Interference of the Hydrogen Atom States (n=2)

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The phenomenon of atomic state interference has been investigated. The frequency of the ($2s_{1/2}$, F=0) - ($2p_{1/2}$, F=1) transition in the hydrogen atom has been measured using atomic interferometry. The Lamb shift was found to be δ = 1057. 8514 ± 0.0019 MHz, where the uncertainty is the statistical standard deviation of a single measurement.

1. Introduction

The develoment of a quantum mechanical approach to nature has given rise to a number of key problems which have as yet no comprehensive solution. Thus, further study of wave properties in microobjects and the comparison of obtained results and theory is necessary. However, one should note that the number of modern experiments demonstrating the phenomena of particle-wave duality is rather limited and they do not allow one to begin to solve vaguely formulated problems. It is probably fair to say, therefore, that some small-scale effects, which nevertheless play an important role, are neglected in many experimental techniques. If one seeks new approaches, it makes sense to study the interference of atomic states, since the interference pattern is extremely sensitive to the characteristics of its components which can manifest themselves in some new, previously unknown ways.

Reference to atomic interference is of interest for other reasons, too. One can study the properties of elementary particles by observing and precisely measuring various fine effects in the bound states. From such measurements, in principle, one can obtain such details as the behaviour of interactions at short distances, which otherwise are manifested only at very high energies. Optical measurements are probably among the most precise. If one adopts this form of measurement it is natural to ask whether it is possible to consider some phenomena of atomic physics within spec-

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troscopy, not in the sense of observing radiation, but, roughly speaking, by considering the atoms as waves and observing the interference of various atomic states. Then we have the following scheme: we can determine the properties of bound states by interference, and thereby determine the characteristics of elementary particles.

2. The Atomic Interferometer

A few years ago I proposed a method (the "atomic interferometer" method) which allows one to observe a stationary interference pattern for a long time while being able to arbitrarily change the phase shift, thus noticeably improving the accuracy of measurement. The interference of atomic states can be observed, in principle, with the aid of a device similar in main details to a standard two beam optical interferometer (e.g. Michelson's interferometer).

Imagine a beam of metastable $2s_{1/2}$ atoms of hydrogen passing in succession through two spatially separated zones I and II (Fig. 1). Inside these zones the atoms are subject to a perturbation that enables them to go to other states, say $2p_{1/2}$ and $2p_{3/2}$. A perturbing factor of this kind might be an electric field E that varies non-adiabatically within each of the zones. The criterion of non-adiabaticity is that the transit frequency $\omega = v/d$ be larger than, or of the order of, the Lamb frequency (for the $2s_{1/2} - 2p_{1/2}$ transition) or of the fine structure frequency (for $2s_{1/2} - 2p_{3/2}$); here v is the velocity of the atom and d is the breadth of the zones I and II, i.e. the distance over which the field changes abruptly. To simplify things, we shall continue our analysis with the two-level $2s_{1/2} - 2p_{1/2}$ system as an example. This is justified for fields that are not too strong; the effect of the $2p_{3/2}$ level can be taken into account by making small corrections.

It follows from the foregoing arguments that in the simplest version of the interferometer the role of zones I and II can be played by the boundaries of the field E, localized in a specified region. Then, when they cross boundary I, the atoms of the beam experience the perturbing influence of a growing field and go into a superposition of eigenstates ϕ_1 and ϕ_2 with energies ϵ_1 and ϵ_2 determined by the value of the field intensity E. At boundary II, where the field decreases to zero, beam components representing both the state $2s_{1/2}$ and the state $2p_{1/2}$ are produced, and each of

