was measured relative to the hyperfine transition in the ground state of hydrogen to search for a diurnal variation of the ratio of these

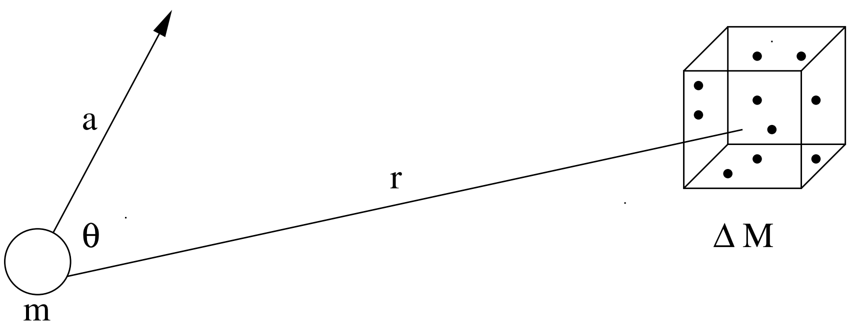

Fig. 6. Body of inertial mass m accelerated relative to a bit of distant matter of mass ΔM

two frequencies. The motivation for the experiment was to test Mach's principle. According to Mach's principle inertial mass of a particle arises from distant matter, and if the distribution of distant matter is anisotropic, there should be a tensor component of mass, *i.e.* $m = m_0 + \Delta m$. Hence Δm would depend on the direction of acceleration of the particle, as indicated in Fig. 6. The ^{7}Li nucleus has spin $J = 3/2$ and according to the nuclear shell model the outer valence proton is in the $P_{3/2}$ state with $L = 1$ and $S = 1/2$. For the $P_{3/2}$ proton an NMR transition with $\Delta m_J = \pm 1$ will have $\Delta m_L = \pm 1$ which implies a change in spatial distribution of the P-state as shown in Fig. 7. According to Mach's

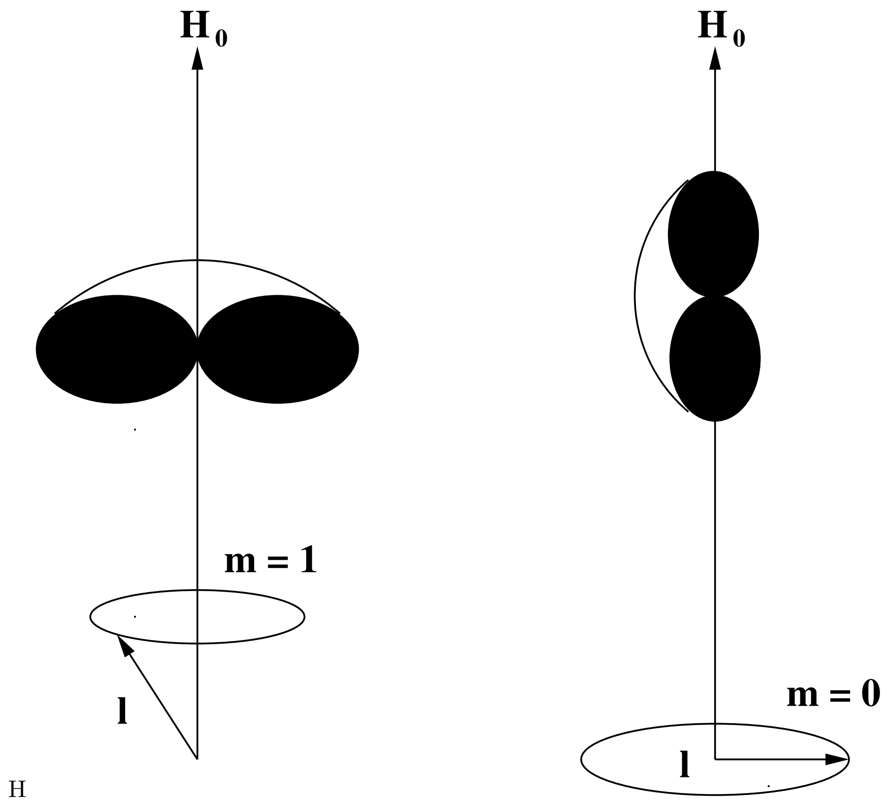

Fig. 7. Spatial distribution of a p state ($l = 1$) in different magnetic substates m relative to an axis in the direction of an external field H_0

principle Δm would be different in the initial and final states and this mass difference contributes to the transition energy or transition frequency. An enormous energy is associated with a mass change as compared to the energy associated with a change in orientation of the nuclear magnetic dipole moment with respect to a magnetic field. Hence the transition frequency would be very sensitive to a change in Δm. Because the earth rotates with respect to a celestial frame of reference the observed NMR transition frequency would vary over a sidereal day.

No variation with time was observed and we concluded $\Delta m/m < 5 \times 10^{-23}$. Also, a limit on the quadrupole term in the mass tensor was evaluated.

In the discussion of Kostelecký *et al.* on testing for Lorentz and CPT invariance using their extension of the standard model, the experiment on ^{7}Li is designated as a clock comparison type experiment which tests for spatial anisotropy. Recent experiments of this type have greatly improved the bounds on parameters for Lorentz violation to $\simeq 10^{-27}$ GeV for the proton (as indicated in Table 1), to $\simeq 10^{-30}$ GeV for the neutron.

In conclusion no violation of CPT or Lorentz invariance has been observed to date. Higher sensitivity tests as well as tests on different physical systems will surely be made [3].

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