Radiation Properties of Diamagnetic Manifolds in Atomic Hydrogen: Line Intensity Dependence on a Magnetic Field

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Abstract. We study the effect of a magnetic field on the probability of radiative transitions of a hydrogen atom from diamagnetic manifold states. The analytical formulae have been derived for the susceptibilities determining the influence of diamagnetic interaction on the probabilities of radiative transitions. To derive the analytic expressions for the higher-order matrix elements, we use the Sturm expansion of the reduced Coulomb Green function. We also examine the frequency dependence of corrections to the radiative matrix elements and its correlation with the structure of the diamagnetic spectrum of excited levels. We discover the selective action of a magnetic field on the diamagnetic components of emission lines: when the field strength increases, an increase in the intensity of some lines is accompanied by a decrease in the intensity of the other lines.

1 Introduction

Calculation of electromagnetic susceptibilities of atoms is an important problem of Atomic Physics. These quantities provide essential information on the fundamental interaction between electromagnetic field and matter. Atomic hydrogen is the only microsystem for which the exact theoretical calculations of amplitudes and electromagnetic susceptibilities are feasible and specifically in the analytical form. So the efforts for deriving analytical formulas and calculating numerical values for diverse quantities describing such processes are worthwhile not only for profound studies of the simplest quantum systems but also for providing possibilities to giving new insights to the fundamental problem of the field-matter interaction.

The problem of diamagnetic states in the Zeeman atomic manifold was studied intensively during the last two decades. The diamagnetic components of the Zeeman spectrum may be optically resolved in rather strong magnetic field. However, due to huge magnitudes of internal atomic fields the perturbation theory, even in the lowest order, is often sufficient for describing the modification of the atomic properties under the action of a very strong field. There exists the field range where the perturbative description is yet possible but for understanding the electromagnetic effect correctly the higher-order amplitudes and susceptibilities of atom should be considered.

The principal optical properties of the diamagnetic sublevels in atomic hydrogen — the shift and splitting in field of the energy levels — have been studied up to the second order in diamagnetic interaction already in the mid-1980s (see the review article [1]). This progress had restarted in late 1990s when the studies of the higher-order corrections have begun [2-4]. This situation is quite simple to understand, since the set of constants of motion in a Coulomb plus magnetic field may be determined approximately, only up to the second order in diamagnetic interaction [5]. That is why the effective Hamiltonian could be constructed [6] only up to the second order and then diagonalized [7] within the set of degenerate states of an excited hydrogen level.

The further progress has become possible for the third-order terms in diamagnetic interaction with the use of another approach, based on the higher-order perturbation theory for degenerate levels and the reduced Coulomb Green function formalism [4,8,9]. This formalism allowed to derive analytical expressions for the third-order diamagnetic matrix in a basis of degenerate spherical Coulomb states which may be used for determining the third-order diamagnetic energies. For a non-degenerate state the corresponding diagonal matrix element itself determines the third-order energy and thus provides analytical expressions for the diamagnetic susceptibilities of the two lowest levels (of opposite parity) in a Zeeman manifold with a given magnetic quantum number m [3].

The recursive algebraic procedure has been developed for calculating numerically very high diamagnetic susceptibilities of the lowest hydrogen levels [2] within the lowest Zeeman manifold $m = 0$. So the diamagnetic effect on atomic eigenfrequencies has been studied to very high orders of perturbation theory.

However, the diamagnetic effect on the other important property of spectrum, namely, on the atomic line strength, has not yet been discussed so in detail. The only result in this respect was derived as a consequence of diagonalization of the diamagnetic Hamiltonian, which is usually carried out together with calculating the corresponding eigenvector of the diamagnetic sublevel. The splitting of a degenerate level into a set of diamagnetic states is accompanied by the splitting of corresponding atomic line into diamagnetic components. The intensities of these components are determined by the contribution of a dipole-accessible state into the diamagnetic sublevel. Thus a distribution of oscillator strengths over diamagnetic components of a Lyman line was derived [1,7]. Since the upper-level diamagnetic states arise from a strictly degenerate basis (all the states have exactly one and the same energy in the zero-field limit) and form an eigenvector of the first-order diamagnetic matrix, corresponding radiative oscillator strengths do not depend on the magnetic field. The field dependence appears only when one allows for the second-order terms in the diamagnetic interaction, thus taking into account all the complete atomic basis of states, including continuum.

In this communication we report on the analytical expressions for the first-order diamagnetic corrections to the matrix elements of radiative dipole transitions, determining the field dependence of the line intensities. Some of the backgrounds and basic aspects of the work may be found in [9].

2 Radiation Matrix Element Dependence on a Magnetic Field

The field dependence for the radiative transition matrix element may be presented in the form of expansion in power series:

$$d_{if}(B) = d_{10}\left(1+\sum_{s=1}^{\infty} q_{10}^{(s)} B^{2s}\right). \tag{1}$$

The coefficients of the series may be calculated on the basis of the perturbation theory for the initial- ($|i\rangle$) and final-state ($|f\rangle$) wave functions of atom in field. The simplest way of calculations is the use of the iteration procedure for the integral equation with the Coulomb Green function $G_{E_{i(f)}}(\mathbf{r},\mathbf{r}')$:

$$\begin{aligned} E_{i(f)} &= E_{1(0)} + \langle 1(0)|V_D|i(f)\rangle, \\ |i(f)\rangle &= |1(0)\rangle - G'_{E_{i(f)}} V_D |i(f)\rangle. \end{aligned} \tag{2}$$

