Quasi-Landau Spectrum of the Chaotic Diamagnetic Hydrogen Atom

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INTRODUCTION

The highly excited hydrogen atom in static magnetic fields has been in recent years a subject of intense experimental [1-4] and theoretical [4-9] studies which have led to substantial progress in the understanding of this previously unsolved elementary problem. Described by the Hamiltonian (in atomic units)

$$H = \frac{1}{2} p^2 + \frac{1}{2} \gamma L_z + \frac{1}{8} \gamma^2 \rho^2 - \frac{1}{r} \tag{1}$$

(cylindrical coordinates, $r = (\rho^2 + z^2)^{1/2}$; field parameter $\gamma = B/B_o$ with $B_o = 2.35 \times 10^5$ Tesla) the magnetized atom is of particular interest in the quasi-Landau regime of strong mixing of the Coulomb and diamagnetic interactions, i.e. where the two forces are of comparable strength. In this regime the motion of the Rydberg electron becomes classically chaotic [10]. It is this aspect which has recently attracted much attention, as the magnetized atom constitutes an ideal model case for detailed experimental studies of the quantum mechanics of a most simple atomic system in classical chaos.

Until recently it has been generally accepted that the physics of atoms in the quasi-Landau regime was essentially represented and determined by the quasi-Landau resonances discovered by Garton and Tomkins in the absorption spectrum of alkaline earth atoms [11] and explained first by Edmonds [12] as resulting from two-dimensional bound motion of the electron on closed classical orbits in the $(z = 0)$-plane perpendicular to the magnetic field axis. It was therefore with great surprise when new quasi-Landau resonances were unexpectedly discovered in first experiments with the hydrogen atom [1, 2] and that they are correlated to three-dimensional periodic orbits through the proton as origin [1, 2, 8]. In this paper we briefly present the state of the experimental investigation on the magnetized hydrogen atom.

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EXPERIMENTAL

Hydrogen atoms are excited by independently tunable pulsed laser light in two steps,

$$H(1s) + h\nu_1(vuv) \rightarrow H(2p) + h\nu_2(uv) \rightarrow H^* \tag{2}$$

in a crossed atom-laser beam arrangement (Fig. 1) at the center of the magnetic field, with the atomic beam directed parallel to the field axis. In the first step single sublevels of the Paschen-Back manifold ($m = 0, \pm 1$) of the 2p-state are selectivly prepared by linearly polarized vuv-laser light tunable in the region of the Lyman-α wavelength (121.6 nm). From there final states are excited with even parity and quantum numbers $|m| = 0, 1,$ or 2 around the ionization limit by scanning the uv-laser, also linearly polarized. Further experimental details can be taken from previous papers [1-3].

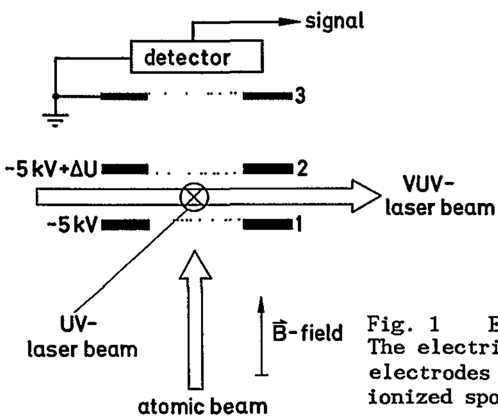

Fig. 1 Experimental crossed laser-atom beam set-up. The electric field arrangement with the two parallel electrodes serves for the detection of H^*- atoms, ionized spontaneously or by the electric field.

RESULTS AND DISCUSSION

Fig. 2 shows a spectrum taken at a magnetic field strength of B = 5.96 T and excited to final even parity states with magnetic quantum number $m = 0$. Also shown in Fig. 2 is a theoretical stick spectrum obtained by quantum mechanical calculation via numerical solution of the Schroedinger equation [4, 5]. Within the precision and resolution limits (1.5 GHz) of the experiment the theoretical and experimental spectra are in good agreement. More recently theoretical spectra have been calculated up to close to the ionization threshold [13].

