Laser Spectroscopy of the 2S Lamb Shift in Hydrogenic Silicon

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Abstract. Transitions in highly charged ions are particularly sensitive to QED effects, which scale rapidly with atomic number, Z. An experiment to determine the 2S Lamb shift in hydrogenic silicon, using ions trapped in the Oxford electron beam ion trap (EBIT) is in progress. The laser system required for the experiment is currently under development at the National Physical Laboratory (NPL); this involves locking a frequency-stabilised Ti:sapphire laser operating at 734 nm to a high finesse build-up cavity. This will be used to drive and measure the $^2S_{1/2}$–$^2P_{3/2}$ transition in Si^{13+}. The transition is much more sensitive to two-loop binding corrections in Si^{13+} than in lower-Z systems. Thus this measurement offers the opportunity to test the uncertainty of theoretical contributions which presently limit the ability of transitions in hydrogen and He^+ to serve as calculable frequency standards. A better understanding of QED effects could also pave the way for calculable X-ray standards based on $\Delta n > 0$ transitions in high-Z hydrogenic systems.

1 Introduction

Transitions in hydrogen and hydrogenic (one-electron) ions are being studied in several laboratories with the aim of improving optical and X-ray frequency standards [1]. Transitions with $\Delta n > 0$ in hydrogen span a large frequency range from radiofrequencies into the vacuum ultraviolet. For hydrogenic ions of higher nuclear charge Z, this range extends into the X-ray region of the spectrum due to the Z^2 scaling of the gross energy level structure. These transitions can be regarded as forming a "natural" frequency scale governed by the Rydberg constant $R_\infty = m_e e^4/(8\varepsilon_0^2 h^3 c)$. Thus hydrogenic systems offer the possibility of a set of frequency standards extending from radiofrequencies into the X-ray region, which are directly related to the fundamental constants via the Rydberg constant.

The two-photon 243 nm $1S_{1/2}$–$2S_{1/2}$ transition and other transitions in the hydrogen atom have recently been included in a new list of approved radiations for the practical realisation of the metre [2]. The particular interest of these transitions lies in the fact that, uniquely among present optical frequency standards, they may be calculated in terms of the Rydberg constant with an accuracy approaching that with which they have been measured. The latest measurement of the 1S-2S transition frequency by phase coherent comparison with a microwave Cs fountain clock reports an uncertainty of 46 Hz [3]. However the determination of the 1S Lamb shift in hydrogen is limited by the knowledge of other hydrogenic transitions, e.g. the 2S–nS or the 2S–nD ($n = 8, 12, ...$) transitions, to 22 kHz [4].

Theoretical uncertainties in the calculation of hydrogen and hydrogenic transition frequencies arise from a number of sources. Of greatest significance for this work are the two-loop QED corrections to the energy levels which are considered in detail later. Other sources of uncertainty not considered further here are: (i) uncertainties which depend on the knowledge of values of the fundamental constants such as the fine structure constant and the electron to proton mass ratio, and (ii) nuclear size corrections which are particularly severe in hydrogen itself because the two most precise measurements of the root mean square charge radius of the proton disagree [5,6]. A new determination of the proton size is underway via a measurement of the Lamb shift in muonic hydrogen [7].

Owing to the rapid scaling of QED effects with atomic number Z, transitions in highly charged ions are much more sensitive than hydrogen to the two-loop binding corrections to the energy levels. Measurements in highly charged hydrogenic ions could therefore test the calculations of these terms, and enable the viability of hydrogen and other hydrogenic systems as calculable frequency standards to be assessed. For example, the $2S_{1/2}$–$2P_{1/2}$ and $2S_{1/2}$–$2P_{3/2}$ transitions in a number of medium-Z hydrogenic ions lie in a wavelength range that is accessible to high-power tuneable lasers. The most sensitive measurement to date in this range of Z is that for P^{14+} [8], which has a fractional precision of 0.14% of the 2S Lamb shift. This is about the same size as the self-energy correction of order $\alpha^2(Z\alpha)^5$, and hence improved precision is really required to obtain a critical test of theory.

This paper describes the progress of a laser resonance experiment which aims to measure the Lamb shift in hydrogenic silicon with an accuracy that will allow it to test the two-loop binding corrections mentioned above. This in turn should allow the viability of calculable frequency standards, based on transitions in lower-Z one-electron systems such as hydrogen and He^+, to be assessed. Following a review of some theoretical contributions to hydrogenic energy levels, the details of the laser resonance experiment are outlined.