The first field-dependent term of expansion (1) $q_{10}^{(1)} = q_{10}(1) + q_{10}(0)$ is determined by the corrections of the first order in the diamagnetic interaction

$$V_D = \frac{B^2}{12} r^2 \left(1 - \mathbf{C}_{20}\left(\frac{\mathbf{r}}{r}\right)\right). \tag{3}$$

to the free-atom wave functions of initial $|1\rangle$ (the term $q_{10}(1)$) and final $|0\rangle$ (term $q_{10}(0)$) states. After integration over angular variables the terms $q_{10}(1(0))$ may be presented as combinations of the radial matrix elements. E.g., for π-transition from $|nlm\rangle$ into $|n'l'm\rangle$ state with $m = l' = l-1$ we have:

$$\begin{aligned} q^{\pi}_{nl,n'l'}(nl) &= -\frac{l}{4(2l+3)} \frac{\langle nl|r^2 g_l^{(n)}(r,r')r'|n'l'\rangle}{\langle nl|r|n'l'\rangle}; \\ q^{\pi}_{nl,n'l'}(n'l') &= -\frac{l}{4(2l+1)} \\ &\times \frac{(2l+3)\langle nl|r g_{l-1}^{(n')}(r,r')r'^2|n'l'\rangle - 2\langle nl|r g_{l+1}^{(n')}(r,r')r'^2|n'l'\rangle}{(2l+3)\langle nl|r|n'l'\rangle}. \end{aligned} \tag{4}$$

With the use of the Sturm-series resolution for the reduced radial Coulomb Green function:

$$\begin{aligned} g_l^{(n)}(r,r') = \frac{4Z}{n}\Bigg\{ &\sum_{k\neq n_r}^{\infty} \frac{k!}{(k+2l+1)!} \frac{f_{kl}\left(\frac{2Zr}{n}\right) f_{kl}\left(\frac{2Zr'}{n}\right)}{k+l+1-n} \\ &+ \frac{n_r!}{(n+l)!n}\left[\frac{5}{2} + r\frac{d}{dr} + r'\frac{d}{dr'}\right] f_{n_r l}\left(\frac{2Zr}{n}\right) f_{n_r l}\left(\frac{2Zr'}{n}\right)\Bigg\} \end{aligned} \tag{5}$$

and the orthogonality of the Sturm function

$$f_{kl}(x) = e^{-x/2} x^l L_k^{2l+1}(x) \tag{6}$$

to the radial wave function

$$R_{nl}(r) = \frac{2Z^{3/2}}{n^2}\sqrt{\frac{n_r!}{(n+l)!}}\, f_{n_r l}\left(\frac{2Zr}{n}\right), \tag{7}$$

with equal argument, we derive the analytical expressions for the matrix elements of numerators and denominators of fractions for the quantities (4) in the form of generalized Gordon's formulae. For small radial quantum numbers of the lower level simple analytical expressions in terms of ratios between the polynomials of the upper-level principal quantum number n can be derived. For the lines of Lyman series ($n' = 1$, $l' = 0$) these expressions reed:

$$\begin{aligned} q^{\pi}_{np,1s} &= \frac{30n^8 - 100n^6 + 74n^4 + 159n^2 - 55}{120Z^4(n^2-1)}; \\ q^{\sigma}_{np,1s} &= \frac{60n^{10} - 260n^8 + 348n^6 + 125n^4 - 208n^2 + 55}{120Z^4(n^2-1)^2}. \end{aligned} \tag{8}$$

Similar expressions for transitions $|np\rangle \to |2s\rangle$ of the Balmer series ($n' = 2$, $l' = 0$) are:

$$\begin{aligned} q^{\pi}_{np,2s} &= \frac{15n^8 - 160n^6 + 164n^4 + 4312n^2 - 6560}{60Z^4(n^2-4)}; \\ q^{\sigma}_{np,2s} &= \frac{15n^{10} - 220n^8 + 804n^6 + 2076n^4 - 12288n^2 + 13120}{30Z^4(n^2-4)^2}. \end{aligned} \tag{9}$$

No more complicated expressions may be written for the transitions $|ns\rangle \to |2p\rangle$:

$$\begin{aligned} q^{\pi}_{ns,2p} &= -\frac{100n^8 - 675n^6 + 1028n^4 - 5088n^2 + 7680}{120Z^4(n^2-4)}; \\ q^{\sigma}_{ns,2p} &= \frac{125n^{10} - 1630n^8 + 5224n^6 + 9280n^4 - 56064n^2 + 61440}{120Z^4(n^2-4)^2}. \end{aligned} \tag{10}$$

And for $|ndm\rangle \to |2pm'\rangle$:

$$\begin{aligned} q^{\pi}_{nd0,2p0} &= \frac{1565n^8 - 20040n^6 + 50512n^4 + 196608n^2 - 344064}{5376(n^2-4)}; \\ q^{\sigma}_{nd1,2p0} &= \frac{15n^{10} - 268n^8 + 1392n^6 - 128n^4 - 8448n^2 + 10752}{42(n^2-4)^2}; \\ q^{\pi}_{nd1,2p1} &= \frac{15n^8 - 208n^6 + 560n^4 + 3008n^2 - 5376}{42(n^2-4)}; \\ q^{\sigma}_{nd2,2p1} &= \frac{45n^{10} - 804n^8 + 4176n^6 + 512n^4 - 34304n^2 + 43008}{84(n^2-4)^2}; \\ q^{\sigma}_{nd0,2p1} &= \frac{835n^{10} - 16580n^8 + 92048n^6 + 70976n^4 - 1081344n^2 + 1376256}{2688(n^2-4)^2}. \end{aligned} \tag{11}$$

It is important to note that the main contribution to these equations is given by the variation in field of the upper-level wave function. So, for the Lyman-transition from the state $n = 5$ the field-induced variation of the ground-state wave function contributes to the coefficients q less than 0.02%. The variation of the lower state in the Balmer transition from the $n = 5$ state does not exceed 3%.

