Illustrating the power of “choose” and “forall”

- generating all and only the pairs $vw \in A^*$ of different words v,w of same length (i.e. $v \neq w$ and $|v| = |w|$)

choose n, i with $i < n$

choose $a,b \in A$ with $a \neq b$

v(i) := a

w(i) := b

forall $j < n$, $j \neq i$

choose $a,b \in A$

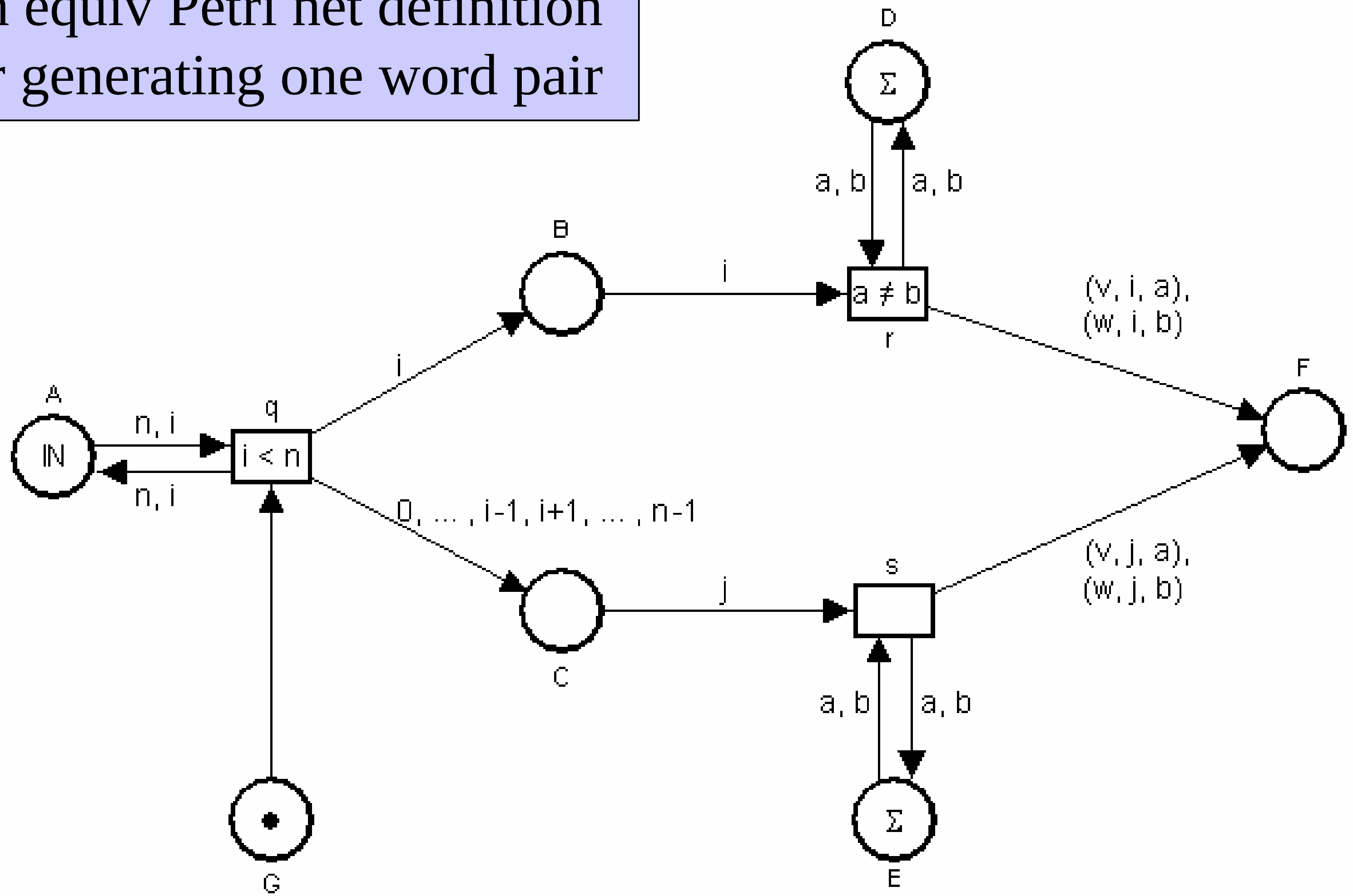
v(j) : = a

w(j) := b

When all possible choices are realized, the set of reachable states vw of this ASM, started with (say) a b for some $a \neq b$, is the set of all vw with $v \neq w$ and $|v| = |w|$.