Fig. 3a shows spectra with magnetic quantum numbers $m = 0$ and $m = -1$ in the chaotic regime from $-30\ cm^{-1}$ up to $+ 30\ cm^{-1}$ in the continuum region. At this resolution the energy quantum spectra have seemingly lost the oscillatory structures of quasi-Landau resonances discovered at lower resolution [1]. Periodic modulations

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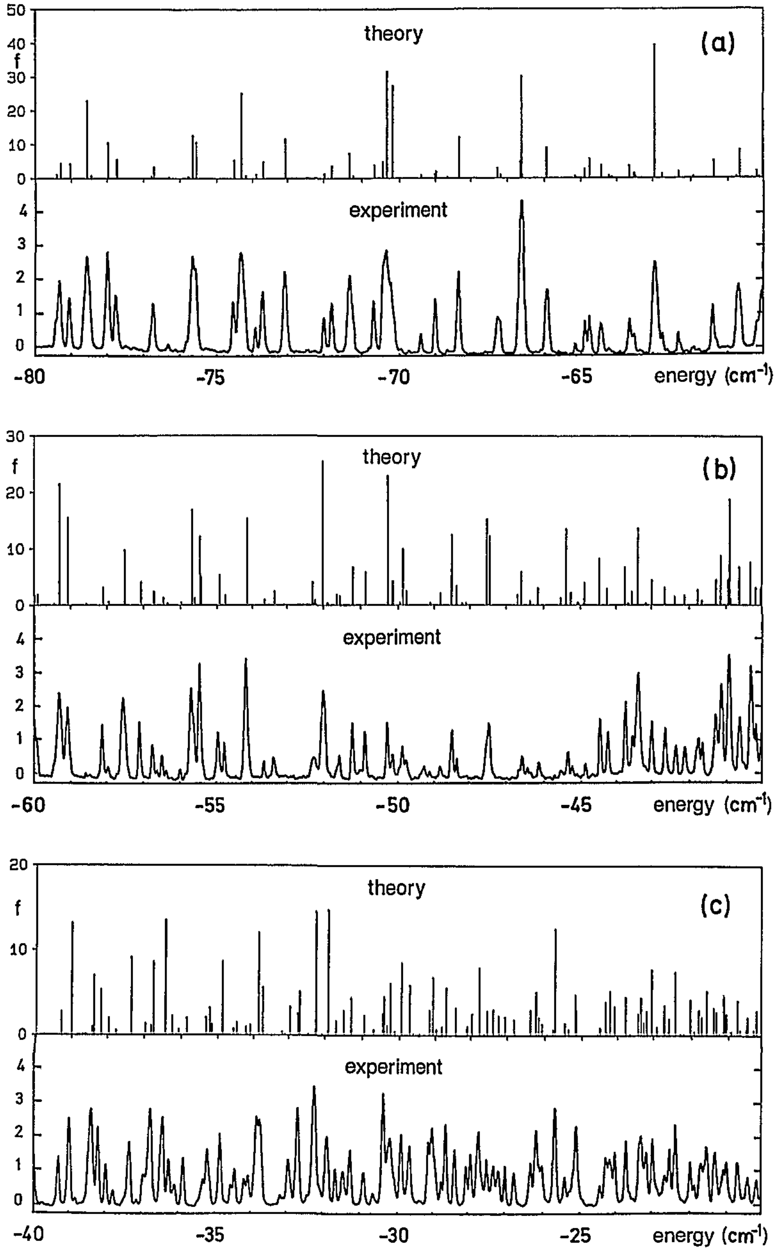

Fig. 2 Rydberg atoms in a magnetic field of 5.96 T: comparison between the theoretical oscillator strength spectrum and the experimental photoabsorption spectrum for $\Delta m = 0$ Balmer transitions to $m = 0$, even parity final states. Oscillator strengths are given in units of 10^{-6}, the experimental intensity scale is in arbitrary units.

(in line density and/or oscillator strength) in high resolution energy spectra can, however, be recovered by Fourier transformation in time-domain. Following Gutzwiller [14] such energy spectrum oscillations can be rationalized by long-range periodic orbits of the electron motion, also in the classically chaotic regime.

