Charged Pion Mass Determination and Energy – Calibration Standards Based on Pionic X-ray Transitions

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Abstract. Recent experiments are aiming at an accuracy of 1 ppm for the mass of the charged pion using the characteristic X-rays from exotic atoms. Once the pion mass is established with that precision, the narrow lines from medium Z pionic atoms can be used as a calibration standard in the few keV range. The precision of this new standard is not limited by the large natural line width of fluorescence X-rays and their complex structure due to multi-hole excitations.

1 Introduction

For X-rays in the few keV range, the energy determination to the ppm level or better is difficult because of the absence of calibration standards. Narrow and intense γ lines from nuclear decay are not available and fluorescence X-rays have a large natural line width owing to fast Auger transitions. Multiple-hole excitation leads to a complex line shape which does, in general, make it complicated to unambiguously relate the center of gravity of the diagram line to a wave length. Furthermore, the creation of vacancies depends on the excitation mechanism itself and strong chemical shifts have to be considered even for K X-rays in low Z elements.

Most of the tabulated values for the X-ray energies trace back to measurements older than 40 years or are, in several cases, obtained from interpolation of neighboring elements [1]. The errors given for the K-transition energies range from a few ppm up to about 50 ppm. For this reason, a redetermination is going on with various methods [2,3].

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Precise values for the X-ray energies are obtained by diffraction methods using Bragg's law $n\lambda = 2d \cdot sin\Theta_B$, where the distance d of lattice planes of a crystal is compared to a wave length λ by measuring the Bragg angle Θ_B. n is the order of reflection. Both absolute measurements of the Bragg angle and relative measurements to known standards were performed.

Several problems arise – besides geometrical aberration – when calibration standards used at higher energies are related to the low-energy range. Different orders of reflection and, therefore, different crystal planes are involved, and for reflection-type spectrometers the correction due to the wave-length dependent index of refraction is the subject of a controversy discussion. Finally, the measured wave length has to be converted to energy by $E = hc/\lambda$. The conversion constant hc was readjusted recently and its precision is now reported to be 0.04 ppm [4].

Crystal lattice spacings are less well known than usually believed. In addition, large variations occur for naturally grown crystals like quartz ($\Delta d/d \approx 5 \cdot 10^{-5}$), which is the only choice for precision spectroscopy at energies below 2.3 keV. Great progress has been made by a method that combines optical and X-ray interferometry. At present, high-precision results are available only for silicon. The accuracy reached is $\Delta d/d = 3 \cdot 10^{-8}$ [5]. Detailed investigations relating the optical standard to X-ray lines are rare except for copper K radiation [6].

2 Calibration using exotic atom X-rays

A different method became available with modern meson factories, where the characteristic X-radiation from exotic atoms can be studied under optimized conditions and with reasonable count rates. Such experiments require the use of high-intensity external beam lines together with a particle concentrator like the cyclotron trap and a high-resolution low-energy crystal spectrometer.

Exotic atoms are produced by stopping a beam of negatively charged particles like muons, pions, or antiprotons in a target, where they are captured in the Coulomb potential of the atoms at high principal quantum numbers n. These systems deexcite mainly by fast Auger emission of electrons in the upper part of the atomic cascade and more and more by X-radiation for lower-lying states.

Studies during the last 20 years proved that exotic atoms become hydrogen-like systems, i. e. without any remaining electrons, when prepared under suitable conditions [7]. Light and medium Z atoms completely emit their electron shell and, when produced in dilute targets, do not capture electrons from the environment during their life time. Consequently, no satellite transitions occur.

For neon it has been found that at pressures below 500 mbar all electrons are emitted when a muon reaches the $n \leq 8$ state [8]. In the case of antiprotons, because of their larger mass, complete ionization has been demonstrated also in argon and in krypton for at least $n \leq 14$ [9].

In the range $Z \approx 2 - 10$ and $n \approx 4 - 8$, there is a window where in pionic and muonic atoms electrons are completely stripped off and the transition energies are accessible by X-ray spectroscopy. For the atomic states with maximum angular momentum $l = n - 1$, where most of the intensity is collected in isolated exotic systems, finite-size effects can be neglected. Therefore, in the case of pions, the level energies are not affected by the strong interaction and hence, binding energies can be calculated with a precision of 1 ppm or better including all relevant QED contributions [10]. In addition, the natural line widths – given exclusively by the radiative transitions – are of the order of 10 meV or less, i. e. much smaller than the resolution achievable by experiment.