An equiv Petri net definition for generating one word pair

Slide courtesy of W. Reisig



const v, w

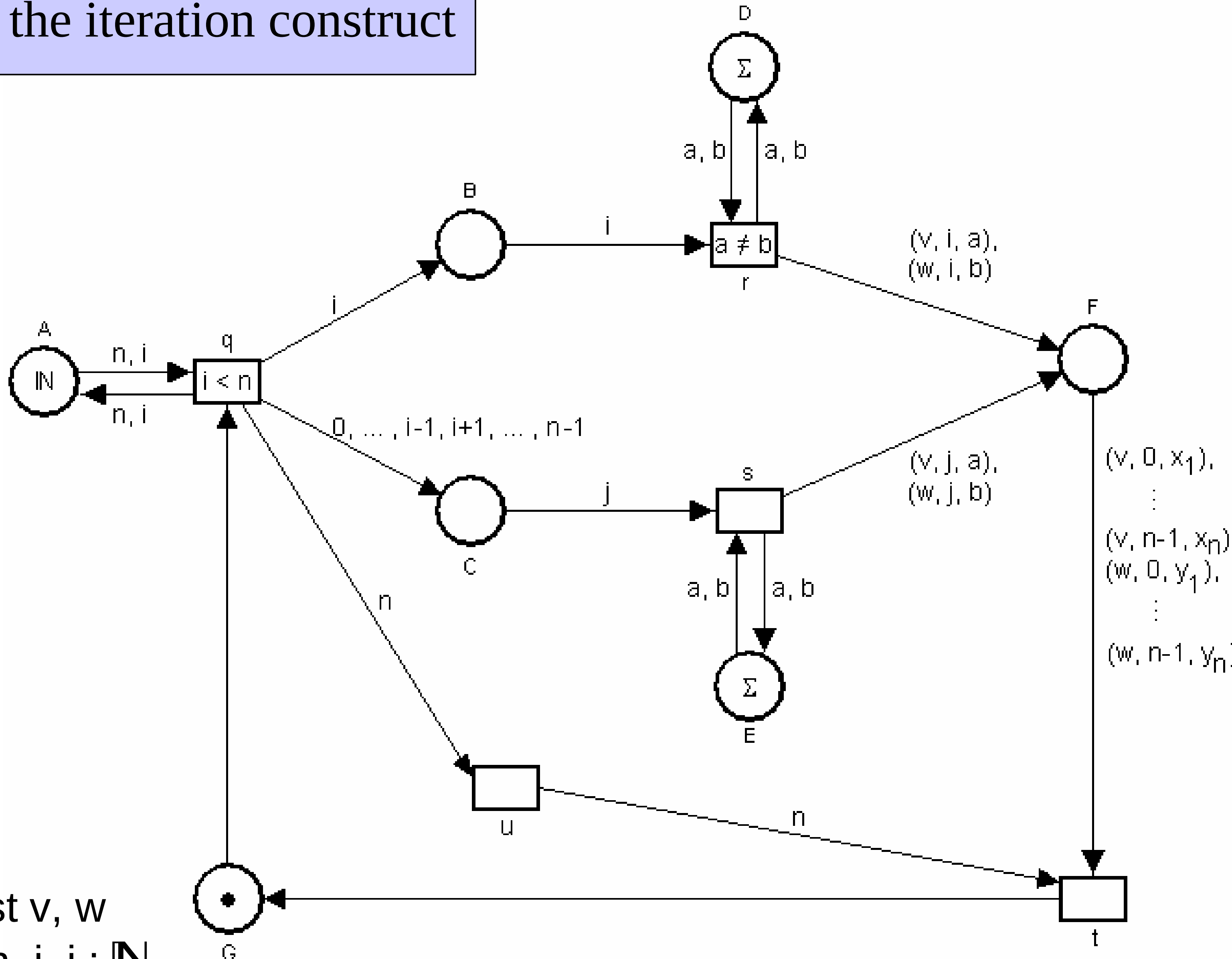
var n, i, j : ℕ

var a, b : Σ

through transition q: at place B 1 mark is produced for a≠b
at place C n-1 marks are produced for arbitrary letters

Adding the iteration construct

Slide courtesy of W. Reisig



const v, w

var n, i, j : $\mathbb{N}$

var a, b, x_1, ..., x_n, y_1, ..., y_n : Σ

when a word has been completed,
it is deleted to produce the next word

Illustrating the power of “forall” and “choose”

- generating the words over alphabet {0,1} of length at least n with a 1 in the n-th place, i.e. the words of form v1w $\in \{0,1\}^{n-1}\, 1\, \{0,1\}^*$
- forall n $\in$ Nat

  choose v $\in \{0,1\}^{n-1}$

  choose w $\in \{0,1\}^*$

  out(n) := v1w

NB. For each n, there is a non-deterministic FSM with O(n) states which accepts the set $\{0,1\}^{n-1}\, 1\, \{0,1\}^*$, but every deterministic FSM accepting this set has at least 2^n states.

The power of non-determinism

- Let $L_n = \{ v \# w \mid v \neq w \text{ and } |v| = |w| = n\}$.
- Exercise. Show that for each n, L_n can be accepted by a non-deterministic finite automaton with $O(n^2)$ states.
- Every unambiguous automaton accepting L_n needs at least 2^n states.
  - See C. M. R. Kintala and K-Y Pun and D. Wotschke: Concise representations of regular languages by degree and probabilistic finite automata. In: Math. Systems Theory 26 (4) 1993, 379—395.