General QED/QCD Aspects of Simple Systems

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1. Bringing you up to date

The honor of addressing this gathering of distinguished atomic physicists came to one of us (VLT) as a shocking surprise. It is true that quite some time in the past VLT too was a member of the "Inverse Millionaires Club" - that circle of people who measure things to a fraction of a ppm - but that was so long ago that it could hardly justify my talking to you now. For a while VLT thought that the invitation was prompted by his fluency in Italian, but that turned out to be wrong, since the talks are to be given in English (presumably largely broken).

The shock of the invitation became even greater when VLT saw the title proposed for his talk: "General Quantum Electrodynamic Aspects Related to the Spectroscopy of Simple Atomic Systems". Only a committee of seasoned sadists could assign such a subject to an experimental physicist, and only an inveterate masochist could volunteer to accept it! Very fortunately the printed program had a vague title: "General Quantum Electrodynamic Aspects", but even that sounded like an impossible challenge.

Under these circumstances, after having foolishly accepted (who can resist a chance to see Pisa again?), VLT decided on the following strategem: a) change the title so as to bring this audience up to date on some modern topics less familiar to this audience than the one proposed, b) get himself a collaborator with impeccable credentials. Stan Brodsky has kindly agreed to assist VLT in an otherwise impossible task.

Paraphrasing what has been said of the famous treatise by Landau and Lifshitz, one could say "This talk will not contain a single formula by Telegdi, and not a single word by Brodsky".

This Conference is devoted to the Hydrogen Atom and its younger relatives like positronium. The latter, composed of (presumably) point-like objects, is the ideal testing ground for QED. It should hence be of interest to this audience to be reminded of the fact that the last decade has led to the discovery and detailed study of new bound particle-antiparticle systems, which we call quarkonia, since they consist of bound quark-antiquark pairs. There can in principle be as many

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such systems as there are "flavors" of quarks (e.g. s, c, b ...) in increasing order of heavyness). The most interesting ones of these are "charmonium" ($c\bar{c}$) and "bottomium" ($b\bar{b}$), since for these heavy quarks a non-relativistic description is quite adequate, ($m_c \cong 1.5$ GeV, $m_b \cong 5$ GeV ; it is amusing to note that the ground state of bottomium has about 10^4 times the mass of positronium!).

Figures 1 and 2 show, respectively, the presently well established levels of charmonium and bottomium. Today more levels are known for these systems than for positronium, and more "spectral lines" (transitions) have been identified than were known for hydrogen in Balmer's days!

What is most remarkable about these levels? Probably two facts: first, although they are hadronic states, they are long-lived; electromagnetic transitions (E1) compete in general appreciably with the emission of mesons. Second, there is really no "series limit" in the sense of ionization into $Q + \bar{Q}$ (Q = c or b).

The Ψ and Υ states are formed as sharp resonances in $e\bar{e}$ collisions. This identifies their spin (J), parity (P) and charge conjugation (C) quantum numbers readily as those of the photon: $J^{PC} = 1^{--}$. The quantum numbers of the states are readily assigned by using well-known (e.m. and hadronic) selection rules. This results in the J^{PC} values given at the bottom of Figs 1 and 2.

From a certain excitation on, the Ψ (and Υ) states can dissociate into two charge-conjugate mesons M, $\bar{M}$ according to the scheme

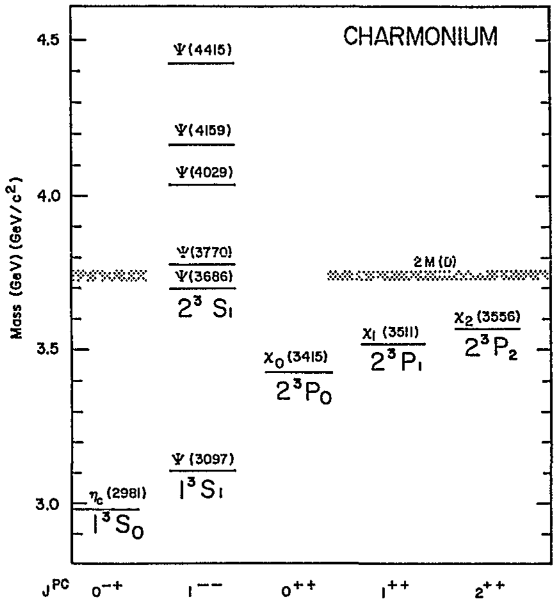

Fig. 1 Charmonium ($c\bar{c}$) spectrum. The band at mass = 2M(D) denotes the flavor threshold, above which levels are broader than those below it.

