

# Highly Excited Hydrogen in Strong Microwaves: Experimental Tests for Classically Chaotic Semiclassical Dynamics and for Quantum Localization

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In recent years the hydrogen atom strongly perturbed by external fields has become a major testing ground in the development of quantum dynamics, the study of general features for the time evolution of nonintegrable quantum systems. We discuss some reasons for this, using for illustration some recent results for the case of an external sinusoidally oscillating electric field.

## 1. Nonlinear dynamics and Quantum Mechanics

In classical Hamiltonian nonlinear dynamics the motion is deterministic and given by Hamilton's equations. On the other hand, quantum time evolution is unitary and controlled by Schrodinger's equation. Very little is known about the general properties of time evolution in quantum systems for which Schrodinger's equation cannot be solved either analytically or by means of perturbation theory. The path integral formulation of quantum mechanics introduced by Feynman is a conceptually useful starting point. When actions are large, a stationary phase situation can lead to a semiclassical regime where the time propagator for the system explicitly reflects the time evolution of the classical action function.

Classical evolution can exhibit interesting nonlinear dynamics. Changes in the character of the evolution can arise with changes in a control parameter, such as period doubling bifurcations and transitions from regular to chaotic

simplicity of the one electron atom, both of these systems are amenable to accurate numerical calculations based on either quantum mechanics or on the mechanics of the system in the hard classical limit. Both systems are within the reach of modern laboratory techniques for direct experimental study. Both exhibit chaotic classical electron trajectories at high external field strengths. The magnetized HEHA is being studied using spectroscopy to ascertain the Wigner energy level separation distribution produced by the classical chaos, as is discussed by others at this Symposium. Again because of underlying chaos, at some microwave frequencies the microwave driven HEHA is predicted to exhibit a microwave energy absorption mechanism associated with a diffusion in electron classical action space that culminates in ionization of the atom [4]. In addition, at some microwave frequencies higher than the initial classical electron orbit frequency for the atom, a second predicted consequence of the chaos is a dynamical type of Anderson localization in quantum number or electron energy space that arises from destructive wave interference presumably produced by the degree of randomness present in the underlying chaos.

### 3. Developments in the Experimental Study of the Microwave Driven Weakly Bound Electron (Microwave Driven HEHA)

Since the initial experimental discovery of microwave ionization of HEHA requiring the absorption of some hundred microwave photons [5] and the initial discovery that classical models can predict the ionization probability [6], the experiments have improved through the development of new techniques. These have hinged upon various uses of applied static electric fields, where the separability of the quantum problem in parabolic coordinates  $n, n_1, m$  that is so special to the hydrogen atom plays a crucial role in maintaining the simplicity of the system. One development was an optical double resonance technique for the laser excitation of atoms into single Stark states, all parabolic quantum numbers being well defined [7]. Here a mixed state fast hydrogen atom beam

was first passed into a region of strong static electric field, where a completely state selective  $m = 0$  to  $m = 0$  infrared CW laser transition to one  $n=10$  Stark substate was made possible by large second order Stark energy level splittings. The atoms then passed into a weak static field, where a second infrared CW laser transition produced atoms in a desired final Stark state having the desired principal quantum number  $n_0$  within the range 30 to 90. This ODR technique has been used to produce HEHA that are electrically polarized or "stretched" maximally along the static electric field direction. When a linearly polarized microwave electric field is also applied along the same direction, the resultant microwave driven HEHA constitutes a nearly one dimensional system, as has been verified by experiment [8]. The verification again crucially involved static electric fields that were used to selectively field ionize atoms with different quantum numbers, thereby separating the probabilities for different types of atoms being produced by the applied microwave field pulse. The measurement of final bound state probability distributions (in a one dimensional system) was made possible only by combining all the static field dependent developments mentioned above for the hydrogen atom [8,9].

### 3.1 Experimental Evidence of an Underlying Role for Classical Chaos in Microwave Driven HEHA

A schematic of the apparatus recently used for microwave driven HEHA studies is shown in Figure 1. A fast atom beam containing laser excited electrically polarized HEHA comes from the right, passes sideways through a WR-62 waveguide with holes electric discharge machined in its narrow sides, and goes to the left for both charge state analysis and final bound state distribution analysis via the differential state-selective static electric field ionization technique. The atom transit time between the planes of the waveguide holes is 7.5 nsec. A microwave source generates a traveling wave in the waveguide that is finally absorbed in a matched load. As the TE<sub>10</sub> mode present in the

