Antiprotonic Helium "Atomcule": Relativistic and QED Effects

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Abstract. We present theoretical calculations for the $(36,35) \to (34,33)$ transition between metastable states in the antiprotonic helium ${}^4\mathrm{He}^+\bar{p}$, which is supposed to be measured in the two-photon high-precision spectroscopy experiment at CERN.

1 Introduction

Metastable states of an exotic atom $\mathrm{He}^+\bar{p}$, where one of the electrons of the usual helium atom is replaced by an antiproton, were of considerable interest in the last few years. After first observation at KEK of the delayed annihilation phenomena, when about 3.6% of antiprotons injected into the helium target [1] survived as long as a few microseconds, a series of spectroscopic measurements of some transition lines both in ^{4}He and ^{3}He atoms has been performed at CERN [2,3]. It was expected that such longevity could be explained by the stability model suggested by Condo [4]. According to this hypothesis antiprotons that occupy nearly circular orbits (with $n \sim 40$) decay by slow radiative transitions only. Further theoretical calculations of the transition energies [5] that brought agreement between theory and experiment to about 5–10 ppm have rigorously confirmed the Condo model.

It is convenient to compare the life-time of these states that is about $\sim 2 \times 10^{-6}$ s with the life-time of those which are more familiar and are known from textbooks, such as the helium 2^3P state, 9.9×10^{-8} s, and the hydrogen $2P$ state, 1.6×10^{-9} s.

2 Relativistic corrections. Radiative corrections. Asymptotic expansion in terms of α

The simplest and the most common way to evaluate relativistic corrections is to calculate the lowest order corrections which are given by the expectation values of the Breit interaction [6]. However, this approach is limited by the accuracy of about $\alpha^4 \approx 0.3 \cdot 10^{-8}$, which is insufficient for our final goal.

To get relativistic corrections of higher order in α we can consider generalization of the Grotch-Yennie equation [7] to the three-body case. Velocity of heavy particles is several orders of magnitude smaller ($v_h^2 \ll v_e^2$), thus the α^2 order relativistic corrections for heavy particles would be enough for the theoretical consideration.

$$\left(\alpha\frac{v_{\bar{p}}}{v_e}\right)^4 \approx (\alpha/40)^4 \approx 1.1\cdot 10^{-15}.$$

On the other hand, inclusion of the first order relativistic correction to the wave function provides the accuracy of $\alpha^6 \approx 1.5\cdot 10^{-11}$. The numerical implementation of the Grotch-Yennie approach is in progress.

The displacement of energy due to the one-loop QED corrections for the bound electron is [8,9] ($\mathbf{r}_i$, $i = 1, 2$ are position vectors of electron with respect to helium nucleus and antiproton)

$$\begin{aligned}\Delta E_{LS}(\text{one-loop}) = \frac{4Z_i\alpha^3}{3}\langle n|\delta(\mathbf{r}_i)|n\rangle 2\text{Ry}\Biggl\{&\left[\ln\frac{1}{\alpha^2} - \ln\frac{k_0(n)}{\text{Ry}} + \frac{5}{6} - \frac{3}{8}\right] \\ &+(Z_i\alpha)3\pi\left(\frac{139}{128} - \frac{1}{2}\ln 2\right) - \frac{3}{4}(Z_i\alpha)^2\ln^2\frac{1}{(Z_i\alpha)^2} \\ &+\left[-\frac{1}{5} + (Z_i\alpha)\pi\frac{5}{64}\right]\Biggr\} - \frac{\alpha}{2\pi}\left[i\,\mu_0\,\langle\beta\boldsymbol{\alpha}\cdot\mathbf{E}\rangle_{nn}\right]2\text{Ry} + O(\alpha^5\ln\alpha).\end{aligned} \quad (1)$$

Here $\ln[k_0(n)/\text{Ry}]$ is the nonrelativistic Bethe logarithm [10] and is the only quantity in Eq. (1) that depends on the whole wave function and requires significant computational efforts. Recently, following Schwartz (see [11]) a new method has been elaborated that is applicable to both one- and two-electron systems of an arbitrary angular momentum [12]. The final accuracy of expansion (1) is $\alpha^5\ln\alpha \approx 1.0\cdot 10^{-10}$.

Eventually, we included the corrections for the finite size of nuclei

$$\Delta E_{\text{nuc}} = \sum\frac{2\pi Z_i(R_i/a_0)^2}{3}\langle\delta(\mathbf{r}_i)\rangle,$$

where R is the root-mean-square radius of the nuclear charge distribution. The RMS radius for the helium nucleus and antiproton were taken, respectively, $R(^4\text{He}) = 1.673(1)$ fm, and $R(\bar{p}) = 0.862(12)$ fm.

3 Results

In Fig. 1 a schematic diagram of a two-photon Doppler-free experiment is shown. It is taken from the ASACUSA proposal [13] for the higher precision measurements of the transition lines between $(36, 35)$ and $(34, 33)$ metastable states, which are expected to be carried out in the nearest future. In what follows results of theoretical consideration for this particular transition are presented.

The Bethe logarithm values for the parent and daughter states were obtained with sufficiently high accuracy,

$$\ln[k_0(36,35)/\text{Ry}] = 4.48962(3)\,, \quad \ln[k_0(34,33)/\text{Ry}] = 4.52396(3)\,.$$

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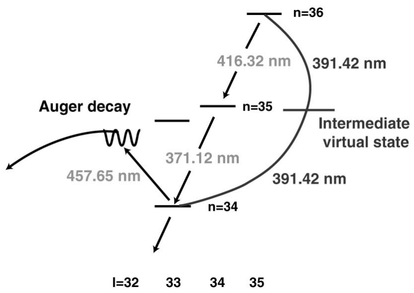

Fig. 1. Schematic diagram of an experiment for the two photon high precision measurement of transition energy between two metastable states

Table 1. Different contributions to the transition energy between metastable states $(36, 35)$ and $(34, 33)$ of the $^4\mathrm{He}^+\bar{p}$ atom

$E_{\text{nonrelativistic}}$	=	1 527 955 769	MHz
$E_{\text{relativistic corrections}}$	=	−50 659	MHz
$E_{\text{self energy}}$	=	7 517	MHz
$E_{\text{vacuum polarization}}$	=	−242	MHz
$E_{\text{finite nuclear size}}$	=	5	MHz
$E_{\text{uncalculated }\alpha^4\text{ order corrections}}$	=	5(5)	MHz
E_{total}	=	1 527 912 390(5)	MHz

Contributions to the spin-independent part of the transition energy $(36, 35) \to (34, 33)$ are summarized below in Table 1.

However, at this level of precision in experiment two distinct lines of the HFS should be observed [14],

$$E(L + \tfrac{1}{2}) = 1\,527\,912\,624 \text{ MHz}, \quad E(L - \tfrac{1}{2}) = 1\,527\,912\,137 \text{ MHz}$$

In calculations presented here the following ratios of heavy particle masses to the electron mass have been adopted: $m_p = 1836.152667(4)\ m_e$, $m_{\mathrm{He}} = 7294.299508(16)\ m_e$. It is obvious now that the uncertainty of 2 ppb in masses will become the main limitation in theoretical predictions in the nearest future.

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