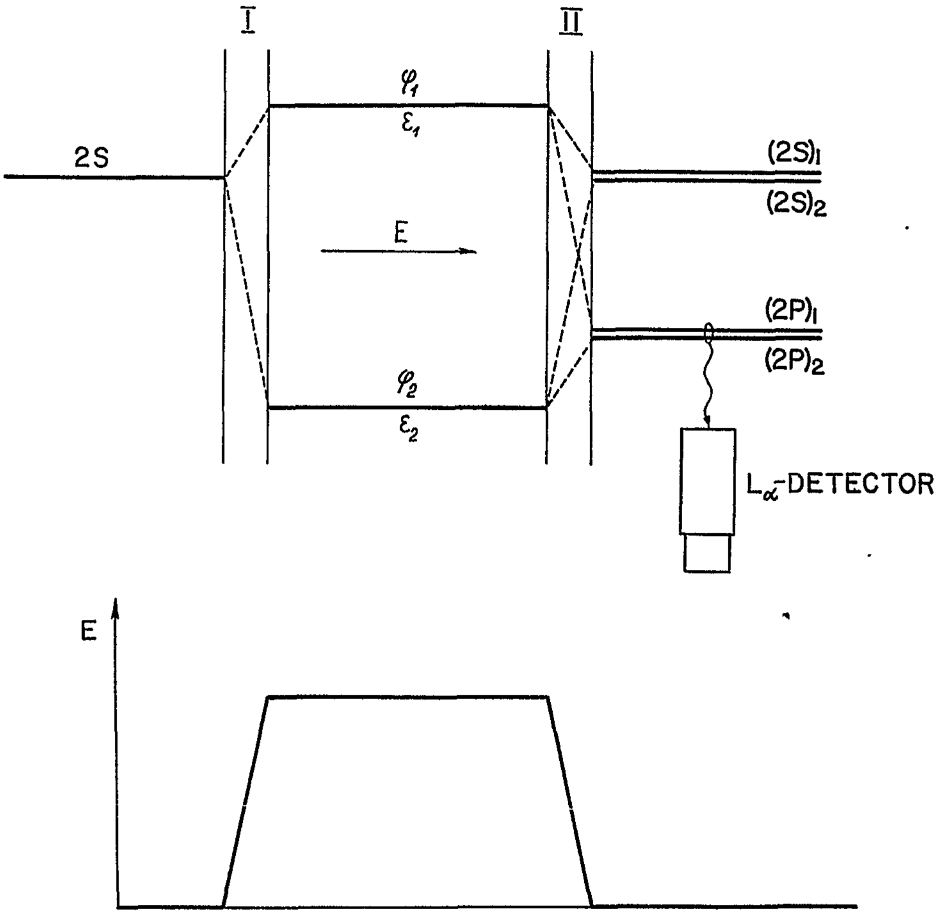

Fig. 1 Schematic diagram of the atomic interferometer

terms ϕ_1 and ϕ_2 initiates a pair of such states: ϕ_1: $(2s)_1$ + $(2p)_1$, and ϕ_2: $(2s)_2$ + $(2p)_2$. Thus, in the field-free region adjacent to boundary II, the state of the atom is described by a superposition of the four components: $(2s)_1$, $(2s)_2$, $(2p)_1$, $(2p)_2$.

Outside the field the amplitudes of the $2s_{1/2}$ and $2p_{1/2}$ eigenstates are defined by the transition amplitudes of, and the phase difference between, the components of each pair $(2s)_1$ - $(2s)_2$ and $(2p)_1$ - $(2p)_2$, which depend on the time of flight in the field and on the transition frequency between the ϕ_1 and ϕ_2 terms split by the electric field. The magnitude of such a splitting is entirely determined by the strength of the field E. Thus, when the field is continuously varied, periodic intensity oscillations of the H_{2s} and H_{2p} atom fluxes (occurring in counterphase) will be observed due to the interference of the $(2s)_1$ - $(2s)_2$ and $(2p)_1$ - $(2p)_2$ waves arising on boundary II. A similar effect is seen when the time of flight T, i.e. the distance between the field boundaries, is changed.

The interference pattern of the $(2p)_1$ - $(2p)_2$ components can be recorded by measuring the intensity of the short lived 2p part of the beam once it has passed through the interferometer. Thus, the detector placed behind zone II (i.e. in the field-free region) must count quanta corresponding to the single-photon transition 2p - 1s, i.e. the resonant line of the Lyman series (λ = 1216 Å). One can also observe the interference of the $(2s)_1$ - $(2s)_2$ components occurring in counterphase, for which purpose the beam should be passed through the additional field, either rf or constant.

To simplify the analysis, it is reasonable to divide the field strength range into "normal" and "strong". Normal should be taken to mean those fields for which the condition $x = \langle d \rangle E/\pi\hbar\delta \propto 1$ is satisfied, i.e. the Stark shift of the $2s_{1/2}$ and $2p_{1/2}$ levels, caused by the fields, proves to be of the same order as the Lamb shift (here $\langle d \rangle$ is the matrix element of the $2s_{1/2}$ - $2p_{1/2}$ transition, E is the field strength and δ is the Lamb shift).