Fourier transformation of the spectra in Fig. 3a yields time-domain spectra (Fig. 3b) clearly exhibiting a number of resonances at times T_i, related to the energy modulation spacing $\Delta\epsilon_i$ by $\Delta\epsilon_i T_i = 2\pi\hbar$. The spectra in Fig. 3b exhibit a multitude of new resonances, in addition to the known Garton-Tomkins one. Each of them can be correlated to a closed classical three-dimensional orbit of the Rydberg electron through the proton as origin, with T_i the recurrence time of the respective orbit. The orbits shown in Fig. 3, plotted in (ρ,z)-projection, are members of a series of resonances ("fundamental" series) discussed and analysed in detail elsewhere [2, 8]. The Fourier spectra in Fig. 3b have recently also been treated theoreti-cally by Delos [9] on the basis of the periodic-orbit theory by Gutzwiller [14].

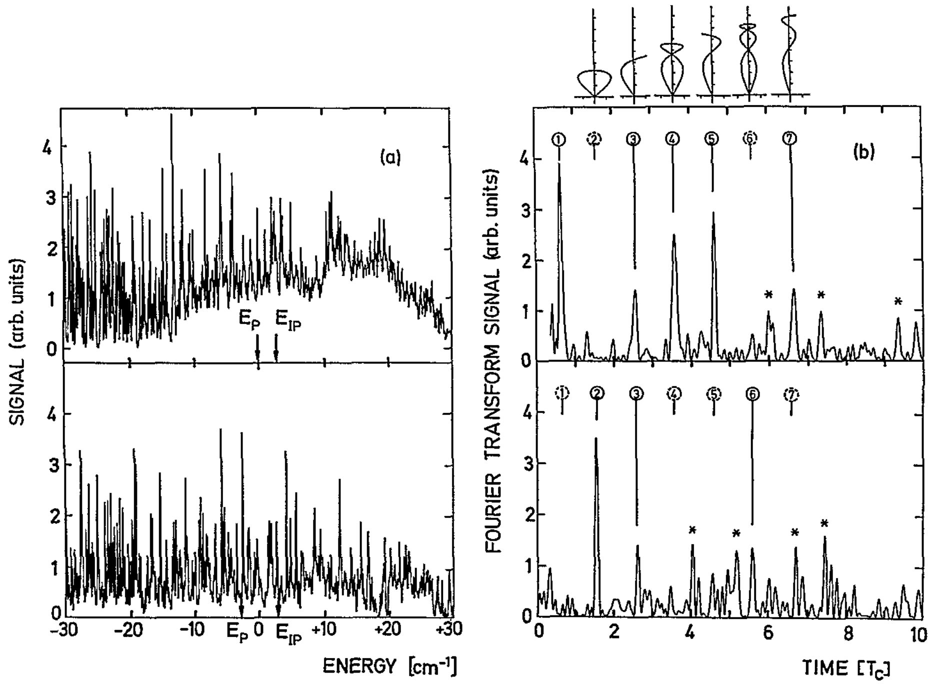

Fig. 3 (a) Excitation ionization spectra of H-atom even parity Balmer series around the ionization limit in a static homogenous magnetic field of B = 5.96 Tesla. Above: magnetic quantum number m = 0; below: m = − 1. (b) Fourier-transformed spectra of (a). Plotted is the absolute value squared. Abscissa with time scale normalized to cyclotron period T_c. Corresponding to resonances ② to ⑦ calculated closed classical orbits of electron motion are shown in (ρ,z)-projection.

The discovery of a multitude of new quasi-Landau resonances raises the basic problem as to the "entire" manifold of resonances arising as a function of the energy (E) and field strength (B) in the excitation of final states with given parity and magnetic quantum number. Spectra like in Fig. 3a taken at constant B as function of E can deliver resonances T_i in the Fourier transform only when the oscillation spacing $\Delta\epsilon_i$ in the energy quantum spectrum is not, or at most weakly, energy dependent within the integration intervall. We have solved this problem by employing "scaled-energy spectroscopy", which allows to observe also strongly energy dependent quasi-Landau resonances. Following theory [10], the technique is based on the scaling property of the Hamiltonian with respect to E and B. With the scaling relations