waveguide has the sine dependence of the electric field shown in the figure, this is the pulse envelope seen by the atoms in their rest frames, ignoring the small loss of microwave power out of the holes which adds small long tails to the envelope. For a microwave frequency of 18 GHz, the nominal microwave total pulse length was 135 microwave periods with a total spread of 6 periods due to the velocity spread of the atom beam and a comparable additional spread expected from the range of different fringe fields seen by various atoms entering the waveguide at different points within the holes. The microwave electric field shown is along the single direction of all the static electric fields seen by the HEHA during their brief microsecond histories in the apparatus. (As spontaneous radiative decay times for HEHA increase with the cube of  $n$ , our HEHA are by themselves essentially stable atoms.) The primary source of microwaves is a frequency synthesizer. A measurement using a microwave spectrum analyzer placed an upper bound on the possible broadband noise r.m.s. power of 0.01% of typical sinewave output powers. The addition of a narrow pass filter tuned to the sinewave frequency that reduced the broadband noise bandwidth by a factor of 100 was found to not change experimental results. Before intersecting the atom beam, the microwave traveling wave passed into the vacuum system through a window; the remainder

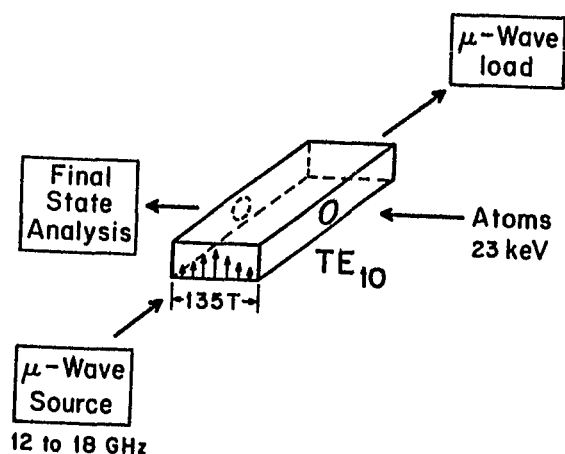


Fig. 1. Schematic of apparatus for HEHA in microwaves

of the microwave system was evacuated to eliminate the need for a second window, thus reducing the possible standing wave power seen by the atoms to less than 2% of the traveling wave power.

For microwave frequencies  $\omega$  below the initial classical electron orbit frequency, experimental "threshold" microwave field strengths for effective ionization are found to be close to classical threshold field strengths for chaos. This is true when the experimental threshold is set for 10% ionization probability, the experimental microwave pulse time is about 300 periods, and the initial quantum number value is well below a cutoff value of  $n=95$  for bound state contributions included in the ionization probability. The classical threshold for chaos is defined by a 1% probability for an ensemble of atoms with initial action  $I_0$  but uniform in action-angle variable to have chaotic classical trajectories while in the microwave field. For ionization and chaos thresholds so defined, Figure 2 shows the comparison [10,11], along with more recent ionization data obtained with the short pulse apparatus of Figure 1 [12]. For comparison purposes, the short pulse threshold field values have been multiplied by a factor of 0.70 in an attempt to account for the differences in pulse length. Although both sets of experimental data are not for the case of a pure beam of electrically polarized atoms, nevertheless the agreement with the predictions of the one dimensional classical model is excellent below  $\omega_0=n^3\omega=1$ . At higher microwave frequencies than this, the experimental thresholds are higher than the classical ones, suggesting a deviation due to quantum mechanics that might be the Anderson localization effect mentioned above. Such a deviation has been predicted to be observable for  $\omega_0 = 1$  and above [13]. The short pulse data of Figure 2 do not definitely confirm this at present, as the normalization factor of 0.70 used for comparison has not been justified for the higher microwave frequencies.

The short pulse ionization data was obtained at a static electric field value of 0.87 V/cm within the microwave waveguide region that was produced by the atoms' motion in an externally applied uniform magnetic field. The dependence

of the ionization threshold field on static field strength has been measured at constant bound state contribution cutoff quantum number value. As the static field is reduced from 6.25 V/cm to 0.87 V/cm, the ionization threshold rises in a way that depends on  $\omega_0$ . Thus the short pulse thresholds in Figure 2 should not be higher than that which would be obtained at the zero static field value assumed for the classical theoretical curve.

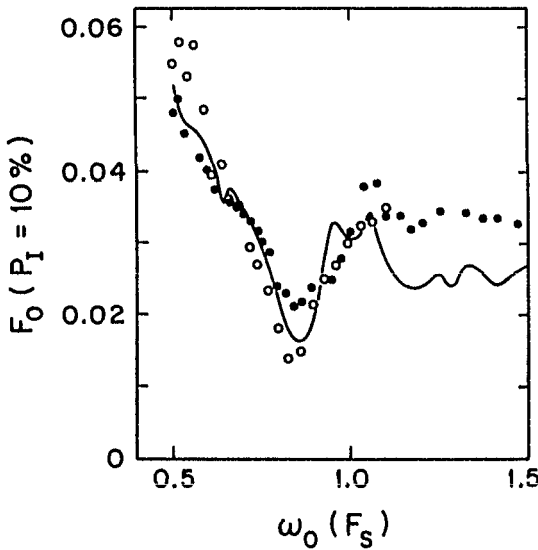


Fig. 2. Microwave field thresholds for classical chaos (solid line) and for observed 10% ionization probability  $\circ$  [10-11],  $\bullet$  [12]

