Questions and Answers

Chapter 5

Questions

(Q1) *In Section 0.2 it is stated that edge detection approaches to FMD computerized analysis are more dependent on several error sources of ultrasound that registration based approaches. Can you explain why?*

(Q2) *The motion model of Section 0.4.2.1 assumes that there is in-plane motion only. Can you comment on this?*

(Q3) *The Kalman filter is causal, which means that its output value is a function of only the inputs that came earlier in time (could also be only later). Comment on the use of non causal tracking strategies like, for instance, non-causal Wiener filtering.*

(Q4) *Derive Equation 11.*

(Q5) *Why do we need to measure the nitroglycerine-mediated dilation (NMD) phase if clinical indexes are only related to the FMD phase?*

Answers

(A1) Edges in ultrasound are often fragmented due to shadowing effects, blurred due to its low resolution, and affected by speckle noise. All this conditions

that vessel segmentation strategies only based on edges are not very robust in general. Several authors have indicated techniques to improve basic edge segmentation by imposing continuity constraints through, for instance, dynamic programming. Methods that use more global information, like the one presented here, are even less sensitive to image artifacts owing to the fact that they rely on integrative measures not depending on low-scale artifacts. On the other hand, if one wants to compute local diameter measurements along the vessel, the registration based approach presented here would not be suitable.

(A2) Indeed, the actual motion is a combination of in-plane and out-of-plane motions. So only gross in-plane motion is compensated for. However, this is an intrinsic problem of the imaging technology and not of the presented FMD analysis technique. It is still an open question what are the effects of the actual 3D motion on our 2D measurements to assess to what extent this error affects the dilation measurements However, it is important to note that the protocol for FMD estimation on the brachial artery presented in this paper is quite standard in clinical research.

(A3) Off-line non-causal filtering on the dilation sequence would be possible. However, if one does not correct for errors on-line, the results would probably not be as good as reported. Our method corrects drifts in the registration parameters as soon as they appear and reduces their propagation to subsequent frames at an early stage. Wiener filtering would possibly be trying to correct for such errors when it is already too late. Finally, and probably even as important, Wiener filtering is not appropriate for non-stationary signals as is most likely the case with FMD time series.

(A4) Derivation of the Kalman gain equation Eq. 11. Be the following state, $x(n)$, and measurement, $y(n)$, models.

$$x(n) = \alpha x(n-1) + w(n) \tag{14}$$

$$y(n) = x(n) + v(n) \tag{15}$$

Where σ_w^2 and σ_v^2 are the variances of two white noise processes

$$\hat{x}(n) = \alpha\, \hat{x}(n-1) + K(n)[y(n) - \alpha\, \hat{x}(n-1)] \tag{16}$$

The equations for computing the Kalman, $K(n)$, gain are

$$P(n|n-1) = \alpha^2 P(n-1|n-1) + \sigma_w^2 \tag{17}$$

$$K(n) = \frac{P(n|n-1)}{P(n|n-1) + \sigma_v^2(n)} \tag{18}$$

$$P(n|n) = [1 - K(n)]P(n|n-1) \tag{19}$$

where $P(n|m)$ is the covariance matrix of the estimation error at simple n given the previous m samples.

From Eqs. 17 and 18 follows that

$$K(n) = \frac{\alpha^2 P(n-1|n-1) + \sigma_w^2(n)}{\alpha^2 P(n-1|n-1) + \sigma_w^2(n) + \sigma_v^2(n)} \tag{20}$$

That can be rewritten for $n+1$ as

$$K(n+1) = \frac{\alpha^2 P(n|n) + \sigma_w^2(n+1)}{\alpha^2 P(n|n) + \sigma_w^2(n+1) + \sigma_v^2(n+1)} \tag{21}$$

$$= \frac{\Gamma(n) + \sigma_w^2(n+1)}{\Gamma(n) + \sigma_w^2(n+1) + \sigma_v^2(n+1)} \tag{22}$$

where

$$\Gamma(n) = \alpha^2 P(n|n) \tag{23}$$

and Eqs. 19 and 23 give

$$\Gamma(n) = \alpha^2(1 - K(n))P(n|n-1) \tag{24}$$

But from Eq. 18 follows that

$$P(n|n-1) = \frac{\sigma_v^2(n)\, K(n)}{1 - K(n)} \tag{25}$$

and therefore

$$\Gamma(n) = \alpha^2 \sigma_v^2(n)\, K(n) \tag{26}$$

Hence, Eq. 22 boils down to

$$K(n+1) = \frac{\alpha^2 \sigma_v^2(n)\, K(n) + \sigma_w^2(n+1)}{\alpha^2 \sigma_v^2(n)\, K(n) + \sigma_w^2(n+1) + \sigma_v^2(n+1)} \tag{27}$$

$$= \frac{\alpha^2\, K(n) + \sigma_w^2(n+1)/\sigma_v^2(n)}{\alpha^2\, K(n) + (\sigma_w^2(n+1) + \sigma_v^2(n+1))/\sigma_v^2(n)} \tag{28}$$

In Eq. 11 we also assumed that $\sigma_v^2(n) = \sigma_v^2$, which further simplifies the previous equation.

(A5) The test is developed to extract information about the endothelium. This vessel layer intervenes in vasodilation caused by flow increasses. That is why this first phase is also known as endothelium-mediated and concentrates the main information of the test. Nitroglycerin is used as an endotelium-independent stimulus to control whether vasodilation is impaired in the first phase by an isolated endothelial dysfunction or by a muscle insensibility to nitrous donnors. Some risk factors altere the muscle relaxation capacity, thus causing a failure of the endothelial valodilation of mixed origin. Applying the method of this article to nitroglycerin-mediated dilation curves might also render some relevant information.