$$\vec{\tilde{p}} = \gamma^{-1/3}\,\vec{p},\quad \vec{\tilde{r}} = \gamma^{2/3}\vec{r} \tag{3}$$

the Hamiltonian (eqn. 1) transforms to

$$\tilde{H} = \frac{1}{2}\,\vec{\tilde{p}}^{\,2} + \frac{1}{2}\,\tilde{L}_z + \frac{1}{8}\,\tilde{\rho}^2 - (\tilde{\rho}^2 + \tilde{z}^2)^{-1/2} \tag{4}$$

which is no more dependent on B and E independently, but on the scaled energy

$$\tilde{E} = E\gamma^{-2/3} \tag{5}$$

only. Transformation of the semi-classical quantization condition with the two non-separable coordinates ρ, z to scaled variables yields

$$\oint_i (\tilde{p}_z d\tilde{z} + \tilde{p}_\rho d\tilde{\rho}) \equiv C_i = n\gamma^{1/3} \tag{6}$$

where i denotes a given closed orbit. For $\tilde{E}$ = constant the action integral is constant, so that a spectrum taken on a scale linear in $\gamma^{-1/3}$ will consist of equidistant lines for each given i. Fourier transformation of the $\gamma^{-1/3}$-spectrum to conjugate coordinates $n\gamma^{1/3}$ thus results in one action resonance C_i only for each i. Fig. 4 shows, as an example, a scaled-energy spectrum taken at $\tilde{E} = -0.45$, and plotted as function of $\gamma^{-1/3}$. In taking this spectrum E and B have been varied simultaneously such that the condition $\tilde{E} = E\gamma^{-2/3}$ = constant was obeyed. The right part of the figure shows the action resonance spectrum obtained by the Fourier transformation of the left spectrum. Also shown are calculated closed orbits correlated to the respective C_i resonances. Such scaled-energy spectra have been measured over a range of $\tilde{E}$ from $\tilde{E} = -0.5$ to $\tilde{E} = +0.2$.
Fig. 5a shows in concise overlay fashion a set of action spectra taken at different scaled energies in steps $\Delta\tilde{E} = 0.01$, representing the evolution of the entire quasi-Landau spectrum as function of $\tilde{E}$ and C from the regular into the chaotic

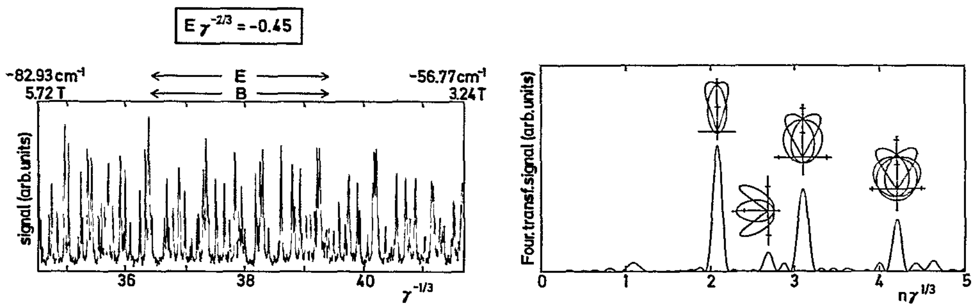

Fig. 4 (a) Scaled-energy spectrum at $\tilde{E} = -0.45$ as a linear function of $\gamma^{-1/3}$.
(b) Fourier-transformed action spectrum of (a); closed orbits correlated to respective resonances in (ρ,z)-projection; z-coordinate vertically.

regime. Seen here for the first time it evolves as a remarkably well-structured, ordered system of branches and clusters of resonances.

To understand the experimental spectrum we have calculated the complete semiclassical action spectrum, that is the position of resonances in the ($\tilde{E}$,C)-plane correlated to periodic orbits with the proton as origin by numerical solution of the scaled quantization integral (6). The result is shown in Fig. 5b where each point represents a calculated resonance and thus a closed orbit.