These formulae and numerical estimates don't take into account the mixing of $|nlm\rangle$-states with equal parity by the diamagnetic interaction.

3 Diamagnetic Corrections to Matrix Elements of Transitions from Degenerate States

Rigorously speaking, the excited hydrogen nlm-levels are degenerate, except for the four states with $m \geq n-3$. For $m < n-3$ only the magnetic quantum number m and parity P remain the integrals of motion. The state mixing occurs in arbitrary weak field, since the states with different l have one and the same energy. To calculate correctly the wave function dependence on field, we use the perturbation theory for degenerate states. The integral equation (2) for them transforms into

$$\psi_{nm\lambda p}(\mathbf{r}) = \sum_{l=m+p}^{l_{max}} a_l(\lambda)[1 + G'_E(\mathbf{r},\mathbf{r}')V_D(\mathbf{r}')]^{-1}|\varphi_{nlm}(\mathbf{r}')\rangle, \tag{12}$$

where $\varphi_{nlm}(\mathbf{r})$ is the wave function of a free atom with fixed orbital momentum; $p = 0(1)$ is the number determining parity $P = (-1)^{m+p}$. The parameter λ is the diamagnetic quantum number. The Green function $G'_E(\mathbf{r},\mathbf{r}')$ is orthogonal to the wave functions of degenerate basis states and accounts for the mixing into the wave function (12) of the complete set of atomic states including continuum and excepting the degenerate basis.

The coefficients of the linear combination a_l are determined from the system of linear equations:

$$(E_n - E)a_l + \sum_{l'=m+p}^{l_{max}} a_{l'} \langle\varphi_{nlm}(\mathbf{r}) \mid \hat{W}(\mathbf{r},\mathbf{r}') \mid \varphi_{nl'm}(\mathbf{r}')\rangle = 0,$$
$$l = m+p, m+p+2, \ldots, l_{max}, \tag{13}$$

with

$$\hat{W}(\mathbf{r},\mathbf{r}') = V_D(\mathbf{r})[1 + G'_E(\mathbf{r},\mathbf{r}')V_D(\mathbf{r}')]^{-1} = V_D(\mathbf{r})\sum_{s=0}^{\infty}[-G'_E(\mathbf{r},\mathbf{r}')V_D(\mathbf{r}')]^s \tag{14}$$

— the integral operator exactly accounting for the diamagnetic interaction. Resolving in power series the matrix element

$$W_{ll'} = \langle\varphi_{nlm}(\mathbf{r}) \mid \hat{W}(\mathbf{r},\mathbf{r}') \mid \varphi_{nl'm}(\mathbf{r}')\rangle = -\sum_{s=1}^{\infty}\frac{v_{ll'}^{(s)}}{2s!}B^{2s}, \tag{15}$$

energy

$$\Delta E_\lambda = E_\lambda - E_n = -\sum_{s=1}^{\infty} \frac{\chi_\lambda^{(s)}}{2s!} B^{2s} \tag{16}$$

($\chi_\lambda^{(s)}$ is the s-order diamagnetic susceptibility), and coefficients

$$a_l = \sum_{s=0}^{\infty} a_l^{(s)} B^{2s}, \tag{17}$$

we may write the system of equations (13) for every term of the last resolution. To calculate the first-order correction to coefficients (17) we have to determine the first- and second-order terms of the matrix elements (15) and energy (16).

Since the zero-order wave function of a diamagnetic state is a linear combination of the degenerate basis wave functions and the diamagnetic interaction allows for extension of dipole selection rules to $\Delta l = \pm 1, \pm 3$, the equation for the coefficient q is modified to read (we consider the lower state to be non-degenerate, with $l' = m'$):

$$q^{\pi}_{n\lambda,n'l'} = q^{\pi}_{nl'+1,n'l'} + \frac{a^{(1)}_{l'+1}(\lambda)}{a^{(0)}_{l'+1}(\lambda)} + \frac{a^{(0)}_{l'+3}(\lambda)}{a^{(0)}_{l'+1}(\lambda)} q^{\pi}_{nl'+3,n'l'}, \tag{18}$$

where $q^{\pi}_{nl'+1,n'l'}$ is the above-described coefficient for the transition between non-degenerate states. The factor $q^{\pi}_{nl'+3,n'l'}$ may also be calculated analytically. For example:

$$\begin{aligned}
q^{\pi}_{nf,1s} &= \frac{n^2\sqrt{3(n^2-4)(n^2-9)/7}}{480(n^2-1)} \left(55n^4 + 225n^2 - 16\right); \\
q^{\pi}_{nf,2s} &= \frac{n^2\sqrt{3(n^2-4)(n^2-9)/7}}{480(n^2-4)^2} \left(55n^6 + 360n^4 - 2832n^2 - 4096\right); \\
q^{\pi}_{ng,2p} &= \frac{n^2\sqrt{2(n^2-9)(n^2-16)/3}}{672(n^2-4)} \left(125n^4 + 780n^2 - 512\right).
\end{aligned} \tag{19}$$

4 Numerical Results for Lyman and Balmer Transitions

The results of numerical calculations of the coefficients $q^{\pi;\sigma}_{nlm,n'l'm'}$ for the first-order diamagnetic corrections to the dipole matrix element of the Lyman ($np \to 1s$) and Balmer ($np \to 2s$, $ns \to 2p$ and $nd \to 2p$) series corresponding to transitions from the excited hydrogen levels with $n \le 8$ are presented in table 1. These results are calculated from equations (8)–(11) without account for diamagnetic mixing of $|nlm\rangle$-states with different orbital momenta l. So they only demonstrate the order of magnitude for the coefficients and the relative contribution of the field-induced variation of the upper-state and lower-state wave functions. The results which take into account the diamagnetic mixing of degenerate states are presented for the π-transition matrix elements in table 2.