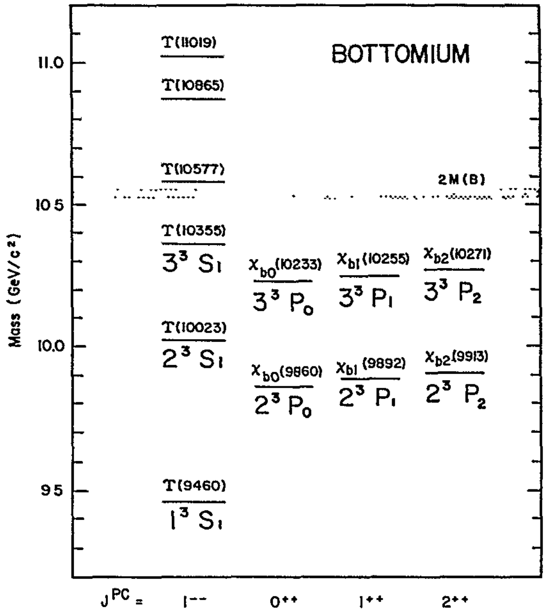

Fig. 2

Spectrum of the upsilon ($b\bar{b}$) family. Levels above flavor threshold [band at mass = 2M(B)] are broader than levels below it.

$$Q\bar{Q} \longrightarrow Q\bar{q} + \bar{Q}q = M + \bar{M},$$

where q is a very light quark (d or u). The combination ($c\bar{q}$) is called a D-meson, the combination ($b\bar{q}$) a B-meson. The corresponding thresholds are indicated in the Figs. by shaded bands. Above these, "hidden charm" turns into "open charm", "hidden beauty" into "open beauty". (The reason for a new name for the flavor "b" should be obvious.) After all the J^{PC} assignments are made, one can - within the framework of the "naive" quarkonium model - assign the standard spectroscopic labels to the levels. This is shown in the overlay. The standard n = 1 and n = 2 positronium levels appear, but in addition many excited $^3S_1$ states. The spacing of the latter indicates that the effective potential (if there is one!) is much softer than the familiar 1/r.

Many authors have proposed phenomenological potentials which yield all the observed states, and predict new ones (e.g. D states) yet to be discovered. The corresponding wave functions yield E1 matrix elements in reasonable agreement with experiment.

The task is to predict the "observed" potentials from first principles. The current theory of strong interactions, quantum chromo-dynamics (QCD), qualitatively succeeds in achieving this. This gauge theory patterned after QED is believed to explain why there is no series limit for quarkonia: quarks are forever "confined" within any hadron. It also explains why the quarkonium states are so narrow. In strict analogy with positronium, the C = -1 states can go only into three, the C = +1 states only into two C = -1 field quanta (called gluons). Indeed the $\chi(={}^3P)$ states are observed to be wider than the Ψ or Υ $({}^3S_1)$ states. We shall return to the QCD-QED analogies later.

Another novelty which deserves your attention is the nature of the beloved fine-structure constant α. It is, as we shall discuss later in more detail, a "constant" only in processes involving very small momentum transfers.

Next, and more importantly, there is the fact that QED has become but part of a broader gauge theory which includes "weak" interactions. Through the discovery of the heavy vector bosons Z^0 and $W^\pm$ at CERN this theory has been brilliantly confirmed. The photon's heavy partner, the Z^0, is exchanged between essentially all particles, not only the charged ones. Atomic parity violation experiments have confirmed this: Laporte's rule is dead. The "weak" analogs of α are also energy dependent, so that at some point the "weak" and electromagnetic forces become comparable, whereby the term "weak" loses its meaning. This is illustrated in Fig. 3.