Modern reflection-type crystal spectrometers reach resolutions of $\approx 10^{-4}$ in the few keV range, i. e., the center of gravity can be determined to a precision of 10^{-6} [11,12,13]. The energies are obtained from the change of the Bragg angle relative to a calibration transition that lies as close as possible. The determination of the energy difference – again – needs the lattice spacing and the conversion constant, but their uncertainties do not contribute significantly to the error for small angle differences. Furthermore, corrections owing to aberration, penetration depth into the crystal, and index of refraction cancel out in leading order.

3 Experimental set-up

The drawback of a high-resolution crystal spectrometer is a rather low efficiency. Therefore, a suitable preparation of the X-ray source is mandatory as are high line yields of the exotic atom transitions by using gas targets with pressures as low as possible.

In order to produce a bright but concentrated X-ray source in dilute targets, beams have to be stopped in a special device – the cyclotron trap [14]. The basic idea of the cyclotron trap is to wind up the range curve in a weakly-focusing magnetic field $\boldsymbol{B}$ (Fig. 1). In the case of pions, deceleration has to be fast because of the short life time. In such experiments, the energy loss of the particles is achieved by degraders and a gas-filled target container made of thin Kapton foils.

At the $\pi E5$ beam of the Paul-Scherrer-Institut (PSI), about 2% of the incoming pions ($\geq 10^9/s$) are stopped in the gas cell with a degrader set-up optimized for pionic atoms. Muons originating from pions decaying shortly before capture are slow enough to be stopped in the gas cell as well. With a set-up optimized for muons, the count rate for muonic atoms is about 4% of the one for pions.

Johann-type crystal spectrometers record an energy interval according to the size of the source simultaneously, if a position-sensitive X-ray detector of corresponding extension is available. The use of spherically-bent crystals allows partial vertical focusing, which increases the count rate by a factor of almost two for Bragg angles around 55°.

For X-ray detection, Charge-Coupled Devices (CCDs) are used. Being pixel detectors, they have a built-in two-dimensional position resolution and an energy resolution even better than conventional semiconductor detectors. They allow an efficient background reduction by analyzing the hit pattern and simultaneously applying a narrow cut on the deposited charge. Such an analysis is essential for low-rate experiments in accelerator environments [13].

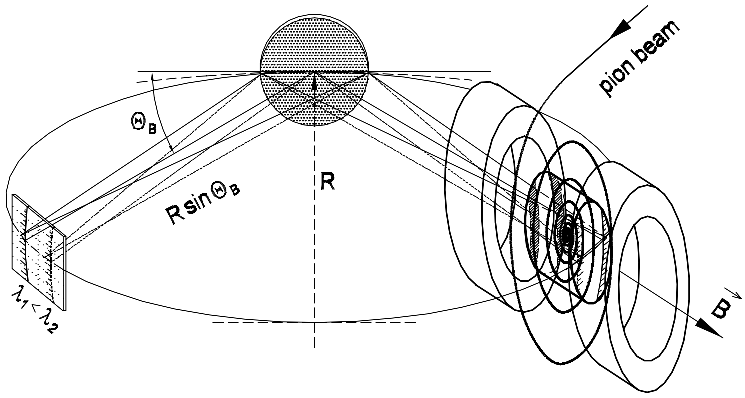

Fig. 1. Principle of the set-up for the pion mass experiment consisting of the cyclotron trap with a gas cell and a curved-crystal spectrometer equipped with a two-dimensional position-sensitive X-ray detector. The focusing condition for a wave length λ reflected under the Bragg angle Θ_B is $R \cdot sin\Theta_B$, where R is the (horizontal) radius of curvature. Typical values for R and the diameter of the spherically bent crystals are $3\,m$ and $100\,mm$

In a set-up optimized for pions, up to 450 events per hour were recorded from the $\pi N(5g-4f)$ transition of 4 keV at an accelerator current of 1.5 mA. In the muon set-up, 15-20 events per hour were achieved for the $\mu O\ (5g-4f)$ transition. The gas pressure was 1.4 bar for both N_2 and O_2. Length and diameter of the gas cell were up to $220\,mm$ and $60\,mm$.