Table 1. The factors $q^{\pi,\sigma}$ for magnetically induced corrections to the matrix elements of the radiative π- and σ-transitions $np-1s$, $2s$ and ndm, $ns-2pm'$ in hydrogen atom; $(k) = 10^k$

q	$n=2$	3	4	5	6	7	8
$q^{\pi}_{np,1s}(n)$	9.926	136.7	876.2	3.543(3)	1.091(4)	2.801(4)	6.315(4)
$q^{\pi}_{np,1s}(1s)$	-1.468	0.0833	0.3815	0.4931	0.5477	0.5788	0.5983
$q^{\sigma}_{np,1s}(1s)$	-2.838	-0.0573	0.4647	0.6580	0.7522	0.8057	0.8391
$q^{\pi}_{np,2s}(n)$		154.3	564.6	2.778(3)	9.368(3)	2.522(4)	5.846(4)
$q^{\pi}_{np,2s}(2s)$		-63.23	35.51	49.90	54.48	56.52	57.60
$q^{\sigma}_{np,2s}(2s)$		-176.4	17.02	45.93	55.38	59.66	61.99
$q^{\pi}_{ns,2p0}(n)$		-137.7	-2.706(3)	-1.149(4)	-3.582(4)	-9.245(4)	-2.090(5)
$q^{\pi}_{ns,2p0}(2s)$		-210.9	-56.89	-28.37	-17.75	-12.50	-9.482
$q^{\sigma}_{ns,2p1}(2s)$		-421.8	-113.8	-56.75	-35.50	-25.01	-18.96
$q^{\pi}_{nd0,2p0}(n)$		55.39	538.2	2.934(3)	1.023(4)	2.806(4)	6.576(4)
$q^{\pi}_{nd0,2p0}(2p)$		-11.65	23.11	28.00	29.50	30.15	30.49
$q^{\sigma}_{nd1,2p0}(n)$		117.6	568.9	3.360(3)	1.206(4)	3.351(4)	7.913(4)
$q^{\sigma}_{nd1,2p0}(2p)$		-54.27	44.44	55.82	58.75	59.82	60.30
$q^{\pi}_{nd1,2p1}(n)$		117.6	568.9	3.360(3)	1.206(4)	3.351(4)	7.913(4)
$q^{\pi}_{nd1,2p1}(2p)$		-51.71	60.44	74.54	78.50	80.08	80.85
$q^{\sigma}_{nd2,2p1}(n)$		176.4	853.3	5.040(3)	1.809(4)	5.027(4)	1.187(5)
$q^{\sigma}_{nd2,2p1}(2p)$		-80.13	74.67	93.09	98.00	99.86	110.72
$q^{\sigma}_{nd0,2p1}(n)$		183.3	345.8	2.533(3)	9.688(3)	2.766(4)	6.630(4)
$q^{\sigma}_{nd0,2p1}(2p)$		-108.5	88.89	111.6	117.5	119.6	120.6

As is evident from the tables, the numerical values of q for the transition matrix elements in the Lyman series are rapidly increasing functions of the upper-level principal quantum number n. All of them are positive and may differ from one another by a factor of several units for transitions from different diamagnetic sublevels (labeled by the diamagnetic quantum number λ which orders the absolute values of their energy [1] represented by the susceptibility $\chi^{(1)}_{nm\lambda}$ [4,8]) within a Zeeman manifold with fixed magnetic quantum number $m = 0, 1$ (we consider only positive values of m as the diamagnetic effects are independent of the m's sign), in the shell of states with one and the same principal quantum number n. The factors q for the Balmer series transitions into $2s$-state are quite similar to those of the Lyman series as one can see from comparison of numerical data of table 2 for $q^{\pi}_{n0\lambda,1s}$ and $q^{\pi}_{n0\lambda,2s}$: the higher n the closer these quantities to

Table 2. The factors $q^{\pi}_{nm\lambda^{\pm},n'l'}$ for magnetic induced corrections to the matrix elements of radiative π-transitions in the Lyman $|n0\lambda^-\rangle \to |1s\rangle$ and Balmer series $|n0\lambda^-\rangle \to |2s\rangle$ and $|nm\lambda^+\rangle \to |2pm\rangle$ with $m = 0, 1$. The upper level index λ is determined by ordering the absolute values of diamagnetic susceptibility, $\chi^{(1)}_{nm\lambda^{\pm}}$, represented in individual columns; $(k) = 10^k$