The coupling constant of the strong interaction (QCD), α_s, decreases with increasing momentum transfer - a point we shall discuss in detail later. There have been proposals for a Grand Unified (gauge) Theory, GUT, where all three interactions become equally "strong" at some very high energy. This is also indicated in Fig. 3.

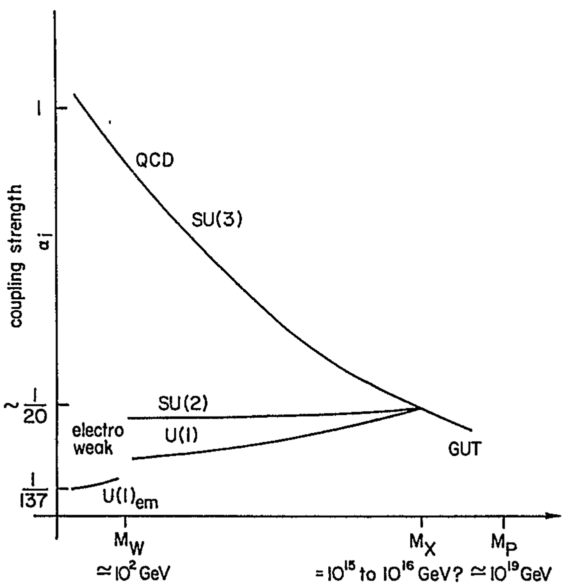

Fig. 3

QED is the model gauge theory after which all others are patterned. We shall divide the discussion of its current status into two parts: Closed subjects, and open subjects. To these one may refer respectively as the "rug" and the "dirt", recalling Feynman's famous statement that he got the Nobel Prize for being better than others in sweeping the dirt under the rug.

2. "Closed" subjects (the "rug")

QED, which is supposed to provide finite answers to all orders of perturbation theory (AOPT), can be represented as resting on a foundation (local gauge invariance) and on three pillars (see Fig. 4). Local gauge invariance implies that the theory is invariant under arbitrary phase transformations of the electron field at each point in space and time. The generalization of this principle to invariance under unitary matrix transformations of the fermion fields leads to the concept of non-Abelian gauge theories which include quantum chromodynamics and the unified electroweak theory. The three pillars are:

2.1 Renormalization theory, and in particular the treatment in terms of the renormalization group. The latter goes back to an idea of Petermann and Stueckelberg, and was formulated quantitatively by Gell-Mann and Low. The essence is that only the observed mass m and the observed charge e of the election (and/or its heavier brother leptons μ and τ) enter into the final results. Ultraviolet infinities ($k \rightarrow \infty$) are consistently eliminated to AOPT. The coupling is characterized by a "running" coupling constant which incorporates vacuum polarization to all orders, viz.

$$\alpha_r(Q^2) = \frac{\alpha(Q_0^2)}{1-\Pi\ (Q^2/Q_0^2)} \tag{1}$$

where Q = (4-momentum) of interest, and Q_0 a "reference" 4-momentum. The function Π is given by

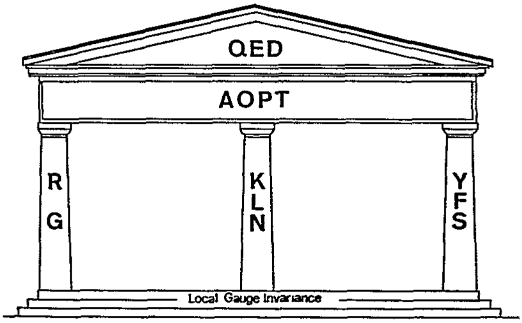

Fig. 4

$$\Pi = \frac{\alpha(Q_0^2)}{3\pi} \ln(Q^2/Q_0^2) + \ldots \tag{1a}$$

where both Q_0^2 and $Q^2 >> m_l^2$ (lepton mass). Reinterpreting things in coordinate space, (1) simply means that the effective coupling decreases with increasing distance: one observes the shielding due to virtual pairs. At extremely small distances where $\Pi(Q^2)$ is of order 1, i.e., ~ 10^{-281} cm, one could have a blow-up ("Landau singularity") where the theory becomes undefined; this may however be "cured" by the unification of QED with other interactions.