2 Theoretical Contributions to Hydrogenic Energy Levels

There have been a number of recent reviews of hydrogenic systems and QED [9]–[12]; these proceedings contain the most extensive and recent information. To calculate transition frequencies in hydrogen to an accuracy comparable with the experimental precision which has been achieved [3], it is necessary to take into account a large number of corrections to the values obtained using the Dirac equation. These include quantum electrodynamic (QED) corrections, pure and radiative recoil corrections arising from the finite nuclear mass, and a correction due to the non-zero volume of the nucleus. The evaluation of these corrections is an extremely challenging task.

Here we briefly mention theoretical sources of uncertainty which arise from higher-order QED terms which were previously neglected or which are in unresolved disagreement. Whilst the one-loop electron self-energy has recently been calculated with high numerical accuracy (2.4 parts in 10^{16} for the 1S state of hydrogen) [13], the situation for the two-loop binding corrections is less satisfactory. The energy shift due to the two-loop corrections may be written in the form [9,11]

$$\Delta E_n = m_e c^2 \,(\alpha/\pi)^2 \left[(Z\alpha)^4/n^3\right] G_n(Z\alpha) \;, \tag{1}$$

where $G_n(Z\alpha)$ is usually written as a double expansion in powers of $(Z\alpha)$ and $\ln(Z\alpha)$,

$$G_n(Z\alpha) = B_{40} + (Z\alpha)B_{50} + (Z\alpha)^2 \left\{B_{63}\ln^3(Z\alpha)^{-2} + B_{62}\ln^2(Z\alpha)^{-2} + \ldots\right\} \;. \tag{2}$$

The leading-order term in this expansion has been known for many years [14], but recent calculations have yielded a larger than expected value for the term B_{50}, of order $\alpha^2(Z\alpha)^5$ [15,16]. Higher-order logarithmic terms (B_{63} and B_{62}) may also be larger than anticipated [17]–[20]; there appears to be disagreement between these calculations which is not yet fully explained. There are indications that the convergence of the series expansion is very poor, and that it would be desirable to carry out a high-accuracy numerical calculation which avoids the expansion in $Z\alpha$, as has been done for the one-loop self energy. Clarification of the theoretical situation would be welcome; meanwhile experimental Lamb shift measurements which are accurate enough to be sensitive to these contributions are most interesting. This is the motivation of the experimental work described below. The approximate size of some of the different contributions to the 2S Lamb shift in Si^{13+} is given in Table 1.

3 Experiment

The technique for studying the $2S_{1/2}$–$2P_{3/2}$ transitions in Si^{13+} by laser spectroscopy is illustrated in Fig. 1. Laser radiation at 734 nm is used to excite ions from the metastable $2S_{1/2}$ state to the $2P_{3/2}$ state, from which they rapidly decay to the ground state via an allowed electric dipole transition. The resonance is monitored by observing the rate of emission of 2 keV Lyman-α X-ray photons as a function of the laser frequency. The 2S Lamb shift may be deduced from such a measurement of the $2S_{1/2}$-$2P_{3/2}$ interval because the $n = 2$ fine structure splitting is more accurately known theoretically.

All previous 2S Lamb shift measurements for medium-Z hydrogen-like ions have been carried out using fast ion beams, and uncertainties associated with Doppler shifts form a significant source of error in all these experiments. Various methods have been employed or suggested for reducing the sensitivity of fast beam experiments to Doppler corrections [22]–[24]. A measurement of the $2S_{1/2}$–$2P_{3/2}$ transition frequency in N^{6+} using a fast ion beam is currently under way at Florida State University [25]. Our approach, however, is to reduce such

Table 1. Size of some contributions to the Lamb shift of the $2S_{1/2}$ state in Si^{13+}. All values are taken from reference 21 except the order $\alpha^2(Z\alpha)^5$ term which is from reference 9. The two-loop logarithmic terms are not included due to the current discrepancy in the values obtained from different calculations (see text)

	Energy shift (meV)
Order α self-energy	67.681
Vacuum polarization	-3.984
Finite nuclear size	0.328 ($\pm$ 0.005)
Relativistic recoil	0.019 ($\pm$ 0.019)
Relativistic reduced mass	-0.003 ($\pm$ 0.003)
Two-loop radiative corrections:	
$\alpha^2(Z\alpha)^4$	0.021
$\alpha^2(Z\alpha)^5$	-0.083
Total	63.97