n	λ	$-\chi^{(1)}_{n0\lambda^-}/2$	$q^{\pi}_{n0\lambda^-,1s}$	$q^{\pi}_{n0\lambda^-,2s}$	$-\chi^{(1)}_{n0\lambda^+}/2$	$q^{\pi}_{n0\lambda^+,2p0}$	$-\chi^{(1)}_{n1\lambda^+}/2$	$q^{\pi}_{n1\lambda^+,2p1}$
3	1	9.0	1.368(2)	9.107(1)	5.171	-1.179(2)	9.0	6.609(1)
	2	–	–	–	1.958(1)	1.920(3)	–	–
4	1	1.335(1)	1.525(3)	1.439(3)	1.868(1)	-1.014(3)	3.60(1)	6.293(2)
	2	3.865(1)	5.396(2)	1.646(2)	6.532(1)	1.044(4)	–	–
5	1	3.425(1)	4.451(3)	4.158(3)	2.834(1)	1.539(2)	4.438(1)	7.788(3)
	2	1.095(2)	2.776(3)	1.701(3)	6.875(1)	-2.033(3)	1.056(2)	2.666(3)
	3	–	–	–	1.654(2)	3.579(4)	–	–
6	1	5.215(1)	2.070(4)	2.025(4)	5.759(1)	-4.188(3)	1.056(2)	2.021(4)
	2	1.150(2)	4.168(3)	3.098(3)	1.710(2)	-3.473(4)	2.454(2)	9.364(3)
	3	2.513(2)	8.941(3)	6.529(3)	3.519(2)	9.876(4)	–	–
7	1	9.064(1)	3.912(4)	3.831(4)	8.631(1)	5.290(3)	1.396(2)	8.490(4)
	2	2.545(2)	2.200(4)	1.964(4)	1.842(2)	-5.906(4)	2.507(2)	3.762(4)
	3	5.001(2)	2.393(4)	1.923(4)	3.636(2)	-1.142(5)	4.918(2)	2.571(4)
	4	–	–	–	6.643(2)	2.349(5)	–	–
8	1	1.322(2)	1.070(5)	1.058(5)	1.352(2)	-6.683(3)	2.444(2)	1.468(5)
	2	2.842(2)	3.637(4)	3.379(4)	3.655(2)	-1.145(5)	4.992(2)	8.790(4)
	3	5.081(2)	4.635(4)	4.168(4)	6.854(2)	-3.249(5)	8.885(2)	6.056(4)
	4	8.995(2)	5.549(4)	4.717(4)	1.150(3)	5.003(5)	–	–

one another. The values of q^{σ} in these transitions equal nearly twice the values of q^{π}. That is why only the quantities for π-transitions are presented.

Thus we may state that the matrix elements of the Lyman series transitions increase with the increase of the magnetic field. Similar increase is also characteristic of matrix elements for π-transitions to the state $2p1$ and π- and σ-transitions to the state $2s$ of the Balmer series. However this property does not hold in general. E.g. for π-transitions to the state $2p0$ from many states of the upper-level diamagnetic manifold the coefficient $q^{\pi}_{n0\lambda,2p0}$ takes negative values (see table 2). So the magnetic field action on the radiative matrix elements is rather selective and depends on the structure of initial and final states and on the type of transition (π or σ).

We have also tested numerically the simple analytical formulae for the correction factor $q^{\pi;\sigma}_{\nu_l\nu_m\nu_{n'}}$ of transitions between nondegenerate states (circular and nearly circular orbits of the atomic electron) $nlm \to n'l-1\,m'$, where $l = n-\nu_l$,

$m = n - \nu_m$, $n' = n - \nu_{n'}$ with $\nu_l \le 2$, $\nu_m \le 3$ and $\nu_{n'} \le 2$; $m' = m$ – for π-transitions, $m' = m - 1$ – for σ-transitions. These formulae are presented explicitly in table 3 together with factor $w^{\pi;\sigma}_{\nu_l \nu_m \nu_{n'}}$, which determines corresponding correction to the transition frequency according to the following equation

$$\omega^{\pi,\sigma}_{\nu_l \nu_m \nu_{n'}}(B) = \omega_{nn'}\left(1 + w^{\pi,\sigma}_{\nu_l \nu_m \nu_{n'}} B^2\right),$$

where $\omega_{nn'}$ is the free-atom eigenfrequency. The data for $w^{\pi;\sigma}_{\nu_l \nu_m \nu_{n'}}$, together with the matrix element (1), may be used to determine the total diamagnetic correction to the probability of transition W_{if} and to the intensity $I_{if}(B)$ of a line emitted by an atom in field:

$$I_{if}(B) \propto \omega_{if} W_{if} \propto \omega^4_{if} \left|d_{if}\right|^2 .$$

The numerical data for the factors q as calculated from expressions of table 3 is presented in table 4 for the upper-level principal quantum number $n = 4 \div 10$. As is seen from the table, the absolute values of q are rapidly increasing functions of n. Another interesting result consists in the change of sign for almost all the quantities from positive at $n = 4$ to negative for higher n. It means that with the increase of n the matrix element of radiative transition between nearly circular states is reduced by the magnetic field. This result also demonstrates the selective action of a magnetic field on the probabilities of radiative transitions in hydrogen.

We have also calculated the numerical values of coefficients $q_{nm\lambda,n'l'm'}$ for the terms of the Lyman and Balmer series corresponding to transitions from highly excited Rydberg states with the principal quantum number $n \gg 1$ and determined the distribution of this quantity over corresponding diamagnetic manifold. We have discovered some singularities in these distributions which are characteristic both of π- and σ-transitions. They are presented in figures 1–3 for the diamagnetic components of a Balmer line corresponding to π-transitions from the state $n = 50$. The values of the correction factor q (given on the vertical axis) for the diamagnetic sublevels are plotted in the figures against the absolute values of the upper level's first-order diamagnetic susceptibility $\chi^{(1)}_{nm\lambda}$ presented on the horizontal axis.