The current, rather successful, theory of strong interactions, QCD, is patterned after QED. It is a scenario where quarks play the rôle of leptons, massless vector gluons the part of the photon (gauge bosons), and "color" that of the charge. The big difference with electromagnetism is that both the sources (quarks) and the fields (gluons) carry color, i.e. charge. One is again led to running coupling constant analogous to α_r, viz.

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1-\Pi(Q^2/Q_0^2)}, \quad \Pi = -\left[(11 - \tfrac{2}{3}n)\frac{\alpha_s(Q_0^2)}{4\Pi} \ln(Q^2/Q_0^2)\right] \tag{2}$$

n = number of flavors

with, however, effectively a plus sign in the denominator. As $Q \to \infty$, i.e. $r \to 0$, the coupling becomes weaker, one has antishielding (in current slang, this plenomenon is called "asymptotic freedom"). It makes it possible to justify the soft potentials corresponding to the observed levels (Figs 1, 2) of the quarkonia. We mention in passing that in virtue of the quark spins and of the vector nature of the gluons one has the fine structures so dear to atomic physicists.

2.2 The Kinoshita-Lee-Nauenberg (KLN) theorem

This theorem, of rather formal character, guarantees that one may (summing over the final states of any inclusive e.m. process) let the lepton mass m_l tend to zero without creating terrible havoc.

2.3 The Yennie - Frautschi - Suura relation

This relation, similar in essence to the old Bloch-Nordsieck theory, guarantees the absence of catastrophes (infra-red divergencies) in the limit $k \to 0$. Such a catastrophe could be anticipated, but obviously does not happen in, say, elastic electron scattering where the final state electron could radiate an infinite number of softer and softer photons.

From these three "pillars" and the "foundation" of local gauge invariance, one can derive - besides the innumerable atomic properties you are all familiar with - many important consequences. These are either interesting in themselves, or through the fact that they are readily generalized to strong interactions (QCD). We discuss a few:

<u>2.3.1 Scale invariance at large momentum transfer</u>

This means that in an inclusive reaction like

$$e + \bar{e} \rightarrow \gamma^* \rightarrow \mu + \bar{\mu} + X \qquad (3)$$

where X = any neutral state composed of leptons and photons

the cross section exhibits, to AOPT, a pointlike behaviour (thus scale invariance meaning that no lengths appear in the formulae):

$$\sigma(e + \bar{e} \rightarrow X) = \frac{4\pi\alpha(Q^2)}{3Q^2}\left(1 + \frac{3}{4}\frac{\alpha(Q^2)}{\pi} + C_2\left(\frac{\alpha(Q^2)}{\pi}\right) + C_3\left(\frac{\alpha(Q^2)}{\pi}\right)^3 + ..\right. \qquad (4)$$

(valid for $Q^2 \gg 4m^2_\mu$).

Note the absence of terms in $\ln m_l$, a consequence of the KLN theorem. The reaction (3) is not one of purely academic interest. In fact, in $e\bar{e}$ colliders the muon pair production is used in practice to monitor the luminosity of the machine, i.e. for normalization purposes. We shall come back to the term in C_3 at a later point.

It is interesting to replace the leptons in (3), either in the initial or the final state, by quarks. We thus consider

$$e + \bar{e} \rightarrow (q + \bar{q}) + X \qquad (5)$$

and

$$q + \bar{q} \rightarrow \mu + \bar{\mu} + X \qquad (6)$$

The brackets in the first reaction represent the fact that the quark and antiquark never appear as isolated physical particles in the final state. They can be produced in a <u>bound state</u> (of spin-parity 1^- equaling that of the γ^*). Such pairs are precisely the ^3S quarkonia shown in Figs 1, 2. Their production cross sections contain factors allowing for the fractional charges of the quarks and for their "color". Process (5) represents muon pair production in the collision of any two hadrons, to the extent that these contain (real or virtual) $\bar{q}$'s. In the jargon it is called the "Drell-Yan" process; it has been the subject of much experimental investigation, and is one of our major sources of information about the quark "wavefunction" of hadrons.