In the two level case and assuming the field terminates abruptly at the boundaries, the yield of the H_{2p} - atoms is proportional to the quantiy I such that /1/

$$I = \sum_i c_i \frac{x_i^2}{1+x_i^2} \left[\cosh \frac{s_i T}{2\tau \sqrt{p_i + x_i^2}} - \cos 2\pi T\delta \, \frac{x}{x_i} \sqrt{1+x_i^2} \right] e^{-\frac{T}{2\tau}(1+\kappa x^2)} \qquad (1)$$

where τ is the lifetime of the H_{2p} atom, T is the time of flight in the field E, and δ is the Lamb shift; c_i, s_i and p_i are constants; $\kappa = \frac{\delta}{2\nu_1}$; ν_1 is the $2p_{3/2}$ - $2p_{1/2}$ splitting frequency. The factor κx^2 is the $2p_{3/2}$ - level effect correction.

In the $2s_{1/2}$ - $2p_{1/2}$ system there are transitions in the electric field between the s and p hyperfine structure sublevels with total angular momentum projections 1, 0

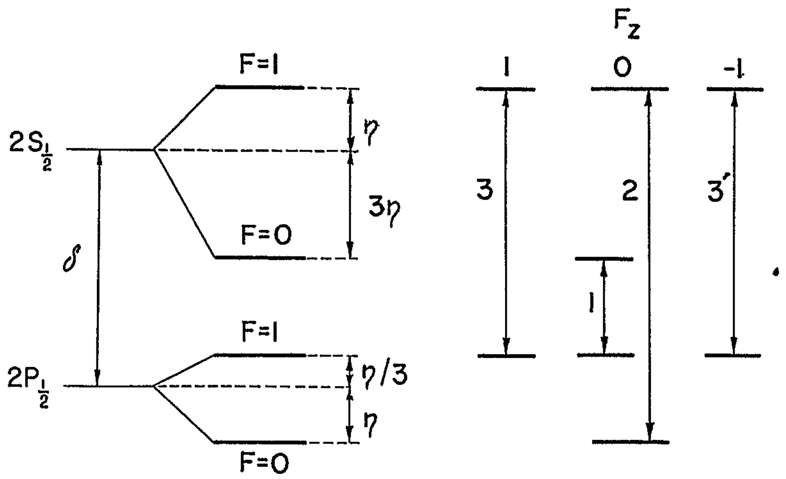

Fig. 2 Hyperfine structure of the 2s levels of the hydrogen atom

and -1 (Fig. 2). The energy differences of 3 and 3′ transitions coincide, so that the summation in (1) is carried out for three components of the hyperfine splitting with the following values of x_i:

$$x_1 = x \bigg/ \left(1 + \frac{2}{3}\,\frac{\eta}{\delta}\right);\ x_2 = x \bigg/ \left(1 - \frac{10}{3}\,\frac{\eta}{\delta}\right);\ x_3 = x \bigg/ \left(1 + 2\,\frac{\eta}{\delta}\right) \qquad (2)$$

where η is the hyperfine splitting frequency.

It follows from (1) that the dependence of the H_{2p} atom yield on E and T found experimentally allows one, in principle, to determine the values of η and δ. It should be noted, however, that the method can not be realised fully in the case of the simple interferometer described. For example the determination of δ to an accuracy of 1 - 2 ppm, entails many practically insurmountable difficulties.

At the same time, a two electrode interferometer, which is convenient in operation, and which can be adjusted relatively easily with respect to a strictly collimated beam of hydrogen atoms, permits the study of many specific features of atomic interference.

Figure 3 shows the two electrode interferometer.