4 Charged Pion Mass

The experiments aiming at the determination of the charged pion mass use X-ray transitions from the intermediate part of the atomic cascade, where the influence of the hadronic interaction can be neglected. The two most recent experiments both measured the $(5g-4f)$ transitions, but from different elements. In one case, a solid state Mg target was taken [15,16], whereas the most recent experiment used an N_2 gas target [12,17].

The recent high-precision measurement of the pion mass was performed in two steps:

- For a high-statistics measurement of the $\pi N(5g-4f)$ transition, the energy calibration was obtained from Cu Kα_1 X-rays. With a description of the Cu Kα line shape, adopted from [18], an accuracy of 4 ppm was reached [12]. The result resolved a 16 ppm ambiguity from the former precision experiment of the $\pi Mg(5g-4f)$ transition [15,16], which stemmed from the unknown K-electron population. The error of the present world average for m_π is close to 3 ppm [4], a value which is consistent with a lower limit for the pion mass derived from an experiments measuring the mass limit for the muon neutrino [19].

- In a second step, the energy calibration is taken from the $\mu O(5g-4f)$ transition using the fact that the mass of the positively charged muon is known to an accuracy of 0.32 ppm [4] together with CPT invariance. To avoid any systematic errors from a change of the set-up, pionic and muonic transitions were measured simultaneously with a new large-area CCD array together with the new cyclotron trap in spring 2000. An O_2/N_2 gas mixture of 90%/10% at 1.4 bar was used in order to achieve equal count rates for the pionic and muonic lines (about 15 per hour each) (Fig. 2). The total statistics accumulated is sufficient to reach the 1 ppm level. The analysis is going on.

5 Energy calibration of fluorescence X-rays

As a first application of narrow exotic-atom transitions to the energy calibration of fluorescence X-rays, preliminary results for scandium and titanium are presented.

- The calibration for the Kα line from metallic scandium was obtained from the $\pi\,^{14}N(5g-4f)$ transition (Fig. 3). The accuracy for the transition energy could be improved from 48 ppm [1] to about 5 ppm. The pronounced satellite structure requires a detailed investigation of the line shape [20].
- In the case of titanium, the $\pi\,^{20}Ne(6h-5g)$ transition energy exactly meets the gap between the Kα_1 and Kα_2 transition (Fig. 4 and 5). The remaining uncertainty for the energy stems mostly from the systematic errors due to the models for the Ti line shape. The accuracy for the transition energy was improved from 11 ppm [1] to 4 ppm.
  The $\pi\,^{20}Ne(6h-5g)$ transition is an ideal case for a calibration line, because no Doppler broadening occurs from Coulomb deexcitation for the noble gas Ne as is the case for diatomic molecules like N_2. Therefore, the line shape reflects exclusively the response of the spectrometer. The resolution achieved is 26" (seconds of arc), which is close to the theoretical limit of 22" for the chosen geometry. The line width of the $\pi N(5g-4f)$ transition, measured to 50", is dominated by Coulomb deexcitation [21].

6 Outlook

Having once established a precise value for the pion mass, the X-ray transitions from pionic atoms can be used to produce a set of calibration lines in the few keV range.