Finally, one may replace the leptons on both sides of Eq.(3) by quarks. Electromagnetism then plays a subordinate rôle, so that the virtual photon γ^* has to be replaced by a virtual gluon g*. Thanks to the gauge structure common to QED and QCD, the essential results remain valid in the latter, with $\alpha_s(Q^2)$ replacing $\alpha(Q^2)$.

2.3.2 Scaling and scaling violation at large momentum transfers ("deeply inelastic" scattering)

Consider (for pedagogical reasons!) the process

$$\mu + e \rightarrow \mu + e\,. \tag{7}$$

One has for the differential cross section without radiation

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha_r^2}{s} f(\vartheta)\,, \qquad (\sqrt{s} = \text{c.m. energy}) \tag{8}$$

which can be generalized to AOPT and to QCD processes. Next consider, to please the tastes of atomic physicists, the inelastic scattering of electrons by muonium

$$e + (\mu\,\bar{\mu}) \rightarrow e' + X\,. \tag{9}$$

Because of the inelasticity, one has now a doubly differential cross section, which can be written as

$$\frac{d\sigma^2}{dQ^2\,dx} = \left(\frac{d\sigma}{dQ^2}\right)_{e\mu} F(x) \tag{10}$$

where x is the dimensionless scaling variable

$$x \equiv \frac{Q^2}{2P\cdot q} \cong \frac{(\text{momentum transfer})^2}{M\ (\text{energy transfer})} \tag{11}$$

with P the 4-momentum and M the mass of the "incident" muonium. Equ. (10) is the basis of the parton model of deeply inelastic scattering of leptons, where the rôle of the muons in our "pedagogical" example is played by the quarks. The elastic collision between quark and lepton is turned into a (deeply) inelastic scattering of the lepton by the hadron, the final state X consisting of real hadrons rather than free partons.

Because of the gauge nature of QCD, entirely similar arguments hold for parton-parton collisions. Radiative corrections are, however, generally more important here, because α_s (s for strong!) is, at given Q^2, larger than $\alpha(Q^2)$: gluons are more easily radiated then photons! Consider reaction (7) with photon radiation by the incident muon. The differential cross section (8) is modified as

$$\frac{d\sigma}{d\Omega} = \frac{\pi\,\alpha_r^2}{s} f(\vartheta)\ [1 + \frac{\alpha}{\pi} \ln \frac{Q^2}{m_\mu^{\,2}} \ln \Delta E/E] \ , \tag{12}$$

where $\Delta E/E$ is an experimental resolution. Similarly, the "structure function" $F(x)$ of the $\mu^+\mu^-$ atom in Equ. (10) becomes

$$F(x, \ln Q^2/Q_0^2) \tag{13}$$

Thereby scale invariance is broken, although no explicit dependence on a length enters. Again, a logarithmic dependence as in (13) is taken over into QCD. All structure functions "evolve", as was shown by Gribov and Lipatov, and by Altarelli and Parisi.

2.3.3 Low-energy theorem in Compton scattering

One can show that the forward scattering anglitude is given, as $\omega \to 0$, to AOPT for any spins by

$$f(0) = -\frac{e^2}{m} \vec{\epsilon}\,' \cdot \vec{\epsilon} - i\omega\mu_a^2\ (\vec{S}/S) \cdot \vec{\epsilon}\,' \mathrm{x}\ \vec{\epsilon} + O(\omega^2), \tag{14}$$

where $\mu_a = \mu - eS/m$ defines the anomalous moment for any spin. This relation, in combination with the optical theorem, enables one to set limits on the composite scale of leptons. It also implies that the normal g-factor $g = (\mu/S)/(e/2m)$ of any pointlike particle is 2. Indeed if the electron or muon were composite, i.e. if they had internal excitations at the mass scale Λ, their anomaly $a = \frac{g-2}{2}$ would be of order (m_e/Λ) or $(m_e/\Lambda)^2$.