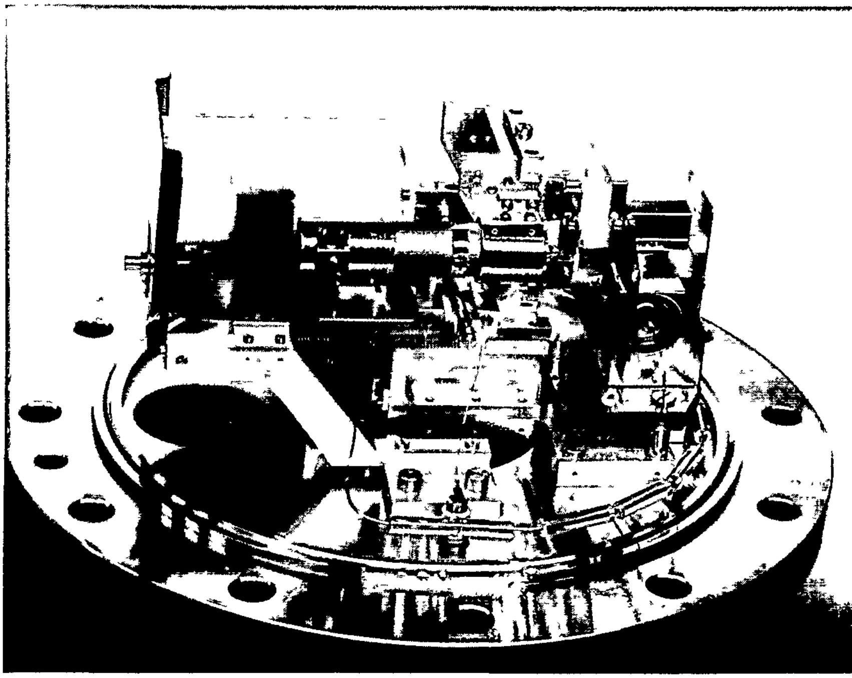

Fig. 3 Interferometer with two electrodes (longitudinal field)

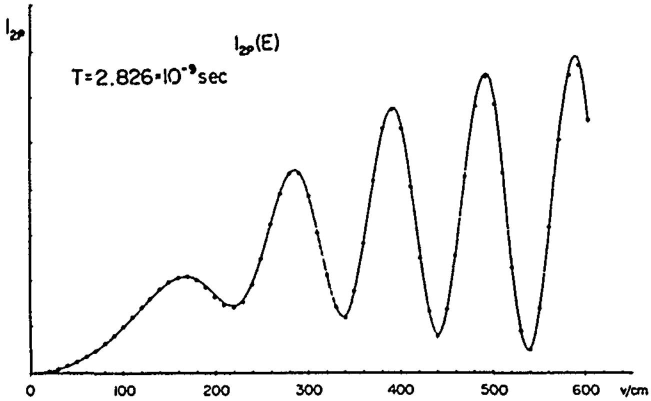

Fig. 4 Plot of $I_{2p}(E)$ $\ell = 0.5$ cm

Figures 4 and 5 show typical plots of $I_{2p}(E)$ and $I_{2p}(\ell)$, where ℓ is the flight distance. Both curves reveal a distinct interference pattern which is the optical analogue of the effect predicted by Pais and Piccioni for the system of K^0 and $\overline{K}^0$ mesons.

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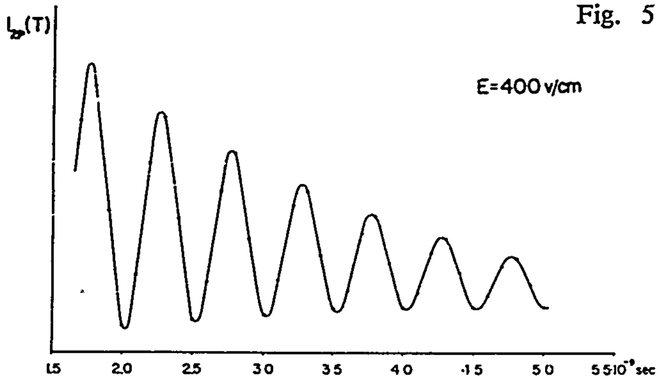