The most important feature in the distribution is a strong relation between the general properties and irregularities of $q_{nm\lambda,n'l'm}$ and those of $\chi^{(1)}_{nm\lambda}$. So the jump in the monotone dependence between the quantities is observed exactly at the boundary between the degenerate (about one fourth of the total number of states) and equidistant parts of the diamagnetic spectrum [1]. The value q^{π} for transition from one level at this boundary is negative while for transitions from all the other levels it is positive. It is useful to note that the quantities $q_{nm\lambda^-,2s}$ and $q_{nm\lambda^-,1s}$ are equal to one another with accuracy of 5 to 6 digit numbers. So Fig.1 may be regarded equivalently as the plot for $q^{\pi}_{nm\lambda^-,1s}$. The plot for the quantity $q^{\sigma}_{nm\lambda^-,2s}$ describing the correction factor of σ-transitions may also be reproduced from Fig.1 by doubling the scale of the vertical axis, in accord with

Table 3. Analytic expressions for the factors $q^{\pi;\sigma}_{\nu_l\nu_m\nu_{n'}}$ and $w^{\pi;\sigma}_{\nu_l\nu_m\nu_{n'}}$ for magnetic induced corrections to the matrix elements of radiative transitions π- and σ between nondegenerate states of the hydrogen atoms $nlm \to n'l-1\,m'$, where $l = n-\nu_l$, $m = n-\nu_m$, $n' = n-\nu_{n'}$; $m' = m$ – for π-transitions, $m' = m-1$ – for σ-transitions

$$q^{\pi}_{121} = -\frac{n(n-1)}{48(2n-1)}\left(18n^5-112n^4+76n^3+4n^2-9n+1\right);$$

$$w^{\pi}_{121} = \frac{(n-1)^3n^3(3n-1)}{4(2n-1)};$$

$$q^{\pi}_{231} = -\frac{(n-2)\left(36n^7-502n^6+1272n^5-2350n^4+2698n^3-1677n^2+521n-64\right)}{48(2n-1)^2};$$

$$w^{\pi}_{231} = \frac{(n-1)^2n^2(n-2)}{4(2n-1)}(3n^2+7n-4);$$

$$q^{\pi}_{232} = -\frac{(n-2)}{24(n-1)}\left(25n^6-210n^5+460n^4-524n^3+400n^2-168n+32\right);$$

$$w^{\pi}_{232} = \frac{n^2(n-2)^3}{8(n-1)}(5n^2-4n+2);$$

$$q^{\sigma}_{111} = -\frac{n}{48(2n-1)^2}\left(48n^7-318n^6+445n^5-271n^4+80n^3+n^2-8n+1\right);$$

$$w^{\sigma}_{111} = \frac{n^3(n-1)^2}{4(2n-1)}(4n^2-3n+1);$$

$$q^{\sigma}_{121} = -\frac{n(n-1)}{48(2n-1)^2}\left(48n^6-306n^5+381n^4-154n^3-6n^2+17n-2\right);$$

$$w^{\sigma}_{121} = \frac{1}{2}n^3(n-1)^3;$$

$$q^{\sigma}_{221} = -\frac{(n-1)\left(48n^7-486n^6+1385n^5-3170n^4+4204n^3-2897n^2+978n-128\right)}{48(2n-1)^2};$$

$$w^{\sigma}_{221} = \frac{n^2(n-1)^3}{2(2n-1)}(2n^2+5n-4);$$

$$q^{\sigma}_{231} = -\frac{48n^8-570n^7+2373n^6-5827n^5+9724n^4-9799n^3+5552n^2-1627n+192}{48(2n-1)^2};$$

$$w^{\sigma}_{231} = \frac{n^2(n-1)^2}{4(2n-1)}(4n^3+3n^2-25n+12);$$

$$q^{\sigma}_{222} = -\frac{57n^8-604n^7+2233n^6-4469n^5+5708n^4-4864n^3+2672n^2-864n+128}{48(n-1)^2};$$

$$w^{\sigma}_{222} = \frac{n^2(n-2)^2}{16}(11n^2-12n+8);$$

$$q^{\sigma}_{232} = -\frac{(n-2)\left(57n^7-540n^6+1623n^5-2563n^4+2550n^3-1612n^2+584n-96\right)}{48(n-1)^2};$$

$$w^{\sigma}_{232} = \frac{n^2(n-2)^3}{16(n-1)}(11n^2-11n+6).$$

Table 4. The factors $q^{\pi,\sigma}$ for magnetic induced corrections to the matrix elements of radiative π- and σ-transitions between nondegenerate states of hydrogen atom $nlm \rightarrow n'l-1\,m'$, where $l = n - \nu_l$, $m = n - \nu_m$, $n' = n - \nu_{n'}$; $m' = m$ – for π-transitions given in the first three lines, $m' = m - 1$ – for σ-transitions presented in the last six lines of the table; $(k) = 10^k$

ν_l	ν_m	$\nu_{n'}$	$n = 4$	5	6	7	8	9	10
1	2	1	1.910(2)	1.942(2)	-6.434(2)	-4.026(3)	-1.324(4)	-3.386(4)	-7.464(4)
2	3	1	5.252(2)	1.718(3)	4.113(3)	7.908(3)	1.265(4)	1.669(4)	1.651(4)
2	3	2	6.293(2)	1.076(3)	-9.947(2)	-1.232(4)	-4.687(4)	-1.295(5)	-3.005(5)
1	1	1	2.121(2)	-1.606(1)	-1.784(3)	-7.743(3)	-2.283(4)	-5.524(4)	-1.177(5)
1	2	1	1.484(2)	-6.460(1)	-1.655(3)	-7.072(3)	-2.093(4)	-5.101(4)	-1.094(5)
2	2	1	4.732(2)	1.069(3)	1.521(3)	4.986(2)	-4.949(3)	-2.026(4)	-5.439(4)
2	3	1	2.106(2)	4.961(2)	4.928(2)	-1.083(3)	-7.057(3)	-2.264(4)	-5.646(4)
2	2	2	9.280(2)	1.269(3)	-2.106(3)	-1.790(4)	-6.361(4)	-1.698(5)	-3.854(5)
2	3	2	6.133(2)	9.106(2)	-1.857(3)	-1.544(4)	-5.579(4)	-1.513(5)	-3.479(5)

the asymptotic (for $n \gg 1$) relation $q^{\sigma}_{nm\lambda^-,2s} = 2q^{\pi}_{nm\lambda^-,2s}$, which holds for all λ, except for two or three at the above-discussed boundary: there is no negative quantity among the total set of $q^{\sigma}_{nm\lambda^-,2s}$.