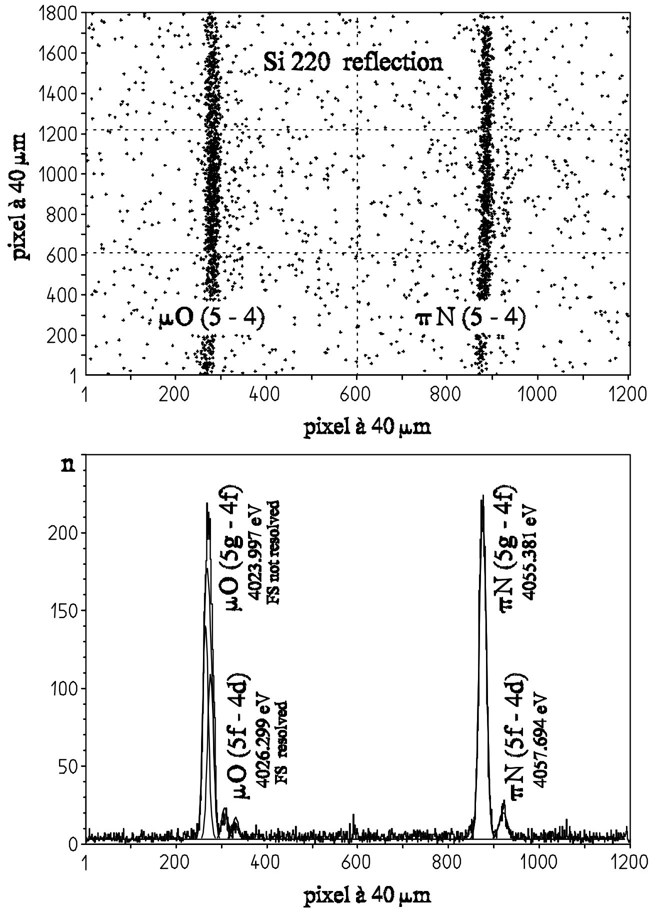

Fig. 2. Reflections of the πN and μO $(5-4)$ transitions recorded simultaneously with a large-area CCD array. (The horizontal and vertical extensions of the CCD array are not in scale for the scatter plot)

A possibility to extend this set comes from the use of an Electron-Cyclotron-Resonance-Ion-Trap (ECRIT), which will be realized using the cyclotron trap itself [22]. Here, hydrogen-like electronic atoms will be produced to obtain narrow calibration lines independent of an accelerator's pion beam. The radiative widths of light elements with $Z \approx 15$ are of the order of a few 10 meV because of the absence of non-radiative inner-shell transitions.

The two sets may be combined by a relative measurement of pairs of transitions belonging to one set each. A first application of this new calibration method will be a precision determination of the hadronic shift and broadening of the ground state in pionic hydrogen [23]. Further possibilities are studies of

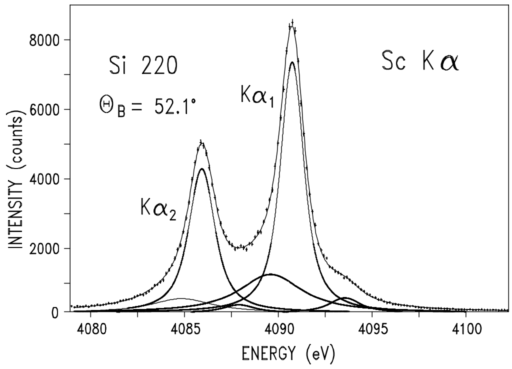

Fig. 3. Scandium Kα doublet

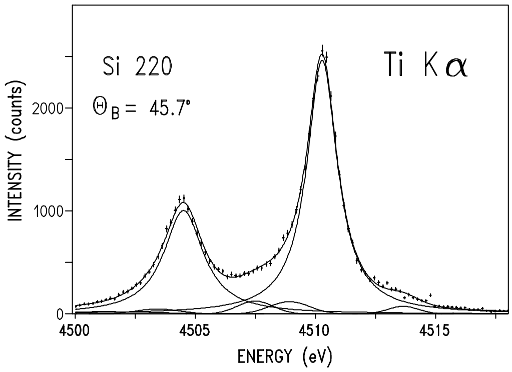

Fig. 4. Titanium Kα doublet

one- and few-electron systems [22] and of the properties of the plasma in the ECRIT itself.

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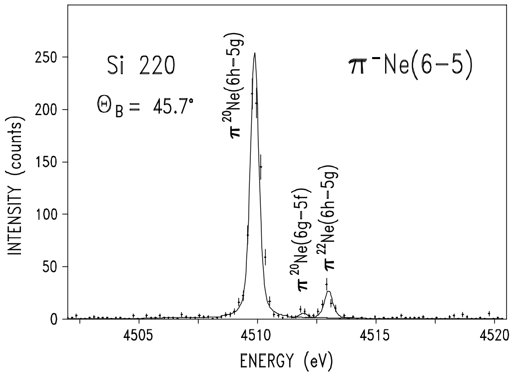

Fig. 5. $\pi Ne(6-5)$ transitions. The line shape is identical with the spectrometer response function because of the small radiative width of 10 meV. A line width of 26" or 550 meV is achieved for a silicon crystal of 95 mm in diameter

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