Fig. 5 Plot $I_{2p}(\ell)$ E = 400 V/cm

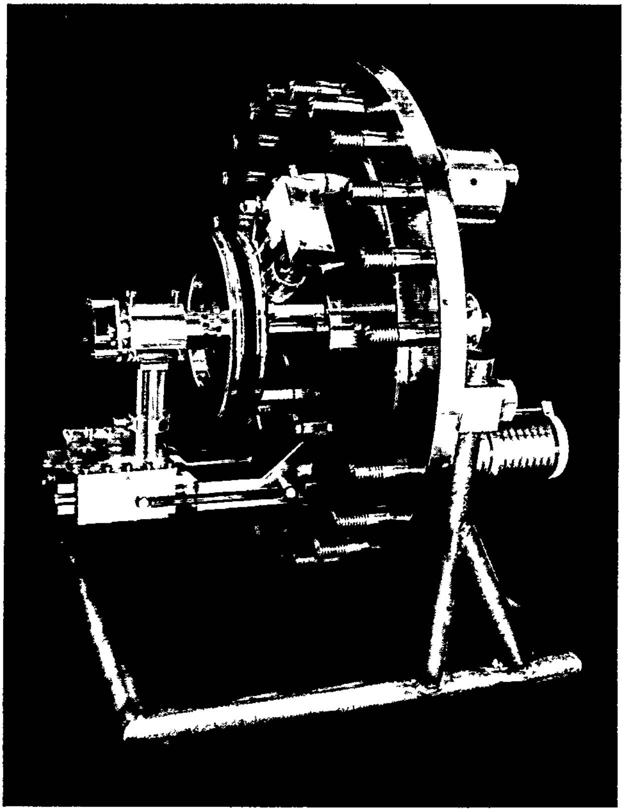

Fig. 6 Interferometer for fields up to 8000 V/cm

Figure 6 shows the interferometer designed for strong fields.

Figure 7 shows the typical interference curve at strong fields.

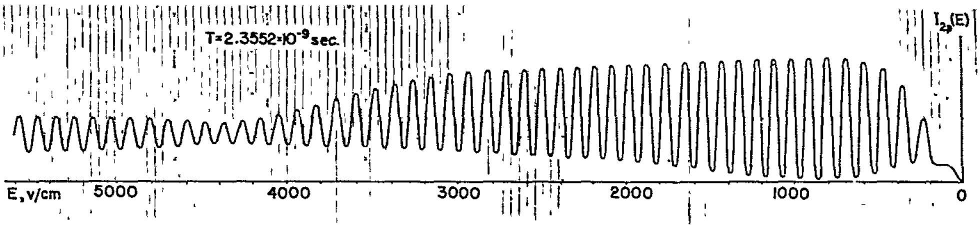

Fig. 7 Interference of $2p_{1/2}$ - state of hydrogen atom

In the experiments described above, it has been shown that when the two phase-shifted components of the 2p (or 2s) atomic hydrogen state interfere, some net curve - the superposition of separate curves corresponding to transitions between the components of the hyperfine 2s and 2p level structure - is registered. Further study of atomic interference has shown that hyperfine splitting can also be obtained in other ways.

The experimental scheme is shown in figure 8. The H_{2s} beam 1 with an energy of about 20 keV passes successively through rf fields A and B, which have frequencies of 1147 and 1088 MHz corresponding to transitions 2 and 3 (see Fig. 2), and then via interferometer 4 with Lyman α detector 5. In such a case only hyperfine structure component I with frequency ν = 909 MHz passes through the interferometer.

The electric field mixes states of opposite parity. Therefore, if the atom entering the interferometer is in a state with definite parity (e.g. in the 2s state), the probability of it emerging in the 2s or 2p state does not depend on the sign of the field.

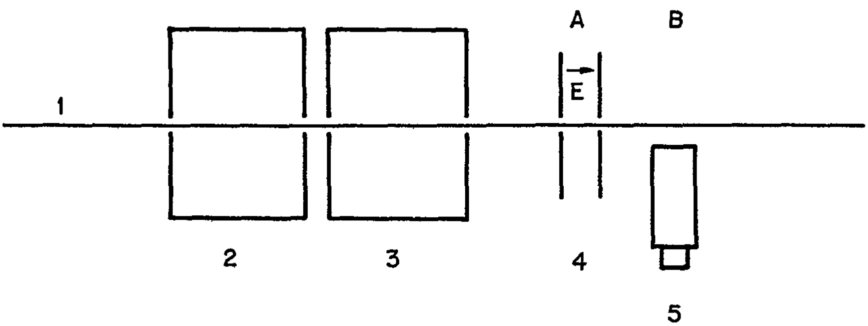

Fig. 8 Experiment on the interferometer field direction

On the other hand, if the initial wave function is a superposition of states of opposite parity (2s and 2p), the emergence probabilities for opposite field directions differ by an amount proportional to the product of the amplitudes of the atomic states 2s and 2p in the initial wave function.

The interferometric curves obtained at the rf fields 2, 3 switched "on", i.e. for the transition 1 at 909 MHz (since the 2s state component with F=1 is absent in this case), are shown in figure 9.

One can see that the yield of 2p atoms really does not depend on the field direction in the interferometer: the curves for +E and -E are in sufficiently good agreement (error bars are shown; experimental points are designated with crosses and circles).

However, when the rf fields 2, 3 are "off", i.e. when the ($2s_{1/2}$, F=1) component also passes through the interferometer, the situation is abruptly changed: the yield of 2p atoms essentially depends on the field sign (Fig. 10).

