Asynchronous ASMs

Consensus in Networks

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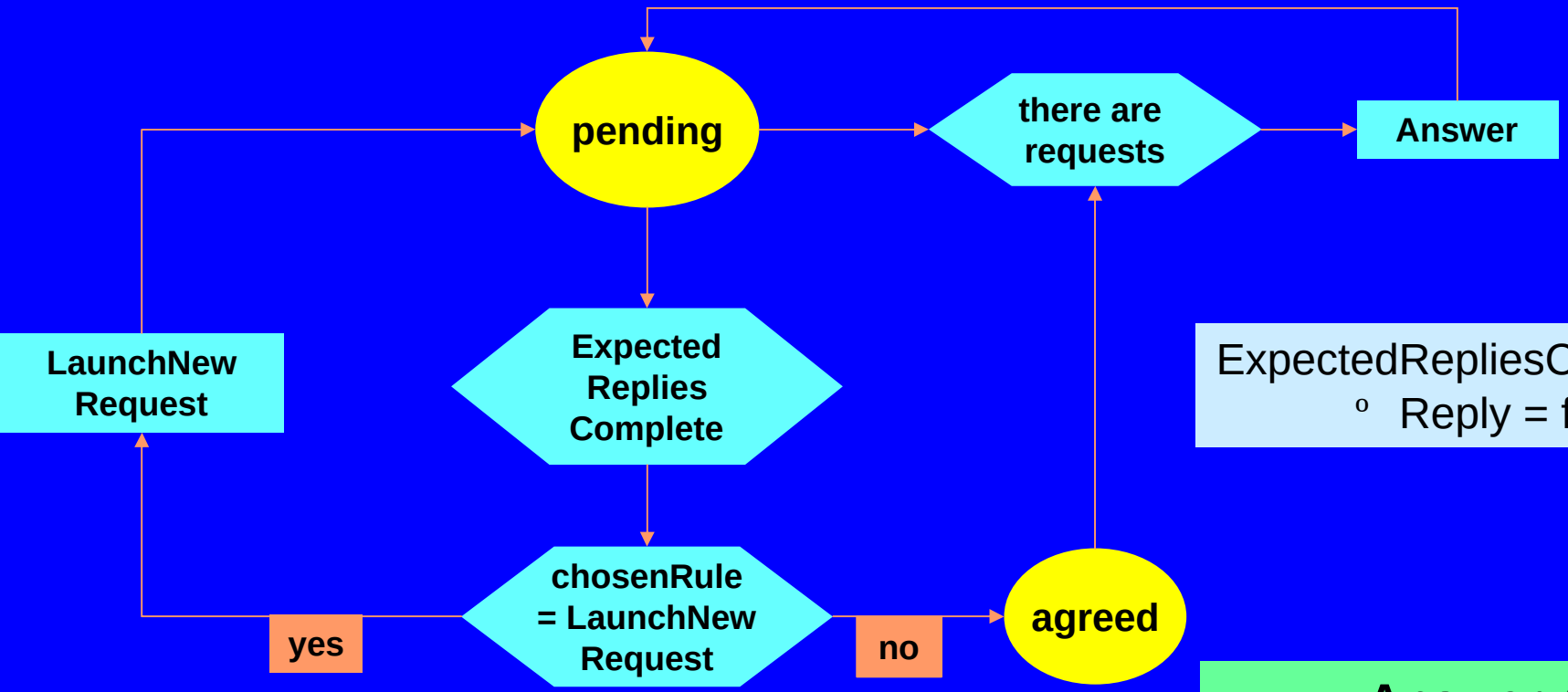
Consensus in Networks: problem statement

- Goal: Design a distributed algorithm for reaching consensus among homogeneous agents of finite connected networks (using only communication between neighbors, without broker or mediator, with abstract requests and answers).
- Algorithmic Idea: every agent (node) may
  - launch a request to his neighbors and wait for the replies
  - agree with the replies received from his neighbors
  - reply to requests received from his neighbors
- until all agents agree (maybe never)

Consensus ASM: Agent Signature

- Agent : finite connected set (of nodes)
- Each agent equipped with:
  - neighb $\subseteq$ Agent (external function)
  - Request $\subseteq$ {RequestFrom(n)|n$\in$ neighb } (controlled function)
  - Reply $\subseteq$ {ReplyFrom(n)|n $\in$ neighb } (controlled function)
  - ctl_state : {pending, agreed}
- Initially ctl_state=pending, Request = empty, Reply = full

Consensus Control State ASM



ExpectedRepliesComplete
- Reply = full

Answer -

forall r ∈ Request **do**
delete r from Request
send answer for r

LaunchNewRequest -
Reply := empty
BroadcastRequest

send answer for RequestFrom(n) -
insert ReplyFrom(self) into Reply(n)

BroadcastRequest - **forall** n ∈ neighb
insert RequestFrom(self) into Request(n)

Consensus ASM : Stability property

- Proposition: In every distributed run of agents equipped with the consensus ASM, if the run terminates, then for every agent holds:
  - ctl_state = agreed
  - Reply = full
  - Request = empty
- Proof (assuming that every enabled agent will eventually make a move): follows from the definition of LaunchNewRequest, Answer.
  - When Reply=full at agent, then there is no RequestFrom(agent) in Request(n) for any n ∈ neighb(agent)

Reference

- W.Reisig: Elements of Distributed Algorithms Springer-Verlag 1998
  - See Section 35 (in particular Fig. 35.1) and Section 80 for a correctness proof.
- E. Börger, R. Stärk: Abstract State Machines. A Method for High-Level System Design and Analysis Springer-Verlag 2003, see http://www.di.unipi.it/AsmBook
  - See Chapter 6.1