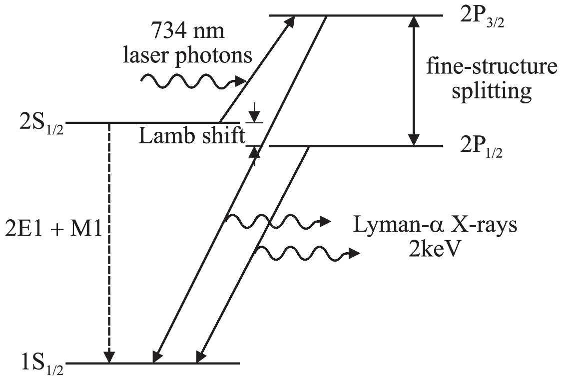

Fig. 1. Partial term diagram of the Si^{13+} ion, showing the transition that would be induced in an $n = 2$ laser resonance experiment. (Not to scale)

systematic errors by using slow ions trapped in the Oxford electron beam ion trap (EBIT) [26], which have no net centre-of-mass motion.

The operation of an EBIT is described in detail elsewhere [27]. Briefly, neutral atoms or low charge state ions are injected into an electron beam which is compressed to high current density by an axial magnetic field, where they are stripped to high charge states by sequential electron impact ionization. The resulting ions are confined radially by a combination of the space charge of the electron beam and the axial magnetic field, whilst axial trapping is achieved using potentials applied to a series of cylindrical electrodes. In this way, the ions are

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confined to a cylindrical volume about 70 μm in diameter and 2 cm in length, where they are further excited by electron impact for spectroscopic measurements. The ionization balance obtained within the trap depends on parameters such as the electron beam energy and background gas pressure.

The $2S_{1/2}$–$2P_{3/2}$ interval in hydrogen-like Si^{13+} has previously been observed using a fast ion beam [28] but there has been no precision measurement of its frequency to date. Our choice of Si^{13+} is based on the availability of high-power lasers at the appropriate wavelength, the ease of ion production in an EBIT, and the efficiency with which the Lyman-α X-rays can be detected. The $2S_{1/2}$–$2P_{3/2}$ transition in Si^{13+} is at a wavelength of approximately 734 nm, and has a natural width of about 7 nm, so can be studied using a Ti:sapphire laser. Silicon ions have been injected into the Oxford EBIT using a metallic vapour vacuum arc (MEVVA) ion source [29], and an electron beam energy of 3–4 keV has been shown to be sufficient to produce the hydrogen-like charge state [30]. The Lyman-α X-rays have an energy of about 2 keV, and may conveniently be observed using a lithium-drifted silicon (Si(Li)) detector.

Although the lifetime of the $2S_{1/2}$ state in Si^{13+} is much longer than that of the $2P_{3/2}$ state, it is still only 16 ns. For this reason, very high laser power (several kW or more) is required to obtain a reasonable transition rate to the $2P_{3/2}$ state [31]. The required intensities may be obtained by using an extremely high finesse enhancement cavity to build up the output power from a frequency-stabilized laser [32].

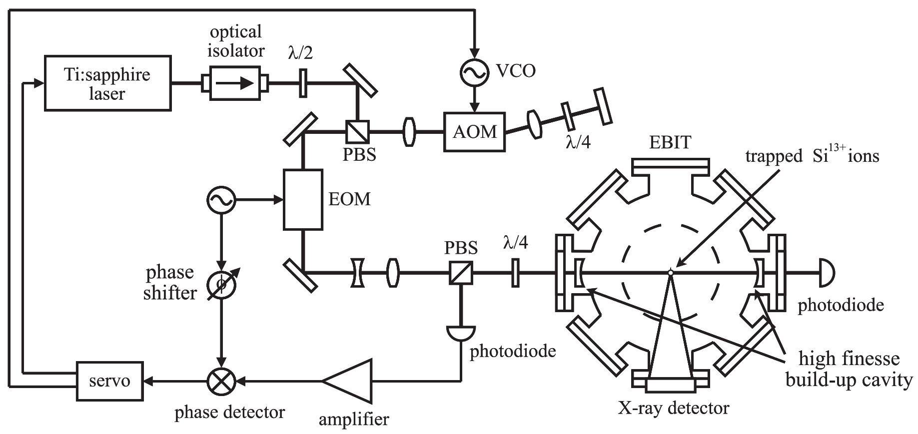

Fig. 2. Schematic diagram of the laser system to be used for a measurement of the $2S_{1/2}$–$2P_{3/2}$ transition frequency in Si^{13+}. ($\lambda/2$: half wave plate, $\lambda/4$: quarter wave plate, PBS: polarizing beamsplitter, VCO: voltage-controlled oscillator, AOM: acousto-optic modulator, EOM: electro-optic modulator)

Radiation at 734 nm from a continuous-wave, single-frequency Ti:sapphire laser, will be coupled into a high finesse cavity constructed around the Oxford

EBIT as shown in Fig. 2. The trapped Si^{13+} ions will lie at the laser beam waist within the enhancement cavity, and to keep the finesse of the cavity as high as possible it is necessary for the high reflectivity mirrors to lie within the EBIT vacuum chamber. The Ti:sapphire laser will be locked to the high finesse cavity using the rf sideband locking technique [33]. Fast frequency fluctuations will be corrected using an acousto-optic modulator in a double-pass configuration, whilst the slower branch of the servo loop will use a piezo-mounted mirror in the laser cavity.