A similar jump in monotone dependence is also observed for the q-values of transitions to the $2p1$-state as may be seen in Fig.2. All q are here positive and the energy dependence for the degenerate part of states (one fourth of the total number) is similar to that of transitions to $2s$-state (see Fig.1). For the equidistant part of the diamagnetic spectrum the energy dependence differs qualitatively from that of the Fig.1: the $q^{\pi}_{nm\lambda^+,2p1}$-values decrease with energy in contrast with the increase for $q^{\pi}_{nm\lambda^-,2s}$. The absolute values of the factors are ranging within one order of magnitude ($(1 \div 12) \times 10^9$ atomic units) whereas corresponding values of the energy factor $\chi^{(1)}_{nm\lambda}$ variate within two orders ($(0.2 \div 19) \times 10^5$ a.u.).

Quite a different kind of the energy dependence is observed for transitions to the $2p0$-state presented in Fig.3. Here only a small oscillation appears at the boundary between the degenerate and equidistant parts of the diamagnetic spectrum on the smooth background of slightly decreasing dependence of $q^{\pi}_{nm\lambda^+,2p0}$ on $\chi^{(1)}_{nm\lambda^+}$. A giant jump from a large negative ($\sim -1.7 \times 10^{11}$ a.u.) to a large positive value ($\sim 1.4 \times 10^{11}$ a.u.) appears approximately at one fifth part of states from the high-energy end of the diamagnetic manifold.

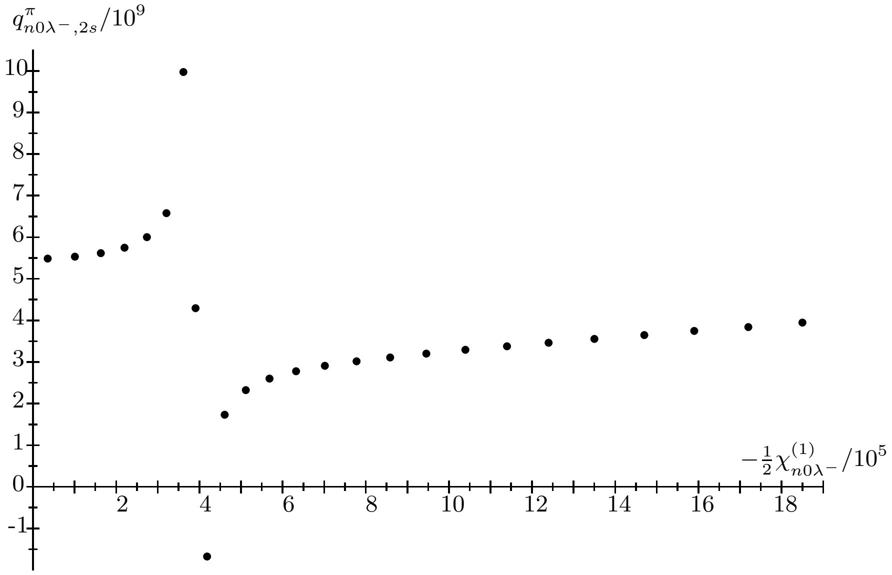

Fig. 1. Numerical values for the coefficients $q^{\pi}_{n0\lambda-,2s}$ of diamagnetic components of the Balmer line corresponding to the radiative decay of the level $n = 50$. The susceptibility $-\frac{1}{2}\chi^{(1)}_{n0\lambda-}$ represents the energy on the horizontal axis

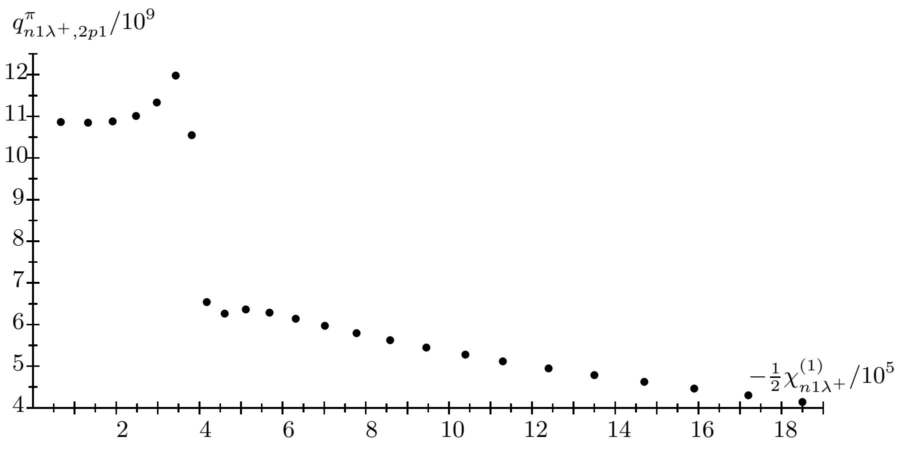

Fig. 2. Numerical values for the coefficients $q^{\pi}_{n1\lambda+,2p1}$ of the diamagnetic components of the Lyman line corresponding to the radiative decay of the level $n = 50$. The energy on the horizontal axis is represented by the susceptibility $-\frac{1}{2}\chi^{(1)}_{n1\lambda+}$