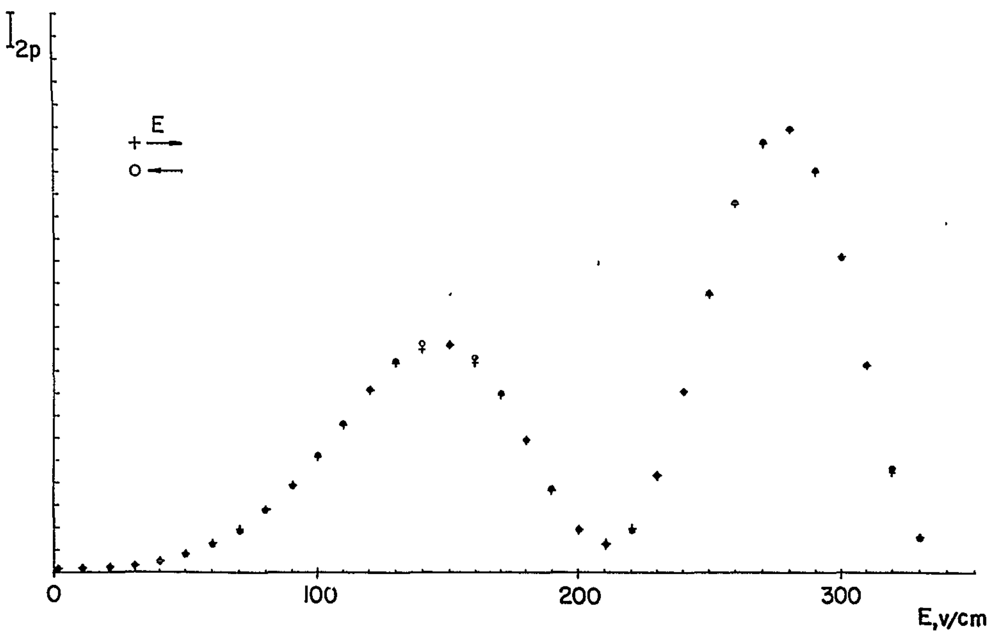

Fig. 9 Interference curves for opposing field directions (for transition 1)

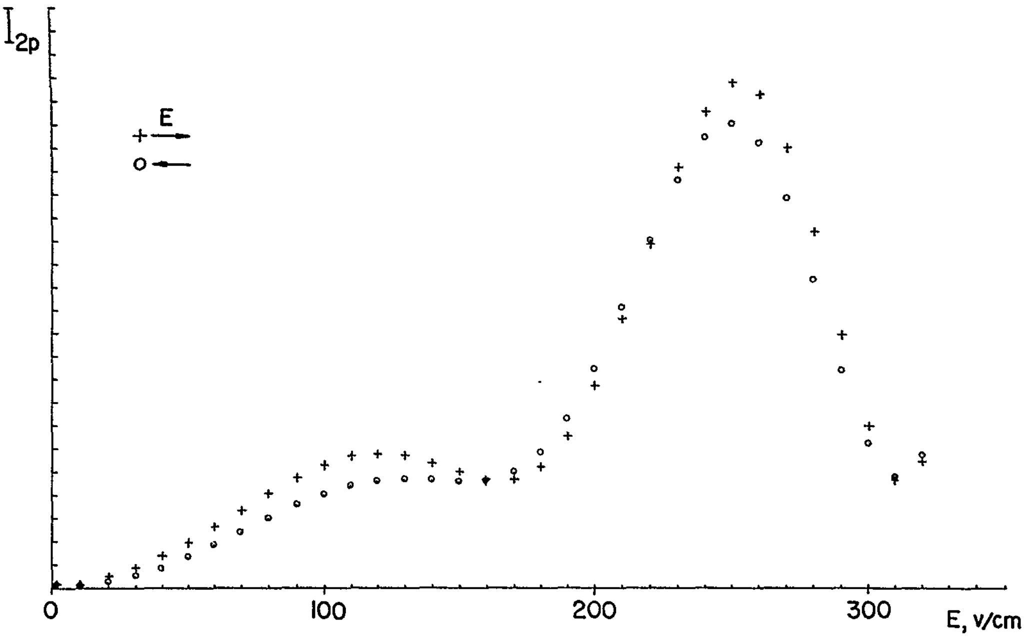

Fig. 10 Interference curves for opposing field directions (for transitions 1, 2, 3)

Although it is distinctly observed, we have not yet managed to describe the nature of the previously mentioned large-scale effect on the basis of quantum mechanical ideas concerning the behaviour of the hydrogen atom in an electric field.