Construction of a prototype enhancement cavity and the associated locking electronics is currently underway. This demonstration cavity will initially operate in air and consists of two identical mirrors with 20 ppm absorption plus scattering loss, and 200 ppm transmission. The cavity dimensions are somewhat constrained by the ion trap geometry; we intend to use mirrors with 0.5 m radius of curvature, positioned in a near-confocal arrangement. This will give a beam waist radius of around 250 μm, sufficiently large to avoid serious difficulties in aligning the laser beam to the trapped ions. Such a cavity has a finesse of around 1.4×10^4, which for an incident laser power of 100 mW would give an average intensity of about $4\,\mathrm{kW\,mm^{-2}}$ at the beam waist, sufficient to obtain a reasonable $2S_{1/2}$–$2P_{3/2}$ transition rate.

4 Hydrogenic Systems for Calculable Standards

The Si^{13+} Lamb shift experiment described above is designed to test QED two-loop binding corrections which presently limit the ability of transitions in hydrogen and He^+ to serve as calculable frequency standards. The limitations in hydrogen due to the proton size uncertainty may be avoided in He^+, where the uncertainty in the nuclear size correction is significantly smaller than the two-loop binding corrections. This has motivated a project at the University of Sussex, which aims to determine the 2S Lamb shift in He^+ via measurements of two-photon 2S–nS,nD transitions [11]. By comparing the results with the measurements in hydrogen, it should be possible to separate out the nuclear size and QED contributions to the Lamb shift. There was also a nine standard deviation discrepancy between the latest theoretical calculations and the best previous measurement of the 2S Lamb shift for He^+, carried out using a quenching anisotropy method [34]. However, an unexpected polarization-related systematic effect in that experiment has been reported and may resolve the situation [35].

A novel approach to He^+ might be to consider the two-photon transition from the ground state. He^+ might be interesting for the scheme proposed by Dehmelt for realising an optical frequency standard using electron shelving [36]. The scheme uses an allowed transition from the ground state for laser cooling and a slow "clock" transition from the ground state to a metastable state as a reference. In such experiments, the fluorescence photons from the cooling transition of a laser-cooled trapped ion are detected using a photo-multiplier, with count rates typically upwards of a few kHz. When a photon is absorbed by the reference transition the fluorescence is extinguished until the metastable state decays and a "quantum jump" is observed. The 30.4 nm $1S_{1/2}$–$2P_{1/2}$ Lyman-α transition in He^+ which would have to be used for laser cooling is very short. The "clock" transition would be the $1S_{1/2}$–$2S_{1/2}$ two-photon transition (analogous to the two-photon transition presently used in hydrogen) for which a 60.8 nm laser would be required. At first sight the helium ion looks like a very difficult candidate for the above scheme because of the very short wavelengths required, however technological developments may make it more tractable in years to come.

Hydrogen and He^+ offer a number of transitions which might serve as calculable frequency standards. At higher Z there are also opportunities to consider. The best measurements of Lyman-α wavelengths in one-electron ions in the range Z=12–28 have reported accuracies of up to 5 ppm [37]–[39] and measurements of lower accuracy have been made right up to U^{91+} [40], for which the energy of the Lyman-α X-rays is about 100 keV. If the measurements are improved and the QED contributions can be understood with sufficient accuracy, such Lyman-α transitions could one day become calculable frequency standards in the X-ray region.

Measurements of transitions in hydrogenic systems are important both as tests of fundamental physics and for obtaining the Rydberg constant from precision spectroscopy of atomic hydrogen. If the QED corrections to the energy levels can be calculated with sufficient accuracy, then hydrogenic systems offer the prospect of "natural" or calculable frequency standards which are directly related to fundamental constants. This paper has reported the progress of a laser resonance experiment which aims to measure the Lamb shift in hydrogenic silicon with an accuracy that will allow it to test QED two-loop binding corrections. This should allow the viability of calculable frequency standards, based on transitions in lower-Z one-electron systems such as hydrogen and He^+, to be assessed.