Figure 3 shows a comparison of final bound state probability distributions for experiments with electrically polarized HEHA [14] and for purely classical predictions based on a one dimensional model [13]. The apparatus of Figure 1 was used for the experiments. A static field of 6.25 V/cm was included in both the theory and experiment. It is seen that the experimental final state distribution is near classical, having a large secondary maximum near  $n=66$  away from the initial value of 72. For  $n$  above 72, the data drop on average exponentially with increasing  $n$ , a behavior predicted by the Fokker-Planck equation for the classical diffusion in action space associated with the random walk character of the chaotic electron trajectories [15].

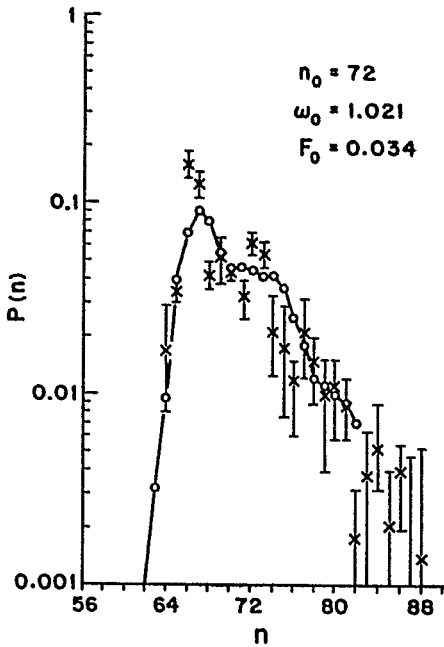


Fig. 3. Classical (circles [13]) and experimental (crosses, [14]) final bound state distributions

### 3.2 Experimental and Quantum Mechanical Localization Lengths in Action Space

The quantum theory of the dynamic Anderson localization predicts that as time evolves, the quantum distribution in action (quantum number) space first follows classical predictions, then differs from it after a quantum "break time", and finally stops changing after a quantum "freezing time" [4,15]. Let us characterize the exponential dependence of the final state distribution on increasing quantum number by the equation

$$P(n) = P_0 \exp [-2(n-n_0)/\ell], \quad n > n_0, \quad (1)$$

where  $\ell(t)$  is the apparent localization length at the time  $t$ . Classically  $\ell(t)$  grows indefinitely with time, while quantum mechanically it freezes to a

constant value after a time that can be tens to hundreds of microwave periods. Figure 4 shows a comparison of experimental and quantum localization lengths obtained for microwave field strength, frequency and pulse time values theoretically expected to be in the quantum Anderson localization "freezing" regime [13, 16]. The experiments again used the apparatus of Figure 1 [14]. The agreement supports the predictions of numerical quantum mechanical calculations and suggests that dynamic Anderson localization may be a reality. However, definitive experimental verification of this requires a direct verification that the localization length freezes in time. New experiments at much longer microwave pulses times are needed to strongly discriminate between classical and quantum predictions for times much longer than the quantum freezing time.

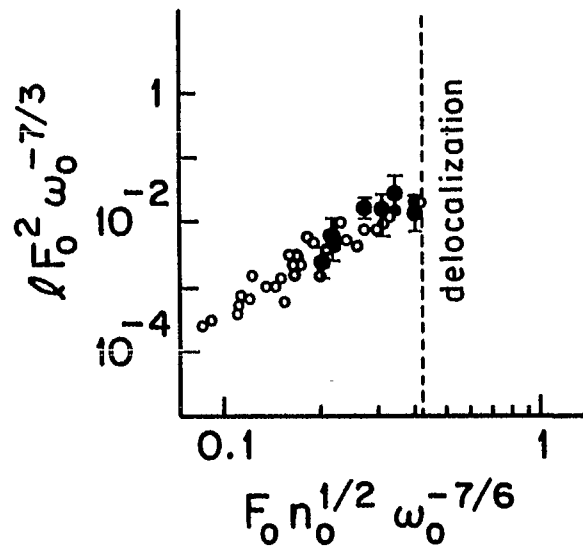


Fig. 4. Quantum mechanical [16] (open circles) and experimental values of scaled localization length [14]

We have seen that the hydrogen atom makes possible the study of three signatures of underlying classical chaos in semiclassical quantum systems,



namely the Wigner nearest neighbor energy level separation distribution, the Fokker-Planck diffusive final state distribution and the dynamic Anderson localization effect. This progress promises major contributions to the development of the field of quantum dynamics.

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