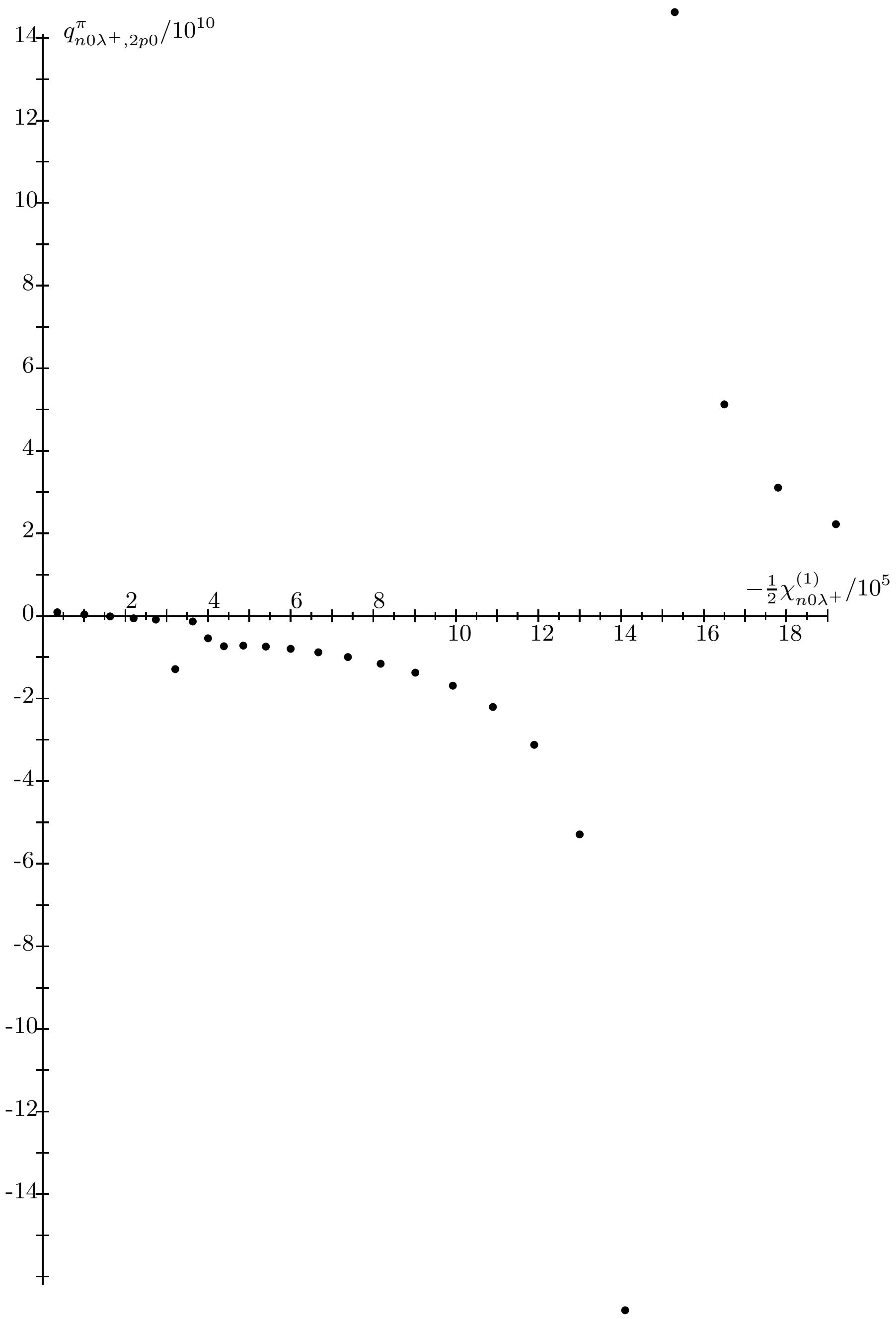

Fig. 3. Numerical values for the coefficients $q^{\pi}_{n0\lambda+,2p0}$ of diamagnetic components of the Balmer line corresponding to the radiative decay of the level $n = 50$. The energy on the horizontal axis is represented by the susceptibility $-\frac{1}{2}\chi^{(1)}_{n0\lambda+}$

5 Conclusion

In a sufficiently strong magnetic field, together with complete splitting of atomic lines, the probability of emission and absorption of radiation may be redistributed dramatically. The intensity of almost all diamagnetic components of the Lyman series increases with field strength. In the Balmer series the same increase in intensity is observed for π-transitions to $2s$- and $2p1$-states. For transitions to the state $2p0$ there is a redistribution of intensities in favor of the decay lines of a small number of states from the low- and high-frequency parts of the diamagnetic states of positive parity. On the other hand, the intensity of the lines of decay of the positive parity states from internal part of the diamagnetic spectrum decreases with increasing field strength.

Our calculations show that the changes in the radiative properties of a hydrogen-like atom in a magnetic field can be described analytically by using spherical Coulomb basis. This way we have arrived at a complete solution of the problem of the hydrogen atom in magnetic field with allowance for diamagnetic corrections not only for the energy but also for the wave functions. The method of determining the corrections of optical characteristics of degenerate states of the atom in a field by solving the system of algebraic equations can be useful in solving other problems of the interaction between atoms and electromagnetic fields and in the theory of atomic collisions.

The analytic expressions and quantitative data listed in the tables 1-4 and figures 1-3 provide information about changes in the intensity of atomic lines. They can be applied not only to hydrogen but also to many-electron atoms in Zeeman states with large magnetic quantum number ($m \geq 3$) and can be used to describe the spectral characteristics of Rydberg atoms in moderate and strong magnetic fields, generated in laboratories and produced in stellar plasma.

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