3. The Lamb Shift Measurement

An interesting problem is the precise calculation and measurement of the Lamb shift δ which we describe here, commenting on the main points of interest. First, there is a disparity - not yet accounted for - both between the at present most precisely known theoretical values of δ, as well as between experiment and theory. Another important point is the opportunity provided to obtain information on the structure and properties of corrections which are not given directly by QED. In contrast to the anomalous magnetic moment, the Lamb shift characterizes the properties of bound electrons, i.e. it takes account of not only the QED effects but the effects arising from the nuclear structure. If the corrections independent of QED are far beyond the error limits of measurements for an anomalous magnetic moment, the corrections related to a finite size of proton for the Lamb shift will turn out to be within the range of modern experiments.

To determine the Lamb shift to within an error of the order of several ppm, the accuracy given by (1) is obviously insufficient. A computer calculation likewise cannot ensure the required accuracy, mainly because of the complicated behaviour of the atom in the interferometer and the uncertainty in the field characteristics at the boundaries, i.e. near the entrance and exit openings in the electrodes.

These difficulties can be eliminated by using an interferometer consisting of two independent systems I and II (Fig. 11), separated by a variable gap ℓ. An atom travelling at a velocity v through such a double interferometer is acted upon by nonadiabatic fields in each system, and thus a mixing of the states 2s and 2p results.

In the gap between the systems, i.e. in the region where there is no field, 2s and 2p are eigenstates, and their evolution can be determined accurately. It follows that an exact expression containing several parameters determined by the action of the fields E_1 and E_2 on the systems can be written for the probability $W(\ell)_{E_1E_2}$ of the

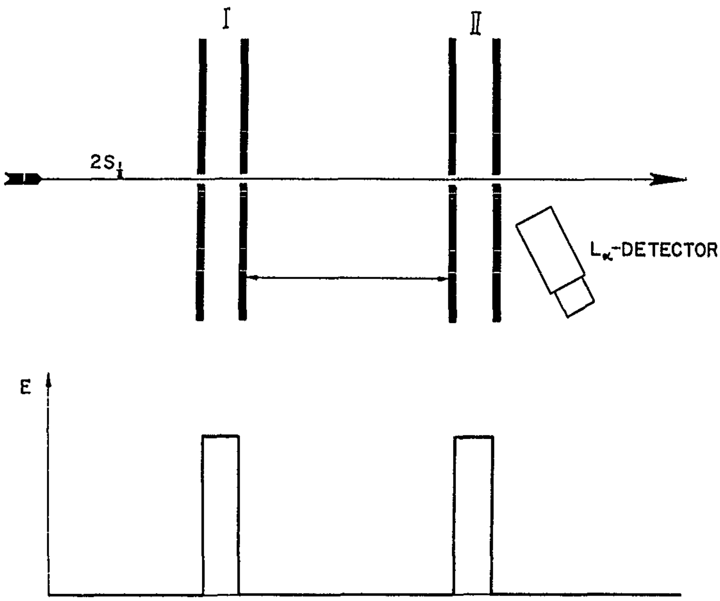

Fig. 11 Diagram of the "dual" atomic interferometer

emission of 2p atoms following the flight through the double interferometer, as a function of the length ℓ (or the time of flight $T=\ell/v$). If the conditions in the systems are kept constant while the length ℓ is varied, these parameters are fixed and need not be calculated (as shown below, the number of these parameters can be decreased to one by suitable reduction of the $W(\ell)$ curve).

It is important that when the function $W(\ell)$ is determined experimentally the variable ℓ can be chosen to be not the absolute value of the flight length, but its increment $\Delta\ell$ reckoned from a certain arbitrary null point.

As a result of the reduction of the experimental curve $W(\ell)$ we obtain an expression that does not contain unknown phases in the arguments of the cosines:

$$\Phi(\Delta\ell) = \cos\left[\frac{\omega}{v}\,(1-v^2/c^2)^{1/2}\,\Delta\ell\right] + \ell\cos\left[\frac{\omega_1}{v}\,(1-v^2/c^2)\,\Delta\ell\right] \qquad (3)$$

The described experimental data reduction procedure was developed by Dr. V.P. Yakovlev and Dr. V.G. Palchikov /2/, /3/.