We summarize the main results and observations derived from the experimental and theoretical quasi-Landau spectra in Fig. 5 and the corresponding trajectory calculation. A detailed discussion is given elsewhere [3]: 1) The experimental and theoretical spectra resemble each other in the overall ($\tilde{E}$,C)-dependence. They evolve from rootes of few "basic" resonances at low $\tilde{E}$ by bifurcation into branches which mix as $\tilde{E}$ increases. In the theoretical spectrum the density of resonances grows rapidly making identification of individual resonances in the experimental spectrum impossible in regions of high $\tilde{E}$ and C. On the other hand, however, the experimental spectrum remains distinctly structured and this despite the comparatively large width of the resonances. This fact shows that even in the classically fully chaotic regime only rather few resonances are preferentially excited or, in other words, correspondingly few periodic orbits dominate there the dynamics of the atom. 2) Three types of "basic" resonances with corresponding types of period orbits are identified: (a) "vibrators", one-dimensional orbits along the (z = 0)-plane, (b) "rotators", two-dimensional orbits in the (z = 0)-plane; and (c) "exotics", genuine three-dimensional orbits. They are labelled in Fig. 5, respectively, by V_μ, R_μ, X_μ (μ = 1, 2 ...). The basic vibrators and rotators constitute sets of resonances harmonic in the action with V_1 and R_1 being the fundamentals. From the V_μ's and R_μ's evolves the spectrum by bifurcation into higher generations of resonances with three-dimensional periodic orbits.