The two cases depend whether or not the interactions of the underlying theory resemble gauge theories and conserve chiral invariance. In either case, the present agreement between theory and experiment for the electron and muon anomalous moments rules out an internal scale below 1 TeV, [see e.g. S. J. Brodsky and J. Primack, Ann. Phys. 52, 315 (1969). S. J. Brodsky and S. D. Drell, Phys. Rev. D22, 2236 (1980).]

2.3.4 Renormalization of the weak angle Θ_W.

The standard theory of electroweak interactions contains two coupling constants but only one free parameter, the Weinberg angle Θ_W. The latter fixes the e.m. - weak connection:

$$e = g \sin\Theta_W = g' \cos\Theta_W \tag{15}$$

as well as the mass ratio of the two heavy gauge bosons:

$$m_W/m_Z = \cos\Theta_W. \tag{16}$$

Since e, i.e. α, is a "running" coupling constant (see above), it is clear that Θ_W itself must be "running". These considerations are of interest for two reasons: (i) they will tell us at which energy e.m. and "weak" interactions will become equally "strong", (ii) by determining Θ_W at two energies, one can experimentally verify the gauge nature of the theory.

2.3.5 The Nambu-Bethe-Salpeter (NBS) equation

This covariant two-body equation, with which this audience is certainly familiar, allows to solve everything in principle, but little in actual practice. This is for two reasons: (i) one needs an infinite number of kernels, (ii) even in the ladder approximation no analytic solution for QED has been produced.

One interesting consequence of the NBS equation is that by its reduction (in the case of two quarks) a Schrödinger equation with a non-local potential emerges.

See also comments below under "open problems".

3. Open problems ("the dirt")

3.1 Does the perturbation series in QED converge?

Nobody knows the answer, but perhaps there is no answer within the old classical framework, i.e. in a world made of leptons and photons alone. Indeed charged leptons interact with each other by both γ and Z^0 exchange, a fact already verified by experiment (μ-pair asymmetry in $e\bar{e}$ collisions). There are "grand" schemes to unify electroweak and strong (QCD) forces, giving them equal strength at some very high (say 10^{14} GeV) energy. In such schemes the "Landau singularity" might be cured.

There exist some exciting warnings from PT that the PT series may not converge. Let us mention two:

3.1.1 The decay rate of $^3S_1$ positronium

The current theoretical prediction is

$$\Gamma = \Gamma_0[1-10.282(\alpha/\pi) + \frac{1}{3}(\alpha^2\ln\alpha)+(300\pm30)(\alpha/\pi)^2]. \tag{17}$$

The unexpectedly large coefficient of the last, experimentally determined term might well be the presage of worse things to come! A similar behavior in QCD, say in the analogous 3-gluon annihilation of $^3S_1$ charmonium, would be a real disaster, since α_S is larger than α.

3.1.2 Radiative corrections to QCD Born cross section

The inclusive cross-section for e+e $\longrightarrow$ hadrons is given by

$$\sigma = \sigma_0 \left[1 + \left(\frac{\alpha_s}{\pi}\right) + 1.41 \left(\frac{\alpha_s}{\pi}\right)^2 f - 64.809 \left(\frac{\alpha_s}{\pi}\right)^3 + ..\right]$$

as reported by Gorishny, Kataev and Larin (Dubna). This may be, if confirmed by independent calculations, an indication of the breakdown of the PT series in gauge theories.

3.2 Progress on the relativistic 2-body equation

There are three methods other than NBS. In the approach of Grotch and Yennie one uses an effective Dirac equation with non-local potentials derived from $e\bar{e}$ scattering. In a more recent method, that of Caswell and Lepage, one starts from an effective Schrödinger equation, again with non-local potentials. Both methods have been used to calculate higher order terms for ep, $e\bar{e}$ and $e\bar{\mu}$ atoms. A third approach, currently being used by S. Brodsky, T. Eller, H.C. Pauli and A. Tang, is that of "discretized light-cone quantization". These authors directly (i.e. numerically) diagonalize the light-cone Hamiltonian, of course with a truncated basis of Fock states. This yields both the mass spectrum (levels) and the wave functions. The method works for any α, but results have only been reported to date for 1 + 1 dimensions.