The values of ω/v and ℓ in (3) were obtained using a least-squares program. In the case where the velocity remains constant, reduction of the initial data yields the values of ω/v and ℓ with an accuracy of not less than 5 ppm. It follows from (3) that to determine the Lamb frequency ν it is necessary to measure the velocity of the 2s atoms, using an independent method.

In all experiments the velocity was measured by observing the decay of the 2p atoms produced from the 2s atoms under the action of a nonadiabatic field. To determine the velocity from the experimentally obtained decay length $\ell_o = v\tau$, we must know τ, i.e. the lifetime of the 2p state. The value of τ was calculated: the error was estimated to be of the order of 1 ppm, which is acceptable for the determination of the Lamb shift with approximately the same accuracy.

It followed from the results obtained, that in 42 cases the velocity of 2s atoms could be regarded as constant. The values of ν obtained by reducing the data corresponding to the constant velocity selection criterion constitute a set of 42 values determined accurately to within several ppm. The equalisation of the accuracy is due to the fact that all the elements of the considered investigation were so constructed that the initial experimental and theoretical data, from which ν was calculated (i.e. ω/v and γ), were determined with practically the same relative error.

Figure 12 shows a histogram of the values of ν. They form a compact group with a mean value

$$\nu = 909.8934 \pm 0.0019 \text{ MHz}$$

(the error is assumed to be equal to one standard deviation). The corresponding value of the Lamb shift is

$$\delta(\text{H},n=2) = 1057.8514 \pm 0.0019 \text{ MHz}$$

If a sufficiently accurate theoretical value for the Lamb shift is available, then, as has been mentioned above, comparison with the highly precise measurements might be used to obtain information on the nuclear structure.

Theoretical values of δ were calculated by Mohr (δ = 1057.864(14) /4/ and by Erickson (δ = 1057.910(10)) /5/. Experimental values of the proton radius were obta-

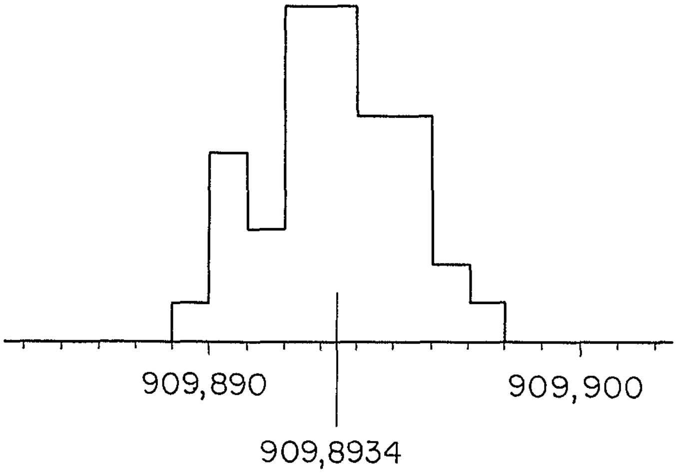

Fig. 12 Histogram of the ν - values

ined by Simon, Schmitt, Borkowsky and Walter ($\langle r^2\rangle^{1/2} = 0.862(12)$ fm) /6/ and by Hand, Miller and Wilson ($\langle r^2\rangle^{1/2} = 0.805(11)$ fm) /7/. Comparison of experimental values of the proton radius with theoretical values of δ were performed by Bhatt and Grotch /8/ and by Kinoshita and Sapirstein /9/. A set of new theoretical corrections introduced by these authors results in

$$\delta = \begin{cases} 1057.852(11), & \langle r^2\rangle^{1/2} = 0.805(11) \\ 1057.870(11), & \langle r^2\rangle^{1/2} = 0.862(12) \end{cases}$$

$$\delta = \begin{cases} 1057.857(14), & \langle r^2\rangle^{1/2} = 0.805(11) \\ 1057.875(13), & \langle r^2\rangle^{1/2} = 0.862(12) \end{cases}$$

We should emphasize the fact that the progress made by us in measuring the Lamb shift to higher precision allows one to determine the radius of the proton within the error limits 0.007 fm from the data obtained. Thus one can conclude that precise atomic spectroscopy is quite competitive in the study of interaction dynamics between electrons and protons. The advantages of such an approach are the opportunity of observing atomic states for a longer period of time and also that the corresponding experimental facilities both in size and cost are considerably more attractive than modern accelerators.

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