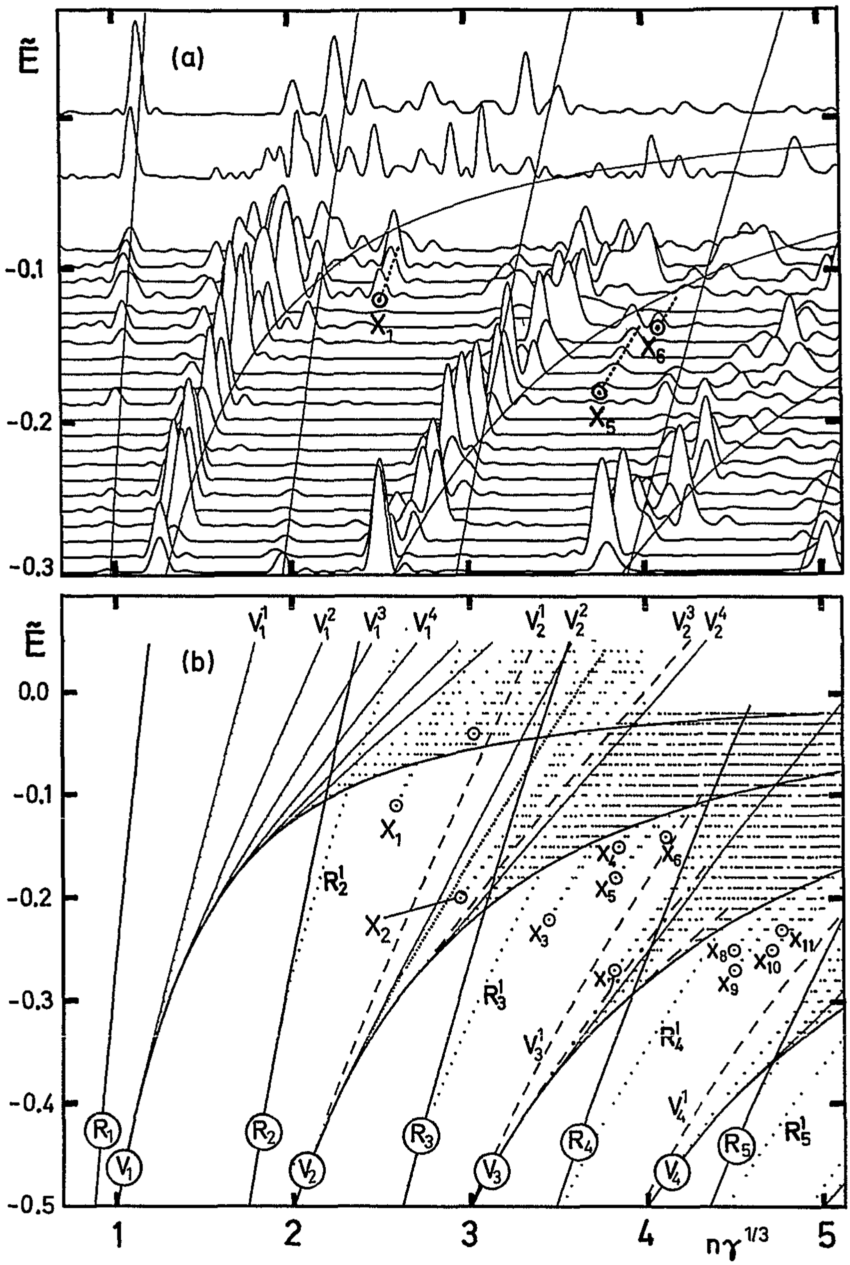

Fig. 5 (a) Experimental quasi-Landau resonance action spectrum as a function of scaled energy $\tilde{E}$ in overlay form. Even-parity, magnetic m = 0 final state. (b) Semiclassically calculated ($\tilde{E}$, C)-spectrum of quasi-Landau resonances correlated to closed classical orbits through origin.

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The existence of "exotics" is experimentally discovered by observation of resonances (e.g. X_1, X_5, X_6) in regions of the ($\tilde{E}$,C)-plane not connected to vibrators or rotators. Main features and properties of them are: a) they occur apparently at random in the chaotic regime of the ($\tilde{E}$,C)-plane, increasing in density with $\tilde{E}$ and C, b) they are born at singular points and bifurcate right at origin, c) the correlated periodic orbits do not show any systematics or symmetry in their topology, which vibrators and rotators do [3], d) the X_1 exotic is peculiar in that it resides in a microscopically regular regime embedded in the otherwise fully chaotic regime.

In summary, studies during the last few years have revealed a much advanced picture of the physics of highly excited, magnetized atoms. Essentially new insight has been gained in the structure and dynamics of these systems under classically chaotic conditions by the discovery of a multitude of new quasi-Landau resonances with correlated three-dimensional periodic orbits and their evolution from the regular into the chaotic regime.

REFERENCES

[1] A. Holle, G. Wiebusch, J. Main, B. Hager, H. Rottke, and K.H. Welge, Phys. Rev. Lett. 56, 2594 (1986)

[2] J. Main, G. Wiebusch, A. Holle, and K.H. Welge, Phys. Rev. Lett. 57, 2789 (1986);
J. Main, A. Holle, G. Wiebusch, and K.H. Welge, Z. Phys. D6, 295 (1987)

[3] A. Holle, J. Main, G. Wiebusch, H. Rottke, and K.H. Welge, Phys. Rev. Lett. 61, 161 (1988)

[4] A. Holle, G. Wiebusch, J. Main, K.H. Welge, G. Zeller, G. Wunner, T. Erd, and H. Ruder, Z. Phys. D5, 279 (1987)

[5] G. Wunner, U. Woelk, I. Zech, G. Zeller, T. Ertl, F. Geyer, W. Schweizer, and H. Ruder, Phys. Rev. Lett. 57, 3261 (1986)

[6] D. Delande and J.C. Gay, Phys. Rev. Lett. 57, 2006 (1986), and 59, 1809 (1987)

[7] D. Wintgen and H. Friedrich, Phys. Rev. Lett. 57, 571 (1986) and Phys. Rev. A 36, 131 (1987)

[8] M.A. Al-Laithy, P.F. O'Mahony, and K.T. Taylor, J. Phys. B 19, L773 (1986)

[9] M.L. Du and J.B. Delos, Phys. Rev. Lett. 58, 1731 (1987)

[10] M. Robnik, J. Phys. A 14, 3195 (1981);
A. Harada and H. Hasegawa, J. Phys. A 16, L259 (1983)

[11] W. R. S. Garton and F. S. Tomkins, Astrophys. J. 158, 839 (1969)

[12] A.R. Edmonds, J. Phys. Pais 31 Colloque C 4, 71 (1970)

[13] G. Wunner and G. Zeller, private communication

[14] M.C. Gutzwiller, J. Math. Phys. 8, 1979 (1967) and 10, 1004 (1969), and 11, 1791 (1970), and 12, 343 (1971);
D. Wintgen, Phys. Rev. Lett. 58, 1589 (1987) and 61, 1803 (1988)