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References

1. J. L. Flowers et al.: 'Hydrogenic systems: calculable frequency standards', to appear in *Proceedings of the 14th European Frequency and Time Forum, Turin, Italy, 14th-16th March 2000*
2. T. J. Quinn: Metrologia **36**, 211 (1999)
3. M. Niering et al.: Phys. Rev. Lett. **24**, 5496 (2000)
4. C. Schwob et al.: Phys. Rev. Lett. **82**, 4960 (1999)

5. L. N. Hand, D. G. Miller and R. Wilson: Rev. Mod. Phys. **35**, 335 (1963)
6. G. G. Simon, Ch. Schmitt, F. Borkowski and V. H. Walther: Nucl. Phys. A **333**, 381 (1980)
7. R. Pohl et al.: *this edition*, pp. 454–466
8. H.-J. Pross et al.: Phys. Rev. A **48**, 1875 (1993)
9. K. Pachucki et al.: J. Phys. B **29**, 177 (1996)
10. M. G. Boshier: 'Hydrogen-like systems and quantum electrodynamics', in *Atomic Physics 15, Proceedings of the 15th International Conference on Atomic Physics, Amsterdam, The Netherlands, 1996*, ed. by H. B. van Linden van den Heuvell, J. T. M. Walraven and M. W. Reynolds (World Scientific, 1997) pp. 328–342
11. M. G. Boshier: Physica Scripta **T86**, 21 (2000)
12. P. J. Mohr, G. Plunien and G. Soff: Phys. Rep. **293**, 227 (2000)
13. U. D. Jentschura, P. J. Mohr and G. Soff: Phys. Rev. Lett. **82**, 53 (1999)
14. T. Appelquist and S. J. Brodsky: Phys. Rev. A **2**, 2293 (1970)
15. K. Pachucki: Phys. Rev. Lett. **72**, 3154 (1994)
16. M. I. Eides and V. A. Shelyuto: Phys. Rev. A **52**, 954 (1995)
17. S. Mallampalli and J. Sapirstein: Phys. Rev. Lett. **80**, 5297 (1998)
18. S. G. Karshenboim: JETP **76**, 541 (1993)
19. I. Goidenko et al.: Phys. Rev. A **83**, 2312 (1999)
20. V. A. Yerokhin: Phys. Rev. Lett. **62**, 012508 (2000)
21. W. R. Johnson and G. Soff: At. Dat. Nucl. Dat. Tab. **33**, 405 (1985)
22. R. A. Holt, S. D. Rosner, T. D. Gaily and A. G. Adam: Phys. Rev. A **22**, 1563 (1980)
23. J. D. Silver: 'The spectroscopy of hydrogen-like highly ionized atoms', in *The Hydrogen Atom*, ed. by G. F. Bassani, M. Inguscio and T. W. Hänsch (Springer-Verlag, 1989) pp. 221–237
24. D. Müller et al.: Z. Phys. D **18**, 249 (1991)
25. E. G. Myers, R. Hankins, J. D. Silver and M. R. Tarbutt: Hyperfine Interactions **127**, 329 (2000)
26. J. D. Silver et al.: Rev. Sci. Instrum. **65**, 1072 (1994)
27. M. A. Levine et al.: Physica Scripta **T22**, 157 (1988)
28. S. N. Lea et al.: Phys. Lett. A **185**, 327 (1994)
29. I. G. Brown, J. E. Galvin, R. A. MacGill and R. T. Wright: Appl. Phys. Lett. **49**, 1019 (1986)
30. T. V. Back et al.: Physica Scripta **T73**, 62 (1997)
31. H. S. Margolis et al.: Hyperfine Interactions **99**, 169 (1996)
32. M. S. Fee et al.: Phys. Rev. A **48**, 192 (1993)
33. R. W. P. Drever et al.: Appl. Phys. B **31**, 97 (1983)
34. A. van Wijngaarden, J. Kwela and G. W. F. Drake: Phys. Rev. A **43**, 3325 (1991)
35. G. W. F. Drake and A. van Wijngaarden: presented at the conference *Hydrogen Atom 2* (unpublished)
36. H. D. Dehmelt: IEEE Trans. Instrum. Meas. **31**, 83 (1982) and references therein
37. G. Hölzer et al.: Phys. Rev. A **57**, 945 (1998)
38. H. F. Beyer, R. D. Deslattes, F. Folkmann and R. E. LaVilla: J. Phys. B **18**, 207 (1985)
39. H. F. Beyer et al.: Phys. Rev. A **43**, 223 (1991)
40. H. F. Beyer et al.: Z. Phys. D **35**, 169 (1995)